The general Compton effect

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ABSTRACT

In the special relativity theory and the quantum theory, the Compton effect is the effect about the standing electron and the moving photon. The general Compton effect is the case that the moving photon impulses the moving electron in the line. In this time, the photon's sense is the opposite sense and the same sense of the moving electron.

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I.Introduction

This article treats that the general Compton effect.
The Compton effect is the case that the moving photon impulses the standing electron.

The photon's energy $h\upsilon$ and the photon's momentum $p_x = \frac{h\upsilon}{c} = \frac{h}{\lambda}, p_y = 0$ and the electron's energy $m_0 c^2$ and the electron's momentum $p_x = 0, p_y = 0$ before the moving photon impulses the standing electron.

The photon's energy $h\upsilon'$ and the photon's momentum $p'_x = (\frac{h}{\lambda'})\cos\theta, p'_y = (\frac{h}{\lambda'})\sin\theta$ and the electron's energy $\gamma m_0 c^2$ and the electron's momentum $p'_x = m_0\gamma\beta c\cos\phi, p'_y = m_0\gamma\beta c\sin\phi$ after the moving photon impulses the standing electron.

In this time, m_0 is the standing electron's mass, $\gamma = 1/\sqrt{v_{f0}{}^2/c^2}$, $\beta = v_{f0}/c$

v_{f0} is the electron's velocity after the moving photon impulses the standing electron.
Therefore,

The energy conservation is

$$h\upsilon + m_0c^2 = h\upsilon' + m_0\gamma c^2 \quad (1), \quad \gamma = 1/\sqrt{v_{f0}^2/c^2}$$

The momentum conservation is

$$\frac{h}{\lambda} = m_0\gamma\beta c\cos\phi + (\frac{h}{\lambda'})\cos\theta \quad (2)$$

$$0 = -m_0\gamma\beta c\sin\phi + (\frac{h}{\lambda'})\sin\theta \quad (3)$$

m_0 is the standing electron's mass, $\gamma = 1/\sqrt{v_{f0}^2/c^2}$, $\beta = v_{f0}/c$

v_{f0} is the electron's velocity after the moving photon impulses the standing electron.
If use Eq(1),Eq(2),Eq(3),

$$\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta) \quad (4), \text{ Compton wavelength } \lambda_C = \frac{h}{m_0c}$$

II. Additional chapter-I

The general Compton effect is the case that the moving photon impulses the moving electron in the line. In this time, if the moving photon's sense is the opposite sense of the moving electron, the general Compton effect is that the photon's energy $h\upsilon$ and the photon's momentum $p_x = \frac{h\upsilon}{c} = \frac{h}{\lambda}, p_y = 0$ and the electron's energy γm_0c^2 and the electron's momentum $p_x = -m_0\gamma\beta c, p_y = 0$ before the moving photon impulses the moving electron in the line.

The photon's energy $h\upsilon'$ and the photon's momentum $p_x = (\frac{h}{\lambda'})\cos\theta, p_y = (\frac{h}{\lambda'})\sin\theta$ and the electron's energy $\gamma' m_0c^2$ and the electron's momentum $p'_x = m_0\gamma'\beta' c\cos\phi$, $p'_y = m_0\gamma'\beta' c\sin\phi$ after the moving photon impulses the moving electron in the line.

m_0 is the standing electron's mass, $\gamma = 1/\sqrt{v_{i0}^2/c^2}$, $\gamma' = 1/\sqrt{v_{f0}^2/c^2}$,

$\beta = v_{i0}/c$, $\beta' = v_{f0}/c$

v_{i0} is the moving electron's velocity before the moving photon impulses the moving electron in the line.

v_{f0} is the moving electron's velocity after the moving photon impulses the moving electron in the line.

Therefore,

The energy conservation is

$$h\upsilon + \gamma m_0c^2 = h\upsilon' + m_0\gamma' c^2 \quad (5), \quad \gamma = 1/\sqrt{v_{i0}^2/c^2}, \quad \gamma' = 1/\sqrt{v_{f0}^2/c^2}$$

The momentum conservation is

$$\frac{h}{\lambda} - m_0\gamma\beta c = m_0\gamma'\beta' c\cos\phi + (\frac{h}{\lambda'})\cos\theta \quad (6)$$

$$0 = -m_0\gamma'\beta' c\sin\phi + (\frac{h}{\lambda'})\sin\theta \quad (7)$$

m_0 is the standing electron's mass,

$\gamma = 1/\sqrt{{v_{i0}}^2/c^2}$, $\gamma' = 1/\sqrt{{v_{f0}}^2/c^2}$, $\beta = v_{i0}/c$, $\beta' = v_{f0}/c$

v_{i0} is the moving electron's velocity before the moving photon impulses the moving electron in the line.

v_{f0} is the moving electron's velocity after the moving photon impulses the moving electron in the line.

Eq(6),Eq(7) is

$$[\frac{h}{\lambda} - m_0\gamma\beta c - (\frac{h}{\lambda'})\cos\theta]^2 = {m_0}^2\gamma'^2\beta'^2c^2\cos^2\phi \quad (8)$$

$$(\frac{h}{\lambda'})^2\sin^2\theta = {m_0}^2\gamma'^2\beta'^2c^2\sin^2\phi \quad (9)$$

Therefore,

$$(\frac{h}{\lambda})^2 + {m_0}^2\gamma^2\beta^2c^2 + (\frac{h}{\lambda'})^2 - 2\frac{h}{\lambda}m_0\gamma\beta c + 2\frac{h}{\lambda'}\cos\theta m_0\gamma\beta c - 2\frac{h^2}{\lambda\lambda'}\cos\theta$$

$$= {m_0}^2\gamma'^2\beta'^2c^2 \quad (10)$$

Eq(5) is

$$\frac{h}{\lambda} - \frac{h}{\lambda'} = m_0c(\gamma' - \gamma) \quad (11)$$

$$(\frac{h}{\lambda} - \frac{h}{\lambda'})^2 = (\frac{h}{\lambda})^2 - 2\frac{h^2}{\lambda\lambda'} + (\frac{h}{\lambda'})^2 = {m_0}^2c^2(\gamma'^2 - 2\gamma'\gamma + \gamma^2) \quad (12)$$

If Eq(10) minus Eq(12),

$$2\frac{h^2}{\lambda\lambda'}(1-\cos\theta) - 2\frac{h}{\lambda\lambda'}m_0\gamma\beta c(\lambda' - \lambda\cos\theta)$$

$$= -{m_0}^2\gamma'^2c^2(1-\beta'^2) - {m_0}^2c^2\gamma^2(1+\beta^2) + 2{m_0}^2c^2\gamma\gamma'$$

$$= -{m_0}^2c^2 - {m_0}^2c^2\gamma^2(1+\beta^2) + 2{m_0}^2c^2\gamma\gamma' = 2{m_0}^2c^2\gamma(\gamma' - \gamma) \quad (13)$$

Therefore,

$$\lambda\lambda' = [h^2(1-\cos\theta) - hm_0\gamma\beta c(\lambda' - \lambda\cos\theta)]/[{m_0}^2c^2\gamma(\gamma' - \gamma)] \quad (14)$$

Eq(11) is

$$\frac{h}{\lambda} - \frac{h}{\lambda'} = \frac{h}{\lambda\lambda'}(\lambda' - \lambda) = m_0c(\gamma' - \gamma) \quad (15)$$

Therefore, if Eq(14) insert Eq(15), the general Compton effect, in this time, if the photon's sense is the opposite sense of the moving electron and the case that the moving photon impulses the moving electron in the line is

$$\lambda' - \lambda = \frac{h(1-\cos\theta)}{m_0c\gamma} - \beta(\lambda' - \lambda\cos\theta) \quad (16)$$

$$\lambda' = \frac{1}{1+\beta}[\frac{h}{m_0c\gamma}(1-\cos\theta) + \lambda(1+\beta\cos\theta)] \quad (17)$$

$$\lambda' - \lambda = \frac{1}{1+\beta}[\frac{h}{m_0c\gamma}(1-\cos\theta) + \lambda(1+\beta\cos\theta) - (1+\beta)\lambda]$$

$$= \frac{1}{1+\beta}[\frac{h}{m_0c\gamma} - \lambda\beta](1-\cos\theta) \quad (18)$$

$\gamma = 1/\sqrt{v_{i0}^2/c^2}$, $\beta = v_{i0}/c$

III. Additional chapter-II

The general Compton effect, in this time, if the photon's sense is the same sense of the moving electron and the case that the moving photon impulses the moving electron in the line is

$$\beta \to -\beta \quad (19)$$

By Eq(19), the general Compton effect is that the photon's energy $h\upsilon$ and the photon's momentum $p_x = \frac{h\upsilon}{c} = \frac{h}{\lambda}, p_y = 0$ and the electron's energy $\gamma m_0 c^2$ and the electron's momentum $p_x = m_0\gamma\beta c, p_y = 0$ before the moving photon impulses the moving electron in the line.

The photon's energy $h\upsilon'$ and the photon's momentum $p_x = (\frac{h}{\lambda'})\cos\theta, p_y = (\frac{h}{\lambda'})\sin\theta$ and the electron's energy $\gamma' m_0 c^2$ and the electron's momentum $p'_x = m_0\gamma'\beta' c\cos\phi$, $p'_y = m_0\gamma'\beta' c\sin\phi$ after the moving photon impulses the moving electron in the line.

m_0 is the standing electron's mass, $\gamma = 1/\sqrt{v_{i0}^2/c^2}$, $\gamma' = 1/\sqrt{v_{f0}^2/c^2}$,

$\beta = v_{i0}/c$, $\beta' = v_{f0}/c$

v_{i0} is the moving electron's velocity before the moving photon impulses the moving electron in the line.

v_{f0} is the moving electron's velocity after the moving photon impulses the moving electron in the line.

The energy conservation is

$$h\upsilon + \gamma m_0 c^2 = h\upsilon' + m_0\gamma' c^2 \quad (20), \quad \gamma = 1/\sqrt{v_{i0}^2/c^2}, \ \gamma' = 1/\sqrt{v_{f0}^2/c^2}$$

By Eq(19), The momentum conservation is

$$\frac{h}{\lambda} + m_0\gamma\beta c = m_0\gamma'\beta' c\cos\phi + (\frac{h}{\lambda'})\cos\theta \quad (21)$$

$$0 = -m_0\gamma'\beta' c\sin\phi + (\frac{h}{\lambda'})\sin\theta \quad (22)$$

m_0 is the standing electron's mass,

$\gamma = 1/\sqrt{v_{i0}^2/c^2}$, $\gamma' = 1/\sqrt{v_{f0}^2/c^2}$, $\beta = v_{i0}/c$, $\beta' = v_{f0}/c$

v_{i0} is the moving electron's velocity before the moving photon impulses the moving electron in the line.

v_{f0} is the moving electron's velocity after the moving photon impulses the moving electron in the line.

By Eq(19), if the photon's sense is the same sense of the moving electron and the case that the moving photon impulses the moving electron in the line is instead of Eq(16),Eq(17)Eq(18)

$$\lambda' - \lambda = \frac{h(1-\cos\theta)}{m_0 c\gamma} + \beta(\lambda' - \lambda\cos\theta) \quad (23)$$

$$\lambda' = \frac{1}{1-\beta}[\frac{h}{m_0 c\gamma}(1-\cos\theta) + \lambda(1-\beta\cos\theta)] \quad (24)$$

$$\lambda'-\lambda = \frac{1}{1-\beta}[\frac{h}{m_0 c\gamma}(1-\cos\theta)+\lambda(1-\beta\cos\theta)-(1-\beta)\lambda]$$

$$= \frac{1}{1-\beta}[\frac{h}{m_0 c\gamma}+\lambda\beta](1-\cos\theta) \quad (25)$$

$$\gamma = 1/\sqrt{{v_{i0}}^2/c^2}, \quad \beta = v_{i0}/c$$

IV.Conclusion

In this time,

$$\theta \to -\theta, \phi \to -\phi \quad (26)$$

Eq(18) and Eq(25) are only the general Compton effect because Eq(5),Eq(6),Eq(7) and Eq(20),Eq (21),
Eq(22) are only invariant about Eq(26).
The Compton effect is that X-ray scattered by the electron in the atom in the experiment. But the general Compton effect is that X-ray scattered by the electron of the electron beam in the experiment.

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