Complete solution of spherically symmetric gravitational field

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Abstract: *There is a paradigm stating that gravitational field is of non-localizable and stationary nature. Contrary, in our model of Expansive Nondecelerative Universe it is hypothesized that gravitational field is always localizable and nonstationary. This assumption allows localizing its energy density. In this contribution, a solution of the issue is offered. This solution can be extrapolated to both the past and future times. Of the solution, the existence of cosmological member and, in turn, the the question of existence of „dark energy“ follows. Moreover, each black hole represents an independent Universe being a subsytem of the infinite Multiverse.*

1: INTRODUCTION:

Any aspect of gravitational field can be expressed by corresponding metrics. A spheric symmetric field can be described by Schwarzschild metric, rotating charged massive objects by Kerr – Newman metric, nonstationary gravitational field by Vaidya metric, singularities of the Schwarzschild metric are eliminated using Finkelstein metric etc. All the mentioned metrics describe strong gravitational fields. Neither of the metrics (excluding Vaidya one) is able to localize gravitational field energy density. Current cosmological models do not comply with the Vaidya metric. We suppose, the model of Expansive Nondecelerative Universe (ENU) might conform to Vaidya metric.
Singularities present in Schwarschild metric may be eliminated using Finkelstein or Kruskal metric, resulting, however, in a metric of flat space (with a body falling freely ito black hole). The body can thus falls to one point located in the centre of the black body which leads to a further singularity. Introducing a time-varying nonstationary gravitational field, the singularities can be eliminated and, also other problems of the current cosmology, such as dark energy issue, can be solved.

2: NONDECELERATIVE UNIVERSE MODEL

Our model of the Universe (Expansive Nondecelerative Universe, ENU) [1], [2] is based on a simple premise that the rate of the Universe expansion is constant and equal to the speed of light. Moreover, the Universe mean energy density is identical to its critical energy density. There are three limiting conditions characterizing the ENU model, namely

$$\Lambda = 0 \tag{1}$$

where Λ is the cosmological constant,

$$k = 0 \tag{2}$$

where k is the curvature, and

$$a = c\, t_U \tag{3}$$

where a is the scale factor, c is the speed of light in the vacuum, t_U is the cosmological time. Their present ENU-based values are following: $a = 1.229 \times 10^{26}$ m; $t_U = 1.373 \times 10^{10}$ yr. Within the classic models of the Universe, the flat Universe is required to gradually decelerate its expansion.

It is a case where the gravitational force affects the Universe GLOBALLY. Contrary, in the ENU, the gravity affects it only LOCALLY.

The dynamic nature of the ENU is described by Friedman equations. Introducing a dimensionless conform time η, the equations can be expressed as follows:

$$\frac{d}{d\eta}\left(\frac{1}{a}.\frac{da}{d\eta}\right) = -\frac{4\pi G}{3c^4}a^2(\varepsilon + 3p) \tag{4}$$

$$\left(\frac{1}{a}.\frac{da}{d\eta}\right)^2 = \frac{8\pi G}{3c^4}a^2\varepsilon - k \tag{5}$$

where ε is the energy density, p is the pressure and the scale factor a is expressed as

$$a = \frac{da}{d\eta} \tag{6}$$

Introducing the conditions (1) to (3) into relations (4) and (5), we get

$$\varepsilon = \frac{3c^4}{8\pi G a^2} \tag{7}$$

$$p = -\frac{\varepsilon}{3} \tag{8}$$

The energy density can be expressed also in the form

$$\varepsilon = \frac{3m_U c^2}{4\pi a^3} \tag{9}$$

where m_U is the mass of the Universe ($m_U \cong 8.673x10^{52}$ kg).
Combining of (7) and (9) one obtains

$$a = \frac{2G\,m_U}{c^2} \tag{10}$$

It follows directly from (10) that a time evolution of the matter must occur. An amount of the mass created in one second is δ

$$\delta = \frac{dm_U}{dt} = \frac{m_U}{t_U} = \frac{c^3}{2G} \tag{11}$$

It means that an amount of the matter created in our Universe in a second is equal to about 10^5 Sun mass. In the inflationary model, the same amount of matter is emerging from beyond the horizon. It is not too much matter if the Universe dimensions are taken into accout. For the sake of illustration, it represents a proton in a cube of 1 km^3 within a year. There is no global scale gravity in the ENU which could decelerate the Universe expansion. The ENU model is this in compliance with a Hawking's statement that the total mass-energy of our Universe must equal precisely to 0. It means that the matter, representing the positive component of the energy, is just compensated with the gravitational field, representing the negative component of the energy. The conservation laws are therefore obeyed.

However, the creation can be understood also differently. We suppose that the mass of the elementary particles decreases in time and the decrease is compansated through an increase in their quantity. We are able to register only this increase which appear as the matter creation in spite of preservation of their total mass. There must thus exist a nonstationary gravitational field in the ENU model. Introducing the transformation

$$m \to m(t_U) \tag{12}$$

The mass depends on the cosmological time ($m_{(t)}$).
It holds:

$$\frac{dm_{(t)}}{cdt} = \frac{m_{(t)}}{a} \tag{13}$$

In this situation it is possible to localize the energy density of gravitational field.

3: LOCALISATION OF WEEK GRAVITATIONAL FIELD

As a starting point, Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{14}$$

is taken. Divergence of this equation leads to gravitational energy density ε_{g} in the form

$$\varepsilon_{\mathrm{g}} = -\frac{c^4}{8\pi G} R \tag{15}$$

where R is the scalar curvature.
Vacuum scalar curvature is equal to zero. It holds:

$$R = \frac{\Delta\varphi}{c^2} = 0 \tag{16}$$

φ is Newton potential.

Gravitational force being a far-reaching force acts in principle up to infinity, it is measurable, however, only to a certain distance called effective range r_{ef}. Its meaning lies in a postulate that in the ENU, the effect of gravitation can be displayed only in such a distance, in which the absolute value of the gravitational energy density is higher than the critical energy density of the Universe.

$$r_{\mathrm{ef}} = \left(r_{\mathrm{g}} a\right)^{\frac{1}{2}} \tag{17}$$

Non-relativistic gravitation potential can be thus express a

$$\Phi = \varphi \exp - \left(\frac{r}{r_{(ef)}}\right) \tag{18}$$

Within the distances shorter that the effective range, this potential is almost identical to Newton potential. At distances $r > r_{\mathrm{ef}}$, the potential approaches zero value.

For weak gravitational field the following members of metric tensor apply:

$$g_{\mu\nu} = diag\left(-1 + \frac{2Gm_{(t)}}{rc^2}, 1 - \frac{2Gm_{(t)}}{rc^2}, 1 - \frac{2Gm_{(t)}}{rc^2}, 1 - \frac{2Gm_{(t)}}{rc^2}\right) \tag{19}$$

It holds:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{20}$$

In our case:

$$\eta_{\mu\nu} = diag(-1,1,1,1) \tag{21}$$

$$h_{\mu\nu} = diag\left(\frac{2Gm_{(t)}}{rc^2}, -\frac{2Gm_{(t)}}{rc^2}, -\frac{2Gm_{(t)}}{rc^2}, -\frac{2Gm_{(t)}}{rc^2}\right) \quad (22)$$

It must then hold for scalar curvature:

$$R = 2\frac{d\Gamma_{00}^0}{dr} = \frac{2Gm_{(t)}}{ar^2c^2} \quad (23)$$

The identical result is obtained using Vaidya metric, and Einstein or Tolman pseudotensor [3], [4].

The scalar curvature can be expressed also by another way applying Yukawa potential Φ. It holds:

$$R = h_{\mu\mu}\frac{\Delta\Phi}{c^2} = \frac{2Gm_{(t)}}{ar^2c^2} \quad (24)$$

where $\mu = (0,1,2,3)$. Combining relations (15), (23) and (24) it follows for the energy density of weak gravitational field:

$$\varepsilon_{(g)} = -\frac{c^2}{8\pi G}h_{\mu\mu}\Delta\Phi = -\frac{c^4}{4\pi G}.\frac{d\Gamma_{00}^0}{dr} = -\frac{m_{(t)}c^2}{4\pi ar^2} \quad (25)$$

At the same time, the identity must hold:

$$\frac{d\Gamma_{00}^0}{dr} = \frac{h_{\mu\mu}}{2c^2}\Delta\Phi \quad (26)$$

or

$$\frac{d\Gamma_{\mu\mu}^0}{2dr} = \frac{h_{\mu\mu}}{c^2}\Delta\Phi \quad (27)$$

4: COMPLETE SOLUTION FOR SPHERICALLY SYMMETRIC FIELD

With regard to (12), Schwarzschild metric can be rewritted either in the form

$$ds^2 = \left(1 - \frac{2Gm_{(t)}}{rc^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm_{(t)}}{rc^2}\right)} - r^2\left(d\vartheta^2 + \sin^2\vartheta.d\varphi^2\right) \quad (28)$$

or

$$ds^2 = e^{\nu_{(t)}}c^2dt^2 - e^{\lambda_{(t)}}dr^2 - r^2\left(d\vartheta^2 + \sin^2\vartheta.d\varphi^2\right) \quad (29)$$

It holds: $m_{(t)} > 0$. Calculating members of Ricci tensor and introducing their in Einstein equations lead to the following relations

$$\frac{8\pi G}{c^4}T_0^0 = -e^{-\lambda_{(t)}}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2} \quad (30)$$

$$\frac{8\pi G}{c^4}T_1^1 = -e^{-\lambda_{(t)}}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} \quad (31)$$

$$\frac{8\pi G}{c^4} T_2^2 \left(T_3^3\right) = -\frac{1}{2} e^{-\lambda} \left(\nu'' + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r} - \frac{\nu' \lambda'}{2} \right) + \frac{1}{2} e^{-\nu} \left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda} \dot{\nu}}{2} \right) \tag{32}$$

The time index at ν and λ was not given in the brackets of equations (30) - (32) for the sake of simplicity.

$$\frac{8\pi G}{c^4} T_0^1 = -e^{-\lambda_{(t)}} \frac{\dot{\lambda}}{r} \tag{33}$$

Outside the central body it holds

$$T_0^0 = T_1^1 = 0 \tag{34}$$

$$T_2^2 = T_3^3 = \frac{c^4 r_{(g)}^2 r}{8\pi G a^2 \left(r - r_{(g)}\right)^3} \tag{35}$$

These members are not significant beyond the horison.

$$T_0^1 = -\frac{c^4}{8\pi G} . \frac{r_{(g)}}{ar^2} \tag{36}$$

The component T_0^1 represents gravitational energy density.

It might be supposed that the positive and negative values of the momentum-energy tensor will mutually cancelled. Then, $m_{(t)} = 0$, $\nu = \lambda = 0$ and $e^{\nu} = e^{\lambda} = 1$.
In such a case the metric on the horison adopts the following form

$$ds^2 = c^2 dt^2 - dr^2 - r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \tag{37}$$

It holds

$$dr \neq 0 \tag{38}$$

It should be pointed out that *r* is proportional to the positive energy component on the horison. In black holes there is a matter creation, in the Universe it is expansion. If, therefore, the Universe is put identical to black hole, the relation (38) must hold and *r* is gradually increasing. Increasing the positive energy component, the negative component must increase as well and the total energy on the horison is thus equal to zero.
Similarly, the components $\nu', \nu'', \lambda', \ddot{\lambda}, \dot{\lambda}$ and $\dot{\nu}$ are equal to zero.
Then $T_0^0 = T_1^1 = T_2^2 = T_3^3 = T_0^1 = 0$
On the horison there is thus no impact of the gravitational field.

In the space under the black hole horison the signs in the metric (28) must be reverse.
Under the horison $r_{(g)} \geq r \geq 0$. This region represents the past and the change of signs is understood as a change in the phase. It is supposed that the energy field density here is positive while beyond the horison it is negative. At the horison all contributions are mutually cancelled and the energy density is thus of zero value. The horison behaves as a flexible body. It tries to resist both stretching (negative field energy density – the effect of gravitation) and compresing (positive field energy density – acting of repulsion).
The metric under horison is then as follows:

$$ds^2 = -\left(1 - \frac{2Gm_{(t)}}{rc^2}\right) c^2 dt^2 + \frac{dr^2}{\left(1 - \frac{2Gm_{(t)}}{rc^2}\right)} + r^2 \left(d\vartheta^2 + \sin^2 \vartheta . d\varphi^2\right) \tag{39}$$

In this case: $T_0^0 = T_1^1 = 0$ and

$$T_2^2 = T_3^3 = -\frac{c^4 r_{(g)}^2 r}{8\pi G a^2 \left(r - r_{(g)}\right)^3} \tag{40}$$

$$T_0^1 = \frac{c^4}{8\pi G} \cdot \frac{r_{(g)}}{ar^2} \tag{41}$$

Under the horison a new scale must be implemented. It holds $r_{(g)} = a$ and then

$$T_0^1 = \frac{c^4}{8\pi G r^2} \tag{42}$$

$$T_2^2 = T_3^3 = -\frac{c^4}{8\pi G} \frac{r}{(r - r_{(g)})^3} \tag{43}$$

In eq. (42) the cosmological member has appeared

$$\Lambda = \frac{1}{r^2} \tag{44}$$

This member exerts a repulsive (antigravitational) effect. At the horison it holds $r_{(g)} = 0$ and this is a reason of why the contribution (35) and (36) will be equal to zero, and the contributions (40) and(41), as well as (42) and (43) will mutually cancelled. The horison enlargement (i. e. the Universe expansion) will occur by a constant velocity. The metric (28) or (29) with solution (36) and the metric (39) with solutions (41) or (42) indicate the accordance with the conception of advanced and retarded waves [5],[6].

The condition $T_0^1 \lt 0$ is in compliance with the existence of advanced waves. An uptake of advanced waves means the expansion of the space-time (the Universe).

The condition $T_0^1 \gt 0$ means the existence of retarded waves.

Each black holes has its own time $t_{(v)}$. It must hold:

$$r = c.t_{(v)} \tag{45}$$

Note: The horison has its own units of time and space.

Relation (45) is valid all the time and it is not altered even by the existence of non-zero cosmological member in the past. An explanation declaring that the Universe expantion decelerated in the past and accelerated at present is not justifiable. I tis not real and rationalizable that the effects of past deceleration are exactly compansated by the present „accelerated“ expansion. Such a coincidence is absolutely unbelievable.

The different values of the component T_0^1 present just a change in phase which is necessary to distinguish the past and the future. These contribution are fully cancelled at the horison. This is why dark energy is just an illusion. The components T_2^2 and T_3^3 exhibit singularities at the horison, but they will be cancelled as well.

5: SINGULARITIES IN THE NONDECELERATIVE MODEL

There is a one issue deserving explanation, specifically elimination of the singularities from Schwarzschildovej metric and singularities in general. In adition, solutions of the metrics (28), (37) and (39) should be unified.

The mentioned issued can be simply explained via application of Yukawa potential (18). In a normal regime, when $r \lt r_{(ef)}$, Yukawa potential is reduced to classic Newton potential. If $r \gt r_{(ef)}$, the potential shaprly decreases to zero.

A different situation relates to the black bole horison. In case when $r \to r_{(g)}$, then $r_{(ef)} \to 0$. It represents a closeness of the space-time (the effective gravitational action radius is equal to zero from the viewpoint of the black hole horison). It means that at the black hole horison also gravitational potential decreases to zero and the metric changes to a flat space metric. This gravitational potential decrease is not gradual but it is realized in a step as an immediate process. All components of momentum-energy tensor will be of zero value and no singularities will exist. However, at the horison, a change in the phase of the advanced and retarded waves characterizing the space-time and matter will occur.

6: CONCLUSION

In the present contribution, Schwarzschild metric with non-stationary gravitational field was solved. Based on the mentioned solution, the following four fundamental conclustions are derived:
1: Gravitational impact is always only of local nature. It applies to the horison and black hole (the Universe) surroundings with regard to the Multiversum.
2: Singularities do not exist. At the horison, the positive and negative parts of energy density are mutually cancelled.
3: The Universe – Multiversum is infinitive both in the time and space. Each black hole represents a local Universe with its own horison and time.
4: Dark energy is just an illusion resulting from non-stationary solution of gravitational equations and the necessity to distinguish the past and future.

NOTE:
There are the following reasons to identify the Universe with black holes.
1: Dimensions of both Nondecelerative Universe and black hole can be described by identical equation.
2: Enlargement of the Universe (expansion) means simultaneously enlargement of black hole (creation).
3: The Nondecelerative Universe and black hole can be described by identical wavefunction.
4: There is identical relation for entropy of Nondecelerative Universe and that of black hole, since logarithm of the Universe phase volume equals to the horison surface devided by the Planck length square.
5: There is a „singularity“ in the black hole centre and apparent „cosmological singularity“ at the beginning of the Universe expansion.
6: In the model of expansit Nondecelerative Universe it holds that the metric tensor represents the phase shift advanced and retarded waves. In gravitational field the phase shift decreases and at the horison is equal to zero. Beyond the horison (the future) and under it (the past) the phase is reversed and there is no space for singularity in the centre of a black hole.

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