The transformation of the power and the constant acceleration in the 2-Dimension inertial system

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ABSTRACT

In the special relativity theory, the acceleration a about the accelerated matter in 2-Dimension inertial coordinate system $S(t,x)$ and the other acceleration a' about the accelerated matter in 2-Dimension inertial coordinate system $S'(t',x')$ are same. Therefore using it, derive the transformation of the power. And the acceleration a' is the constant acceleration a_0, the acceleration in 2-Dimension inertial coordinate system $S(t,x)$ and in 2-Dimension inertial coordinate system $S'(t',x')$ is the constant acceleration a_0.

PACS Number:**03.30.+p**

Key words:The special relativity theory,

The 2-dimension inertial system,

The acceleration,

The transformation,

The power,

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I.Introduction

Use following the formula about the constant accelerated matter.

$$x+\frac{c^2}{a_0}=\frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c}),t=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \quad (1)$$

x and t is the coordinate and the time in the inertial system about the constant accelerated matter. a_0 is the constant acceleration, τ is invariable time about the constant accelerated matter, c is light speed in the inertial system in the free space-time.

In the special relativity, the formula about 2-Dimension inertial coordinate system $S(t,x)$ and $S'(t',x')$ is

$$V=\frac{u+v_0}{1+\frac{u}{c^2}v_0} \ , \ V=\frac{dx}{dt},u=\frac{dx'}{dt'} \ , \ dx=\frac{dx'+v_0dt'}{\sqrt{1-\frac{v_0^2}{c^2}}}, \ dt=\frac{dt'+\frac{v_0}{c^2}dx'}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}), \ a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (2)$$

II. Additional chapter-I

The velocity V and the velocity u and the acceleration a are

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}})=\frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{v_0^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}})=\frac{1}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{u^2}{c^2}}})$$

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (3)$$

In this time , if the acceleration a' about the velocity u is

$$a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \ , \ u=\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \quad (4)$$

Eq(3) is

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}})=a'+v_0\frac{d}{dt'}(\sqrt{1+\frac{1}{c^2}[\int a'dt']^2})$$

$$= a' + v_0 \frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \frac{a'}{c^2} = a'(1+\frac{v_0}{c^2} \frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}})$$

$$= a'(1+\frac{v_0}{c^2}u) \qquad (5)$$

Therefore, the acceleration a about the accelerated matter in 2-Dimension inertial coordinate system $S(t,x)$ and the other acceleration a' about the accelerated matter in 2-Dimension inertial coordinate system $S'(t',x')$ are same.

$$a' = \frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) = a = \frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}) \qquad (6)$$

The total energy E', the kinetic energy KE' in 2-Dimension inertial coordinate system $S'(t',x')$ and the total energy E ,the kinetic energy KE in 2-Dimension inertial coordinate system $S(t,x)$ are

$$a' = \frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}), u = \frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}}, a = \frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}), V = \frac{\int adt}{\sqrt{1+\frac{1}{c^2}[\int adt]^2}} \qquad (7)$$

$$E' = \frac{m_0c^2}{\sqrt{1-\frac{u^2}{c^2}}} = m_0c^2\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}, \quad E = \frac{m_0c^2}{\sqrt{1-\frac{V^2}{c^2}}} = m_0c^2\sqrt{1+\frac{1}{c^2}[\int adt]^2} \qquad (8)$$

$$KE' = \frac{m_0c^2}{\sqrt{1-\frac{u^2}{c^2}}} - m_0c^2 = m_0c^2\sqrt{1+\frac{1}{c^2}[\int a'dt']^2} - m_0c^2 \qquad (9)$$

$$KE = \frac{m_0c^2}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{m_0c^2}{\sqrt{1-\frac{v_0^2}{c^2}}} = m_0c^2\sqrt{1+\frac{1}{c^2}[\int adt]^2} - \frac{m_0c^2}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (10)$$

The power $P' = \frac{d(KE')}{dt'} = \frac{dE'}{dt'}$ in 2-Dimension inertial coordinate system $S'(t',x')$ and the power $P = \frac{d(KE)}{dt} = \frac{dE}{dt}$ in 2-Dimension inertial coordinate system $S(t,x)$ are

$$P' = \frac{d(KE')}{dt'} = \frac{dE'}{dt'} = m_0 c^2 \frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \frac{a'}{c^2} = m_0 a'u = F'u \quad (11)$$

$$P = \frac{d(KE)}{dt} = \frac{dE}{dt} = m_0 c^2 \frac{\int adt}{\sqrt{1+\frac{1}{c^2}[\int adt]^2}} \frac{a}{c^2} = m_0 aV = FV = m_0 a\left(\frac{u+v_0}{1+\frac{v_0}{c^2}u}\right) \quad (12)$$

Eq(11),Eq(12) is equal to the classical power's definition.

In Eq(6),

$$a' = \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = a = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right)$$

$$F' = m_0 a' = \frac{d}{dt'}\left(\frac{m_0 u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = m_0 a = \frac{d}{dt}\left(\frac{m_0 V}{\sqrt{1-\frac{V^2}{c^2}}}\right) = F \quad (13)$$

Therefore, the transformation of the power is

$$P = \frac{d(KE)}{dt} = \frac{dE}{dt} = m_0 a\left(\frac{u+v_0}{1+\frac{v_0}{c^2}u}\right) = m_0 a'\left(\frac{u+v_0}{1+\frac{v_0}{c^2}u}\right) = \frac{m_0 a'u + m_0 a'v_0}{1+\frac{v_0}{c^2}u}$$

$$= \frac{\frac{dE'}{dt'} + F'v_0}{1+\frac{v_0}{c^2}u} = \frac{\frac{d(KE')}{dt'} + F'v_0}{1+\frac{v_0}{c^2}u} = \frac{P'+F'v_0}{1+\frac{v_0}{c^2}u},$$

$$F' = m_0 a' \quad , P' = \frac{d(KE')}{dt'} = \frac{dE'}{dt'} = m_0 a'u \qquad (14)$$

The inverse-transformation of the power is

$$P' = \frac{d(KE')}{dt'} = \frac{dE'}{dt'} = m_0 a'u = m_0 au = m_0 a\left(\frac{V-v_0}{1-\frac{v_0}{c^2}V}\right) = \frac{m_0 aV - m_0 av_0}{1-\frac{v_0}{c^2}V}$$

$$= \frac{\frac{dE}{dt} - Fv_0}{1-\frac{v_0}{c^2}V} = \frac{\frac{d(KE)}{dt} - Fv_0}{1-\frac{v_0}{c^2}V} = \frac{P - Fv_0}{1-\frac{v_0}{c^2}V}$$

$$F = m_0 a, \; P = \frac{d(KE)}{dt} = \frac{dE}{dt} = m_0 aV \qquad (15)$$

III. Additional chapter-II

In this time, if the acceleration a' is the constant acceleration a_0, the acceleration in 2-Dimension inertial coordinate system $S(t,x)$ and in 2-Dimension inertial coordinate system $S'(t',x')$ is the constant acceleration a_0.

$$a_0 = a' = \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = a = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right) \quad (16)$$

Therefore,

$$V = \frac{dx}{dt} = \frac{a_0 t + C}{\sqrt{1+\frac{1}{c^2}(a_0 t + C)^2}}, \quad u = \frac{dx'}{dt'} = \frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}, x' = \frac{c^2}{a_0}\left(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1\right)$$

$$= \frac{\gamma a_0 (t' + \frac{v_0}{c^2}x') + C}{\sqrt{1+\frac{1}{c^2}(a_0 \gamma (t' + \frac{v_0}{c^2}x') + C)^2}} \quad , \ C \text{ is the constant.}$$

$$= \frac{\gamma a_0 (t' + \frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)) + C}{\sqrt{1+\frac{1}{c^2}(a_0 \gamma (t' + \frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)) + C)^2}}$$

$$= \frac{\gamma a_0 t' + \gamma v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C}{\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C)^2}}$$

$$= \frac{u + v_0}{1 + \frac{u}{c^2}v_0} = \frac{\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}} + v_0}{1 + \frac{v_0}{c^2}\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}} = \frac{a_0 t' + v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2}}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2} + \frac{v_0}{c^2}a_0 t'} \quad (17)$$

In this time,

$$\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0 \sqrt{1+\frac{1}{c^2}(a_0 t')^2})^2} = \gamma\left(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} + \frac{v_0}{c^2}a_0 t'\right) \quad (18)$$

Therefore,

$$C = \gamma v_0 \quad (19)$$

Hence,

$$x=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t+\gamma v_0)^2}-\sqrt{1+\frac{1}{c^2}(\gamma v_0)^2})$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t+\gamma v_0)^2}-\gamma)=\frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}-\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}})\ (20)$$

$$V=\frac{a_0t+\gamma v_0}{\sqrt{1+\frac{1}{c^2}(a_0t+\gamma v_0)^2}}\ (21)$$

$$x'=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1)=\frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}-1)\ ,\ u=\frac{a_0t'}{\sqrt{1+\frac{1}{c^2}(a_0t')^2}},$$

$$\gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}}\qquad (22)$$

And

$$d\tau=\sqrt{1-V^2/c^2}\,dt=\frac{dt}{\sqrt{1+\frac{1}{c^2}(a_0t+\gamma v_0)^2}},\ d\tau=\sqrt{1-u^2/c^2}\,dt'=\frac{dt'}{\sqrt{1+\frac{1}{c^2}(a_0t')^2}}$$

$$\tau=\frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c})-\frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c})=\frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c})-\tau_0$$

$$\tau+\tau_0=\frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c}),\qquad \tau=\frac{c}{a_0}\sinh^{-1}(\frac{a_0t'}{c})$$

$$\tau_0=\frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}),\ \gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}}\qquad (23)$$

Therefore,

$$t+\gamma\frac{v_0}{a_0}=\frac{c}{a_0}\sinh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)$$

$$=\frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c})+\cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \qquad (24)$$

In this time,

$$\tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0 t'}{c}) \rightarrow \sinh(\frac{a_0\tau}{c}) = \frac{a_0 t'}{c},$$

$$x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1) \rightarrow \cosh(\frac{a_0\tau}{c}) = \sqrt{1+\frac{{a_0}^2 t'^2}{c^2}} = 1+\frac{a_0}{c^2}x'$$

$$\tau_0 = \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}) \rightarrow \sinh(\frac{a_0\tau_0}{c}) = \frac{\gamma v_0}{c},\ \cosh(\frac{a_0\tau_0}{c}) = \sqrt{1+\frac{\gamma^2 {v_0}^2}{c^2}} = \gamma,$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \qquad (25)$$

Therefore, Eq(24) is

$$t+\gamma\frac{v_0}{a_0} = \frac{c}{a_0}\sinh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)$$

$$= \frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c})+\cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$= \frac{c}{a_0}[\gamma\sinh(\frac{a_0\tau}{c})+\cosh(\frac{a_0\tau}{c})\frac{\gamma v_0}{c}]$$

$$= \frac{c}{a_0}[\frac{a_0 t'}{c}\cdot\gamma+(1+\frac{a_0}{c^2}x')\cdot\frac{\gamma v_0}{c}] = \gamma(t'+\frac{v_0}{c^2}x')+\gamma\frac{v_0}{a_0},\ \gamma = \frac{1}{\sqrt{1-\frac{{v_0}^2}{c^2}}} \qquad (26)$$

IV.Conclusion

Therefore, Eq(20) is

$$x = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}-\gamma), \quad x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1)$$

$$= \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}x')+\gamma v_0)^2}-\gamma)$$

$$= \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1))+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(\gamma a_0 t'+\gamma v_0\sqrt{1+\frac{1}{c^2}(a_0t')^2})^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{(\gamma\sqrt{1+\frac{1}{c^2}(a_0t')^2}+\gamma a_0\frac{v_0}{c^2}t')^2}-\gamma)$$

$$=\gamma\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1)+\gamma v_0 t'=\gamma(x'+v_0t')\ ,\ \gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \qquad (27)$$

or by Eq(25),Eq(26)

$$x=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\sinh^2(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)}-\gamma)=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\cosh(\frac{a_0}{c}\tau_0)+\sinh(\frac{a_0}{c}\tau)\sinh(\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\gamma+\sinh(\frac{a_0}{c}\tau)\frac{\gamma v_0}{c}-\gamma)$$

$$=\frac{c^2}{a_0}\gamma\sqrt{1+\frac{1}{c^2}(a_0t')^2}+\gamma v_0 t'-\frac{c^2}{a_0}\gamma,\quad \gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}} \qquad (28)$$

The transformation of the power and the inverse-transformation of the power are

$$P=\frac{d(KE)}{dt}=\frac{P'+F'v_0}{1+\frac{v_0}{c^2}u},P'=\frac{d(KE')}{dt'}=\frac{P-Fv_0}{1-\frac{v_0}{c^2}V}$$

$$F'=m_0a'\ \ ,P'=\frac{d(KE')}{dt'}=\frac{dE'}{dt'}=m_0a'u\,,\ \ F=m_0a\,,\ \ P=\frac{d(KE)}{dt}=\frac{dE}{dt}=m_0aV \qquad (29)$$

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