On the Influence of Air Resistance and Wind during Long Jump

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Abstract. In this article we perform theoretical analysis of long jumps with the purpose to find contribution of air resistance and wind into final results. It appears that in the absence of wind the drag effect during a long-jump would reduce the jump by no more than 1%. The wind has a significant effect mainly because of changes in take-off values. The faster the athlete runs, the greater the horizontal velocity at the instant he/she touches the take-off board and the greater the take-off velocity. The model predicts an increase in jump distance up to 23 cm from a jump on a still day to a jump by the same athlete with 2 ms^{-1} tailwind (the allowable limit for records).

Keywords: biomechanical analysis, air resistance, long jumps.

Introduction

Wind affects the performance of long-jumpers. The International Athletics Union acknowledges this by imposing a special rule relating to wind during long-jump performances. The average wind component parallel to the track is measured near the jumping pit during an interval encompassing the run-up and the jump. If this measurement exceeds 2 ms^{-1} , no record-breaking jump is recognized.

In making his record jump, Beamon enjoyed a number of advantageous environmental factors. At an altitude of 2240 m (7349 ft), Mexico City's air had less resistance than air would have at sea level. This allows runners to run faster and jumpers to jump farther. In addition to Beamon's record, world records were broken in most of the sprinting and jumping events at the 1968 Olympic Games. Beamon also benefited from a trailing wind of 2 meters per second on his jump, the maximum allowable for record purposes. It has been estimated that the trail wind and altitude may have improved Beamon's long jump distance by 31 cm (12.2 inches) [1]. During the same hour Lee Evans set the world record for 400 meters that lasted for almost 20 years.

In this paper we investigate how the aerodynamic forces such as drag, lift and wind will affect the sportsman during long jump.

Theoretical Model

To facilitate the study of long jumps, it has been proposed to split the total distance jumped into partial distances, and then to identify the determining factors for each. For the long jump, Hay (1981) classifies the following partial distances as shown in Figure 1.

L_0: Take-off distance: the horizontal distance between the anterior edge of the take-off board and the vertical projection of the centre of gravity (CG) at the instant of take-off.

L_1: Flight distance: the horizontal distance covered by the CG while the athlete is free in the air.

L_2: Landing distance: the horizontal distance between the vertical projection of the centre of gravity at the instant the heels touch the sand and the mark from where the jump will be measured.
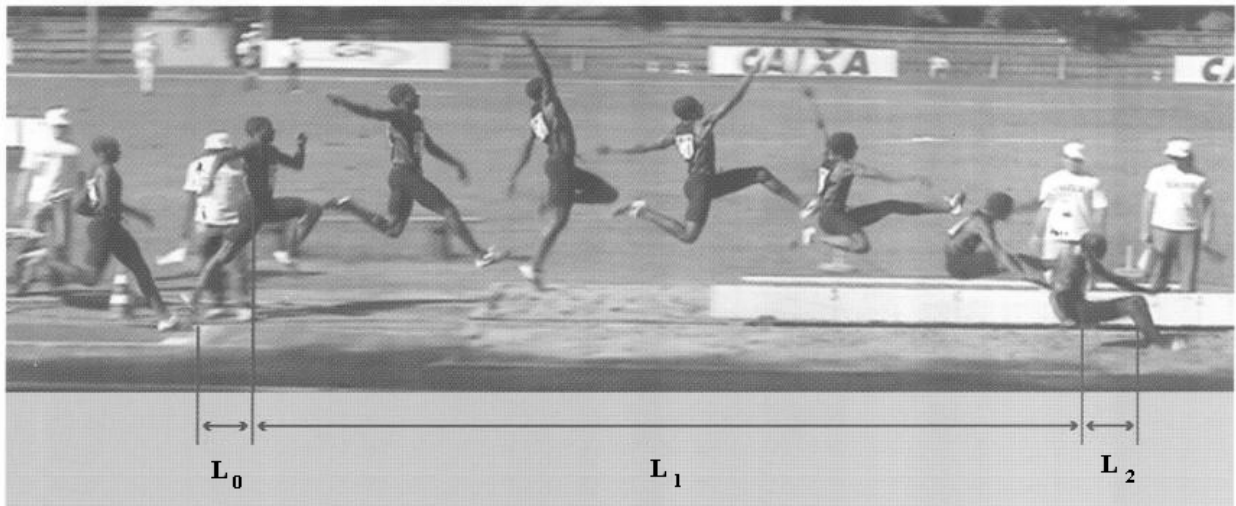

Figure 1. Partial distances in the long jump.

The distance L_1 represents more than 85% of the total distance of a jump and thus has the highest relationship with the final result. We can say that L_1, and thus performance in the horizontal jumping events, is determined by the same four factors affecting movement of all projectiles: take-off height, angle and velocity, and air resistance.

In the absence of wind, the velocity of the jumper relative to the ground is the same as the velocity relative to the air. The long-jumper is modeled as a projectile acted on by constant gravity plus the two components of the total aerodynamic force (drag and lift), which at jumpers' speeds are usually assumed to be proportional to the square of the air speed. With the origin at the position of the jumper's centre of mass at take-off, the x direction chosen as parallel to the run-up track and the y direction chosen as vertically upwards, the governing equation of motion is [2]

$$m\frac{d^2\bar{r}}{dt^2} = -mg\hat{k} - \frac{1}{2}\rho SC_D \mid \bar{v}\mid^2 \hat{\tau} + \frac{1}{2}\rho SC_L \mid \bar{v}\mid^2 \hat{n} \qquad (1)$$

Here ρ denotes the density of the air, S is a typical cross-sectional area of the jumper, m denotes the jumper's mass, $\bar{r}$ denotes the position vector of the jumper at any time t during the jump, $\bar{v}$ denotes the corresponding velocity vector, while C_D and C_L denote the drag and lift coefficients respectively. The unit vectors $\hat{k}$, $\hat{\tau}$ and $\hat{n}$ are respectively in the directions vertically upwards, parallel to the jumper's velocity, and perpendicular to the jumper's velocity but lying in the vertical plane through the athlete's centre of mass.

At sea level, air density ρ is about 1.226 kg/m³; air density at 3,000 meters is about 0.905 kg/m³. Since air density changes in a roughly linear fashion with altitude, you can use the formula ρ = 1.226 - Altitude x [(1.226 - 0.905)/3,000].

Typical values of S range from 0.4 to 0.7; larger jumpers and less aerodynamic positions result in higher values. Precise calculation is not possible, but you can obtain an estimate by assuming that a jumper weighing 50 kg. will have a frontal area of 0.4 and that his frontal area increases by 0.0033 square meters with each pound of body weight. So the sportsman's frontal area, S, will equal approximately 0.0033 x W, where W is his body weight.

To include the aerodynamic forces more precisely would require knowledge of the drag and lift coefficients at each stage of the motion of the jumper through the air. In addition the typical area S is usually chosen as the projected area of the jumper in a plane normal to the jumper's velocity. This also changes during the long jump from a maximum value in the take-off position to a much smaller value just before landing. In this paper the usual assumption is made that SC_D and SC_L are each some average constant for the duration of each long jump. The value for SC_D has been estimated as 0.36 from measured values on sprinters, cyclists and speed-skaters quoted in Ward-

Smith [3]. A value for SC_L also needs to be estimated. The lift to drag ratio (C_L/C_D) for skijumpers has been well documented (Krylov and Remizov [4], Ward-Smith and Clements [5]), and can be as high as 0.25, but it varies with angle of incidence and would never be this large for long-jumpers. To obtain some idea of the effect of lift, a representative value 0.04 is chosen for SC_L.

When a wind $\overline{w}$ is blowing, the air speed of any projectile is given by

$$\overline{v}^* = \overline{v} - \overline{w} \qquad (2)$$

The drag and lift effects will depend on $\overline{v}^*$, and only on $\overline{v}$ in the absence of wind. Therefore, with the addition of wind, the basic equation (1) for the projectile part of the long-jumper's motion becomes [2]

$$m\frac{d^2\overline{r}}{dt^2} = -mg\hat{k} - \frac{1}{2}\rho SC_D \, |\overline{v}^*|^2 \, \hat{\tau}^* + \frac{1}{2}\rho SC_L \, |\overline{v}^*|^2 \, \hat{n}^* \qquad (3)$$

where $\hat{\tau}^*$ is a unit vector in the direction of $\overline{v}^*$, and $\hat{n}^*$ is a unit vector perpendicular to $\hat{\tau}^*$ and lying in the vertical plane.

Disscusion

It seems take-off velocity is the most important factor affecting L_1 and it has a very high relationship with the velocity at the touchdown of the take-off foot at take-off, which in turn is dependent on the approach velocity. In other words, the faster the athlete runs, the greater the horizontal velocity at the instant he/she touches the take-off board and the greater the take-off velocity. We can easily monitor approach velocity using photoelectric cells.

From Figure 2 you can see how the distance L depends on take-off velocity $\overline{v}_0$, α, r and β (where α is the angle between OX and $\overline{v}_0$, r is the length of the radius-vector of the center of gravity at the push-off moment, β is the angle between r and OX).

From our analysis of (1) it appears that in the absence of wind the drag effect during a long-jump would reduce the jump by no more than 1% (7-9 cm).
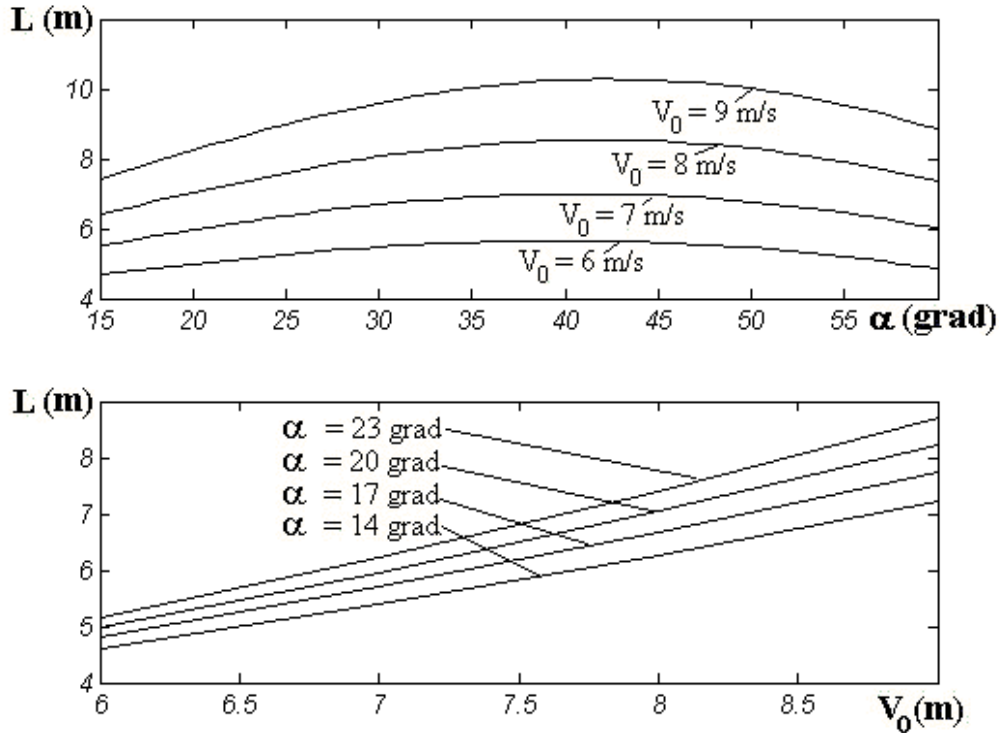
Figure 2. Dependencies of the result L on $V_0 = |\bar{v}_0|$ and α (r=1.5 m, β=60 grad, L₂=0.5 m).

There are a number of papers [6-9] which discuss the impact of wind and altitude in the 100 m race. The general consensus of these researchers is that the maximum legal tail wind of +2.0 ms⁻¹ provides a 0.10-0.12 second advantage over still conditions at sea level and with no wind every 1000 meters of elevation will improve a performance by roughly 0.03-0.04 seconds. According to the simplest gravitational model the increase of take-off velocity will improve the distance by 12-16 cm from a jump on a still day to a jump by the same athlete with 2 ms⁻¹ tailwind (at sea level). Altitude also may improve a performance by 8-12 cm at 2240 m (Mexico City's air density).

The addition of drag to the analysis reduces the time of flight for all values of the wind considered, but only by an amount of the order of 0.1%. The addition of lift reverses this trend, so that for all headwinds and the lesser tailwinds the model predicts that the jumper will be held up in the air just slightly longer than when drag and lift are neglected. The inclusion of drag reduces the gravity-only distances by 12 cm for the strongest headwind considered down to 2 cm for the strongest tailwind. Since jumpers are mainly interested in the tailwind cases (because these give

longer jumps) it appears that even for the presence of winds, a gravity-only analysis will suffice for the aerial-phase calculations [2].

Conclusion

Our analysis shows that the wind has a significant effect mainly because of changes in take-off values. The faster the athlete runs, the greater the horizontal velocity at the instant he/she touches the take-off board and the greater the take-off velocity. The addition of drag to the analysis reduces the time of flight for all values of the wind considered, but only by an amount of the order of 0.1%. The model predicts an increase in jump distance of 23 cm from a jump on a still day to a jump by the same athlete with 2 ms^{-1} tailwind (the allowable limit for records). The maximum legal tail wind of +2.0 ms^{-1} provides a 12-16 cm advantage over still conditions at sea level and with no wind every 1000 meters of elevation will improve a performance by roughly 3-5 cm.

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