Three Non-linear α-Discounting MCDM-Method Examples

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Abstract.

In this paper we present three new examples of using the α-Discounting Multi-Criteria Decision Making Method in solving non-linear problems involving algebraic equations and inequalities in the decision process.

Introduction.

We have defined [see 1] a new procedure called *α-Discounting Method for Multi-Criteria Decision Making (α-D MCDM)*, which is as an alternative and extension of Saaty's Analytical Hierarchy Process (AHP). We have also defined the *degree of consistency* (and implicitly a *degree of inconsistency*) of a decision-making problem [1].

The α-D MCDM can deal with any set of preferences that can be transformed into an algebraic system of linear and/or non-linear homogeneous and/or non-homogeneous equations and/or inequalities.

We discuss below three new examples of non-linear decision making problems.

Example 1.

The Set of References is $\{C_1, C_2, C_3\}$.

1. C_1 is as important as the product of C_2 and C_3.
2. The square of C_2 is as important as C_3.
3. C_3 is less important than C_2.

We denote $C_1 = x,\ C_2 = y,\ C_3 = z$, and we'll obtain the following non-linear algebraic system of two equations and one inequality:

$$\begin{cases} x = yz \\ y^2 = z \\ z < y \end{cases}$$

and of course the conditions

$$\begin{cases} x + y + z = 1 \\ x, y, z \in [0,1] \end{cases}$$

From the first two equations we have:

$$x = yz = y \cdot y^2 = y^3$$

and

$$z = y^2$$

whence the general priority vector is

$$\langle y^3 \ y \ y^2 \rangle.$$

We consider $y \neq 0$, because if $y = 0$ the priority vector becomes $\langle 0 \ 0 \ 0 \rangle$ which does not make sense.

Dividing by y we have $\langle y^2 \ 1 \ y \rangle$, and normalized:

$$\left\langle \frac{y^2}{y^2+y+1} \quad \frac{1}{y^2+y+1} \quad \frac{y}{y^2+y+1} \right\rangle.$$

From $y^2 = z$ and $z < y$

we have

$$y^2 < y \text{ or } y^2 - y < 0 \text{ or } y(y-1) < 0,$$

hence $y \in (0,1)$.

For $y \in (0,1)$ we have the order:

$$\frac{1}{y^2+y+1} > \frac{y}{y^2+y+1} > \frac{y^2}{y^2+y+1}$$

so

$$C_2 > C_3 > C_1.$$

Example 2.

The Set of References is also $\{C_1, C_2, C_3\}$.

1. C_1 is as important as the square of C_2.
2. C_2 is as important as double C_3.
3. The square of C_3 is as important as triple C_1.

We again denote $C_1 = x,\ C_2 = y,\ C_3 = z$, and we'll obtain the following non-linear algebraic system of three equations:

$$\begin{cases} x = y^2 \\ y = 2z \\ z^2 = 3x \end{cases}.$$

If we solve it, we get:

$$\left.\begin{array}{l} x = 4z^2 \\ x = \dfrac{z^2}{3} \end{array}\right\} \Rightarrow 4z^2 = \frac{z^2}{3} \Rightarrow z = 0 \Rightarrow y = 0 \Rightarrow x = 0.$$

Algebraically the only solution is $\langle 0 \ 0 \ 0 \rangle$, but the null solution is not convenient for MCDM.

Let's parameterize, i.e. "discount" each equality:

$$\left.\begin{cases} \left.\begin{array}{l} x = \alpha_1 y^2 \\ y = 2\alpha_2 z \end{array}\right\} \Rightarrow x = \alpha_1 \left(2\alpha_2 z\right)^2 = 4\alpha_1\alpha_2^2 z^2 \\ z^2 = 3\alpha_3 x \Rightarrow x = \dfrac{1}{3\alpha_3} z^2 \end{cases}\right\} \Rightarrow 4\alpha_1\alpha_2^2 = \frac{1}{3\alpha_3}$$

or

$$12\alpha_1\alpha_2^2\alpha_3 = 1,$$

which is the characteristic parametric equation needed for the consistency of this algebraic system.

The general algebraic solution in this parameterized case is:

$$<4\alpha_1\alpha_2^2 z^2 \quad 2\alpha_2 z \quad z>.$$

Using the Fairness Principle as in the α-Discounting MCDM Method, we set all parameters equal:

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha$$

whence, from the characteristic parametric equation, we obtain that

$$12\alpha^4 = 1,$$

therefore

$$\alpha = \sqrt[4]{\frac{1}{12}} = \frac{1}{\sqrt[4]{12}} \approx 0.537.$$

Thus the general solution for the Fairness Principle is:

$$<4\alpha^3 z^2 \quad 2\alpha z \quad z>$$

and, after substituting $\alpha \approx 0.537$, it results:

$$\begin{cases} x = 4\alpha^3 z^2 \simeq 0.619 z^2 \\ y = 2\alpha z = 1.074 z. \end{cases}$$

Whence the general solution for the Fairness Principle becomes:

$$\left\langle 4\alpha^3 z^2 \quad 2\alpha z \quad z \right\rangle \simeq \left\langle 0.619 z^2 \quad 1.074 z \quad z \right\rangle$$

and dividing by $z \neq 0$ one has:

$$\left\langle 0.619z,\ 1.074,\ 1 \right\rangle.$$

But y = 1.074 > 1 = z, hence y > z.

Discussion:

a) If $z < \dfrac{1}{0.619} \approx 1.616$, then y > z > x.

b) If $z = \dfrac{1}{0.619} \approx 1.616$, then y > z = x.

c) If $\dfrac{1}{0.619} < z < \dfrac{1.074}{0.619}$ or 1.616 < z < 1.735, then y > x > z.

d) If $z = \dfrac{1.074}{0.619} \approx 1.735$, then x = y > z.

e) If $z > \frac{1.074}{0.619} \approx 1.735$, then x > y > z.

From the orders of x, y, and z it results the corresponding orders between the preferences C_1, C_2, and C_3.

Example 3.

Let's suppose that the sources are not equally reliable

First source is five times less reliable than the second, while the third source is twice more reliable that the second one. Then the parameterized system:

$$\begin{cases} x = \alpha_1 y^2 \\ y = 2\alpha_2 z \\ z^2 = 3\alpha_3 x \end{cases} \text{ becomes } \begin{cases} x = 3\alpha_2 y^2 \\ y = 2\alpha_2 z \\ z^2 = 3\frac{\alpha_2}{4} x \end{cases}$$

since $\alpha_1 = 3\alpha_2$ which means that we need to discount the first equation three times more than the second, and $\alpha_3 = \frac{\alpha_2}{4}$ which means that we need to discount the third equation a quarter of the second equation's discount.

Denote $\alpha_2 = \alpha$, then:

$$\begin{cases} x = 3\alpha y^2 \\ y = 2\alpha z \\ z^2 = \frac{3\alpha}{4} x \end{cases}$$

whence

$$x = 3\alpha\left(2\alpha z\right)^2 = 12\alpha^3 z^2$$

and

$$x = \frac{4}{3\alpha} z^2$$

therefore

$$12\alpha^3 z^2 = \frac{4}{3\alpha} z^2, \text{ or } 36\alpha^4 = 4$$

Thus

$$\alpha = \frac{1}{\sqrt[4]{9}} \simeq 0.485.$$

The algebraic general solution is:

$$\left\langle 12\alpha^3 z^2 \;\; 2\alpha z \;\; z \right\rangle = \left\langle 1.557z^2 \;\; 0.970z \;\; z \right\rangle = \left\langle \frac{1.557z}{1.557z + 1.854} \;\; 0.854 \;\; 1 \right\rangle.$$

And in a similar way, as we did for Example 2, we may discuss upon parameter z > 0 the order of preferences C_1, C_2, and C_3.

Reference:

1. Florentin Smarandache, *α-Discounting Method for Multi-Criteria Decision Making (α-D MCDM)*, presented at the Fusion 2010 International Conference, Edinburgh, Scotland, 26-29 July, 2010; published in "Review of the Air Force Academy / The Scientific Informative Review", No. 2, 29-42, 2010;
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and the whole paper at http://xxx.lanl.gov/pdf/1002.0102v4 .