# Faster than the Brighter-Light beacon

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Official version at : Phys. Educ. 44. pp.296-300, (2009)

## Abstract

We analyse the motion of a spot of light projected onto a flat screen by a rotating source. We find that the motion of the spot has many interesting features such as spot splitting and superluminal effects. Our discussion is well suited for undergraduates and can be an interesting add-on to their curriculum, giving them new insights into the effects of kinematics.

## Introduction

In astronomy it is well know that the apparent speeds of various phenomena are greater than the speed of light[1-4]. One such phenomenon is due to the light beacon: a beam of light produced by a rotating source. The light projected onto a flat screen by such a source has superluminal features. In studying the nature of this optical effect, we have discovered other interesting features produced by rotating sources. Even though the mathematics involved is simple, we have not found any accessible study of such effects in the literature[5-7]. We therefore present a simple and comprehensive study of light beacon effects. The level of our discussion is appropriate for college or undergraduate students and the subjects presented can easily be integrated as an exercise in a kinematics or optics course.

## Light beacon setup

A rotating light source is positioned at a distance $D$ from a flat screen. The source will be treated as a laser producing a "beam" of light[8]. The source rotates in the clockwise direction with a constant angular speed $\omega > 0$. The source will therefore sweep a spot of light across the screen as in Fig. 1. We wish to find the position of the spot on the screen as a function of time.

Since the source can only produce a light spot on the screen when it is (was) aimed at the screen, the angle $\theta$ is contained in the interval $[-\pi/2, \pi/2]$ *rad* where the screen is taken to be infinite in extent. Fig.2.

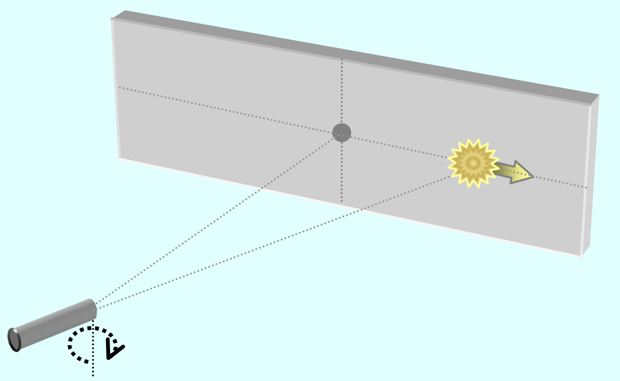

Fig. 1. Perspective view of the light bacon setup. Clockwise rotating source producing a travelling spot of light on a flat screen.

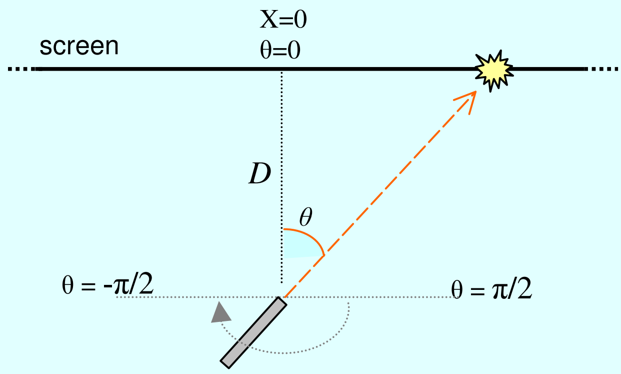


Fig. 2. Top view. Clockwise rotating source producing a travelling spot of light on the screen.

## Analysis of the position and speed of the light spot projected onto the screen

Our goal is to find at what time T a particular point X of the screen will be illuminated. Both X and T can be expressed as a function of $t$, the time at which the light "signal" emanated from the source. This generates two parametric equations X($t$) and T($t$). Solving these two equations will permit us to express X as a function of T. We opt however to express T as a function of X since T(X) can be written in a simpler form than X(T).

Consider the light source which rotates from $\theta = -\pi/2$ to $\pi/2$ at an angular speed $\omega$. Without loss of generality, the source starts rotating at time $t = 0$ as it is oriented at an angle of $\theta = -\pi/2$. The angle of emission of the light as a function of time is thus $\theta(t) = -\pi/2 + \omega t$ . For $t$, we need to only consider the time interval $[\,0, \pi/\omega\,]$ since outside this time interval the source is no longer aimed at the screen .

At time $t$, $\theta = -\pi/2 + \omega t$. Thus the source is *aimed* at a point X of the screen, but does not yet illuminate this point since it takes time for the light to reach the screen. Referring to Fig. 2, basic geometry yields

$$X = D\tan(\theta) = D\tan(-\pi/2 + \omega t) \tag{1}$$

Since $t \in [\,0, \pi/\omega\,]$, X therefore has a range of $\pm\infty$. The physical significance of this result is that at emission angles $-\pi/2$ and $\pi/2$, it is the (infinitely) far edges of the screen that will be illuminated.

The transit time $\tau$ for the light signal to go from the source to the point X is found by considering the distance traveled by the light signal divided by the speed of the light signal. Basic kinematics yields

$$\tau = \frac{\sqrt{X^2 + D^2}}{c} \tag{2}$$

Therefore, the time T at which the point X of the screen becomes illuminated is the time $t$ at which the light was sent out by the source plus the light transit time $\tau$ :

$$T = t + \frac{\sqrt{X^2 + D^2}}{c} \tag{3}$$

Note that T is greater than zero since the source starts emitting at time zero ($t = 0$) and it takes time ($\tau > 0$) for the light to reach the screen. Isolating $t$ in (1) and replacing in (3), we express T as a function of X. This yields

$$T(X) = \frac{\pi}{2\omega} + \frac{1}{\omega}\arctan(X/D) + \frac{1}{c}\sqrt{X^2 + D^2} \tag{4}$$

Figure 3 plots T(X) for values of $\omega = \infty$, 0.5, 0.1, and 0.025. We have set $D = c = 1$ in these graphs.

From Eq.(4) we can express the average speed $\overline{V}$ of the spot travelling between the two points $X_1$ and $X_2$. The average speed being the distance covered by the time, we get

(5) $$\overline{V} = \frac{X_2 - X_1}{\left(\frac{1}{\omega}\arctan(X_2 / D) + \frac{1}{c}\sqrt{X_2^2 + D^2}\right) - \left(\frac{1}{\omega}\arctan(X_1 / D) + \frac{1}{c}\sqrt{X_1^2 + D^2}\right)}$$

In the case where the two points $X_1$ and $X_2$ are arbitrarily close, Eq.(5) gives the instantaneous speed *V* of the travelling spot when it is located at $X_1 = X_2 \equiv X$. One can also use calculus and compute $V(X) = 1/(dT/dX)$. Both calculations yield

(6) $$V(X) = \frac{c\omega(X^2 + D^2)}{X\omega\sqrt{X^2 + D^2} + cD}$$

Figure 4 plots V(X) for $\omega = \infty$, 0.1, and 0.025. We have set $D = c = 1$ in these graphs. The Eqs.(4) and (6) suffice to give an extensive description of the travelling spot, which is what we present in the following sections.

## Discussion of a particular case

Since the motion of the spot has many features, we begin the discussion by analyzing a particular case. Let $\omega = 0.1$ and without loss of generality, the parameters *D* and *c* are set to unity. Equations (4) and (6) become, respectively

(7) $$T(X) = 5\pi + 10\arctan(X) + \sqrt{X^2 + 1}$$

(8) $$V(X) = \frac{(X^2 + 1)}{X\sqrt{X^2 + 1} + 10}$$

See Figs. 3 and 4 for the graphs of these two functions where $\omega = 0.1$. Figure 5 shows the "actual" spot on the screen at four different times.

The first point of the screen to be illuminated is the point which takes the least time T to be reached. By minimizing Eq.(7) or simpler yet, by consulting the graphs of Eqs.(7) and (8) we see that the discontinuity point of Eq.(8) corresponds to the first point to be illuminated. Solving for the roots of the denominator of Eq.8 yields $X \approx -3.08$ and occurs at time $T \approx 6.38$. Consulting the graph of T(X) in Fig.3 we see that the earliest point to be illuminated is at time $T \approx 6$ and the point is $X \approx -3$. Fig. 5(a) depicts this case. From the initial point $X \approx -3$, the spot splits. There is a "splitting" of the initial spot because locations near $X \approx -3.08$ take a greater time to be illuminated than the point $X = -3.08$, and thus are illuminated after the initial spot. Mathematically, for any $T > 6.38$, there are two solutions to Eq.(7), as seen in Fig.3.

As the spot splits, the speeds of the two new spots are arbitrarily large. This can bee seen from Fig.3 where the slope of the curve near $X = -3$ is near zero, which corresponds to very high speeds ( Fig.3 is the graph of T(X) and not X(T), therefore a large speed corresponds to a horizontal slope ). To the right of the minimum, the slope of the curve is positive. This indicates that the speed of the spot is positive, i.e., it is travelling to the right. To the left of our minimum, the slope is negative, and therefore the spot is travelling to the left. The same conclusions can be found by considering Fig. 4, which represents the speed *V* of the spot when it is at position X. The discontinuity of V(X) at X ≈ -3 corresponds to the first bright spot appearing on the screen. Note that all points to the left of the initial spot have speeds greater than 1, i.e. are superluminal. However, not all points to the right of X ≈ -3 are superluminal.

Eventually the spot travelling to the right will reach the center of the screen. The time it takes for the center of the screen to be illuminated is (5 π + 1) ≈ 16.7 as found by setting X = 0 in Eq.(7). Consulting Fig. 3, we can see that the curve appropriately passes through the coordinates X = 0 and T ≈ 16.7. This case is depicted in Fig. 5(c). The speed of the bright spot near the center of the screen is among the lowest (in magnitude), as seen in Fig. 4.

The spots continue moving. The speed of the spot travelling to the right continuously increases and tends toward the speed of light $c$ = 1. The speed of the spot travelling to the left "decreases" and tends toward the speed of light $c$ = 1.

## Discussion of the general case

From the above discussion and consulting Eqs. (4) and (6), we can generalize the main features of the motion of the spot.

- The first point of the screen to be illuminated is $X = -\sqrt{\sqrt{D^4 + 4c^2 D^2 / \omega^2} - D^2} \Big/ \sqrt{2}$ which is found by considering the discontinuity point (denominator equals zero) of Eq.(6), which turns out to be a simple quadratic equation.
- From the initial light spot on the screen, the spot splits in two, the spots travelling in opposite directions. Eventually the spot travelling to the right will cross the center point of the screen.
- The speed at which a spot travels on the screen is the tangent to the curve X(T), or the cotangent to the curve T(X). The initial speed of the split spot(s) is ±∞, and hence is superluminal.
- The spot which crosses the X = 0 point of the screen does so at time T(X=0) = $\pi/(2\omega) + D/c$.
- The speed at which the spot crosses the X = 0 point is given by V(X=0) = $\omega D$ . This is what is expected when one considers the tangential speed of a rotating disc.
- In the hypothetical case where the angular speed of the source is arbitrarily large ($\omega = \infty$), the source is nothing more than a source aiming at all angles at the same time. In essence it is a flash of light propagating in all directions. Therefore the first point of the screen to be illuminated is the closest point, the X=0 point. From there, the spot splits and travels symmetrically in opposite directions. The curve of $\omega = \infty$ in Fig. 3 depicts this case.
- The spot traveling to the left is "quicker" than the rightward traveling spot as one can see by consulting Fig.5.
- For points of the screen that are far from the center point, the speeds of the spots approach the speed of light $c$.
- Even though the source is rotating in the clockwise direction (Fig. 2), there is a spot which will travel in the counterclockwise direction ( decreasing X ). These are the points X of the screen where the tangent to the curve X(T) or T(X) is negative.
- If earth is considered as a screen for distant rotating sources (pulsars), without loss of generality, we observers are located at X=0. The speed at which we observe the spot sweeping us is given by the expression V(0) = $\omega D$ . This is in the clockwise direction, the direction in which the source is rotating. For sufficiently large $D$, $V$ can be greater than the speed of light and we therefore observe a superluminal speed.
- Consulting the graph of V(X) in Fig. 4, we see that the travelling spots do not have a uniform speed. For example, the spot traveling to the right begins with an arbitrarily high speed, quickly decelerates to a much lower speed, then slowly regains speed to tend toward the speed of light $c$.

Numerous other features are produced by the light beacon. For instance, the “beam” of light produced by the source (as drawn in Figs. 1 and 2) is actually not a straight line but a spiral, as previously pointed out. Although we present a simple setup of the light beacon, in practice many other factors are to be considered, as the dimensions, velocity, and intensity of the source, as discussed in refs. 6 and 7.

It is worth mentioning that the superluminal features discussed in this text are not at odds with the theory of special relativity. A particular position of the spot is unrelated to its previous position(s). The travelling spot is simply an optical and non-causal effect.

An early acquaintance with these unexpected effects will give the aspiring scientist a clearer understanding of such phenomena and perhaps aid in the formulation of better models. For the interest of our readers, we have produced an animation file which simulates the travelling spot(s) on the screen[9].

## Conclusion

The kinematical analysis of a rotating light source yields many interesting and unexpected effects. When approached correctly, the mathematical analysis remains in the realm of simple algebra and thus accessible to students in a kinematics course. The present topic is particularly interesting to aspiring astronomers but its scope can be extended to many areas of physics as in high-speed photography. The study of the effects produced by the rotating source may serve as interesting homework and make the students realize that even within basic physics there is more than meets the eye.


1. Wikipedia: Superluminal motion. http://en.wikipedia.org/wiki/Superluminal_motion
2. M. H. Cohen *et al*., “Radio sources with superluminal velocities”, *Nature*, vol. 268, 405-409 (Aug. 4, 1977)
3. D.F. Falla *et al*., “Superluminal light echoes in astronomy”, *Eur. J. Phys*. **24**, 197-213 (2003),
4. M. J. Chodorowski, “Superluminal apparent motions in distant radio sources”. *Am. J. Phys* . **73** (7), 639-643 (2005)
5. We did find a few texts that treat subjects similar to our own and are referenced in refs. 6 and 7.
6. N. Meyer-Vernet, “Ghost's shadows and superluminal occultations”, *Am . J. Phys*, **54** (10), 916-918 (1986).
7. K. L. Sala, “Anticausal behavior in the imaging of a moving object: The implications for high-speed photography”, *Phys. Rev. A*, **19** (6), 2377 - 2391 (1979)
8. Actually, the shape of the light beam path is spiral due to the rotating source. This however is of no consequence to our calculations.
9. http:// (TBA)


PACS codes: 42.15.-i, 98.62.Tc, 95.10.Gi

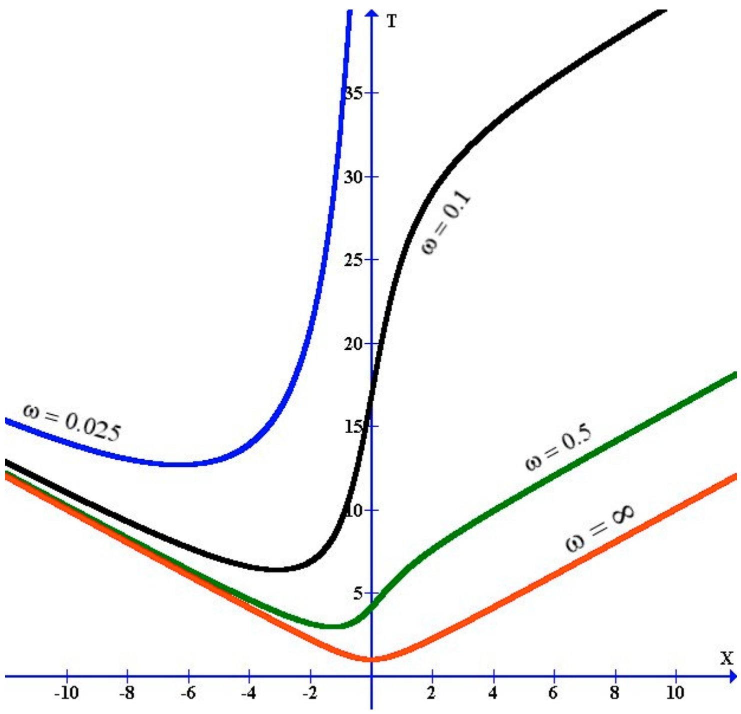


Fig. 3. Curves of $T(X)$ , the time at which the point $X$ is illuminated. $\omega = \infty$, 0.5, 0.1, and 0.025

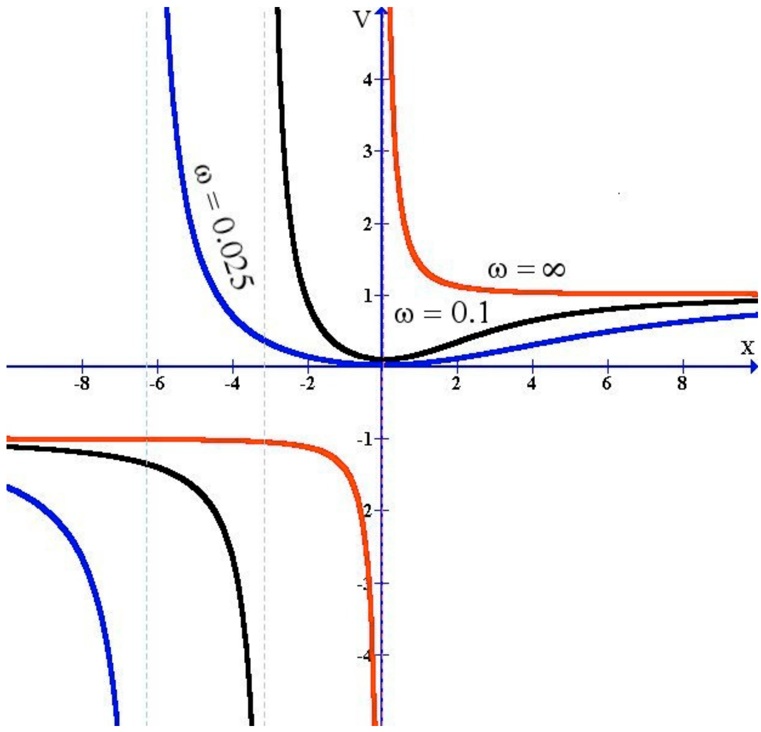


Fig. 4. Curves of $V(X)$, the speed of the spot located at point $X$. $\omega = \infty$, 0.1, and 0.025

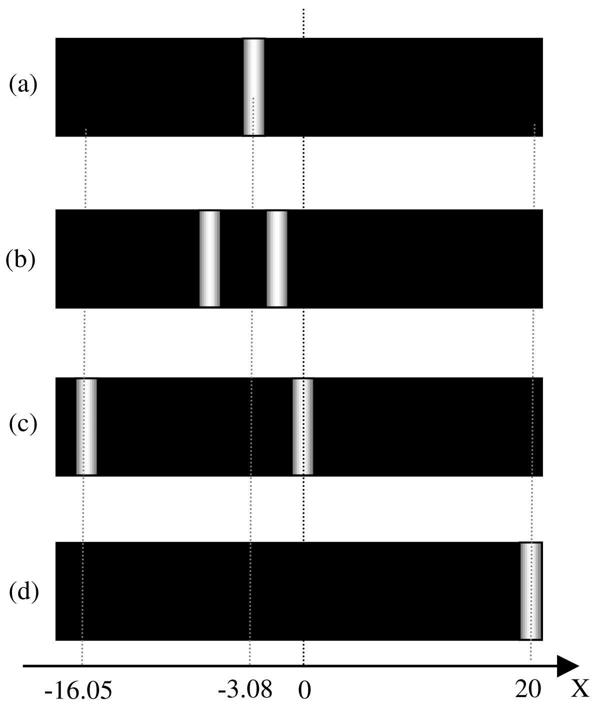


Fig. 5. Four "snapshots" of the screen. $\omega = 0.1$, $D = c = 1$. (a) Initial spot appearing at time $T = 6.38$, at position $X = -3.08$, (b) Traveling spots, $T = 7.7$ (c) $T = 16.7$, time at which the rightward traveling spot crosses the center of the screen, (d)Traveling spot(s), $T = 50.9$