Article 34:

Propagation of Light in the Gravitational Field in the Context of the Theory of Gravitational Relativity I

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Propagation of light in the gravitational field is studied in the context of the theory of gravitational relativity (TGR). A local gravitational red shift relation is derived as a consequence of the gravitational time dilation formula in the context of TGR. A light ray emitted at a given position in a gravitational field, or a light ray emitted elsewhere, which is momentarily passing through the given position, suffers local gravitational red shift at that position. And when the light ray has propagated to another position, it suffers the local gravitational red shift at the new position. The gravitational red shift as a light ray passes through two positions of different gravitational potentials is inferred from the local gravitational red shifts at the two positions. The invalidity of the underlying assumptions of the two theories of gravitational red shift encompassed by the general theory of relativity (GR), which invalidates those theories, are are shown, while upholding the theory of gravitational red shift in the context of TGR as the valid theory. The prediction of gravitational red shift for a terrestrial light ray in the context of TGR is in agreement with the result of the Pound, Rebka and Snider experiment (PRS) to within 99.94%. Although the prediction of Einstein's theory of gravitational red shift of light (in GR) is in agreement with the result of this experiment to within 99.94%, it is shown that this does not imply the validity of that theory.

1 Introduction

Thus far we have considered the motion of material particles with non-zero rest mass, (that is with non-zero gravitational rest mass and/or non-zero electromagnetic (or dynamical) rest mass), on flat spacetime in gravitational fields in the contexts of the theory of gravitational relativity (TGR). We have included even the most general situation where the test particle is in motion relative to a fixed frame through a point in space at the neighborhood of several gravitational field sources that are scattered in the Euclidean 3-space about the moving test particle. This is the subject of Articles 12 – 16 in parts 3A and 3B and Articles 23 – 25 in part 5 of this first volume of this monograph series.

We have also considered the propagations on flat spacetime in gravitational fields in the context of TGR of massless fields namely, electric field $\vec{E}$, magnetic field $\vec{B}$, gravitational field $\vec{g}$ and the newly isolated partner to the gravitational field $\vec{d}$ (but which is non-detectable experimentally). These are the subjects of Articles 29 – 31 in part 7 of this first volume of this monograph series. We shall consider the propagation of the massless particles of light (or photons) with zero rest mass, (that is, zero electromagnetic rest mass and zero gravitational rest mass), on flat spacetime in gravitational fields in the context of TGR in this article and its second part in this final part 8 of volume one of this monograph series.

There is no first-order interaction of light with the Newtonian (or classical) gravitational field. There is however a second-order interaction of light with gravitational field, which results in a repulsive central force $\vec{F}_\gamma$ on the particle of light (or photon) from a gravitational field source in the context of TGR. Although the gravitational force on photon is small, it causes the path of a light pulse colliding with a gravitational field source at nonzero impact parameter to be slightly bent in a gravitational field of arbitrary strength. It also causes a light pulse that starts at speed c_γ at infinity and propagates towards a gravitational field source, to be slowed down to a radially varying speed $v_\gamma(r')$, which is slightly lower than c_γ, even in vacuo, as the light pulse approaches the field source.

The second-order interaction of light with the gravitational field shall be neglected in this first part of this article, thereby allowing a light ray to propagate at constant speed c_γ in vacuo in a gravitational field (without interacting with the field), on the flat four-dimensional spacetime of the theory of gravitational relativity. This first part of this article shall be be devoted to theory of gravitational red shift of light in the context of TGR, while the second part shall be devoted to the effects of the second-order intrinsic geometry-induced interaction of light with a gravitational field in the context of TGR. Those effects shall include the slowing down of light to speeds lower than c_γ in vacuo, the bending of light ray and radar time-delay in a gravitational field in the context of TGR.

2 Local gravitational red shift relations in the context of the theory of gravitational relativity

Let us start by writing the well known Doppler frequency expression in the context of the special theory of relativity (or the special-relativistic Doppler effect) as follows

$$\nu = \frac{(1 - v^2/c_\gamma^2)^{1/2}\,\nu_0}{1 - (v/c_\gamma)\cos\theta} \tag{1}$$

The well illustrated derivation of relation (1) on pages 116 – 119 of [1], for instance, may be contacted.

In Eq. (1), ν_0 is the frequency of the light measured in an inertial reference frame F′ in which the source of light is at rest, ν is the frequency measured in an observer's frame F, relative to which the light source is moving with velocity $\vec{v}$ along the common $x-$axis of frames F and F′, and θ is the angle between the direction of motion of the light source (or velocity $\vec{v}$) and the direction of light emitted towards the receiver (the line-of-sight). The frequency ν shall be referred to as the special-relativistic Doppler frequency, while ν_0 is the natural frequency.

The Doppler effect (1) arises by virtue of possession of dynamical velocity $\vec{v}$ relative to the observer of the source of light. There is a corresponding effect in the context of the theory of gravitational relativity (TGR), to be called gravitational Doppler effect, which arises by virtue of possession of gravitational speed $V_g(r')$ of a light source, that is, due to the momentary propagation of a light source through a position with gravitational speed $V_g(r')$. However, since the gravitational speed is not made manifest in translation of the light source that possesses it relative to any observer, the expression for the gravitational-relativistic Doppler effect (in the context of TGR) is bound to take on a different form from the relation (1) for the special-relativistic effect.

Let us rewrite the relation (1) for Doppler effect in SR as follows

$$\nu = \frac{\nu'_0}{1-(v/c_\gamma)\cos\theta} \tag{2a}$$

$$\nu'_0 = \nu_0(1-\frac{v^2}{c_\gamma^2})^{1/2} \tag{2b}$$

Equation (2a) can be recognized as the classical Doppler effect for a wave of natural frequency ν'_0 and speed c_γ, which is emitted from a source in motion at velocity $\vec{v}$ along a direction which makes angle θ with the line-of-sight of the observer, while Eq. (2b) clearly arises as a consequence of special-relativistic time dilation. Thus the expression for Doppler effect in SR can be seen as combination (or product) of the classical Doppler effect and red shift due to special-relativistic time dilation.

For propagation of a sound wave, say, emitted from a source in motion at a low speed, $v << c$, relative to an observer, there is no time dilation. Hence Eq. (2b) simplifies as $\nu'_0 = \nu_0$. Then the speed of light c_γ must be replaced by the speed of sound u in Eq. (2a), thereby reducing Eqs. (2a) and (2b) to a single expression for classical Doppler effect.

Now let a light ray be emitted by a source at rest relative to an observer, which is located at radial distance r from the center of the inertial mass M of a gravitational field source. The source of light thereby possesses gravitational speed $V_g(r')$. Let us by sheer analogy between the dynamical velocity $\vec{v}$ of SR and gravitational velocity

$\vec{V}_g(r')$ of TGR, write an expression for gravitational Doppler effect (in TGR), like relation (1) in SR as follows

$$\nu = \frac{\overline{\nu}}{1 - (V_g(r')/c_g)\cos\theta} \tag{3a}$$

$$\overline{\nu} = \nu_0(1 - V_g(r')^2/c_g^2)^{1/2} \tag{3b}$$

Again Eq. (3a) must be seen as expressing classical Doppler effect but in terms of the gravitational speed possessed by a stationary light source in a gravitational field. Since $V_g(r')$ in Eq. (3a) is not made manifest in translation of the light source as does the dynamical speed v in Eq. (2a), and since the motion of the source of a wave is required in the classical Doppler theory, $V_g(r')$ must be allowed to vanish in Eq. (3a), yielding $\nu = \overline{\nu}$. Equation (3b) must be retained, on the other hand, since it arises from gravitational time dilation, which actually exists in the context of TGR. Thus Eq. (3b) solely expresses gravitational Doppler effect in the context of TGR.

Equation (3b) expresses local (at a point in space) gravitational Doppler effect at radial distance r from the center of a gravitational field source of inertial mass M in the relativistic Euclidean 3-space Σ, in the context of TGR. The frequency $\overline{\nu}$ is the gravitational-relativistic frequency in the context of TGR, while frequency ν_0 is the natural frequency in the absence of TGR and SR.

The gravitational-relativistic frequency $\overline{\nu}$ in the context of TGR serves as the proper frequency in the special-relativistic Doppler effect in a gravitational field. Thus when both SR and TGR are present, such as happens when when a light ray is emitted by a source in motion at a velocity $\vec{v}$ relative to an observer on the flat relativistic spacetime (Σ, ct) of TGR in a gravitational field, the proper frequency ν_0 in the special-relativistic Doppler effect (1), must be replaced by the gravitational-relativistic frequency $\overline{\nu}$ of Eq. (3b). The Doppler effect in the context of combined SR and TGR is therefore given as follows

$$\nu = \frac{(1 - v^2/c_\gamma^2)^{1/2}\,\overline{\nu}}{1 - (v/c_\gamma)\cos\theta}$$

$$= (1 - \frac{V_g(r')^2}{c_g^2})^{1/2}\frac{(1 - v^2/c_\gamma^2)^{1/2}\,\nu_0}{1 - (v/c_\gamma)\cos\theta} \tag{4}$$

$$= (1 - \frac{2GM_{0a}}{r'c_g^2})^{1/2}\frac{(1 - v^2/c_\gamma^2)^{1/2}\,\nu_0}{1 - (v/c_\gamma)\cos\theta} \tag{5}$$

Relation (3b) for gravitational-relativistic frequency in the context of TGR, corresponds to the relation $m = m_0(1 - 2GM_{0b}/r'c_g^2)$ for mass, and several such local relations for physical parameters in the context of TGR summarized in Table I of Article 17 [2]. It can be observed in Eq. (3b) that a light-ray emitted by a massive body, such as the Sun, suffers larger local gravitational red shift at the surface of the body than when it has propagated to a large distance from the surface. A light ray emitted by a star will be observed in its proper (or natural) frequency ν_0 upon traveling to infinity where the effect of gravitational relativity on it vanishes.

The light source does not have to be located at distance r from the center of the field source for Eq. (3b) to be applicable. A light-ray of proper (or natural) frequency ν_0, which is momentarily propagating through radial distance r from the center of the field source along any direction in space, possesses gravitational frequency $\overline{\nu}$ at the instant it is propagating through the point. (This is so because the gravitational time dilation from which the gravitational frequency relation (or local gravitational Doppler effect) originates, is the same along every direction in space at a give position in a gravitational field). As the light ray propagates to another position of different radial distance r_1, say, from the center of the field source, it possesses gravitational frequency, $\overline{\nu}_1 = (1 - 2GM_{0\mathrm{a}}/r_1'c_g^2)^{1/2}\nu_0$, at the instant it is passing through distance r_1.

If the light source is constrained to be at rest at a point in space, which is of distances r_1 and r_2 from the centers of gravitational field sources of rest masses M_{01} and M_{02} respectively, or if a light ray of classical (or natural) frequency ν_0 emitted from elsewhere is momentarily passing through this point, then as derived and expressed by Eq. (38) or (42) of Article 25 [3], the resultant gravitational time dilation at this location in the context of TGR is given as follows, no matter how the gravitational field sources are located in space about the point,

$$\begin{aligned} \delta t = \overline{\gamma}_g^t \delta t' &= \gamma_{g1}\gamma_{g2}\delta t' \\ &= (1 - \frac{2GM_{0\mathrm{a}1}}{r_1'c_g^2})^{-1/2}(1 - \frac{2GM_{0\mathrm{a}2}}{r_2'c_g^2})^{-1/2}\delta t' \end{aligned} \tag{6}$$

The gravitational-relativistic frequency $\overline{\nu}$ of the light-ray emitted by the light source or of the light momentarily passing through is then related to the natural frequency ν_0 in this case as follows

$$\overline{\nu} = \nu_0(1 - \frac{2GM_{0\mathrm{a}1}}{r_1'c_g^2})^{1/2}(1 - \frac{2GM_{0\mathrm{a}2}}{r_2'c_g^2})^{1/2} \tag{7}$$

Generalizing this result, the relativistic frequency in the context of TGR at a point P in space at the common neighborhood of N gravitational field sources that are arbitrarily scatted in 3-space about this point is the following

$$\bar{\nu} \quad = \quad \prod_{i=1}^{N}(1 - \frac{2GM_{0\mathrm{a}i}}{r_i' c_g^2})^{1/2}\nu_0 \tag{8}$$

where $M_{0\mathrm{a}i}$ is the gravitational charge, which is equal to the rest mass M_{0i} of the ith gravitational field source in magnitude and r_i' is the radial distance from the center of the rest mass M_{0i} of ith field source to point P in the proper Euclidean 3-space Σ'.

2.1 *Local gravitational red shift on earth's surface in the context of the theory of gravitational relativity*

The resultant gravitational speed at the surface of the earth is due to the earth, the Sun, the other planets, as well as other stars in the Milky Way and stars in other galaxies. However the gravitational speeds due to stars in the Milky Way and stars in other galaxies at the surface of the earth are negligible compared to the gravitational speeds due to the earth, the Sun and the other planets, because of their large distances from the earth. Hence to an excellent approximation, the gravitational-relativistic frequency $\bar{\nu}$ in the context of TGR, of a light-ray of proper (or natural) frequency ν_0, which is emitted on the earth's surface, or which is passing through the surface of the earth is given as follows

$$\bar{\nu} = \nu_0(1 - \frac{2GM_{0\mathrm{a}S}}{r_{ES}' c_g^2})^{1/2}(1 - \frac{2GM_{0\mathrm{a}E}}{r_E' c_g^2})^{1/2}\prod_{P=1}^{8}(1 - \frac{2GM_{0\mathrm{a}P}}{r_P' c_g^2})^{1/2} \tag{9}$$

where M_{0S} is the rest mass of the Sun; r_{ES}' is the radial distance in Σ' from the center of the Sun to the surface of the earth; M_{0E} is the rest mass of the earth; r_E' is the radius in Σ' of the earth; M_{0P} is the rest mass of the pth planet and r_P' is the radial distance in Σ' from the center of the pth planet to the surface of the earth.

By substituting numerical values for the various quantities we find that Eq. (9) is given to a very good approximation still by the following

$$\bar{\nu} = \nu_0(1 - \frac{2GM_{0\mathrm{a}S}}{r_{ES}' c_g^2})^{1/2}(1 - \frac{2GM_{0\mathrm{a}E}}{r_E' c_g^2})^{1/2} \tag{10}$$

Equation (10) shall be approximated further, because of the weakness of the gravitational fields of the earth and the Sun at the surface of the earth by the following

$$\begin{aligned} \overline{\nu} &\approx \nu_0(1 - \frac{GM_{0aS}}{r'_{ES}c_g^2})(1 - \frac{GM_{0aE}}{r'_E c_g^2}) \\ &\approx \nu_0(1 - \frac{GM_{0aS}}{r'_{ES}c_g^2} - \frac{GM_{0aE}}{r'_E c_g^2} + \frac{G^2 M_{0aS} M_{0aE}}{r'_{ES} r'_E c_g^4}) \end{aligned} \quad (11)$$

or

$$\overline{\lambda} \approx \lambda_0(1 + \frac{GM_{0aS}}{r'_{ES}c_g^2} + \frac{GM_{0aE}}{r'_E c_g^2} - \frac{G^2 M_{0aS} M_{0aE}}{r'_{ES} r'_E c_g^4}) \quad (12)$$

This corresponds to fractional local gravitational red shift on earth of

$$\frac{\delta\overline{\nu}}{\nu_0} = \frac{\overline{\nu} - \nu_0}{\nu_0} = -\frac{GM_{0aS}}{r'_{ES}c_g^2} - \frac{GM_{0aE}}{r'_E c_g^2} + \frac{G^2 M_{0aS} M_{0aE}}{r'_{ES} r'_E c_g^4} \quad (13)$$

By substituting the numerical values: $M_{0S} \approx M_S = 1.99 \times 10^{30}$ kg; $M_{0E} \approx M_E = 5.98 \times 10^{24}$ kg; $r'_{ES} \approx r_{ES} = 149.6 \times 10^9$ m and $r'_E \approx r_E = 6.378 \times 10^6$ m, into Eq. (13) we have

$$\delta\overline{\nu}/\nu_0 \approx -9.858 \times 10^{-9} - 6.949 \times 10^{-10} + 6.850 \times 10^{-18} \quad (14)$$

A light ray (or electromagnetic wave) of proper (or classical) frequency ν_0, which is emitted at the surface of the earth, or which is emitted from elsewhere and is traveling through the surface of the earth, should suffer the local fractional gravitational red shift of Eq. (14), which is different from the familiar gravitational red shift as a light ray propagates between two positions of different gravitational potentials known in general relativity, which shall be discussed in the context of TGR later in this article.

If a gravitational field source, such as a distant star, which is in motion at a velocity $\vec{v}$ relative to an observer on earth, emits a light-ray of proper (or natural) frequency ν_0, then the light-ray suffers both special-relativistic Doppler red shift and local gravitational red shift relative to the earth-observer. The frequency ν of the light-ray observed by the earth-observer when it reaches the earth is given by replacing the natural frequency ν_0 in Eq. (1) in the absence of gravitational field by the gravitational frequency $\overline{\nu}$, (which serves as the proper frequency in special-relativistic Doppler effect as mentioned earlier), due to the gravitational fields of the earth, the Sun and the distant star on earth's surface. Thus

$$\nu = \frac{\overline{\nu}(1 - v^2/c_\gamma^2)^{1/2}}{1 - (v/c_\gamma)\cos\theta}$$

$$= \nu_0(1 - \frac{2GM_{0aE}}{r'_E c_g^2})^{1/2}(1 - \frac{2GM_{0aS}}{r'_{ES} c_g^2})^{1/2}(1 - \frac{2GM_{0a}}{r' c_g^2})^{1/2} \times \frac{(1 - v^2/c_\gamma^2)^{1/2}}{1 - (v/c_\gamma)\cos\theta} \quad (15)$$

where M_{0E}, M_{0S}, r'_{ES} and r'_E have been defined previously; M_0 is the rest mass of the moving distant star; r' is the radial distance from the center of the rest mass M_0 of the moving distant star to the surface of the rest mass of the earth in the proper Euclidean 3-space Σ'; v is the assumed constant speed of the emitting distant star relative to the observer on earth and θ is the angle between the light-ray reaching the earth and the direction of motion of the moving distant star emitter of the light-ray.

Actually the distant r from the center of the moving emitter to the surface of the earth is very large, hence $(1 - 2GM_{0a}/r'c_g^2) \approx 1$, thereby leaving only the Doppler effects of the gravitational fields of the Sun and the earth and of special relativity due to the motion of the emitting star relative to the earth-observer in Eq. (15).

3 Gravitational red shift as light propagates between two positions in a gravitational field in the context of the theory of gravitational relativity

Now let us consider the propagation of a light ray of natural frequency ν_0 from the surface of the Sun where it is emitted into outer space. The local gravitational frequency $\bar{\nu}_S$ at the surface of the Sun, (due to local gravitational red shift at the surface of the Sun by the Sun's gravitational field), of the light-ray at the moment it is emitted is,

$$\begin{aligned} \bar{\nu}_S &= \nu_0(1 - \frac{2GM_{0aS}}{r'_S c_g^2})^{1/2} \\ &\approx \nu_0 - \frac{GM_{0aS}}{r'_S c_g^2}\nu_0 - \frac{G^2 M_{0aS}^2 \nu_0}{4r'^2_S c_g^4} - \frac{G^3 M_{0aS}^3 \nu_0}{8r'^3_S c_g^6} - \cdots \end{aligned} \quad (16)$$

where M_{0S} is the rest mass of the Sun and r'_S is its radius. Upon the light ray propagating to radial distance r_1 from the center of the Sun, its gravitational frequency becomes $\bar{\nu}_1$ given as follows

$$\begin{aligned} \bar{\nu}_1 &= \nu_0(1 - \frac{2GM_{0aS}}{r'_1 c_g^2})^{1/2} \\ &\approx \nu_0 - \frac{GM_{0aS}}{r'_1 c_g^2}\nu_0 - \frac{G^2 M_{0aS}^2 \nu_0}{4r'^2_1 c_g^4} - \frac{G^3 M_{0aS}^3 \nu_0}{8r'^3_1 c_g^6} - \cdots \end{aligned} \quad (17)$$

The change in local gravitational frequency, (or in gravitational-relativistic frequency in the context of TGR), $\Delta\overline{\nu}_{1\mathrm{S}}$ as the light ray propagates from the surface of the Sun to radial distance r_1 from the center of the Sun is then given as follows

$$\begin{aligned}\Delta\overline{\nu}_{1S} &= \overline{\nu}_1 - \overline{\nu}_{\mathrm{S}} = \nu_0\left[(1-\frac{2GM_{0\mathrm{a}S}}{r'_1 c_g^2})^{1/2} - (1-\frac{2GM_{0\mathrm{a}S}}{r'_S c_g^2})^{1/2}\right] \\ &= \frac{\nu_0}{c_g^2}\left(-\frac{GM_{0\mathrm{a}S}}{r'_1} - (-\frac{GM_{0\mathrm{a}S}}{r'_S})\right) + \frac{\nu_0}{4c_g^4}\left(-\frac{G^2M_{0\mathrm{a}S}^2}{r'^2_1} - (-\frac{G^2M_{0\mathrm{a}S}^2}{r'_2})\right) \\ &\quad + \frac{\nu_0}{8c_g^2}\left(-\frac{G^3M_{0\mathrm{a}S}^3}{r'^3_1} - (-\frac{G^3M_{0\mathrm{a}S}^3}{r'^3_S})\right) + \cdots \end{aligned} \tag{18}$$

Since $-GM_{0\mathrm{a}S}/r'_1 = \Phi'_S(r'_1)$ and $-GM_{0\mathrm{a}S}/r'_S = \Phi'_S(r'_S)$, where $\Phi'_S(r')$ is the Newtonian gravitational potential of the Sun at radial distance r from the center of the inertial mass M_{S} $(= M_\odot)$ of the Sun in the relativistic Euclidean 3-space Σ of TGR, Eq. (18) can be written as follows

$$\begin{aligned}\Delta\overline{\nu}_{1\mathrm{S}} = \overline{\nu}_1 - \overline{\nu}_{\mathrm{S}} &= \frac{\nu_0}{c_g^2}(\Phi'_S(r'_1) - \Phi'_S(r'_S)) + \frac{\nu_0}{4c_g^2}(\Phi'_S(r'_1)^2 - \Phi'_S(r'_S)^2) \\ &\quad + \frac{\nu_0}{8c_g^2}(\Phi'_S(r'_1)^3 - \Phi'_S(r'_S)^3) + \cdots \end{aligned} \tag{19}$$

Equation (19) is given to a good approximation in the weak gravitational field of the Sun as follows

$$\begin{aligned}\Delta\overline{\nu}_{1\mathrm{S}} = \overline{\nu}_1 - \overline{\nu}_{\mathrm{S}} &\approx \frac{\nu_0}{c_g^2}(\Phi'_S(r'_1) - \Phi'_S(r'_S)) \\ &\approx \nu_0\Delta\Phi'_S/c_g^2 \end{aligned} \tag{20}$$

or

$$\Delta\overline{\nu}_{1\mathrm{S}} = \overline{\nu}_1 - \overline{\nu}_{\mathrm{S}} \approx \frac{GM_{0\mathrm{a}\mathrm{S}}\nu_0}{c_g^2}(\frac{1}{r'_S} - \frac{1}{r'_1}) \tag{21}$$

However $\Delta\overline{\nu}_{1\mathrm{S}}$ in Eq. (20) or (21) is just the difference between local gravitational frequencies (or between gravitational-relativistic frequencies) at radial distance r_1 from the center of the Sun and at the surface of the Sun in the context of TGR. It is not yet in the form it can be referred to as a red shift relation, since a red shift is the difference between measured or observed frequency and the natural frequency. That is, the difference between gravitational frequency $\overline{\nu}$ and natural frequency ν_0 in the present case. In order to obtain the gravitational red shift at radial distance r_1 from the center of the Sun, of a light-ray, as the light-ray propagates from the surface of the Sun to radial distance r_1 from the center of the Sun, we must obtain expression for $\overline{\nu}_1 - \nu_0$, since $\overline{\nu}_1$ is the observed frequency at r_1 and ν_0 is the natural frequency.

Now by replacing $\overline{\nu}_S$ by the right-hand side of Eq. (16) in $\overline{\nu}_1 - \overline{\nu}_S$ we have

$$\begin{aligned} \Delta\overline{\nu}_{1S} &= \overline{\nu}_1 - \nu_0(1 - \frac{2GM_{0as}}{r'_S c_g^2})^{1/2} \\ &\approx \overline{\nu}_1 - \nu_0(1 - \frac{GM_{0as}}{r'_S c_g^2}) \\ &= \overline{\nu}_1 - \nu_0 + \frac{GM_{0as}}{r'_S c_g^2} \end{aligned}$$

Hence

$$\delta\overline{\nu}_1 = \overline{\nu}_1 - \nu_0 = \Delta\overline{\nu}_{1S} - \frac{GM_{0as}\nu_0}{r'_S c_g^2} \qquad (22)$$

Using Eq. (21) to eliminate $\Delta\overline{\nu}_{1S}$ in Eq. (22) we have

$$\delta\overline{\nu}_{1S} = \overline{\nu}_1 - \nu_0 \approx -\frac{GM_{0as}\nu_0}{r'_1 c_g^2} \qquad (23)$$

Equation (23) expresses the difference between the frequency $\overline{\nu}_1$ of the light ray from the Sun, which is observed at radial distance r_1 from the center of the Sun and the natural frequency ν_0. Thus $\delta\overline{\nu}_{1S}$ is the gravitational red shift suffered by the light ray upon traveling from the surface of the Sun to radial distance r_1, and not $\Delta\overline{\nu}_{1S}$ as mentioned earlier.

Likewise a light-ray of natural frequency ν_0 emitted at radial distance r_1 from the center of the Sun will, upon traveling to radial distance r_2 from the center of the Sun, suffer a change in local gravitational frequency in the context of TGR of

$$\Delta\overline{\nu}_{21} = \overline{\nu}_2 - \overline{\nu}_1 \approx \frac{GM_{0as}\nu_0}{c_g^2}(\frac{1}{r'_1} - \frac{1}{r'_2})$$

or

$$\Delta\overline{\nu}_{21} \approx \overline{\nu}_2 - \nu_0(1 - \frac{GM_{0as}}{r'_1 c_g^2}) \approx \frac{GM_{0as}\nu_0}{c_g^2}(\frac{1}{r'_1} - \frac{1}{r'_2})$$

Thus the gravitational red shift suffered at radial distance r_2 by the light-ray upon propagating from r_1 to r_2 is

$$\delta\overline{\nu}_{21} = \overline{\nu}_2 - \nu_0 \approx -\frac{GM_{0as}\nu_0}{r'_2 c_g^2} \qquad (24)$$

We observe from the foregoing results that the gravitational red shift suffered at a point P_2 by a light-ray upon the light ray moving from point P_1 to point P_2 in a gravitational field in the context of TGR, depends upon the gravitational potential at P_2 only. The gravitational potential at P_1 where the light-ray is emitted or initially observed while passing through, does not count in the gravitational red shift at P_2. As shall be found shortly in this article, this represents a major break from the existing theory of gravitational red shift in the context of the general theory of relativity (GR), in which a light ray is known to be emitted in its natural frequency ν_0 and suffers gravitational red shift as it propagates away. Consequently the gravitational potential at the point P_1 of emission of a light ray appears in the gravitational red shift relation at another point P_2 as the light ray propagates from point P_1 to point P_2 in the exiting theory of gravitational red shift in GR.

By taking the natural frequency to the right-hand sides of equations (23) and (24) we have

$$\bar{\nu} \approx \nu_0 - \frac{GM_{0as}\nu_0}{r_1' c_g^2} = \nu_0(1 - \frac{2GM_{0S}}{r_1' c_g^2})^{1/2} \qquad (25)$$

and

$$\bar{\nu}_2 \approx \nu_0 - \frac{GM_{0as}\nu_0}{r_2' c_g^2} = \nu_0(1 - \frac{2GM_{0as}}{r_S' c_g^2})^{1/2} \qquad (26)$$

We find from these results that what we have called gravitational red shift at radial distance r_1 of a light-ray, upon the light ray traveling from the surface of the Sun to radial distance r_1 from the center of the Sun in Eq. (23), is actually the local gravitational red shift at radial distance r_1, and what has been called gravitational red shift at radial distance r_2 of a light-ray upon the light ray traveling from radial distance r_1 to radial distance r_2 from the center of the Sun in Eq. (24), is the local gravitational red shift at r_2. We conclude from the these that the gravitational red shift suffered by a light-ray, which propagates from any position P_1 to any other position P_2 in a gravitational field, is equal to the local gravitational red shift at position P_2. That is, $\delta\bar{\nu}_{21} = \delta\bar{\nu}_2$, in the context of TGR. The concept of non-local gravitational red shift between two positions at different gravitational potentials does not arise, and only local gravitational red shift at the location of a light-ray in a gravitational field at any given instant exists and can be measured in the context of TGR. The change in local gravitational frequency $\Delta\bar{\nu}_{12}$ as a light ray propagates from position P_1 to position P_2 in a gravitational field, although not a gravitational red shift, can also be measured, as shall be found later in this article.

According to relation (23) or (24), a light-ray of proper (or natural) frequency ν_0 emitted from the surface of the Sun, which propagates to the surface of the earth, will suffer local fractional gravitational red shift at the surface of the earth due to the gravitational field of the Sun solely on the surface of the earth of,

$$\frac{\delta\overline{\nu}_{ES}}{\nu_0} = \frac{\overline{\nu}_E - \nu_0}{\nu_0} = -\frac{GM_{0aS}}{r'_{ES}c_g^2} = -0.858 \times 10^{-9} \tag{27}$$

However this light-ray will actually suffer gravitational red shift due to the gravitational fields of both the earth and the Sun upon reaching the earth's surface, while neglecting the red shift at the surface of the earth of the gravitational fields of the other planets and distant stars. Hence the complete expression for $\delta\overline{\nu}_{ES}/\nu_0$ is the same as given for $\delta\overline{\nu}/\nu_0$ in Eq. (13) or (14).

The theory of gravitational red shift in the context of the general theory of relativity shall be discussed in detail in the next section. However for ease of comparison of the results of gravitational red shift in GR with the above results in the context of TGR, the corresponding results in GR shall be written here. The gravitational red shift relation known in GR depends on the difference between the gravitational potentials at the position where the light-ray is emitted and the position where it is received, as mentioned above. For a light ray emitted from the surface of the Sun, which propagates to radial distance r_1 from the center of the Sun, the gravitational red shift at r_1 in the context of GR is

$$(\delta\nu_{1S})_{GR} = \nu_1 - \nu_0 \approx -\frac{GM_S\nu_0}{c^2}\left(\frac{1}{r_S} - \frac{1}{r_1}\right) \tag{28}$$

And for a light ray emitted from the surface of the Sun, which is received on earth, the fractional gravitational red shift on earth in GR is

$$\left(\frac{\delta\nu_{ES}}{\nu_0}\right)_{GR} = \frac{\nu_E - \nu_0}{\nu_0} \approx -\frac{GM_S}{c^2}\left(\frac{1}{r_S} - \frac{1}{r_{ES}}\right) \approx -2.1 \times 10^{-6} \tag{29}$$

where M_S is the inertial mass of the sun sometimes denoted by $M_\odot$.

The relation (28) in GR corresponds to relation (23) in TGR, while relation (29) in GR corresponds to Eq. (27) in TGR. One observes from equations (28) and (29) that gravitational red shift is sensitive to the gravitational potential at the position where light is emitted in the context of GR, whereas this does not count at all in the gravitational red shift in the context of TGR, which depends on the gravitational potential at the location where the light is received only, as mentioned above. A critique of the red shift relation in GR shall be given later in this article, where it shall be concluded that the red shift relations in the context of TGR are the authentic red shift relations.

Now let us consider a terrestrial light ray of classical frequency ν_0 emitted at a vertical height H above the surface of the earth, and which propagates to the surface of the earth. This light ray is actually emitted at radial distances $(r_{\text{es}} - H)$ from the center of the sun and $(r_E + H)$ from the center of the earth, (assuming the sun is vertically above the earth), and upon reaching the earth's surface, it is at radial distance r_{ES} from the center of the sun and radial distance r_E from the center of the earth. The effects of the other planets and distant stars are negligible. The local gravitational frequencies $\overline{\nu}_H$ and $\overline{\nu}_E$ of the light ray at height H and at the surface of the earth in the context of TGR are given respectively as follows

$$\begin{aligned}
\overline{\nu}_H &= \nu_0(1 - \frac{2GM_{0\text{a}S}}{(r'_{ES} - H')c_g^2})^{1/2}(1 - \frac{2GM_{0\text{a}E}}{(r'_E + H')c_g^2})^{1/2} \\
\overline{\nu}_H &\approx \nu_0(1 - \frac{GM_{0\text{a}S}}{(r'_{ES} - H')c_g^2})(1 - \frac{GM_{0E}}{(r'_E + H')c_g^2}) \\
&\approx \nu_0(1 - \frac{GM_{0\text{a}S}}{(r'_{ES} - H')c_g^2} - \frac{GM_{0\text{a}E}}{(r'_E + H')c_g^2} + \frac{G^2M_{0\text{a}S}M_{0\text{a}E}}{(r'_{ES} - H')(r'_E + H')c_g^4}) \quad (30\text{a}) \\
&\approx \nu_0(1 - \frac{GM_{0\text{a}S}}{(r'_{ES} - H')c_g^2} - \frac{GM_{0\text{a}E}}{(r'_E + H')c_g^2}) \quad (30\text{b}) \\
\overline{\nu}_E &= \nu_0(1 - \frac{2GM_{0\text{a}S}}{r'_{ES}c_g^2})^{1/2}(1 - \frac{2GM_{0\text{a}E}}{r'_E c_g^2})^{1/2} \\
&\approx \nu_0(1 - \frac{GM_{0\text{a}S}}{r'_{ES}c_g^2} - \frac{GM_{0\text{a}E}}{r'_E c_g^2} + \frac{G^2M_{0\text{a}S}M_{0\text{a}E}}{r'_{ES}r'_E c_g^4}) \quad (30\text{c}) \\
&\approx \nu_0(1 - \frac{GM_{0\text{a}S}}{r'_{ES}c_g^2} - \frac{GM_{0\text{a}E}}{r'_E c_g^2}) \quad (30\text{d})
\end{aligned}$$

If we consider the approximations to the first order as done in Eqs. (30b) and (30d), then the change in frequency of the light ray as it propagates from height H to the surface of the earth is given as follows

$$\begin{aligned}
&\Delta\overline{\nu}_{EH} = \overline{\nu}_E - \overline{\nu}_H \\
&\approx \frac{GM_{0\text{a}E}\nu_0}{c_g^2}(\frac{1}{r'_E + H'} - \frac{1}{r'_E}) \quad (30\,\text{e})
\end{aligned}$$

since $r_{ES} >> H$. Hence

$$\Delta\overline{\nu}_{EH} = \overline{\nu}_E - \overline{\nu}_H \approx -\frac{GM_{0aE}H'\nu_0}{r_E'^2 c_g^2}(\frac{1}{1 + H'/r_E'}) \quad (30\,f)$$

or

$$\Delta\overline{\nu}_{EH} \approx -\frac{g'H'\nu_0}{c_g^2}(\frac{1}{1 + H'/r_E'}) \quad (30\,g)$$

On the other hand, the change in frequency of the light ray as it propagates from the surface of the earth to a vertical height H above the earth's surface in the relativistic Euclidean 3-space Σ of TGR is given to the first order approximations (30b) and (30d) as,

$$\Delta\overline{\nu}_{HE} = \overline{\nu}_H - \overline{\nu}_E \approx \frac{GM_{0aE}H'\nu_0}{r_E'^2 c_g^2}(\frac{1}{1 + H'/r_E'}) \quad (30\,h)$$

or

$$\Delta\overline{\nu}_{HE} \approx \frac{g'H'\nu_0}{c_g^2}(\frac{1}{1 + H'/r_E'}) \quad (30\,i)$$

The changes in frequency (30f) or (30g) and (30h) or (30i) are not to be referred to gravitational red shift and gravitational blue shift relations, since the shifts in frequency they express have not been measured relative to the proper (or natural) frequency ν_0. They can be measured experimentally as shall be described later in this paper.

Now,

$$\Delta\overline{\nu}_{EH} = \overline{\nu}_E - \overline{\nu}_H \approx \nu_0(\frac{GM_{0aE}}{(r_E' + H')c_g^2} - \frac{GM_{0aE}}{r_E' c_g^2})$$

or

$$\Delta\overline{\nu}_{EH} \approx \overline{\nu}_E - \nu_0(1 - \frac{GM_{0aE}}{(r_E' + H')c_g^2}) \approx \nu_0(\frac{GM_{0aE}}{(r_E' + H')c_g^2} - \frac{GM_{0aE}}{r_E' c_g^2})$$

Hence the gravitational red shift suffered by the light ray upon reaching the surface of the earth from height H is

$$\delta\overline{\nu}_{EH} = \overline{\nu}_E - \nu_0 \approx -\frac{GM_{0aE}}{r_E' c_g^2}\nu_0 \quad (31\,a)$$

or

$$\frac{\delta\overline{\nu}_{EH}}{\nu_0} = \frac{\overline{\nu}_E - \nu_0}{\nu_0} \approx -\frac{GM_{0aE}}{r_E' c_g^2} \quad (31b)$$

Equation (31b) gives the fractional red shift on the surface of the earth (to the first oder approximations expressed by Eqs. (30b) and (30d)), for a light ray of natural frequency ν_0, which is emitted at height H above the surface of the earth and propagates to the surface of the earth.

On the other hand, a light ray of classical frequency ν_0 emitted at the surface of the earth, which propagates to height H above the earth's surface, will suffer local fractional gravitational red shift at height H (to the first oder approximations expressed by Eqs. (30b) and (30d)) of,

$$\frac{\delta\overline{\nu}_{HE}}{\nu_0} = \frac{\overline{\nu}_H - \nu_0}{\nu_0} \approx -\nu_0\left(\frac{GM_{0aE}}{(r'_E + H')c_g^2} - \frac{GM_{0aE}}{r'_E c_g^2}\right)$$

or

$$\Delta\overline{\nu}_{HE} \approx \overline{\nu}_H - \nu_0\left(1 - \frac{GM_{0aE}}{(r'_E + H')c_g^2}\right) \approx -\nu_0\left(\frac{GM_{0aE}}{(r'_E + H')c_g^2} - \frac{GM_{0aE}}{r'_E c_g^2}\right)$$

Hence the gravitational red shift suffered by the light ray upon propagating to height H above the earth's surface is

$$\delta\overline{\nu}_{HE} = \overline{\nu}_H - \nu_0 \approx -\frac{GM_{0aE}}{(r'_E + H')c_g^2}\nu_0 \tag{32 a}$$

or

$$\frac{\delta\overline{\nu}_{HE}}{\nu_0} = \frac{\overline{\nu}_H - \nu_0}{\nu_0} \approx -\frac{GM_{0aE}}{(r'_E + H')c_g^2} \tag{32b}$$

Again Eq. (32b) gives the fractional red shift (to the first oder approximations expressed by Eqs. (30b) and (30d)), at height H above the surface of the earth, for a light ray of natural frequency ν_0, which is emitted on the surface of the earth, which propagates to height H above the surface of the earth.

The effect of the gravitational potential of the Sun must actually be added to equations (31b). This will be accomplished by using the approximations (30 a) and (30 c) and neglecting no terms in the derivations above to have the fuller form of Eq. (31b) as Eq. (13) and its numerical value (14), while the corresponding fuller form of Eq. (32b) is the following

$$\frac{\delta\overline{\nu}_{HE}}{\nu_0} \approx -\frac{GM_{0aE}}{(r'_E + H')c_g^2} - \frac{GM_{0aS}}{(r'_{ES} - H')c_g^2} + \frac{G^2 M_{0aS} M_{0aE}}{(r'_E + H')(r'_{ES} - H')c_g^4} \tag{33}$$

Equations (13) and (33) are the more complete expressions for fractional gravitational red shifts at the surface of the earth and at a height H above the earth's surface, for a terrestrial light ray emitted at height H, which propagates to the earth's surface and the converse respectively. We have now found that the fractional red shift relations (13) at the surface of the earth and (33) at height H above the earth's surface, are valid for a light ray emitted from anywhere in the universe at the instants it reaches the surface of the earth and height H above the earth's surface respectively.

Finally although the primed distances and primed parameters in the proper Euclidean 3-space Σ' must appear in Eqs. (30a-i) – (33), they can be approximated by the respective gravitational-relativistic distances and gravitational-relativistic parameters in the relativistic Euclidean 3-space Σ of TGR, without significant loss of accuracy in numerical results, due to the weaknesses of the gravitational fields of the Sun and the earth at the surface of the earth. In other words, we can let $M_{0\mathrm{a}E} \approx M_E$; $M_{0\mathrm{as}} \approx M_{\mathrm{S}}$ $(= M_\odot)$; $r'_E \approx r_E$; $r'_{ES} \approx r_{ES}$; $H' \approx H$ and $\vec{g}' \approx \vec{g}$ in obtaining numerical values in Eqs. (30a-i) – (33).

The terrestrial gravitational red shift relation in the context of GR for a light ray emitted from height H, which propagates to the surface of the earth is the following

$$(\delta \nu_{EH})_{GR} = \bar{\nu}_E - \nu_0 = \frac{gH\nu_0}{c^2}\left(\frac{1}{1 + H/r_E}\right) \tag{34}$$

$$= \frac{GM_E}{c^2 r_E^2}\left(\frac{1}{1 + H/r_E}\right) \tag{35}$$

Equation (34) or (35) in GR are exact because $M_{0\mathrm{a}E} = M_0 = M_E$; $r'_E = r_E$; $H' = H$ and $\vec{g}' = \vec{g}$ are exact relations (and not approximations) in classical gravitation. Eqs. (34) or (35) corresponds to the completely different relation (31 a) in TGR.

As can be seen from the red shift relations (28) and (29) and relations (34) and (35) in GR, which are based on first-order interaction of light with the Newtonian gravitational field, as shall be discussed shortly, a light ray is emitted in its natural frequency ν_0 where ever it is emitted in a gravitational field in the context of GR, (because it has not interacted with the external gravitational field and consequently has not lost energy at the point of emission). This implies that the local gravitational red shift (in the context of TGR) of the light ray at the point of emission is not taken into consideration in the red shift relation in GR. It shall be shown in the next section that the red shift relations (28) and (29), as well as relations (34) or (35) in GR are not valid, while the corresponding relations (23), (27), (31a) and (32a) in TGR are the valid relations.

The reason the gravitational red shift suffered by a light ray in propagating from an initial position to a final position in a gravitational field depends solely on the gravitational potential of the final position in the context of TGR, is that there is no first-order interaction between light and Newtonian gravitational field. Else the red shift of a light ray in TGR will depend on the difference between the gravitational potentials of the initial and final positions, as is the case in the red shift relation inherently based on a first-order interaction of light with the Newtonian gravitational field in GR, to be shown shortly.

That light has no first-order interaction with the Newtonian gravitational field follows from the fact that light possesses photonic rest mass $m_{0\gamma}$, which is equivalent to zero rest mass, (i.e. zero gravitational rest mass and zero electromagnetic rest mass), as discussed and presented as Eq. (16) in Article 11 [4]. Hence the Newtonian gravitational potential energy $-GM_{0\mathrm{a}}m_{0\gamma}/r$ of a photon is zero gravitational potential energy, and the difference in gravitational potential energy of a photon between two positions in a Newtonian gravitational field is zero energy. Moreover intrinsic curvature parameter $k_g(r')$ is zero everywhere in a Newtonian gravitational field, hence there is no intrinsic geometry-induced passive gravitational mass $-k_g(r')^2 m_{0\gamma}$ of photon that could interact with gravitational field in Newtonian gravitation. The minute second-order interaction of light with the gravitational field of the Sun in the context of TGR in which intrinsic curvature parameter $k_g(r')$ is small but non-zero, is being neglected in the derivation of the red shift relation in TGR in this first part of this article, but will be considered in explaining other phenomena in the second part.

One observes from equations (23), (24) and (27) that the local gravitational red shift due to a field source in the context of TGR, decreases from a maximum value at the surface of the field source where the gravitational-relativistic effect of the field is highest to zero value at infinity where the gravitational-relativistic effect of the gravitational field is zero. The light ray suffers no permanent red shift (or permanent "damage") as it is emitted from the surface of the field source and propagates away to infinity in the context of TGR. This shows that gravitational red shift is a relativity phenomenon and not a fundamental interaction phenomenon in the context of TGR, which is certainly so since it arises as a consequence of gravitational time dilation. On the other hand, a light ray emitted from a gravitational field source suffers gravitational red shift (or permanent "damage") upon propagating to infinity in GR, because red shift relation in GR is based on fundamental interaction of light with the (Newtonian) gravitational field, leading to permanent loss of energy of photon. Gravitational red shift in GR is hence not a relativity phenomenon.

It will be shown in section 5 of this paper that the Mössbauer-effect-based experiment of Pound, Rebka and Snider [5, 6], which is usually interpreted to have tested the terrestrial red shift relation (34) or (35) in GR to within 99.94%, can also be interpreted to have tested the corresponding red shift relations (31a) and (32a) in TGR to the same accuracy. As shall be shown, this does not mean that the two theories are both correct.

4 The theories of gravitational frequency shift of light in general relativity

4.1 Einstein's theory of gravitational red shift

The approach originated by Albert Einstein himself [7] does not rely on his field equations and their solution, but applies the principles of equivalence (the weak principle) and conservation of energy to the propagation of light in a Newtonian gravitational field effectively. In his approach, Einstein prescribes a gravity-free frame K with uniform upward acceleration $\vec{a}$ and a second frame K′, which is stationary relative to an observer in a uniform gravitational field, where the acceleration due to gravity $\vec{g}$ is equal in magnitude but oppositely directed to the acceleration $\vec{a}$ of frame K.

Einstein then allows light to be emitted along the upward direction of motion of the frame K at the instant, $t = 0$, when this frame starts to move from rest relative to the observer. Upon the light traveling distance H after time t, the frame K has acquired speed, $v = at$. But $t = H/c$, (to a first approximation). Hence $v = aH/c$. Now according to the special-relativistic Doppler effect, the observed frequency ν of a light ray of natural frequency ν_0 emitted from a source moving towards the observer at velocity v, is given as follows to the first order in $v/c \approx 0$,

$$\nu = \nu_0(1 - v/c) \tag{36}$$

Equation (1) reduces as Eq. (36) to a first order in v/c by letting $\theta = \pi$ in that equation, for a source a light ray that is moving towards the observer along the line-of-sight. This is the case of the light ray propagating in the accelerated frame K above. We must therefore replace v by aH/c, so that the observed frequency of the light after time t of motion of the frame is

$$\nu = \nu_0(1 + aH/c^2) \tag{37}$$

Einstein then allows this light to be emitted at a point A and to propagate vertically downward (in the direction of gravitational acceleration $\vec{g}$) to a point B in the frame K′, where the distance from point A to point B is H. He applies the equivalence of inertial acceleration and gravitational acceleration, ($\vec{a} \equiv \vec{g}$) to write the observed frequency of the light ray at point B as follows

$$\nu_B = \nu_0(1 + gH/c^2) \tag{38}$$

If, on the other hand, the light ray is emitted at the point B and propagates vertically upwards to the point A in frame K′, then its frequency ν_A at point A will be

$$\nu_A = \nu_0(1 - gH/c^2) \tag{39}$$

It must be noted that the foregoing arguments and derivations by Albert Einstein are based within the framework of classical mechanics and classical gravitation, as Eqs. (36) – (39) show. This, in the light of the present theory, implies that his arguments and derivations pertain to the flat proper spacetime (Σ', ct') containing the rest masses M_0 of gravitational field sources and m_0 of test particle, but where the equivalences $M = M_0$, $m = m_0$ and $-GM_{0\mathrm{a}}/r' = -GM/r$ are exact. Thus inherent in Einstein's derivation of Eq. (38) and (39) is the assumption of interaction of light with the Newtonian gravitational field, thereby converting its (radiation) energy to changes in its gravitational potential energy. For by multiplying through by the Planck constant and rearranging, Eq. (38) or (39) can be written as follows

$$h\nu_A - h\nu_0 = \pm\, m_{0\gamma} gH = \pm\frac{h\nu_0}{c^2} gH \tag{40}$$

where, $m_{0\gamma} = h\nu_0/c^2$, is mass equivalent (or the photonic rest mass) of the radiation energy $h\nu_0$. Also since $-gH = \Phi_B - \Phi_A = \Delta\Phi$, Eq. (40) can be written as

$$h\nu_A - h\nu_0 = h\Delta\nu = \pm\Delta\Phi h\nu_0/c^2 \tag{41}$$

Thus the increase or decrease in energy of the photon in Eq. (38) or (39) is provided by the decrease or increase in the "Newtonian gravitational potential energy" of the photon as it "falls" from point A to point B. In its original form, Einstein's argument in support of the interaction of light with the Newtonian gravitational field (or what is tantamount to this) is as follows [7],

Let two material systems S_1 and S_2 be situated a distance H apart along the z-axis of frame K′. The following cyclic process is carried out

1. Energy E is emitted in the form of radiation in S_2 towards S_1, where, by the result of Eq. (38) or (39), energy $E(1 + gH/c^2)$ is absorbed.
2. A body W of mass M is lowered from S_2 to S_1, work MgH being done in the process.

3. The energy E is transferred from S_1 to the body W, while W is in S_1. Let the gravitational mass of W be thereby changed so that it acquires the value M'.
4. Let W be again raised to S_2, work $M'gH$ being done in the process.
5. Let E be transferred from W to S_2.

Going further in his argument, Einstein remarked that the effect of the cycle is simply that S_1 has undergone the increase of energy EgH/c^2, and that the quantity of energy $M'gH - MgH$ has been conveyed to the system in the form of mechanical work. By the principle of conservation of energy, we therefore have

$$EgH/c^2 = M'gH - MgH$$

or

$$\Delta M = M' - M = E/c^2 \tag{42}$$

The import of Einstein's argument is that a radiation energy, $E = h\nu_0$, possesses equivalent inertial mass, $\Delta M = h\nu_0/c^2$. Thus as a radiation propagates in a Newtonian gravitational field, its positive equivalent inertial mass interacts with the field and it is attracted like a material particle. Thus the above argument and the further argument that a body of mass M suspended on a spring balance in system K′ will indicate a weight of Mg, and when a radiation energy E is transferred to this body, then by virtue of the law of inertia of energy, will indicate a weight of $(M + E/c^2)g$, establishes for Einstein the validity of the first-order interaction of radiation (or light) with the Newtonian gravitational field and, consequently the validity of the red shift relation (38) inherently based on this notion.

Other approaches in GR, such as the one that starts with the gravitational time dilation formula on page 135 of [8], all end up in the red shift relation (41) derived originally by Albert Einstein. For a light ray of natural frequency ν_0 emitted at position A at the surface of the Sun, which propagates to another position B of radial distance r_1 from the center of the Sun, the gravitational red shift relation (41) is given explicitly as follows

$$\begin{aligned}(\delta\nu_{1S})_{GR1} = \nu_B - \nu_0 &= -\frac{\nu_0}{c^2}(\Phi_B - \Phi_A)\\ &= \frac{GM_S\,\nu_0}{c^2}\left(\frac{1}{r_1} - \frac{1}{r_S}\right)\end{aligned} \tag{43}$$

The subscript GR1 is used to denote the present theory due to Einstein as the first theory of gravitational frequency shift in general relativity. There is a second theory of gravitational frequency shift in general relativity to be discussed in the next subsection, which shall be denoted by GR2.

One should actually write, $(\Delta\nu_{1S})_{GR1} = \nu_B - \nu_A$, instead of $(\delta\nu_{1S})_{GR1} = \nu_B - \nu_0$ for the change in frequency between emission of light at A and its reception at B. However a light ray has not suffered gravitational red shift at the moment of emission in the context of the theory of gravitational red shift due to Albert Einstein, thereby making $\nu_A = \nu_0$. Thus in the context of the theory of gravitational red shift based on a first-order interaction of light with the Newtonian gravitational field, a light ray emitted anywhere in a gravitational field is emitted in its natural frequency ν_0, (assuming no motion of its source relative to the observer). Whereas a light ray is emitted in its gravitational-relativistic (or local gravitational) frequency, $\overline{\nu} = \nu_0(1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{1/2}$, at radial distance r from the center of the inertial mass M in Σ of a gravitational field source of rest mass M_0 in Σ', in the context of the theory of gravitational red shift in (TGR).

4.2 Local gravitational frequency shift based on propagation of light in Schwarzschild geometry

By considering the motion of a test particle to take place along a geodesic in the Schwarzschild geometry, the laws of conservations of energy and angular momentum for equatorial orbits have been derived respectively as follows, see pages 655-657 of [9] for example.

$$E = |g_{00}(r)|^{1/2}E_{\mathrm{local}}(r) = \mathrm{constant} \qquad (44)$$

$$p_\phi = (E_{\mathrm{local}}(r)/c^2)v_\phi r = \mathrm{constant} \qquad (45)$$

where $E_{\mathrm{local}}(r)$ is the energy measured locally (in the absence of special relativity) at r; E is the "energy at infinity", that is, E is the energy measured locally (in the absence of special relativity) at $r = \infty$, and v_ϕ is the component tangent to the equatorial orbit of the particle's velocity.

As commented on page 659 of [9], the conservation law (44) is valid for any time-independent metric tensors for which $g_{0j} = 0$, and for particles with both zero and nonzero rest masses. It is sometimes called the "law of energy red shift". It describes how the locally measured energy of a particle or photon changes, (i.e, are "red- shifted" or "blue-shifted") as it climbs out or falls into a static gravitational field.

If we consider a test particle of rest mass m_0, which is at rest at infinity relative to an observer, then its total energy is, $E = m_0c^2$, in Eq. (44). Then if we let its local total energy at radial distance r from the center of the field source be mc^2, that is, if we let $E(r) = \overline{m}c^2$, then Eq. (44) becomes the following

$$mc^2(g_{00}(r))^{1/2} = m_0c^2$$

or

$$m = m_0(g_{00}(r))^{-1/2} = \frac{m_0}{(1 - 2GM/rc^2)^{-1/2}}; \quad \text{(in GR)} \tag{46}$$

Although the mass of the test particle is generally assumed to be independent of position in a gravitational field, since this is required along with the validity of local Lorentz invariance for the strong equivalence principle to be valid in GR, the mass relation (46) − which relates the inertial mass m of the test particle observed or measured at radial distance r from the center of a gravitational field source to its rest mass m_0 (in the absence of SR) − is implied by the "law of energy red shift" of Eq. (44). Equation (46) has also been derived by a different approach in [10]. It is to be recalled that the corresponding mass relation derived in the context of the theory of gravitational relativity (TGR) in Articles 14 and 15 [11, 12] is the following

$$m = m_0(1 - \frac{2GM_{0a}}{r'c_g^2}); \quad \text{(in TGR)} \tag{47}$$

Now let us replace the test particle in the above by the massless photon. A photon of proper (or natural) frequency ν_0 possesses energy $h\nu_0$ at infinity, $(r = \infty)$, from the center of a gravitational field source. Hence for a photon (or a light ray) we must let $E = h\nu_0$ in Eq. (44). If we let the local energy (in the absence of SR) measured while it is passing through radial distance r from the center of the gravitational field source be $h\nu$, then Eq. (44) becomes the following for a light ray propagating in a gravitational field

$$h\nu|g_{00}(r)|^{1/2} = h\nu_0$$

or

$$\nu = |g_{00}(r)|^{-1/2}\nu_0 = \nu_0(1 - \frac{2GM}{rc^2})^{-1/2}; \quad \text{(in GR)} \tag{48}$$

Equation (48) is the local frequency shift relation (or gravitational Doppler effect) implied by the energy conservation law (44) derived from the study of propagation of light in Schwarzschild geometry in GR. It states that the gravitational frequency $\overline{\nu}$ of light ray is larger than its natural frequency ν_0, (that is, it is blue-shifted), at all finite radial distances from the center of a gravitational field source.

The relation in TGR, which corresponds to relation (48) in GR is the local gravitational red shift relation (8) for $N = 1$, derived earlier in this article.

The first point of departure of the theory of gravitational frequency shift posited on propagation of light in Schwarzschild geometry in the foregoing, from Einstein's theory of gravitational frequency shift inherently posited on a first-order interaction of light with Newtonian gravitational field, while upholding the principle of equivalence and conservation of energy, is that the Einstein's theory predicts redshift of a light ray that is emitted from the surface of a gravitational field source and propagates to a position away from the surface, while the theory of gravitational frequency shift based on propagation of light in Schwarzschild geometry of this sub-section predicts blue shift. The second point of departure is that the latter predicts that a light ray of natural frequency ν_0, which is emitted at radial distance r from the center of a gravitational field source, will be observed in its local frequency $\overline{\nu}$ given by Eq. (44) at the position of emission in the theory of this sub-section, while Einstein's theory (or the former) states that a light ray is not red-shifted at the position of emission. Thus the two theories of gravitational frequency shift in GR are mutually contradictory.

Now let us calculate the change in frequency $(\Delta\overline{\nu}_{1S})_{GR2}$ as a light ray emitted from the surface of the Sun propagates to radial distance r_1 from the center of the Sun according to relation (48). This is given as follows

$$\begin{aligned}(\Delta\overline{\nu}_{1S})_{GR2} &= \overline{\nu}(r_1) - \overline{\nu}(r_S) \\ &\approx \nu_0(1 + \frac{GM_S}{r_1c^2}) - \nu_0(1 + \frac{GM_S}{r_Sc^2}) \\ &\approx \frac{GM_S\nu_0}{c^2}(\frac{1}{r_1} - \frac{1}{r_S}) \qquad (49)\end{aligned}$$

where the subscript GR2 is used to denote the theory of gravitational frequency shift of this sub-section, posited on propagation of light in Schwarzschild geometry. Although Eq. (49) looks the same as Eq. (43) derived earlier in the context of gravitational frequency shift posited on a first-order interaction of light with the Newtonian gravitational field, they are not the same. This is so because $(\delta\nu_{1S})_{GR1} = \nu(r_1) - \nu_0$ in Eq. (43), whereas $(\Delta\nu_{1S})_{GR2} = \nu(r_1) - \nu(r_S)$ in Eq. (49). Hence Eq. (43) expresses red shift while Eq. (49) expresses change in local gravitational frequencies in the context of gravitational red shift theory based on propagation of light in the Schwarzschild geometry (GR2).

In order to derive a relation in the context of GR2, which can be compared with Eq. (43) of GR1, we must replace $\nu(r_S)$ by ν_0 in the definition, $(\Delta\nu_{1S})_{GR2} = \nu(r_1) - \nu(r_S)$, in Eq. (49) as follows

$$\begin{aligned}(\Delta\nu_{1S})_{GR2} &= \nu(r_1) - \nu(r_S) = \nu(r_1) - \nu_0(1 - \frac{2GM_S}{r_S c^2})^{-1/2} \\ &\approx \nu(r_1) - \nu_0(1 + \frac{GM_S}{r_S c^2}) \approx \frac{GM_S \nu_0}{c^2}(\frac{1}{r_1} - \frac{1}{r_S})\end{aligned}$$

Hence

$$(\delta\nu_{1S})_{GR2} = \nu(r_1) - \nu_0 \approx \frac{GM_S \nu_0}{c^2}(\frac{1}{r_1} - \frac{1}{r_S}) + \frac{GM_S}{r_S c^2}$$

or

$$(\delta\nu_{1S})_{GR2} = \bar{\nu}(r_1) - \nu_0 \approx \frac{GM_S \nu_0}{r_1 c^2} \tag{50}$$

It is $(\delta\bar{\nu}_{1S})_{GR2}$ of Eq. (50) that can be compared with $(\delta\nu_{1S})_{GR1}$ of Eq. (43), from which we again find departure from one another of the two theories of gravitational frequency shift in GR.

The relation in TGR that corresponds to Eq. (50) in GR is Eq. (23), from which we find that (local) gravitational frequency shift $\delta\bar{\nu}$ in TGR differs from (local) gravitational frequency shift $\delta\bar{\nu}_{GR2}$ only in sign. Irrespective of where a light ray is emitted and irrespective of what direction in space it is moving, the gravitational frequency shift that can be measured at radial distance r from the center of a gravitational field source depends only on the gravitational potential at r as in Eq. (23) in TGR and Eq. (50) in the context of gravitational frequency shift theory based on propagation of light in Schwarzschild geometry (GR2). On the other hand, the frequency shift $(\delta\nu)_{GR1}$ at r depends on the difference between the gravitational potentials at position r and position r_1 where the light was emitted as in Eq. (43) in the context of Einstein's theory of gravitational frequency shift (GR1), inherently based on a first-order interaction of light with the Newtonian gravitational field and satisfaction of equivalence principle.

Now let us consider the propagation of light ray near the surface of the earth in the context of GR2. Let a light ray propagate from height H above the surface of the earth to the surface of the earth. The change in gravitational frequency of light is given as follows

$$\begin{aligned}(\Delta\nu_{EH})_{GR2} &= \nu_E - \nu_H \\ &\approx \frac{GM_E \nu_0}{c^2}(\frac{1}{r_E} - \frac{1}{r_E + H})\end{aligned}$$

$$\approx \quad \frac{GM_E H\nu_0}{r_E^2 c^2}\left(\frac{1}{1+H/r_E}\right)$$

or

$$(\Delta\nu_{HE})_{GR2} = \nu_H - \nu_E \approx \frac{gH\nu_0}{c^2}\left(\frac{1}{1+H/r_E}\right) \tag{51}$$

On the other hand, if the light ray is emitted from the surface of the earth and propagates to vertical height H above the earth's surface, then the change in frequency is,

$$(\Delta\nu_{HE})_{GR2} = \nu_H - \nu_E \approx -\frac{gH\nu_0}{c^2}\left(\frac{1}{1+H/r_E}\right) \tag{52}$$

Equations (51) and (52) express changes in local frequencies in traveling from vertical height H to the surface of the earth, and in traveling from the surface of the earth to vertical height H respectively. They are not to be referred to as gravitational blue shift and gravitational red shift relations. Hence they cannot be compared with the corresponding relations implied by equations (40) and (41) implied by relation(38) and the corresponding relation implied by Eq. (39), which are derived in the context of the theory of gravitational frequency shift posited on a first-order interaction of light with the Newtonian gravitational field (GR1). However equations (51) and (52) are to be compared with the corresponding equations (30 g) and (30 i) in TGR.

What is to be referred to as gravitational frequency shift of a light ray as it propagates from vertical height H, (or from anywhere for that matter), to the surface of the earth in the context of GR2 is given from Eq. (51) as follows

$$(\Delta\nu_{EH})_{GR2} \approx \nu_E - \nu_0\left(1 + \frac{GM_E}{(r_E+H)c^2}\right) \approx \frac{GM_E\nu_0}{c^2}\left(\frac{1}{r_E} - \frac{1}{r_E+H}\right)$$

Hence

$$(\delta\nu_{EH})_{GR2} = \nu_E - \nu_0 \approx \frac{GM_E\nu_0}{r_E c^2} \tag{53}$$

And the frequency shift at height H above the earth's surface of a light ray emitted from the surface of the earth or from elsewhere is

$$(\delta\nu_{HE})_{GR2} = \nu_H - \nu_0 \approx \frac{GM_E\nu_0}{(r_E+H)c^2} \tag{54}$$

Equations (53) and (54) in the context of the theory of gravitational frequency shift (GR2) based on propagation of light in Schwarzschild geometry in GR, corresponds to Eqs. (30a) and (32a) respectively in TGR.

We have isolated three distinct theories of gravitational frequency shift of light in this article namely, the gravitational frequency shift theory in the context of TGR (Theory 1) of the preceding section; the gravitational frequency shift theory due to Albert Einstein, in which light inherently has first-order interaction with the newtonian gravitational field in GR (Theory 2) and the gravitational frequency shift theory based on propagation of light in Schwarzschild geometry in GR (Theory 3). The results of the three theories are summarized in Table I for ease of comparison.

One finds from Table I that the two theories in existence in GR namely, Theory 2 and Theory 3, differ significantly from one another, whereas Theory 3 differs from Theory 1 only in sign. Interestingly the shifts in frequency as light propagates from the surface of the earth to a height H predicted by Theory 1 (in the context of TGR) and Theory 2 (Einstein's theory posited on a first-order interaction of light with Newtonian gravitational field) are the same. Clearly this singular point of agreement of the two theories does not mean that they are both correct. A formal critique of the conceptual and geometrical foundations of Theory 2 and Theory 3 shall now be given.

4.3 Critique of the theories of gravitational frequency shift of light in general relativity

Let us start with Theory 2 in Table I. In Theory 2 is the inherent assumption that light interacts with the Newtonian gravitational field. Einstein's argument in support of this assumption presented earlier in sub-section 4.1 culminates in Eq. (42), which relates radiation energy, $E = h\nu_0$, to increase in mass $\Delta M = M' - M$. However the mass-equivalent of proper radiation energy, $m_{0\gamma} = h\nu_0/c_\gamma^2$, referred to as photonic rest mass, which evolves in the proper Euclidean 3-space Σ' in the context of the theory of absolute intrinsic gravity (ϕAG), at the first stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in a gravitational field is absolutely massless. It is equivalent to zero rest mass of a material particle or body. That is, it is equivalent to zero gravitational rest mass and zero electromagnetic (or dynamical) rest mass, as stated explicitly by Eq. (16) of Article 11 [4].

It follows from the foregoing paragraph that the radiation energy, $E = h\nu_0$, transferred to the body W of inertial mass M, upon lowering this body from frame S_1 through height H to frame S_2 in Einstein's thought experiment presented earlier in sub-section 4.1, cannot increase the mass of W to M′ at the classical gravitation regime in which Einstein had argued. There is however a second-order interaction

Table I: Results of the theories of gravitational frequency shift in TGR and GR.

	Theory 1 (TGR)	Theory 2 (GR1)	Theory 3 (GR2)
Mass relation	$m = m_0(1 - \frac{2GM_0\mathrm{a}}{r' c_g^2})$	– –	$m = m_0(1 - \frac{2GM}{rc^2})^{-1/2}$
Local frequency relation	$\bar{\nu} = \nu_0 \,(1 - \frac{2GM_0\mathrm{a}}{r' c_g^2})^{1/2}$	– –	$\nu = \nu_0 \,(1 - \frac{2GM}{rc^2})^{-1/2}$
Change in gravitational frequency as light propagates from r_1 to r_2	$\Delta\bar{\nu}_{21} = \frac{GM_0\mathrm{a}\,\nu_0}{c^2} \times (\frac{1}{r'_1} - \frac{1}{r'_2})$	– –	$\Delta\nu_{21} = -\frac{GM\,\nu_0}{c^2} \times (\frac{1}{r_1} - \frac{1}{r_2})$
Shift in frequency as light propagates from r_1 to r_2	$\delta\bar{\nu}_{21} = -\frac{GM_0\mathrm{a}\,\nu_0}{r'_2 c^2}$	$\delta\nu_{21} = \frac{GM\,\nu_0}{c^2} \times (\frac{1}{r_2} - \frac{1}{r_1})$	$\delta\nu_{21} = \frac{GM\,\nu_0}{r_2 c^2}$
Change in frequency as light propagates from earth's surface to height H	$\Delta\bar{\nu}_{HE} = \frac{g' H'\,\nu_0}{c^2}$ $\Delta\bar{\nu}_{EH} = -\frac{g' H'\,\nu_0}{c^2}$	– –	$\Delta\nu_{HE} = -\frac{gH\,\nu_0}{c^2}$ $\Delta\nu_{EH} = \frac{gH\,\nu_0}{c^2}$
Shift in frequency as light propagates from earth's surface to height H	$\delta\bar{\nu}_{HE} = -\frac{GM_0\mathrm{a}_E\,\nu_0}{(r'_E+H')c^2}$	$\delta\nu_{HE} = -\frac{GM_E\,\nu_0}{(r_E+H)c^2}$	$\delta\nu_{HE} = \frac{GM_E\,\nu_0}{(r_E+H)c^2}$

of light with gravitational field that causes photon to possess negative geometry-induced passive gravitational mass $-k_g(r')h\nu_{0\gamma}/c_\gamma = -2GM_{0\mathrm{a}}h\nu_{0\gamma}/r'c_g^2c_\gamma^2$, in the context of TGR, as found in Article 27 [13], see Eqs. (71) – (75) of that article, which will cause the weight of the body W to decrease in Einstein's thought experiment. The mass of W will remain unchanged from its value M upon transferring radiation energy E to it classically. This then invalidates Einstein's argument and Eq. (42) derived from it. The conclusion of the argument that light has first-order interaction with the Newtonian gravitational field is consequently invalid.

The relativistic gravitational potential and gravitational-relativistic frequency relations derived in the context of TGR namely,

$$\Phi(r) = \Phi'(r')(1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{1/2} \text{ and } \nu = \nu_0(1 - 2GM_{0\mathrm{a}}/r'c_g^2)^{1/2},$$

which must be employed in a gravitational field of arbitrary strength, were unknown to Einstein. Rather the equivalence $\Phi(r) = \Phi'(r')$ and the equivalence $\nu = \nu_0$ in the absence of special relativity, which the principle of equivalence implies at a point in a gravitational field, were applied by Einstein in a gravitational field or arbitrary strength in his argument. Thus Einstein's argument is tantamount to allowing light of classical (or natural) frequency ν_0 to interact with the classical gravitational potential $\Phi'(r') \equiv \Phi(r)$ from the point of view of the present theory (TGR).

The Theory 2 of gravitational frequency shift inherently assumes that a light ray possesses positive equivalent inertial mass that interacts with the Newtonian gravitational field, according to the foregoing. According to Theory 2, this interaction leads to a decrease or increase in energy (or frequency) of the light ray, as the case may be, as it propagates between two positions of different gravitational potentials in a Newtonian gravitational field. Theory 2 is definitely not valid because of this inherent underlying invalid assumption.

In the case of Theory 3 in Table I, it shall simply be noted that the asymptotic definition of local energy relation in a gravitational field in general relativity given by Eq. (44), described as "law of energy red-shift" earlier, is not a valid relation when the particle in motion on curved spacetime in a gravitational field in the context of GR is photon. The reason for this is found in the new spacetime/intrinsic spacetime geometry for the theories of relativity and gravitation in the four-world picture of the present theory. As has been established in [1], the light cone concept does not exist in the new geometry. Thus light does not propagate along the surface of the light cone in the new geometry. Rather the absolutely massless particulate nature of the photon—the photonic mass m_γ—naturally propagates at speed c_γ along the light-axis χ that lies parallel to the time dimension ct, while the wave nature (or radiation) $h\nu$ of photon naturally propagates at speed c_γ along a coordinates $c_\gamma \tau$ that can be along any direction in Σ, as already introduced in [2], and shall be embarked upon and developed fully in an article later in this volume. On the other hand, the three-dimensional inertial mass m of a material particle or body exists and moves in the Euclidean 3-space Σ, while its one-dimensional inertial mass ε/c^2 exists and moves in the time dimension ct, as has been shown graphically in the previous articles.

Thus when a material particle is in motion on the proposed curved spacetime in a gravitational field in GR, its inertial mass m moves on curved 3-space, while its one-dimensional mass ε/c^2 moves in curved time dimension ct. On the other hand, the absolutely massless photonic mass m_γ moves along the curved time dimension ct, while the wave nature of photon $h\nu$ moves on curved 3-space Σ. An implication of these is that, while the components of the Schwarzschild metric g_{00} and g_{11} retain their usual roles when describing the motion of material particles on curved spacetime in a gravitational field in the context of GR, they must be interchanged when describing the motion of photon (or light) on curved spacetime in the context of GR. In other words, while the metric tensor with signature $[+---]$ is valid for the description of the motion of material particles on curved spacetime in GR, the signature $[---+]$ must be adopted for the description of the motion of photon (or light).

As follows from the foregoing paragraph, the component of the metric tensor g_{00} in the "law of energy red-shift" (44) must be replaced by g_{11} when applying that relation to the motion of photon (or light) on curved spacetime in GR. In other words, that relation must be modified as follows in the case of photon (or light) as the moving particle,

$$E = |g_{11}(r)|^{1/2} E_{\text{local}}(r) = \text{constant} \tag{55}$$

The frequency relation (48) implied by Eq. (44) becomes the following implied by Eq. (55)

$$\nu = |g_{11}(r)|^{-1/2}\, \nu_0 = (1 - \frac{2GM}{rc^2})^{1/2}\, \nu_0 \tag{56}$$

Relation (56) is the same as the local gravitational frequency relation (or gravitational-relativistic frequency) (8), for $N = 1$, implied by gravitational time dilation in the context of TGR, except the difference in the notations of GR adopted in (56) and the notation of TGR in (8). The notation in Eq. (56) can be adopted in Eq. (8)

of TGR in the weak gravitational field limit for which $2GM_{0a}/r'c_g^2 >> 1$. Thus once the "law of energy red shift" (44) has been modified as Eq. (55) when being applied to photon (or light), leading to the frequency relation (56), the second theory of gravitational red shift in GR (Theory 3), becomes the same as the valid theory of gravitational red shift in TGR (Theory 1), at least in the case of one spherical gravitational field source of weak gravitational field, for which $2GM_{0a}/r' \approx 2GM/r$ is a good approximation. On the other hand, the theory of gravitational red shift implied by "law of energy red shift" (44) or the implied frequency relation (44), described earlier, is an invalid theory.

Theory 1 of gravitational red shift in Table I, derived in the context of the theory of gravitational relativity in section 2 of this article, is the authentic theory. In this theory, the period of oscillation of an electromagnetic wave, which is emitted at radial distance r from the center of a gravitational field source by an emitter at rest relative to the laboratory, or which is emitted by a 'stationary' source elsewhere and is propagating through this position, possesses local gravitational-relativistic frequency of Eq. (8), for $N = 1$, as a consequence of gravitational time dilation in the context of TGR.

5 The experimental aspect

The Pound, Rebka and Snider experiment (PRS), [5, 6], has been remarked to be the most accurate test of gravitational red shift relation of Theory 2 so far [8]. The experiment applies the Mössbauer effect. The ^{57}Fe has a line near 14.4 KeV with fractional width of 10^{-17}. If a radioactive sample of ^{57}Fe is placed near a non-radioactive sample of the same material, a large fraction of the gamma radiation falling on the sheet is resonantly absorbed. If however either the absorber or the emitter is moved at a velocity of only a few millimeter per second, the consequent Doppler shift becomes as large as the line width, and resonant absorption falls off very rapidly. This is the Mössbauer effect used to detect quite small frequency shifts.

In the PRS, the emitter and absorber were placed at opposite ends of a 22.5 meter helium-filled bag in a shaft in the Jefferson Physical Laboratory of Harvard University. Resonant absorption of gamma from the emitter was lost due to gravitational red shift of Theory 2 in the context of the general theory of relativity. Now let the emitter be moved vertically upward towards the absorber at a small velocity v. Then the frequency ν of the emitted light pulse is given relative to the absorber from the theory of Doppler effect in special relativity as follows in the situation where light is emitted along the direction of motion of the emitter, which is along the line-of-sight of the absorber,

$$\nu = \nu_0(\frac{1 + v/c}{1 - v/c})^{1/2} \approx \nu_0(1 + v/c) \tag{57}$$

It is the Doppler blue-shifted frequency ν that is then gravitationally red-shifted upon reaching the absorber. That is, it is the frequency ν that must serve as the proper (or natural) frequency in the gravitational red shift relation in the context of Theory 2. By moving the emitter at the appropriate velocity, the Doppler blue-shift due to motion can be made to completely nullify gravitational red shift, thereby restoring resonant absorption in the absorber at the top end of the shaft. This basically is the principle of the experiment.

Now let us again summarize the three theories of Table I, but now for the purpose of determining whether the PRS has tested any other theory apart from Theory 2, which the experiment is usually known to have tested until now. Let the top end of the shaft where the absorber is placed be A, while the base where the emitter is located be B. Then Figs. 1(a) - 1(c) illustrate the experiment for Theory 1, Theory 2 and Theory 3 respectively.

As indicated in Fig. 1, the gravitational potentials at positions A and B appear in Theory 1 (in the context of TGR) and Theory 3 (in GR), while gravitational potential at A and B measured relative to position B, (the base of the shaft), appear in Theory 2. Moreover gravitational frequencies $\overline{\nu}_A$ and $\overline{\nu}_B$ at positions A and B appear in Theory 1 and Theory 3, while the natural frequency ν_0 appears at position B and the gravitational red-shifted frequency ν_A due to passage through height H of the light ray, while interacting with the Newtonian gravitational field, appears at position A in Theory 2. Then we must replace the natural frequency ν_0 in the red-shifted frequency ν_A at A, given earlier by Eq. (39) in the context of Theory 2, by the Doppler-shifted frequency ν of Eq. (57) to have the following in the context of Theory 2

$$\begin{aligned} \nu_A &= \nu(1 - gH/c^2) \\ &\approx \nu_0(1 + v/c)(1 - gH/c^2) \\ &\approx \nu_0(1 - gH/c^2 + v/c) \end{aligned} \tag{58}$$

Hence

$$(\frac{\delta\nu}{\nu_0})_{GR1} = \frac{\nu_A - \nu_0}{\nu_0} \approx -gH/c^2 + v/c \tag{59}$$

Resonant absorption of gamma will occur in the absorber at the top end of the shaft when the resultant shift in frequency is zero. Hence the velocity at which

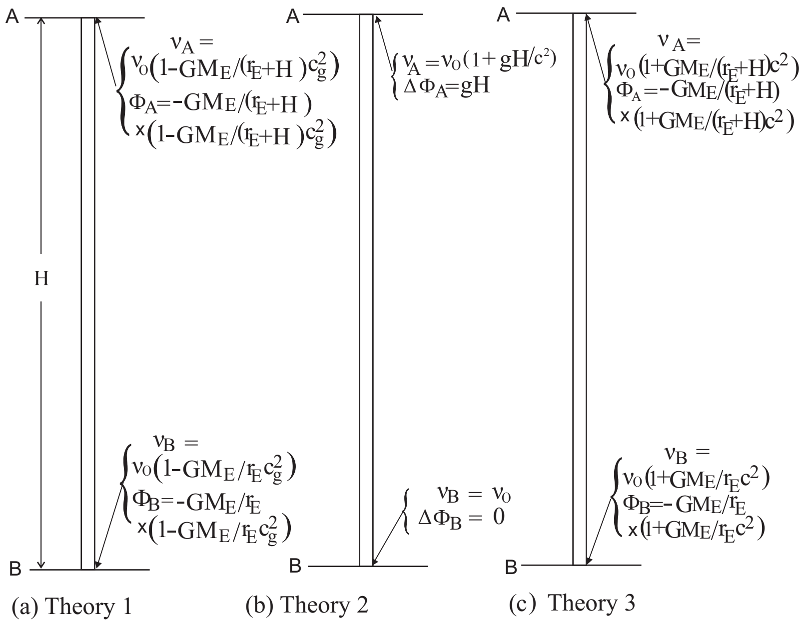

Fig. 1: The Pound, Rebka and Snider experiment illustrated for three theories of gravitational frequency shift in TGR and GR.

the emitter must be moved upward for Doppler blue shift to completely nullify the gravitational red shift predicted in the context of Theory 2 is given by letting $\delta\nu$ to vanish in Eq. (59) yielding the following

$$v/c \approx gH/c^2; \quad \text{(in Theory2 or GR1)} \tag{60}$$

Thus for H=22.5 m and g=9.81 m/s^2, the predicted emitter velocity in the context of Theory 2 is 7.3575×10^{-7} m/s or 7.3575×10^{-4} mm/s. In the PRS, an emitter velocity of 7.353×10^{-4} mm/s, which represents 0.9994 times the predicted value was reported. The gravitational red shift relation (39) in the context of Theory 2 must indeed be adjudged to be confirmed by PRS.

Let us now interpret the result of the PRS in the context of Theory 1. Let the emitter at the base B of the shaft be moved upward at a low velocity v' towards the absorber at the top end of the shaft in Fig. 1(a). Then the frequency ν_B of the gamma rays leaving the emitter towards the stationary absorber suffers both local gravitational red shift and Doppler blue shift due to the motion of the emitter at velocity v' towards the absorber, and is given relative to the absorber as follows

$$\begin{aligned} \nu_B &\approx \nu_0(1 + v'/c)(1 - GM_{0\mathrm{a}E}/r'_E c_g^2) \\ &\approx \nu_0(1 - GM_{0\mathrm{a}E}/r'_E c_g^2 + v'/c_\gamma) \end{aligned} \tag{61}$$

It is possible by moving the emitter towards the absorber at the appropriate velocity for Doppler blue shift to completely nullify gravitational red shift, so that the gamma emitted towards the absorber has the natural frequency ν_0 at emission. Let the velocity of the emitter towards the absorber when this occurs be v_0. Then

$$\nu_B = \nu_0 \approx \nu_0(1 - GM_{0\mathrm{a}E}/r'_E c_g^2 + v_0/c_\gamma) \tag{62}$$

Thus by moving the emitter towards the absorber at velocity v_0, natural condition, that is, absence of gravitational frequency shift field and absence of special-relativistic Doppler shift due to relative motion), is effectively attained at the emitter; the emitter thereby effectively emits gamma of natural frequency ν_0.

The gamma ray of natural frequency ν_0 leaving the emitter that is being moved at velocity v_0 towards the absorber in Eq. (62), propagates through height H to arrive at the absorber, and suffers local gravitational red shift to frequency $\overline{\nu}_A$ in the context of TGR at the location of the absorber where,

$$\overline{\nu}_A \approx \nu_0(1 - \frac{GM_{0\mathrm{a}E}}{(r'_E + H')c_g^2}) \tag{63}$$

The change in frequency of the gamma ray in propagating from the emitter to the absorber is then given as follows

$$\begin{aligned} \delta\overline{\nu}_{AB} &= \overline{\nu}_A - \nu_0 \\ &= \nu_0(1 - \frac{GM_{0\mathrm{a}E}}{(r'_E + H')c_g^2}) - \nu_0(1 - \frac{GM_{0\mathrm{a}E}}{r'_E c_g^2} + \frac{v_0}{c_\gamma}) \end{aligned} \tag{64}$$

where ν_0 in in the first line has been replaced by the right-hand side of Eq. (62) to have Eq. (64).

Equation (64) expresses gravitational red shift since $\delta\overline{\nu}_{AB}$ is the difference between the frequency $\overline{\nu}_A$ observed (or measured) at A and the natural frequency ν_0. In order for resonant absorption to occur, $\delta\overline{\nu}_{AB}$ must vanish, and this can be achieved by controlling the velocity of the emitter towards the absorber to an appropriate velocity v. That is, by letting, $v_0 = v$, we must let $\delta\overline{\nu}_{AB} = 0$ in Eq. (64) to have

$$\nu_0(1 - \frac{GM_{0aE}}{(r'_E + H')c_g^2}) - \nu_0(1 - \frac{GM_{0aE}}{r'_E c_g^2} + \frac{v}{c_\gamma}) = 0 \tag{65}$$

Hence

$$\begin{aligned} \frac{v}{c} &= \frac{GM_{0aE}}{r'_E c_g^2} - \frac{GM_{0aE}}{(r'_E + H')c_g^2} \\ &\approx \frac{GM_{0aE}H'}{c_g^2 r'^2_E}(\frac{1}{1 + H'/r'_E}) \end{aligned} \tag{66}$$

or

$$v/c \approx g'H'/c_g^2; \quad \text{(in Theory 1 or TGR)} \tag{67}$$

since $r'_E >> H'$.

Thus the velocity at which the emitter must be moved towards the absorber for resonant absorption to occur predicted in the context of Theory 1, (or in the context of TGR), expressed by Eq. (67), is the same as that predicted in the context of Theory 2 expressed by Eq. (60). It follows then that the velocity of the emitter towards the absorber to restore resonant absorption of 7.353×10^{-4} mm/s reported in the PRS experiment represents 0.9994 times the predicted value in the context of Theory 1, or in the context of the theory of gravitational red shift in the theory of gravitational relativity.

The fact that Theory 1 and Theory 2 lead to the same expression for the velocity of the emitter required to restore resonant absorption in the absorber namely, Eqs. (60) and (67), does not mean that the two theories are identical or that they are both valid theories. The condition for restoration of resonant absorption in the context of Theory 2, which follows from Eq. (60) is the following

$$-gH/c^2 + v/c = 0 \quad \text{(Theory 2)} \tag{68}$$

And the corresponding condition in Theory 1 is given from Eq. (66) or (67) as follows

$$-\frac{GM_{0aE}}{(r'_E + H')c_g^2} - (-\frac{GM_{0aE}}{r'_E c_g^2}) - \frac{v}{c_\gamma} = 0$$

or

$$\frac{GM_{0aE}H'}{c_g^2 r'^2_E}(\frac{1}{1 + H'/r'_E}) - \frac{v}{c_\gamma} = 0$$

or

$$g'H'/c_g^2 - v/c_\gamma = 0; \quad \text{(Theory 1)} \tag{69}$$

since $r'_E >> H'$.

The gravitational red shift at the location of the absorber depends on the gravitational potential at the location of the absorber solely in the context of Theory 1, whereas it depends on the difference between the gravitational potentials at the locations of the emitter and absorber in the context of Theory 2. Hence the two theories, (Theory 1 and Theory 2), diverge as regards the prediction of red shift, as can be seen from the expressions for $\delta\overline{\nu}_{21}$ in the two theories in Table I. However as regards the prediction of the velocity of the emitter towards the absorber that restores resonant absorption, the two theories are in agreement, because the required emitter velocity depends on the difference between the gravitational potentials at the locations of the emitter and absorber in both theories, as Eqs. (68) and (69) show. It can also be observed from Table I that the expressions for the shift in frequency as a light ray propagates from the surface of the earth to a height H namely, $\delta\nu_{EH}$ are the same in both theories.

In Theory 2 is the inherent assumption of fundamental (or a first-order) interaction of light with gravitational field, thereby converting its (radiation) energy to changes in its gravitational potential energy. Thus in Theory 2, gamma ray is emitted at its natural frequency ν_0 from the stationary emitter towards the absorber, and upon climbing height H, has gained gravitational potential energy $gH\,h\nu_0/c^2$. Since energy is conserved, gamma ray has lost equal energy, $h\nu_A = h\nu_0 - gH\,h\nu_0/c^2$, at the location of the absorber.

As concluded at the end of the preceding section, Theory 2 is not an authentic theory because of the invalidity of the underlying assumption of fundamental (or first-order) interaction of light with the Newtonian gravitational field that is inherent in it. The fact that the PRS confirms relation (60) derived in the context of this theory does not alter this conclusion. It should be of some interest in epistemology that an authentic experiment supports an invalid theory. On the other hand, Theory 1 is the authentic theory, as concluded at the end of the preceding section, and the confirmation of relation (67) derived in the context of this theory by the PRS is one experimental support for the theory already.

The interpretation of the PRS in the context of Theory 3 with Fig. 1(c) is the same as the interpretation of the experiment in the context of Theory 1 concluded above. The condition for restoration of resonant absorption in the context of Theory 3, which corresponds to relations (68) and (69) of Theory 2 and Theory 1 respectively is the following

$$gH/c^2 + v/c = 0; \quad \text{(Theory 3)} \tag{70}$$

Thus Theory 3 predicts that the emitter at the base of the shaft in the PRS must be moved away (i.e downward) from the absorber at the top of the shaft at a velocity of 7.3575×10^{-4} mm/s for resonant absorption of gamma to be restored in the absorber, which is opposite the direction of motion of the emitter in the experiment. The PRS confirms the invalidity of Theory 3 that has been concluded in the preceding section.

This first part of this article shall be ended at this point, while the second part shall be devoted to the second-order interaction of photon (or light) with the gravitational field and its implications. The recommendation shall also be made that experiments must be performed to test the predicted gravitational red shift due to the combined gravitational fields of the Sun and the earth in the context of TGR of Eq. (13) or (14).

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