Gordon Decomposition and Chiral Properties of Yang Mills Magnetic Monopole Baryons

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Abstract: *In a separate paper [1], the author advanced the thesis that baryons are Yang-Mills magnetic monopoles. Here, we apply the Gordon decomposition to the final results from [1] to understand what is "inside" a Yang-Mills magnetic monopole baryon. We develop a local chiral duality formalism inherent in the Dirac algebra which appears to have lain undiscovered for decades. We also discover a "vector/axial inversion" that is inherent in hadron physics rooted in Dirac's $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$ which may explain the observed chiral asymmetries in hadrons including the many vector and axial mesons clearly observed in phenomenological data. Finally, to define what is "inside" a baryon in experimental terms, in (5.1) we specify the form from which predicted cross sections of magnetic monopole baryons can be computed and then used to experimentally confirm the thesis that baryons are Yang-Mills magnetic monopoles.*

1. Introduction

In a recent paper [1], the author advanced the thesis that magnetic monopole densities which come into existence in non-Abelian Yang-Mills gauge theory are synonymous with baryon densities. It particular, it was shown that Yang-Mills magnetic monopoles naturally confine their gauge fields, naturally contain exactly three fermions which we identify with colored quarks in a color-neutral singlet, and that the only particles crossing their surface or observed as decay products are mesons in color-neutral singlets. In particular, it was shown in [4.3] of [1] that the flow of mesons across any closed surface surrounding a magnetic monopole baryon is given by:

$$\iiint P = \iiint P^{\sigma\mu\nu} dx_\sigma dx_\mu dx_\nu = -2\oiint\left(\frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{"\not{p}_R - m_R"}+\frac{\overline{\psi}_G\sigma^{\mu\nu}\psi_G}{"\not{p}_G - m_G"}+\frac{\overline{\psi}_B\sigma^{\mu\nu}\psi_B}{"\not{p}_B - m_B"}\right)dx_\mu dx_\nu = \oiint F = -i\oiint G^2\,, \tag{1.1}$$

where, by virtue of the color singlet $\overline{R}R+\overline{G}G+\overline{B}B$ above as well as the decomposition

$$\begin{aligned}\overline{\psi}\sigma^{\mu\nu}\psi &= \tfrac{i}{2}\overline{\psi}\gamma^\mu\gamma^\nu\psi - \tfrac{i}{2}\overline{\psi}\gamma^\nu\gamma^\mu\psi = \tfrac{i}{2}\overline{\psi}\gamma^\mu\gamma^\nu\psi + \tfrac{i}{2}\overline{\psi}_C\gamma^\mu\gamma^\nu\psi_C \\ &= \tfrac{1}{2}\left(\overline{\psi}\sigma^{\mu\nu}\psi + \overline{\psi}_C\sigma^{\mu\nu}\psi_C\right) + \tfrac{i}{2}g^{\mu\nu}\left(\overline{\psi}\psi + \overline{\psi}_C\psi_C\right)\end{aligned} \tag{1.2}$$

developed in [4.2] of [1], we interpret (1.1) as saying that spin 2 mesons are the only particles allowed by the spacetime geometry to flow across any closed surface of a baryon. There is no coupling to the geometry in (1.1) that allows a spin 1 meson to pass, spin 1 gluons are not permitted to pass for the same geometric reasons that there are no magnetic monopoles in Abelian gauge theory, and the spin 0 mesons in (1.2) are filtered by $g^{\mu\nu}dx_\mu \wedge dx_\nu = 0$.

While (1.1) tells us what does and does not flow across the closed surface of a baryon, it says nothing about what is found "*inside*" the volume represented by $\iiint P$. We show here how the "Gordon decomposition" can be applied to (1.1) to decompose and look "inside" the baryon., and what it actually means, experimentally, to be able to "see" what is "inside" the baryon. Because the Dirac relationship $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ will be integral to this baryon decomposition, we shall along the way learn a great deal about the chiral properties of hadrons. All this may assist us to experimentally test the thesis that baryons are Yang-Mills magnetic monopoles.

2. The Gordon Decomposition of a Yang-Mills Magnetic Monopole Baryon

We start with [3.8] of [1], which we rewrite using (1.2) as such:

$$P^{\sigma\mu\nu} = -\begin{pmatrix} \partial^\sigma \dfrac{\left(\overline{\psi}_R \sigma^{\mu\nu}\psi_R + \overline{\psi}_{CR}\sigma^{\mu\nu}\psi_{CR}\right) + ig^{\mu\nu}\left(\overline{\psi}_R\psi_R + \overline{\psi}_{CR}\psi_{CR}\right)}{"\not{p}_R - m_R"} \\ + \partial^\mu \dfrac{\left(\overline{\psi}_G \sigma^{\nu\sigma}\psi_G + \overline{\psi}_{CG}\sigma^{\nu\sigma}\psi_{CG}\right) + ig^{\nu\sigma}\left(\overline{\psi}_G\psi_G + \overline{\psi}_{CG}\psi_{CG}\right)}{"\not{p}_G - m_G"} \\ + \partial^\nu \dfrac{\left(\overline{\psi}_B \sigma^{\sigma\mu}\psi_B + \overline{\psi}_{CB}\sigma^{\sigma\mu}\psi_{CB}\right) + ig^{\sigma\mu}\left(\overline{\psi}_B\psi_B + \overline{\psi}_{CB}\psi_{CB}\right)}{"\not{p}_B - m_B"} \end{pmatrix}. \tag{2.1}$$

If we put this into the form of (1.1), its meson character is manifest not only via the $\overline{R}R + \overline{G}G + \overline{B}B$ color singlets, but also via the "particle plus conjugate particle" structure of each term. Now, we start the Gordon decomposition of (2.1).

The first step employs Reinich-Wheeler duality [2], [3] based on the totally antisymmetric Levi-Cevita tensor $\varepsilon^{\mu\nu\alpha\sigma}$. As Misner, Wheeler and Thorne make clear in [4] at pages 87-89, not only is there a second (even) rank duality specified generally by $*A^{\mu\nu} \equiv \frac{1}{2!}\varepsilon^{\mu\nu\alpha\sigma}A_{\alpha\sigma}$, but also a first / third (odd) rank duality via $*B^{\alpha} \equiv \frac{1}{3!}\varepsilon^{\sigma\mu\nu\alpha}B_{\sigma\mu\nu}$ and $*B^{\sigma\mu\nu} \equiv \varepsilon^{\alpha\sigma\mu\nu}B_\alpha$. It is this odd-rank duality that we now wish to apply, which enables us to compact (2.1) to:

$$*P^\alpha = \frac{1}{3!}\varepsilon^{\sigma\mu\nu\alpha}P_{\sigma\mu\nu} = -\frac{1}{3!}\sum\nolimits_{N=R,G,B}\partial_\sigma \frac{\overline{\psi}_{VN}\varepsilon^{\sigma\mu\nu\alpha}\sigma_{\mu\nu}\psi_{VN} + \overline{\psi}_{CVN}\varepsilon^{\sigma\mu\nu\alpha}\sigma_{\mu\nu}\psi_{CVN}}{"\not{p}_N - m_N"}. \tag{2.2}$$

To get from (2.1) to (2.2) we have lowered spacetime indexes, multiplied both sides by $\frac{1}{3!}\varepsilon^{\sigma\mu\nu\alpha}$, renamed all summed indexes which removes the cyclic index combination in (2.1), and used a sum Σ over *N*=*R*,*G*,*B* to save having to write everything in triplicate. We use *N* rather than *C* for color, because to use *C* would cause notational confusion with the use of *C* for conjugates. Because $\varepsilon^{\sigma\mu\nu\alpha}g_{\mu\nu} = 0$, all scalar terms in (2.1) are filtered out, just as they were in (1.1) by $g^{\mu\nu}dx_\mu \wedge dx_\nu = 0$. Finally, we label each wavefunction with a *V* subscript, to indicate these are implicitly vector wavefunctions ψ_V *defined such that* the current density $J^\mu = \overline{\psi}_V\gamma^\mu\psi_V$ transforms as a Lorentz four-vector in spacetime. Axial (*A*) wavefunctions are then $\psi_A \equiv \gamma^5\psi_V$, and the adjoint $\overline{\psi} \equiv \psi^\dagger\gamma^0$ then implies $\overline{\psi}_A = -\overline{\psi}_V\gamma^5$.

Now, in a fateful step that casts the dye for hadron interactions to be definitively *not chiral symmetric*, we write the Dirac relationship $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in the covariant form $\varepsilon^{\sigma\mu\nu\alpha}\sigma_{\mu\nu} = 2i\sigma^{\alpha\sigma}\gamma^5$, then rewrite (2.2) as:

$$*P^\alpha = \frac{1}{3!}\varepsilon^{\sigma\mu\nu\alpha}P_{\sigma\mu\nu} = -\frac{2}{3!}i\sum\nolimits_{N=R,G,B}\partial_\sigma \frac{\overline{\psi}_{VN}\sigma^{\alpha\sigma}\gamma^5\psi_{VN} + \overline{\psi}_{CVN}\sigma^{\alpha\sigma}\gamma^5\psi_{CVN}}{"\not{p}_N - m_N"}. \tag{2.3}$$

The appearance of $\gamma^5 \psi_{VN} = \psi_{AN}$ above means that the tensor mesons in (2.2) have now turned into *axial tensor* mesons in (2.3). There is also now an overall factor of *i* that comes from the *i* in $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$.

The next step is to specify three quark spinors $q_N(p)$ generally according to $\psi_N \equiv q_N e^{-ip_N{}^\alpha x_\alpha}$ and $\overline{\psi}_N \equiv \overline{q}_N e^{ip'_N{}^\alpha x_\alpha}$ and then use those in (2.3). We also set $\psi_{AN} = \gamma^5 \psi_{VN}$. Thus we obtain:

$$*P^\alpha = \frac{1}{3!}\varepsilon^{\sigma\mu\nu\alpha}P_{\sigma\mu\nu} = -\frac{2}{3!}i\sum_{N=R,G,B}\partial_\sigma e^{i(p'-p)_N{}^\alpha x_\alpha}\frac{\overline{q}_{VN}\sigma^{\alpha\sigma}q_{AN} + \overline{q}_{CVN}\sigma^{\alpha\sigma}q_{CAN}}{"\not{p}_N - m_N"}. \tag{2.4}$$

We now apply $\partial_\sigma e^{i(p'-p)^\alpha{}_N x_\alpha} = i(p'-p)_{\sigma N}e^{i(p'-p)^\alpha{}_N x_\alpha}$, then reabsorb $e^{i(p'-p)^\alpha{}_n x_\alpha}$ into the wavefunctions to obtain:

$$*P^\alpha = \frac{1}{3!}\varepsilon^{\sigma\mu\nu\alpha}P_{\sigma\mu\nu} = -\frac{2}{3!}i\sum_{N=R,G,B}\frac{\overline{\psi}_{VN}\left(i\sigma^{\alpha\sigma}(p'-p)_{\sigma N}\right)\psi_{AN} + \overline{\psi}_{CVN}\left(i\sigma^{\alpha\sigma}(p'-p)_{\sigma N}\right)\psi_{CAN}}{"\not{p}_N - m_N"}. \tag{2.5}$$

The emergence of the vertex term $i\sigma^{\alpha\sigma}(p'-p)_\sigma$ now allows us to apply the Gordon decomposition: (see, e.g., [5] at 343-345) which we express in terms of wavefunctions in the general form:

$$\overline{\psi}\gamma^\alpha\psi = \frac{1}{2m}\overline{\psi}\left((p'+p)^\alpha + i\frac{g}{2}\sigma^{\alpha\sigma}(p'-p)_\sigma\right)\psi. \tag{2.6}$$

Above, *g* is the gyromagnetic "g-factor" for the fermion in question and *m* is its mass. For a bare Dirac electron, $g/2 = 1$. Note from the above reference to [5] that (2.6) implicitly incorporates Dirac's equation $\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0$ and its adjoint $i\partial_\mu\overline{\psi}\gamma^\mu + m\overline{\psi} = 0$, and is also based on the identity $\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}$ derived from combining the two fundamental Dirac relationships $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right]$ and $g^{\mu\nu} = \frac{1}{2}\left\{\gamma^\mu,\gamma^\nu\right\}$. To use (2.6) in (2.5), we rewrite (2.6) with the $\sigma^{\alpha\sigma}$ term on the left, and use *A* wavefunctions and *V* adjoints, thus:

$$\overline{\psi}_V\left(i\sigma^{\alpha\sigma}(p'-p)_\sigma\right)\psi_A = \frac{4m}{g}\overline{\psi}_V\gamma^\alpha\psi_A - \frac{2(p'+p)^\alpha}{g}\overline{\psi}_V\psi_A. \tag{2.7}$$

So, placing (2.7) into (2.5) we obtain:

$$*P^\alpha = -\frac{2}{3}i\sum_{N=R,G,B}\left(\frac{2m_N}{g_N}\frac{\overline{\psi}_{VN}\gamma^\alpha\psi_{AN} + \overline{\psi}_{CVN}\gamma^\alpha\psi_{CAN}}{"\not{p}_N - m_N"} - \frac{(p'+p)^\alpha{}_N}{g_N}\frac{\overline{\psi}_{VN}\psi_{AN} + \overline{\psi}_{CVN}\psi_{CAN}}{"\not{p}_N - m_N"}\right). \tag{2.8}$$

This is the *first rank dual* of the third rank magnetic monopole baryon $P^{\sigma\mu\nu}$ in (2.1). Now, we apply the inverse $P^{\sigma\mu\nu} = \varepsilon^{\alpha\sigma\mu\nu} * P_\alpha$, employ the fact that for odd-rank duality $** = 1$ (see [4], equation [3.53]), and then set this equal to the original $P^{\sigma\mu\nu}$ in (2.1) with *V* wavefunctions explicitly denoted, to obtain:

$$
P^{\sigma\mu\nu}=-\begin{pmatrix}\partial^{\sigma}\dfrac{\left(\overline{\psi}_{V\,R}\sigma^{\mu\nu}\,\psi_{V\,R}+\overline{\psi}_{CV\,R}\sigma^{\mu\nu}\,\psi_{CV\,R}\right)+ig^{\mu\nu}\left(\overline{\psi}_{V\,R}\psi_{V\,R}+\overline{\psi}_{CV\,R}\psi_{CV\,R}\right)}{"\,p\!\!\!/_R-m_R\,"}\\ +\partial^{\mu}\dfrac{\left(\overline{\psi}_{V\,G}\sigma^{\nu\sigma}\,\psi_{V\,G}+\overline{\psi}_{CV\,G}\sigma^{\nu\sigma}\,\psi_{CV\,G}\right)+ig^{\nu\sigma}\left(\overline{\psi}_{V\,G}\psi_{V\,G}+\overline{\psi}_{CV\,G}\psi_{CV\,G}\right)}{"\,p\!\!\!/_G-m_G\,"}\\ +\partial^{\nu}\dfrac{\left(\overline{\psi}_{V\,B}\sigma^{\sigma\mu}\,\psi_{V\,B}+\overline{\psi}_{CV\,B}\sigma^{\sigma\mu}\,\psi_{CV\,B}\right)+ig^{\sigma\mu}\left(\overline{\psi}_{V\,B}\psi_{V\,B}+\overline{\psi}_{CV\,B}\psi_{CV\,B}\right)}{"\,p\!\!\!/_B-m_B\,"}\end{pmatrix}. \qquad (2.9)
$$

$$
=-\frac{2}{3}i\varepsilon^{\alpha\sigma\mu\nu}\sum\nolimits_{N=R,G,B}\left(\frac{2m_N}{g_N}\frac{\overline{\psi}_{V\,N}\gamma_\alpha\psi_{A\,N}+\overline{\psi}_{CV\,N}\gamma_\alpha\psi_{C\,A\,N}}{"\,p\!\!\!/_N-m_N\,"}-\frac{(p'+p)_{\alpha N}}{g_N}\frac{\overline{\psi}_{V\,N}\psi_{A\,N}+\overline{\psi}_{CV\,N}\psi_{C\,A\,N}}{"\,p\!\!\!/_N-m_N\,"}\right)
$$

Finally, mirroring (1.1), we apply Gauss' / Stokes' law, and write (2.9) in integral form as:

$$
\oiint F=-i\oiint G^2=\iiint P=-\oiint\left(\sum\nolimits_{N=R,G,B}\frac{\overline{\psi}_{V\,N}\sigma^{\mu\nu}\,\psi_{V\,N}+\overline{\psi}_{CV\,N}\sigma^{\mu\nu}\,\psi_{CV\,N}}{"\,p\!\!\!/_R-m_R\,"}\right)dx_\mu dx_\nu \qquad .(2.10)
$$

$$
=-\frac{2}{3}i\varepsilon^{\alpha\sigma\mu\nu}\iiint\sum\nolimits_{N=R,G,B}\left(\frac{2m_N}{g_N}\frac{\overline{\psi}_{V\,N}\gamma_\alpha\psi_{A\,N}+\overline{\psi}_{CV\,N}\gamma_\alpha\psi_{C\,A\,N}}{"\,p\!\!\!/_N-m_N\,"}-\frac{(p'+p)_{\alpha N}}{g_N}\frac{\overline{\psi}_{V\,N}\psi_{A\,N}+\overline{\psi}_{CV\,N}\psi_{C\,A\,N}}{"\,p\!\!\!/_N-m_N\,"}\right)dx_\sigma dx_\mu dx_\nu
$$

<u>The alternative formulations (2.8), (2,9) and (2.10) represent the Gordon decomposition of a magnetic monopole baryon</u>, and they have four features of immediate interest. First, (2.8) and the bottom line terms in (2.9) and (2.10) have become *imaginary*. This is because of the *i* in $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$. We shall examine in the next two sections how to reformulate this so that all of these terms are real and represent real, observable mesons. Second, the baryon color factor of 2/3 has naturally emerged from the Gordon decomposition. Third, one may think of and apply (2.10) as the "Maxwell's equation" for a baryon, wherein the field strength F uses the polarization and magnetization bivectors **P** and **M** in place of the electric and magnetic bivectors **E** and **B**.

But finally, and most importantly, as we have previously seen, flowing through the closed baryon surface as $\oiint F=-i\oiint G^2$ are *tensor mesons* $\sum_{N=R,G,B}\left(\overline{\psi}_{V\,N}\sigma^{\mu\nu}\,\psi_{V\,N}+\overline{\psi}_{CV\,N}\sigma^{\mu\nu}\,\psi_{CV\,N}\right)$. However, inside the volume of $\iiint P$ are *axial vector mesons* $\sum_{N=R,G,B}\left(\overline{\psi}_{V\,N}\gamma^\alpha\psi_{AN}+\overline{\psi}_{CV\,N}\gamma^\alpha\psi_{CAN}\right)$ and *pseudoscalar mesons* $\sum_{N=R,G,B}\left(\overline{\psi}_{V\,N}\psi_{AN}+\overline{\psi}_{CV\,N}\psi_{CAN}\right)$. This too is because of the Dirac relation $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$, which at (2.3) has flipped the vector (V) wavefunctions of (2.1) into axial (A) wavefunctions in (2.8). We shall refer to this phenomenon as "vector/axial inversion" or "$V\leftrightarrow A$ inversion." *This inversion, which says that hadron physics in general is inherently <u>not chiral symmetric</u>, will be central to how we shall propose to confirm the thesis that baryons are Yang-Mills magnetic monopoles, and will clearly affect any cross section expressions we derive.*

3. Second Rank Continuous, Local, Chiral Duality; and SU(2) Vector / Axial Doublets

We now return to the second rank duality relationship written as $*\sigma^{\mu\nu}=\frac{1}{2!}\varepsilon^{\mu\nu\alpha\sigma}\sigma_{\alpha\sigma}$. We also recall that in (2.3) we used $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$ in the form $\varepsilon^{\mu\nu\alpha\sigma}\sigma_{\alpha\sigma}=-2i\sigma^{\mu\nu}\gamma^5$. Combining these together and using the even-rank duality relationship $**=-1$ (see [4], equation [3.53]) immediately enables us to write $\sigma^{\mu\nu}=i*\sigma^{\mu\nu}\gamma^5$. If we then sandwich this between a vector wavefunction and a vector adjoint, we find the alternative relations:

$$\begin{aligned}\overline{\psi}_V\sigma^{\mu\nu}\psi_V &= i*\overline{\psi}_V\sigma^{\mu\nu}\gamma^5\psi_V = i*\overline{\psi}_V\sigma^{\mu\nu}\psi_A \\ \overline{\psi}_V\sigma^{\mu\nu}\psi_A &= \overline{\psi}_V\sigma^{\mu\nu}\gamma^5\psi_V = i*\overline{\psi}_V\sigma^{\mu\nu}\psi_V\end{aligned}. \tag{3.1}$$

This says that tensor $\overline{\psi}_V\sigma^{\mu\nu}\psi_V$ is the imaginary dual of axial tensor $\overline{\psi}_V\sigma^{\mu\nu}\psi_A$. Alternatively, axial tensor $\overline{\psi}_V\sigma^{\mu\nu}\psi_A$ is the imaginary dual of tensor $\overline{\psi}_V\sigma^{\mu\nu}\psi_V$. But (3.1) is also just another way of saying $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$, using duality. So now, $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$ sits at the heart not only of hadron chiral asymmetry as seen in (2.9) and (2.10), but at the center of chiral duality relationships endemic to Dirac algebra. (3.1) above are another way of saying $\gamma^5=i\gamma^0\gamma^1\gamma^2\gamma^3$.

Next, because the second rank duality operator $**=-1$, it is possible to specify a duality angle θ (referred to by Wheeler in [3] as the "complexion" angle) which generates a *continuous* duality rotation between duals $\overline{\psi}_V\sigma^{\mu\nu}\psi_V$ and $*\overline{\psi}_V\sigma^{\mu\nu}\psi_V$. Using $**=-1$ in a series expansion, one may readily form $e^{*\theta}=\cos\theta+*\sin\theta$. If we now apply this to each of $\overline{\psi}_V\sigma^{\mu\nu}\psi_V$ and $*\overline{\psi}_V\sigma^{\mu\nu}\psi_V$ and use $**=-1$, we deduce the paired equations:

$$\begin{aligned}e^{*\theta}\overline{\psi}_V\sigma^{\mu\nu}\psi_V &= \cos\theta\overline{\psi}_V\sigma^{\mu\nu}\psi_V+\sin\theta*\overline{\psi}_V\sigma^{\mu\nu}\psi_V \\ e^{*\theta}*\overline{\psi}_V\sigma^{\mu\nu}\psi_V &= \cos\theta*\overline{\psi}_V\sigma^{\mu\nu}\psi_V-\sin\theta\overline{\psi}_V\sigma^{\mu\nu}\psi_V\end{aligned}. \tag{3.2}$$

Then via $\overline{\psi}_V\sigma^{\mu\nu}\psi_V=i*\overline{\psi}_V\sigma^{\mu\nu}\psi_A$ from (3.1) , these can be refashioned into the matrix equations:

$$\begin{pmatrix}\overline{\psi}_V\sigma^{\mu\nu}\psi_V(\theta_2)\\ i\overline{\psi}_V\sigma^{\mu\nu}\psi_A(\theta_2)\end{pmatrix}=\begin{pmatrix}\cos\theta_2 & \sin\theta_2\\ -\sin\theta_2 & \cos\theta_2\end{pmatrix}\begin{pmatrix}\overline{\psi}_V\sigma^{\mu\nu}\psi_V(\theta_2=0)\\ i\overline{\psi}_V\sigma^{\mu\nu}\psi_A(\theta_2=0)\end{pmatrix}=\exp\left(iT^2\theta_2\right)\begin{pmatrix}\overline{\psi}_V\sigma^{\mu\nu}\psi_V(\theta_2=0)\\ i\overline{\psi}_V\sigma^{\mu\nu}\psi_A(\theta_2=0)\end{pmatrix}. \tag{3.3}$$

Alternatively, equivalently, (3.2) may be represented as (using $(\theta_0)\equiv(\theta=\theta_0)$ generally to denote evaluation at θ_0):

$$\begin{pmatrix}\overline{\psi}_V\sigma^{\mu\nu}\psi_V(\theta_1)\\ \overline{\psi}_V\sigma^{\mu\nu}\psi_A(\theta_1)\end{pmatrix}=\begin{pmatrix}\cos\theta_1 & i\sin\theta_1\\ i\sin\theta_1 & \cos\theta_1\end{pmatrix}\begin{pmatrix}\overline{\psi}_V\sigma^{\mu\nu}\psi_V(0)\\ \overline{\psi}_V\sigma^{\mu\nu}\psi_A(0)\end{pmatrix}=\exp\left(iT^1\theta_1\right)\begin{pmatrix}\overline{\psi}_V\sigma^{\mu\nu}\psi_V(0)\\ \overline{\psi}_V\sigma^{\mu\nu}\psi_A(0)\end{pmatrix}. \tag{3.4}$$

The above, (3.3) and (3.4), are simply *alternative representations* of duality relationship (3.2) with $\theta\to\theta_2$ and $\theta\to\theta_1$ respectively to denote the axis of rotation via $\exp\left(iT^2\theta_2\right)$ and $\exp\left(iT^1\theta_1\right)$. They both appear to occur in a chiral duality SO(3) space with SU(2) rotation generators T^i, i=1,2,3. In the former "T^2 representation," the SU(2) doublet contains real tensor $\overline{\psi}_V\sigma^{\mu\nu}\psi_V(0)$ and imaginary axial tensor $i\overline{\psi}_V\sigma^{\mu\nu}\psi_A(0)$, with a real rotation via $\exp\left(iT^2\theta_2\right)$. In the latter "$T^1$ representation," both members of the doublet are real, but the rotation $\exp\left(iT^1\theta_1\right)$ has imaginary elements. For most of the subsequent development we will find it more convenient to use the T^2 representation.

But before we do, the T^1 representation reveals one very important result: If we write (3.4) in terms of wavefunctions, one may readily show with $\overline{\psi}=\psi^\dagger\gamma^0$ and trigonometric double-angle identities, the double covering:

$$\begin{pmatrix}\psi_V(\theta_1)\\ \psi_A(\theta_1)\end{pmatrix}=\begin{pmatrix}\cos(\theta_1/2) & i\sin(\theta_1/2)\\ i\sin(\theta_1/2) & \cos(\theta_1/2)\end{pmatrix}\begin{pmatrix}\psi_V(0)\\ \psi_A(0)\end{pmatrix}=\exp\left(iT^1\theta_1/2\right)\begin{pmatrix}\psi_V(0)\\ \psi_A(0)\end{pmatrix} \tag{3.5}$$

which will identically generate tensor rotation (3.4). Thus, $\psi_V = |T^3 = +1\rangle$ and $\psi_A = |T^3 = -1\rangle$ are revealed to be the upper and lower eigenstates of an SU(2) doublet. Once we establish this doublet, a T^2 rotation using this doublet is:

$$\begin{pmatrix} \psi_V(\theta_2) \\ \psi_A(\theta_2) \end{pmatrix} = \begin{pmatrix} \cos(\theta_2/2) & \sin(\theta_2/2) \\ -\sin(\theta_2/2) & \cos(\theta_2/2) \end{pmatrix} \begin{pmatrix} \psi_V(0) \\ \psi_A(0) \end{pmatrix} = \exp(iT^2\theta_2/2) \begin{pmatrix} \psi_V(0) \\ \psi_A(0) \end{pmatrix}. \tag{3.6}$$

Consequently, rotation to $\theta_2 = -\pi/2$ reveals a rotated left / right-handed SU(2) doublet:

$$\begin{pmatrix} \psi_V(-\pi/2) \\ \psi_A(-\pi/2) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \psi_V(0) \\ \psi_A(0) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_V(0) - \psi_A(0) \\ \psi_V(0) + \psi_A(0) \end{pmatrix} = \sqrt{2} \begin{pmatrix} \psi_L(0) \\ \psi_R(0) \end{pmatrix} \tag{3.7}$$

So not only do we see SU(2) in Dirac algebra chiral duality, but we now see a path to obtain the left-handed or right-handed chiral projections based on orientation of the vector / axial doublet in duality space.

This noted, we now return to the T^2 representation of (3.3) to deduce that:

$$\begin{aligned} \bar{\psi}_V \sigma^{\mu\nu} \psi_V(\pi/2) &= i\bar{\psi}_V \sigma^{\mu\nu} \psi_A(0) \\ \bar{\psi}_V \sigma^{\mu\nu} \psi_A(\pi/2) &= i\bar{\psi}_V \sigma^{\mu\nu} \psi_V(0) \end{aligned}. \tag{3.8}$$

These tell us that a *real tensor* disposed at $\theta_2 = \pi/2$ is the same thing as an *imaginary axial tensor* disposed at $\theta_2 = 0$, and that a *real axial tensor* disposed at $\theta_2 = \pi/2$ is the same thing as an *imaginary tensor* disposed at $\theta_2 = 0$. And, it is very important to observe, *this chiral duality is an <u>inherent</u> feature of Dirac's algebra, independent of any discussion about hadron physics, as is the appearance of an SU(2) group with vector and axial wavefunction eigenstates.*

Next, let us regard θ_2 not as global, but as a *local* duality angle $\theta_2(x^\mu)$ varying in spacetime. This is important because (2.1) and (2.9) contain multiple terms $\partial^\sigma \bar{\psi} \sigma^{\mu\nu} \psi$ and if θ_2 is local, then this locality will introduce new terms with $\partial^\sigma \theta_2$. Specifically, applying ∂^σ to (3.3) with θ_2 regarded as a *local* angle, the derivative $\partial^\mu \exp(iT^2\theta_2)$ rotates the mesons in the associated term by $\pi/2$. So following reduction and consolidation, and also using (3.8) plus $-\bar{\psi}_V \sigma^{\mu\nu} \psi_V(0) = \bar{\psi}_V \sigma^{\mu\nu} \psi_V(\pi)$ derived from (3.3), we obtain:

$$\partial^\sigma \begin{pmatrix} \bar{\psi}_V \sigma^{\mu\nu} \psi_V(\theta_2) \\ \bar{\psi}_V \sigma^{\mu\nu} \psi_V(\theta_2 + \pi/2) \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \left[\partial^\sigma \begin{pmatrix} \bar{\psi}_V \sigma^{\mu\nu} \psi_V(0) \\ \bar{\psi}_V \sigma^{\mu\nu} \psi_V(\pi/2) \end{pmatrix} + \partial^\sigma \theta_2 \begin{pmatrix} \bar{\psi}_V \sigma^{\mu\nu} \psi_V(\pi/2) \\ \bar{\psi}_V \sigma^{\mu\nu} \psi_V(\pi) \end{pmatrix} \right]. \tag{3.9}$$

This type of result should be familiar from local gauge theory. So we now absorb $\partial^\sigma \theta_2$ into a new gauge boson ${B_2}^\sigma$ which SU(2)-transforms as $B_i^\sigma \to B_i^\sigma - \varepsilon_{ijk} \theta_j B_k^\sigma + \partial^\sigma \theta_i$ with i=2, see, e.g., [6] eq. IV.5[4], as part of a triplet B_i^σ of gauge bosons. So, not only does $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ introduce duality and chiral asymmetry, but when that duality is taken continuously and locally, it introduces SU(2) chiral duality gauge bosons. For now, we do not address the physical meaning of these B_i^σ, and we take $\theta_2(x^\mu)$ to simply be a mathematical parameter.

Comparing (3.3) and (3.9) with (2.9), we see that as the tensor doublet $\bar{\psi}_V \sigma^{\mu\nu} \psi_V(\theta_2)$, $i\bar{\psi}_V \sigma^{\mu\nu} \psi_A(\theta_2)$ is rotated using $\exp(iT^2\theta_2)$ in the SU(2) duality space, so too are vectors and scalars according to:

$$\begin{pmatrix} \bar{\psi}_V \gamma^\alpha \psi_V(\theta_2) \\ i\bar{\psi}_V \gamma^\alpha \psi_A(\theta_2) \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \bar{\psi}_V \gamma^\alpha \psi_V(0) \\ i\bar{\psi}_V \gamma^\alpha \psi_A(0) \end{pmatrix}, \text{ and} \tag{3.10}$$

$$\begin{pmatrix} \bar{\psi}_V \psi_V(\theta_2) \\ i\bar{\psi}_V \psi_A(\theta_2) \end{pmatrix} = \begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \begin{pmatrix} \bar{\psi}_V \psi_V(0) \\ i\bar{\psi}_V \psi_A(0) \end{pmatrix}. \tag{3.11}$$

All except (3.10) and (3.11) above are inherent features of Dirac algebra, without any consideration of hadron physics.

4. Gordon Decomposition Redux, With Local Chiral Duality

Now we return to hadrons. What we learned about local chiral duality in section 3 lets us rewrite (2.1) as:

$$\begin{pmatrix} P^{\sigma\mu\nu}(\theta_2) \\ P^{\sigma\mu\nu}(\theta_2+\pi/2) \end{pmatrix} = -\begin{pmatrix} \partial^{\sigma}\dfrac{\left(\overline{\psi}_{VR}\sigma^{\mu\nu}\psi_{VR}(\theta_2)+\overline{\psi}_{CR}\sigma^{\mu\nu}\psi_{CVR}(\theta_2)\right)+ig^{\mu\nu}\left(\overline{\psi}_{VR}\psi_{VR}(\theta_2)+\overline{\psi}_{CVR}\psi_{CVR}(\theta_2)\right)}{"p_R-m_R"} \\ \partial^{\sigma}\dfrac{\left(\overline{\psi}_{VR}\sigma^{\mu\nu}\psi_{VR}(\theta_2+\pi/2)+\overline{\psi}_{CVR}\sigma^{\mu\nu}\psi_{CVR}(\theta_2+\pi/2)\right)+ig^{\mu\nu}\left(\overline{\psi}_{VR}\psi_{VR}(\theta_2+\pi/2)+\overline{\psi}_{CVR}\psi_{CVR}(\theta_2+\pi/2)\right)}{"p_R-m_R"} \end{pmatrix}$$
$$-\begin{pmatrix} \partial^{\mu}\dfrac{\left(\overline{\psi}_{VG}\sigma^{\nu\sigma}\psi_{VG}(\theta_2)+\overline{\psi}_{CVG}\sigma^{\nu\sigma}\psi_{CVG}(\theta_2)\right)+ig^{\nu\sigma}\left(\overline{\psi}_{VG}\psi_{VG}(\theta_2)+\overline{\psi}_{CVG}\psi_{CVG}(\theta_2)\right)}{"p_G-m_G"} \\ \partial^{\mu}\dfrac{\left(\overline{\psi}_{VG}\sigma^{\nu\sigma}\psi_{VG}(\theta_2+\pi/2)+\overline{\psi}_{CVG}\sigma^{\nu\sigma}\psi_{CVG}(\theta_2+\pi/2)\right)+ig^{\nu\sigma}\left(\overline{\psi}_{VG}\psi_{VG}(\theta_2+\pi/2)+\overline{\psi}_{CVG}\psi_{CVG}(\theta_2+\pi/2)\right)}{"p_G-m_G"} \end{pmatrix}$$
$$-\begin{pmatrix} \partial^{\nu}\dfrac{\left(\overline{\psi}_{VB}\sigma^{\sigma\mu}\psi_{VB}(\theta_2)+\overline{\psi}_{CVB}\sigma^{\sigma\mu}\psi_{CVB}(\theta_2)\right)+ig^{\sigma\mu}\left(\overline{\psi}_{VB}\psi_{VB}(\theta_2)+\overline{\psi}_{CVB}\psi_{CVB}(\theta_2)\right)}{"p_B-m_B"} \\ \partial^{\nu}\dfrac{\left(\overline{\psi}_{VB}\sigma^{\sigma\mu}\psi_{VB}(\theta_2+\pi/2)+\overline{\psi}_{CVB}\sigma^{\sigma\mu}\psi_{CVB}(\theta_2+\pi/2)\right)+ig^{\sigma\mu}\left(\overline{\psi}_{VB}\psi_{VB}(\theta_2+\pi/2)+\overline{\psi}_{CVB}\psi_{CVB}(\theta_2+\pi/2)\right)}{"p_B-m_B"} \end{pmatrix} . \quad (4.1)$$

We then take the dual and compact this to rewrite via duality (2.2) as the doublet equation:

$$\begin{pmatrix} *P^{\alpha}(\theta_2) \\ *P^{\alpha}(\theta_2+\pi/2) \end{pmatrix} = -\frac{1}{3!}\sum_{N=R,G,B}\partial_{\sigma}\begin{pmatrix} \dfrac{\overline{\psi}_{VN}\varepsilon^{\sigma\mu\nu\alpha}\sigma_{\mu\nu}\psi_{VN}(\theta_2)+\overline{\psi}_{CVN}\varepsilon^{\sigma\mu\nu\alpha}\sigma_{\mu\nu}\psi_{CVN}(\theta_2)}{"p_N-m_N"} \\ \dfrac{\overline{\psi}_{VN}\varepsilon^{\sigma\mu\nu\alpha}\sigma_{\mu\nu}\psi_{VN}(\theta_2+\pi/2)+\overline{\psi}_{CVN}\varepsilon^{\sigma\mu\nu\alpha}\sigma_{\mu\nu}\psi_{CVN}(\theta_2+\pi/2)}{"p_N-m_N"} \end{pmatrix} . \quad (4.2)$$

This expresses $*P^{\alpha}(\theta_2)$ as a function of θ_2 and incorporates the chiral SU(2) doublet uncovered in (3.3) and (3.9) in the T^2 representation. Now, we factor out $\exp(iT^2\theta_2)$ and repeat all the same steps that were used to go from (2.2) to (2.9). Along the way, we write the Gordon decomposition (2.7) in terms of θ_2. But via *local* duality we add a new term that contains the SU(2) gauge boson B_2^{τ} via (3.9) and $B_i^{\sigma} \to B_i^{\sigma} - \varepsilon_{ijk}\theta_j B_k^{\sigma} + \partial^{\sigma}\theta_i$. And from (3.8), (3.10) and (3.11), we derive and then employ in the various doublets, the relationships:

$$\begin{aligned} &i\overline{\psi}_V\sigma^{\mu\nu}\psi_A(\pi/2) = -\overline{\psi}_V\sigma^{\mu\nu}\psi_V(0); \quad i\overline{\psi}_V\sigma^{\mu\nu}\psi_A(\pi) = -i\overline{\psi}_V\sigma^{\mu\nu}\psi_A(0) = -\overline{\psi}_V\sigma^{\mu\nu}\psi_V(\pi/2) \\ &i\overline{\psi}_V\gamma^{\alpha}\psi_A(0) = \overline{\psi}_V\gamma^{\alpha}\psi_V(\pi/2); \quad i\overline{\psi}_V\gamma^{\alpha}\psi_A(\pi/2) = -\overline{\psi}_V\gamma^{\alpha}\psi_V(0) = \overline{\psi}_V\gamma^{\alpha}\psi_V(\pi) \\ &i\overline{\psi}_V\psi_A(0) = \overline{\psi}_V\psi_V(\pi/2); \quad i\overline{\psi}_V\psi_A(\pi/2) = -\overline{\psi}_V\gamma^{\alpha}\psi_V(0) = \overline{\psi}_V\psi_V(\pi) \end{aligned} \quad . \quad (4.3)$$

all of this leads to:

$$\begin{pmatrix} P^{\sigma\mu\nu}(\theta_2) \\ P^{\sigma\mu\nu}(\theta_2+\pi/2) \end{pmatrix} = \varepsilon^{\alpha\sigma\mu\nu}\begin{pmatrix} *P_{\alpha}(\theta_2) \\ *P_{\alpha}(\theta_2+\pi/2) \end{pmatrix}$$
$$= -\frac{2}{3}\varepsilon^{\alpha\sigma\mu\nu}\begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix}\sum_{N=R,G,B}\left[\begin{gathered} \frac{1}{2}B_2^{\tau}\begin{pmatrix} -\dfrac{\overline{\psi}_{VN}\sigma_{\alpha\tau}\psi_{VN}(0)+\overline{\psi}_{CVN}\sigma_{\alpha\tau}\psi_{CVN}(0)}{"p_N-m_N"} \\ -\dfrac{\overline{\psi}_{VN}\sigma_{\alpha\tau}\psi_{VN}(\pi/2)+\overline{\psi}_{CVN}\sigma_{\alpha\tau}\psi_{CVN}(\pi/2)}{"p_N-m_N"} \end{pmatrix} \\ +\frac{2m_N}{g_N}\begin{pmatrix} \dfrac{\overline{\psi}_{VN}\gamma_{\alpha}\psi_{VN}(\pi/2)+\overline{\psi}_{CVN}\gamma_{\alpha}\psi_{CVN}(\pi/2)}{"p_N-m_N"} \\ \dfrac{\overline{\psi}_{VN}\gamma_{\alpha}\psi_{VN}(\pi)+\overline{\psi}_{CVN}\gamma_{\alpha}\psi_{CVN}(\pi)}{"p_N-m_N"} \end{pmatrix} \\ -\frac{(p'+p)_{\alpha N}}{g_N}\begin{pmatrix} \dfrac{\overline{\psi}_{VN}\psi_{AN}(\pi/2)+\overline{\psi}_{CVN}\psi_{CAN}(\pi/2)}{"p_N-m_N"} \\ \dfrac{\overline{\psi}_{VN}\psi_{VN}(\pi)+\overline{\psi}_{CVN}\psi_{CVN}(\pi)}{"p_N-m_N"} \end{pmatrix} \end{gathered}\right] . \quad (4.4)$$

This is the SU(2) chiral space counterpart of the bottom line of (2.9), *and it is equal to (4.1)*. So, we set (4.1) with $\exp(iT^2\theta_2)$ factored out, equal to (4.4). We then take the Gauss' / Stoke' integral as in (2.10), to obtain the *final result*:

$$\iiint\begin{pmatrix} P(\theta_2) \\ P(\theta_2+\pi/2) \end{pmatrix} = -\begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \oiint \sum_{N=R,G,B} \begin{pmatrix} \dfrac{\overline{\psi}_{VN}\sigma^{\mu\nu}\psi_{VN}(0)+\overline{\psi}_{CN}\sigma^{\mu\nu}\psi_{CVN}(0)}{"p_N - m_N"} \\ \dfrac{\overline{\psi}_{VN}\sigma^{\mu\nu}\psi_{VN}(\pi/2)+\overline{\psi}_{CVN}\sigma^{\mu\nu}\psi_{CVN}(\pi/2)}{"p_N - m_N"} \end{pmatrix} dx_\mu dx_\nu$$

$$= -\frac{2}{3}\varepsilon^{\alpha\sigma\mu\nu}\begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix} \times \iiint \sum_{N=R,G,B} \begin{bmatrix} \dfrac{1}{2}B_2{}^{\tau}\begin{pmatrix} -\dfrac{\overline{\psi}_{VN}\sigma_{\alpha\tau}\psi_{VN}(0)+\overline{\psi}_{CVN}\sigma_{\alpha\tau}\psi_{CVN}(0)}{"p_N - m_N"} \\ -\dfrac{\overline{\psi}_{VN}\sigma_{\alpha\tau}\psi_{VN}(\pi/2)+\overline{\psi}_{CVN}\sigma_{\alpha\tau}\psi_{CVN}(\pi/2)}{"p_N - m_N"} \end{pmatrix} \\ +\dfrac{2m_N}{g_N}\begin{pmatrix} \dfrac{\overline{\psi}_{VN}\gamma_\alpha\psi_{VN}(\pi/2)+\overline{\psi}_{CVN}\gamma_\alpha\psi_{CVN}(\pi/2)}{"p_N - m_N"} \\ \dfrac{\overline{\psi}_{VN}\gamma_\alpha\psi_{VN}(\pi)+\overline{\psi}_{CVN}\gamma_\alpha\psi_{CVN}(\pi)}{"p_N - m_N"} \end{pmatrix} \\ -\dfrac{(p'+p)_{\alpha N}}{g_N}\begin{pmatrix} \dfrac{\overline{\psi}_{VN}\psi_{AN}(\pi/2)+\overline{\psi}_{CVN}\psi_{CAN}(\pi/2)}{"p_N - m_N"} \\ \dfrac{\overline{\psi}_{VN}\psi_{VN}(\pi)+\overline{\psi}_{CVN}\psi_{CVN}(\pi)}{"p_N - m_N"} \end{pmatrix} \end{bmatrix} dx_\sigma dx_\mu dx_\nu \quad . \tag{4.5}$$

Via (4.3), we have rewritten every *imaginary axial* meson as *real vector* meson which has undergone a $\pi/2$ rotation about the y axis in the SU(2) duality space. Thus in contrast to (2.9), the overall factor of *i* is removed, all terms are real, and everything is written in terms of vector (*V*) wavefunctions with duality space rotations of the mesons. If we consider this further in light of (3.7), we see how rotation in the SU(2) / SO(3) chiral duality space allows us to express what is a vector (*V*) object in one orientation, as an axial (*A*) object in a different orientation, and even as left or right chiral projections in yet other orientations.

We now interpret (3.8) and the similar relationships derivable from (3.10) and (3.11) such as (4.3) as saying that experimentally-observed axial (*A*) mesons are *real* non-axial (*V*) mesons which have been rotated through the duality space from $\theta_2 = 0$ to $\theta_2 = \pi/2$. So for example, when we observe pseudoscalar mesons such as the π mesons, we are actually observing *imaginary* objects $i\overline{\psi}_V\psi_A(0)$ which are identical to and simply another name for duality-rotated *real* objects $\overline{\psi}_V\psi_V(\pi/2)$, i.e., we are observing real scalar mesons disposed at $\theta_2 = \pi/2$.

Equation (4.5) is the central mathematical result of this paper. If ever there was statement that baryons and hadrons do not exhibit chiral symmetry, (4.5) with its mix of $\theta = 0$ and $\theta = \pi/2$ and $\theta = \pi$ dispositions is such a statement. Now let us see what (4.5) teaches about hadrons, and how it can be experimentally tested.

5. Specifying Deep Inelastic Scattering Cross Sections for Yang-Mills Magnetic Monopole Baryons

First, we note that every tensor, vector and scalar object in (4.5) is a meson. This is indicated by both the $\overline{R}R + \overline{G}G + \overline{B}B$ color singlet that appears throughout (compacted via the sum Σ) and the conjugate wavefunctions which accompany all particle wavefunctions. Second, we see that the <u>only</u> mesons which flow through the closed surface of integration are second rank tensor mesons of the basic spacetime form $\overline{\psi}\sigma^{\mu\nu}\psi$. These have spin 2, so as already reported in [1], spin 2 is the "passport" in and out of a baryon surface. For a meson of any other spin, it must either excite or decay into a spin 2 meson to pass. Third, inside the baryon (and we shall momentarily seek to give a more precise definition to the phrase "inside the baryon"), as a function of θ_2, are also second rank tensor and axial tensor mesons coupled with the SU(2) gauge boson $B_2{}^\tau$ which arose from *local* duality, in the form $B_2{}^\tau\overline{\psi}\sigma_{\alpha\tau}\psi$. These of course also have spin 2. But there are also spin 1 vector and axial vector mesons of the form $\overline{\psi}\gamma_\alpha\psi$ and spin 0 scalar and pseudoscalar mesons $\overline{\psi}\psi$ coupled in spacetime through momentum in the form $(p'+p)_\alpha\overline{\psi}\psi$.

Finally, and most importantly in terms of experimental confirmation, the "$V \leftrightarrow A$ inversion" found at the end of section 2 is further developed in (4.5). *No matter what the orientation of the duality angle* θ_2*, tensor mesons are always* $\pi/2$ *out of synch with vector and scalar mesons.* Under like conditions (meaning for the same θ_2 which is local and so varies from one event in spacetime to another), wherever there is a preponderance of spin 2 tensor mesons there will be a proportionate preponderance of spin 1 *axial* tensor mesons and spin 0 *pseudoscalar* mesons, and vice versa. Careful experimentation should be able to reveal a pattern in which spin 2 mesons are always $V \leftrightarrow A$ inverted in relation to spin 1 and spin 0 mesons under otherwise identical conditions as specified by θ_2, and this inversion will be central to how we propose to confirm the thesis that baryons are Yang-Mills magnetic monopoles.

Now, as a prelude to discussing cross sections, we turn to the question, what does it really mean to talk about what is happening "inside the baryon"? Equation (4.5) is a volume integral with spin 0 scalar and pseudoscalar mesons, spin 1 vector and axial vector mesons, and spin 2 tensor and axial tensor mesons "inside" the closed volume. This is mathematically identical in nature to the integration volume in the Maxwell's charge equation $\iiint {}^*J^{\sigma\mu\nu}dx_\sigma dx_\mu dx_\nu = \oiint {}^*F^{\mu\nu}dx_\mu dx_\nu$. But in Maxwell's equation, we are generally considering macroscopic surfaces enclosing a large number of charges, so there is a very definite, colloquial understanding of "inside" the volume. Not so for (4.5) however. Here the volume is that of a baryon, and the only known way to discern the "inside" of a baryon is via scattering experiments such as $ep \rightarrow ep$ and more generally, $ep \rightarrow eX$. So, how do we do "see" "inside"?

To facilitate discussion, we have reproduced below, Figures 8.2 and 8.5 from Halzen and Martin's [7], and refer to the accompanying discussion in Section 8.2 and 8.3 of this same reference [7]:

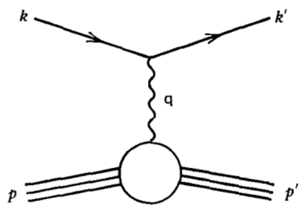

Fig. 8.2 Lowest-order electron–proton elastic scattering.

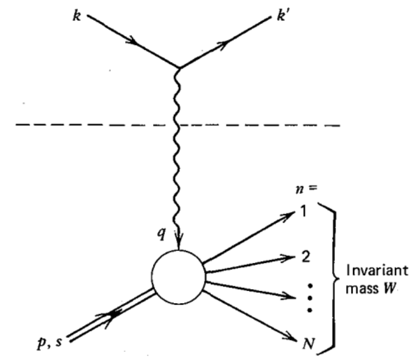

Fig. 8.5 Lowest-order diagram for ep → eX.

Figure 1: Comparison of elastic verus inelastic proton scattering diagrams from [7]

When we consider baryons (which we shall take to be protons for this discussion), we "see" "inside" the proton by probing with photons (or other mediating field quanta) and then studying the scattering debris. The differential cross sections $d\sigma \propto L^{e}{}_{\mu\nu} L^{p\ \mu\nu}$ are calculated from the tensor $L^{e}{}_{\mu\nu} \propto J^{e}{}_{\mu} J^{e}{}_{\nu}{}^{*}$ based on the electron vertex $J^{e}{}_{\mu} = \bar{u}(k')\gamma_{\mu} u(k) e^{i(k'-k)\cdot x}$ summed with an analogous tensor $L^{p\ \mu\nu}$ based on the proton vertex. For large wavelength (small $q^2 = (k'-k)^2$) photons, we are able to approximate the proton as a structureless point particle, see [7] eqs. [8.11] to [8.13]. We do so using "the most general four-vector form [$J^{p\ \mu}$] that can be constructed from p, p', q and the Dirac γ-matrices sandwiched between $\bar{u}$ and u." ([7] page 176.) This $J^{p\ \mu}$ is then used to form $L^{p\ \mu\nu} \propto J^{p\ \mu} J^{p\ \nu} *$.

But for large $q^2 = (p'-p)^2$ "the debris becomes so messy that the initial state proton loses its identity completely and a new formalism must be devised to extract information from these measurements." ([7] page 179.) The customary approach is to form a hadron tensor $W^{\mu\nu} \propto L^{p\ \mu\nu}$ out of $g^{\mu\nu}, p, p', q$, directly on general considerations, see [8.24] of [7]. In large part, this is because there is no four-vector heir-apparent that allows us to replace the vertex [8.13] of [7] when the proton debris gets really messy and the proton loses its identity. So we use $W^{\mu\nu}$ to "parameterize our total ignorance of the form of the current at the other end of the propagator" ([7] page 180) in Figure 8.5 of Figure 1 above. But this is where (4.5) now comes into play, because (4.5) reduces this ignorance.

En route to (4.5), embedded directly in (4.4), we obtained the magnetic monopole baryon dual $*P_{\alpha}(\theta_2)$ which is itself a four-vector containing all spin 0, 1 and 2 objects, both vector (V) and axial (A) that can be constructed using elements of Dirac algebra (see also (2.8) deduced before we developed local chiral duality), with *V* and *A* terms mixed via parameter θ_2. This is exactly what we need to specify $L^{p\ \mu\nu}$ to calculate deep scattering cross sections. Specifically, we now form $L^{p\ \mu\nu} \propto *P^{p\ \mu} * P^{p\ \nu} *$ from the $*P_{\alpha}(\theta_2)$ contained in (4.4) (the first two * in the foregoing represent duality; the final * represents conjugation), so the deep scattering cross section is now specified using:

$$d\sigma \propto \mathfrak{M} \propto L^{e}{}_{\mu\nu} L^{p\ \mu\nu} \propto \left(J^{e}{}_{\mu} J^{e}{}_{\nu} *\right)\left(* P^{p\ \mu} * P^{p\ \mu} *\right). \tag{5.1}$$

Consider the following reasons in support of calculating cross sections using the dual vector $*P^{\nu}(\theta_2)$ in this way. First, we need a four-vector vector to represent the baryon for large q^2. The problem with the vector [8.12], [8.13] in [7] is that this represents a structureless point particle, and for large q^2 we must account for a structure that is lacking in [8.12], [8.13] of [7]. Second, while Figure 8.5 of Figure 1 shows the proton breaking up into "messy" debris, we do know some things about the nature of this mess. We know that this mess consists of mesons, never of free quarks or gluons. We know that this mess always contain spin 0 scalar and pseudoscalar mesons, spin 1 vector and axial vector mesons, spin 2 tensor and axial tensor mesons (and via duality and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, spin 3 tensor mesons transforming as spin 1 axial vectors and spin 4 tensor mesons transforming as spin 0 pseudoscalars, and A/V vice versa). This "mess" is well catalogued, for example, in the PDG meson data [8]. Third, in contrast to the vector [8.12], [8.13] in [7], the dual vector $*P^{\alpha}(\theta_2) = \frac{1}{3!}\varepsilon^{\sigma\mu\nu\alpha} P_{\sigma\mu\nu}(\theta_2)$ is not only an alternative vector, but it contains all the essential structural information about the magnetic monopole baryon $P_{\sigma\mu\nu}(\theta_2)$ and the "mess" catalogued in [8] that this baryons would produce in deep inelastic scattering experiments. $*P^{\alpha}(\theta_2)$ naturally contains all the different spin 0, 1 and 2 vector and axial mesons that can naturally be formed in spacetime from elements of the Dirac algebra, and by duality one can readily inject spins 3 and 4. We do not need to "construct" anything. Everything is already there.

Fourth, and most importantly, by using the first rank baryon dual $*P^{\alpha}(\theta_2)$ to specify cross sections as in (5.1), *we provide the most direct means possible to experimentally confirm the thesis that Yang-Mills magnetic monopole really are baryons: we specify how to calculate cross sections based on this thesis*.

Finally, (5.1) answers the question "what does it really mean to talk about what is happening 'inside the baryon'?" in the most direct, experimentally-couched way possible: We look "inside" the baryon with deep inelastic scattering experiments, by taking what is shown to be "inside" the baryon in the Maxwell sense of (4.5), and using those same "innards" via $*P^{\alpha}(\theta_2)$ of (4.4) to specify scattering cross sections via (5.1). This is how we may discuss what is "inside" the baryon in a way that can be experimentally tested.

6. Conclusion

The complete calculation of differential cross sections based on what is specified in (5.1) is a substantial exercise which is left for a future paper. But it is through (5.1) that we can most directly confirm, invalidate, or perhaps fine tune with experimental data, the thesis that baryons are Yang-Mills magnetic monopoles.

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