Strong interaction and electromagnetism

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Abstract – The basic structure of the classical electromagnetism is used as a means to study the strong interaction. The strong coupling constant and the strong field (expessed in terms of an equivalent electric field) inside the nucleon (proton or neutron) are estimated. It is also evaluated the vacuum pressure on the boundary of the nucleon. Quark confinement is briefly discussed. Simplified versions of the MIT bag model are also considered in the present treatment

I – Introduction

Under the guidance of Rutherford, Geiger and Marsden [1] performed the experiment of scattering alpha particles through thin metallic foils. This experiment leads to the discovery of the atomic nucleus. According to this experiment, the atomic nucleus occupies a very small fraction of the atomic volume, although it contains the whole positive charge of matter. Today we know that the atomic nucleus is constituted of particles carrying positive electric charge called proton and of null electric charge the so called neutron. It is worth to point out that almost all the mass of the atom is contained in the atomic nucleus. The very instable situation, represented by the positive charge confined in a so small space volume, is balanced by the presence of a second fundamental force of nature, namely, the strong interaction of attractive character which compensates the coulomb repulsion between protons [2]. Therefore, the nuclear gluing is warranted by the strong interaction between protons, neutrons and proton- neutron pairs.

On the other hand, the synthesis of the classical electromagnetism is contained in the Maxwell equations [3], a set of differential equations in the space-time, making the relationship between electric and magnetic fields and also linking these fields to their respective sources: charges and currents. Two important constants which appear in the Maxwell equations are ε_o e μ_o, representing respectively the electric permittivity and the magnetic permeability of vacuum. So, the light velocity in vacuum can be expressed as the inverse of the square root of the product of these constants. One of the possible solutions of the Maxwell equations describes the propagation of electromagnetic waves in vacuum, being the speed of propagation given by the light velocity defined above.

Two important characteristics of the strong interactions are the following

i) If we consider a proton pair which distance of separation is comparable to the nuclear size, the attractive force between them is very much greater than their mutual electric repulsion.

ii) The nuclear interaction is short ranged, working only at distances comparable to the nuclear sizes, while the electromagnetic force has an infinite range.

One of the first trying to establish a model for strong interactions between nucleons (proton, neutron) was proposed by Yukawa [4]. The Yukawa equation differs from that which describes the propagation of the electromagnetic waves by the adding of a term non-homogeneous which captures justly the short range character of the strong interaction.

Today we know that the model of Yukawa does not correspond to the more fundamental manifestation of the strong interactions, being considered as a residual effect of it in binding nucleons to constitute the atomic nucleus.

A deepest investigation of the inner of matter, as it happens in the case of the strong interaction, requires the use of the quantum physics, once we are working in regions of sizes where classical physics is no more a good description. The best theories to dealing with the fundamentals forces of nature in the elementary particles scales are the quantum field theories. Making things explicit, the electromagnetic interactions are treated by the quantum electrodynamics (QED), and the strong interactions by the quantum chromodynamics (QCD) [5]. In the QED the interaction between electric charges is provided by the exchange of virtual particles called photons, whereas in the QCD the color charges interact by the exchange of gluons. However a remarkable difference makes a distinction between these two theories. Photons do not carry electric charges and therefore do not interact, while gluons may be endowed with color charges which lead to interactions between them. In other words, QED is a linear theory (the fields obey the superposition principle), whereas QCD is a non –linear theory (non-abelian).

In this essay we are mainly interested in make an evaluation of the strong interaction inside the nucleons (protons and neutrons). But for educational proposes we are working within the framework of the classical electromagnetism, exploring some basic concepts of the strong interaction. In this way we introduce the idea of an equivalent strong electric field, stated in Volts per meter (V/m) units. We also will make evaluations of the strong coupling constant and of the vacuum pressure at the nucleon boundary. The concept of quark confinement also will be workout.

Thinking in terms of its educational function, this work has a double proposal. The first one is to make wider the universe of applications of some basic tools of the classical electromagnetism, as for instance, the use of the Gauss law and of the density of energy stored in the electromagnetic field. Meanwhile this essay intends to introduce some basic concepts of the particle physics, with respect to the strong interaction.

II – Quarks and the strong interaction

The nucleons are constituted of particles called quarks (endowed with color charges), interacting by the exchange of virtual gluons [6].At low energies it is sufficient to take in account the up quark having a electric charge equal to 2/3 of the quantum of elementary charge (e) and the down quark which electric charge equal to – 1/3e. The proton is constituted of two up quarks and one down quark and the neutron is constituted of two down quarks and one up quark. The energetic balance inside nucleons is largely dominated by the strong interaction between color charges of quarks as compared with the electromagnetic interaction between them. Quarks behave as free particles as investigated by high energy probes, although remains confined inside the nucleons. It is elucidating to compare the strong interaction with the electromagnetic one through the following examples. In an electric dipole [7] the lines of force emerge from the positive charges ending into the negative ones. However adjacent lines do not interact reflecting the linear character of the theory. On the other hand the lines of the strong force between a quark-antiquark pair suffer a mutual attraction leading to the generation of a flux tube, in this way reflecting the non-linear character of the theory [7]. As a resulting of the interaction between the gluons (or between the lines of force in the Faraday's picture), an uniform color electric field emerges. This field between the quark-antiquark pair, resembles the uniform electric field which occurs between the parallel plates of a capacitor. Adding energy to the quark-antiquark pair leads to the stretching of the flux tube until a new pair to be generated, instead of the occurrence of the breaking of the pair. This feature will be commented later on when examining the quark confinement phenomenon.

III – The nucleon as a cubic bag of quarks

Inspired in the analogies between the flux tube and the parallel plate capacitor let us propose a model for the nucleon. It will be though as a cubic region of the space of edge L which confines the color fields. This "capacitor" will have plates of area L^2 spaced by the distance L. Indeed we are considering here the averaged color field, in this way neglecting edges effects. A relevant feature of this work is that the color electric field and its source will be evaluated in terms of equivalents electric field (Volts/meter) and electric charge (Coulomb).

The energy density u, of the electric field between the plates of a capacitor can be expressed as [8]

$$u = \tfrac{1}{2}\, \varepsilon_o E^2. \qquad (1)$$

In the case of the strong interaction we are going to use the same relation for the energy density of the strong field, but in terms of an equivalent electric field. In this way we are considering that the strong field is confined inside the nucleon of size L. Besides this we think that the mass energy of the nucleon can be accounted by the energy stored in color field of the strong interaction. We write

$$\tfrac{1}{2}\, \varepsilon_o E^2 L^3 = m_n c^2, \qquad (2)$$

where E is the strong field given in units of volts per meter, m_n the nucleon mass, and c is the light speed in vacuum.

By using Gauss' law [9], we get the following relation for the strong uniform field E

$$E = Q_F / (\varepsilon_o L^2) = \sigma_F / \varepsilon_o, \qquad (3)$$

where Q_F is the strong (or color) charge, and σ_F is its respective surface density.

Relations (2) and (3) lead to

$$Q_F = \sigma_F L^2 = (2\, \varepsilon_o m_n c^2 L)^{1/2}, \qquad (4)$$

and

$$E = [(2m_n c^2) / (\varepsilon_o L^3)]^{1/2}. \qquad (5)$$

Given $m_n = 1{,}67 \times 10^{-27}$ Kg e $L = 10^{-15}$ m, we have

$$Q_F = 1{,}63 \times 10^{-18}\ C \approx 10\, e, \qquad (6)$$

and

$$E = 1{,}84 \text{ x } 10^{23} \text{ V/m}. \quad (7)$$

Gottfried e Weisskopf [10] were able to estimate the nuclear charge (e_N), the color charge of nucleons. We may think in the deuteron, proton and neutron interacting through a Coulomb-like potential each particle having a charge equal to e_N, which was evaluated as

$$e_N \approx 3{,}7 \text{ e}. \quad (8)$$

If we compare (8) with (6), we verify that in a certain sense, the interaction between nucleons may be though as a residual effect of the strong interaction. The strong interaction in its integrality happens only inside the hadrons (nucleons, in particular). The intensity of color electric field inside the nucleon, given in terms of an equivalent electric (see(7)), can be compared with the electric field at the surface of the Uranium nucleus which order of magnitude is 10^{21} V/m [11].

However the comparison in intensities of the various fundamentals forces of the nature is usually done in terms of their respective coupling constants. Let us consider in the hydrogen atom the electric energy between proton and electron, when separated by the distance r. We have

$$U_{p\text{-}e}(r) = - e^2/(4\pi\varepsilon_o \, r) = - (\alpha\hbar c)/r. \quad (9)$$

In (9), α is the electromagnetic coupling, a non-dimensional quantity approximately equal to 1/137. Analogously we can write

$$Q_F^2/(4\pi\varepsilon_o) = \alpha_F \, \hbar c, \quad (10)$$

where

$$\alpha_F \approx 100 \, \alpha. \quad (11)$$

Therefore, while the electric interaction coupling has a order of magnitude of 10^{-2}, the strong interaction inside the nucleon has a order of magnitude of 1.(see [2]).

It would be interesting to continue using the tools of the classical electromagnetism as a means to explore a bit more the strong interaction physics. Next we are going to do this. Turning to the parallel plates capacitor, let us suppose their plates displayed parallel to xy plane of a cartezian coordinate system with the negative charged plate sited at the z=0 plane. The energy stored in the electric field of the capacitor is given by

$$U = (\tfrac{1}{2} \, Q_F^2)/C = \tfrac{1}{2} \, [(Q_F^2 \, z)/(\varepsilon_o \, L^2)]. \quad (12)$$

In (12) the capacitance C was written as :

$$C = (\varepsilon_o \, L^2)/z, \quad (13)$$

where z stems for the plates separation. The force on the positive plate 12] is

$$F = -\, dU/dz = -\tfrac{1}{2}\, [\, (Q_F^2)/(\varepsilon_o L^2)\,]. \qquad (14)$$

The negative signal implies that this force has an attractive character. From (14) we can also evaluate the force per unit of area (electrostatic tension) acting on the capacitor plates, namely

$$|\, F\, |/L^2 = \tfrac{1}{2}\, \varepsilon_o E^2 . \qquad (15)$$

Taking in account the analogy between strong interaction and electromagnetism, we consider that in the cubic bag model of quarks the capacitor plates would represent the interface between matter and vacuum. In the quantum field theory the vacuum is endowed with physical properties so that it behaves like a material media. In this way, quarks confined inside the nucleon of size L, acquires a perpetual motion due to the Heisenbrerg uncertainty principle, exerting a pressure over its interface. This pressure is balanced by the vacuum pressure over the nucleon's walls. In the present model the vacuum pressure is the analogous of the electrostatic tension and is given by

$$p_{vac} = \tfrac{1}{2}\, \varepsilon_o E^2. \qquad (16)$$

Inserting (16) into (2), we obtain

$$p_{vac} = \rho\, c^2, \qquad (17)$$

where ρ is the average density of matter inside the nucleon. Order of magnitude estimates give: $p_{vac} \sim 10^{30}$ atm and $\rho \sim 10^{18}$ Kg/m^3.

IV – The MIT bag model

The MIT bag model [7,14,15] considers that in hadrons (nucleons in particular) quarks and gluons (in the particle-like description) or the electric color fields and their sources are confined inside a bag. The interfaces between hadronic matter and the vacuum are represented by the walls of the bag. As was pointed out by Jaffe [7] it is possible to distinguish a certain region of space in a way consistent with the relativity theory , by submitting the frontier of that region to a constant pressure B exerted by the neighborhood vacuum over the interface of the hadron. In an idealized picture the hadron looks similar to a gas bubble immersed in an isotropic and uniform perfect fluid. The bubble's dynamics is determined by the balance between external pressure exerted by the fluid (vacuum) and the thermodynamic pressure of the confined gas of quarks and gluons. However the number of particles in the bag is small, rather than the great number of molecules of gas contained in the bubble. In the next step we propose a potential to describe this model. First consider a confining potential represented by a harmonic oscillator. We suppose that the averaged quark kinetic energy corresponds to half of the mass energy of the quark. Therefore the other half will correspond in average to its potential energy. We write

$$\tfrac{1}{2}\, m_q c^2 = (\alpha_F\, \hbar c)/R, \qquad (18)$$

where m_q is the constituent mass of the quark. Each quark contributes to the potential with a term given by

$$V_q(R) = (2\alpha_F \hbar c)/R. \tag{19}$$

On the other hand the contribution of the vacuum to the potential can be written as

$$V_{vac}(R) = 4/3\ \pi\ R^3\ B. \tag{20}$$

Taking in account that the nucleon is constituted by three quarks we finally obtain

$$V_{bag}(R) = 4/3\ \pi\ R^3\ B + 3[\ (2\alpha_F \hbar c)/R]. \tag{21}$$

Making the differentiation of (21) with respect to R, we get at the minimum radius R_n:

$$B = (6\alpha_F \hbar c)/(4\pi\ R_n{}^4), \tag{22}$$

and

$$V_{bag}(R_n) = (8\alpha_F \hbar c)/R_n = m_n\ c^2. \tag{23}$$

In (23) we identified the bag potential evaluated at its minimum value with the mass energy of the nucleon. Relations (22) and (23) lead to

$$B\ 4/3\ \pi\ R_n{}^3 = ¼\ m_n\ c^2, \tag{24}$$

where $4/3\pi R_n{}^3$ is the volume of the nucleon. Therefore the vacuum pressure can be expressed as

$$B = ¼\ \rho\ c^2. \tag{25}$$

Making an analogy with the cubic bag it is also possible to write B in terms of an equivalent strong electric field, namely

$$B = ½\ \varepsilon_o\ (E_{new})^2. \tag{26}$$

Comparing (16), (17), (25), e (26), we obtain

$$E_{new} = ½\ E \sim 10^{23}\ V/m. \tag{27}$$

It is also possible to evaluate the strong coupling constant within the formalism of the MIT bag model. From equation (23), we get

$$\alpha_F \big|_{\ MIT\text{-}\ bag} = (m_n\ c\ R_n)/(8\hbar). \tag{28}$$

We observe that the relation for the strong coupling obtained in (28) is comparable to that deduced before for the cubic bag

$$\alpha_F \Big|_{\text{cubic bag}} = (m_n\, c\, L)/(2\pi\hbar). \qquad (29)$$

In obtaining (29) we considered (4) and (10).

VI – Explaining quark confinement

It would be interesting to consider the arguments by Jaffe [7] explaining why free quarks have not been observed. Let us think about a flux tube of length L and cross-sectional area A. The field lines of the strong interaction should start at quarks and stop at anti-quarks and will be treated approximately as the lines of an uniform field. The energy stored in the field of the system is

$$U = \tfrac{1}{2}\, \varepsilon_o\, E^2\, A\, L = \tfrac{1}{2}\, (Q_F^2\, L)/(\varepsilon_o\, A). \qquad (30)$$

The breaking of the quark-antiquark pair implies in performing the fission of the flux tube. This procedure leads to a formation of a neck in the tube ($A \rightarrow 0$). Meanwhile to separate the quark-antiquark pair by an infinite distance requires $L \rightarrow \infty$. An examination of relation (30) reveals that this task leads to an infinity consumption of energy and therefore cannot be done. When a sufficient amount of energy is delivered to the system, we have a creation of a new quark antiquark pair, making the duplication of the flux tube. The field lines however remain confined inside each tube.
A deeper discussion of these features requires the use of the QCD, and as is well known that in the study of that theory the underlying symmetries play a fundamental role [16, 17].

VII – Conclusion

In this work we have analyzed some features of the strong interaction using the theoretical tools of the classical electromagnetism. We have evaluated the orders of magnitude of the strong coupling constant and that of the vacuum pressure at the boundary of the nucleon, besides that of the of the color electric field inside the nucleon. The vacuum pressure confines the sources of the strong force (quarks and gluons) at the inner of the nucleon. The analysis is based in great extent in simplified versions of the MIT bag model.

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