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Maximum Force Derived from Special Relativity, the Equivalence Principle and the Inverse Square Law

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Abstract Based on the work of Jacobson [1] and Gibbons, [2] Schiller [3] has shown not only that a *maximum force* follows from general relativity, but that general relativity can be *derived* from the principle of maximum force. In the present paper an alternative derivation of maximum force is given. Inspired by the equivalence principle, the approach is based on a modification of the well known special relativity equation for the velocity acquired from uniform proper acceleration. Though in Schiller's derivation the existence of gravitational *horizons* plays a key role, in the present derivation this is not the case. In fact, though the kinematic equation that we start with does exhibit a horizon, it is not carried over to its gravitational counterpart. A few of the geometrical consequences and physical implications of this result are discussed.

Keywords maximum force · general relativity · special relativity · equivalence principle · Newtonian gravity · horizons

PACS 04. General relativity and gravitation · 04.80.CcExperimental tests of gravitational theories

1 Introduction

In a recent paper in this journal, Schiller has shown how the maximum force in nature, $c^4/4G$, "plays the same role for general relativity as the maximum speed plays for special relativity." In the present paper we show that the same force can be derived from a novel combination of special relativity's speed limit, Einstein's equivalence principle, and the inverse-square law of gravity. Use of the speed limit as a maximum serves to compliment Schiller's thesis. The present derivation diverges from Schiller's thesis, however, with regard to

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the significance of horizons. Gravitational horizons play a key role in Schiller's argument. Whereas, though the present derivation arrives at exactly the same maximum force, it actually implies an absence of gravitational horizons.

Insofar as our derivation is based on well established principles and agrees with the maximum force prediction, it is appropriate to explore a few of its other consequences. It implies, for example, that for most observationally accessible circumstances, spacetime is curved almost exactly as predicted by general relativity. For extreme cases, however, i.e., for large m/r ratios, the present result is significantly different from general relativity. (See Appendix.) Specifically, the predicted absence of gravitational horizons naturally also means an absence of gravitational singularities, i.e., black holes. According to the present result, what are now thought to be physical black holes would thus instead be more properly called, "dim compact massive objects." The collapse of stars or collections of large masses in the centers of stellar systems need not result in any singularities. The line of thought leading to this result also leads to a possible test by laboratory experiment.

2 Hyperbolic motion

Let's begin by considering a body undergoing uniform proper acceleration with respect to an inertial system, I. The equation for the velocity of the body is well known to be

$$v = \frac{at}{\sqrt{1 + a^2t^2/c^2}} \ , \tag{1}$$

where a is the acceleration given by an accelerometer attached to the body, t is the time given by a clock in I, and c is the light speed constant. As $t \to \infty, v \to c$. This is often called hyperbolic motion because the track on

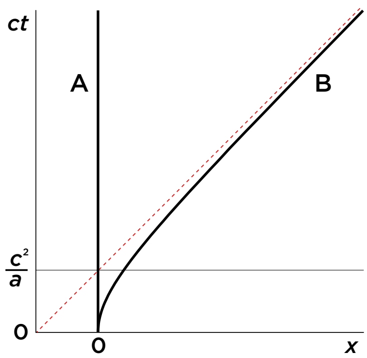

Fig. 1 Hyperbolic motion: The asymptote defines a light cone that B's time track never reaches because B's speed will never reach the speed of light. The asymptote also represents a horizon, a communication barrier, because B will never receive signals from A after the time c/a.

a spacetime diagram is a hyperbola whose asymptote represents the speed of light. This is shown in Figure 1, which also illustrates another important property of constant proper acceleration, that is, a *horizon.* In the figure the vertical track of A represents an observer who remains at rest in I, while the hyperbolic track of B reflects B's acceleration. The asymptote to B's trajectory also represents a light cone and therefore a horizon. B will never receive signals from A emitted after the time, c/a. These are elementary consequences of special relativity.

3 Equivalence principle

Appealing now to Einstein's equivalence principle, we note that if body B has an extent, h in the direction of motion, then observers who exchange signals from the ends of h can detect a shift in light frequency, f. If B1 and B2 represent the leading and trailing ends of h, respectively, then an observer at B1 would see B2's signal red-shifted according to

$$f_{\mathrm{B2}} \approx f_{\mathrm{B1}}(1 - ah/c^2) \ . \tag{2}$$

And B2 would see a signal from B1 correspondingly blue-shifted. This result is often used by analogy ("equivalence") to derive the variation of clock rates found at different heights near a gravitating body. The reasoning behind (2) appeals to the Doppler effect, which makes sense in the kinematic circumstance. In the time between emission and reception, B acquires the speed $\approx ah/c$, which produces the shift. In a stationary gravitational field, however, the expression "gravitational Doppler effect" is a bit of a misnomer because the observed frequency difference isn't due to a spectral shift caused by a change in motion between emitter and receiver. It is due to the difference in frequency between two clocks, neither of whose speeds change while the signals are en route. Another obvious, though important, distinction, i.e., *non*-equivalence, between these circumstances is that, over the course of its accelerated journey through a real universe such as ours, system B would find light from sources in its direction of acceleration to get increasingly hotter, while light from sources in the opposite direction would get correspondingly colder. This doesn't happen on a gravitating body.

What is important here is that effects that are found in the flat space of a uniformly accelerating system permit deducing similar effects near a gravitating body. In the latter case one cannot consistently ascribe the effects to kinematics because the system is stationary. Since the effects nevertheless exist, one is led to the conclusion that time is curved by massive bodies. The spirit of the equivalence principle is thus to deduce this curvature and to not worry too much about the differences between the kinematic and gravitational circumstances.

4 Modified kinematic equation

In this spirit, then, we note that what makes B's circumstance unlike life on a gravitating body is, in terms of (1), the *time* variable. The speed of light is approached with increasing time. We can replace the time variable and also the explicit acceleration a, with a stationary gravitational quantity. If not clearly analogous, this is at least mathematically permissible. Specifically, we replace (at) by $\sqrt{2GM/r}$. This gives

$$V_{\mathrm{S}} = \frac{\sqrt{\frac{2GM}{r}}}{\sqrt{1+\frac{2GM}{rc^2}}} = \sqrt{\frac{2GM}{r+\frac{2GM}{c^2}}}\,. \tag{3}$$

The only obvious physical meaning we could attach to this velocity is that it is (at least approximately) the relative speed of the surface at r, with respect to a geodesic trajectory "from infinity." Two things adding to its possible signifcance are: 1) For any physical values of M and r, it remains that $V_{\mathrm{S}} < c$. And 2) It leads to a maximum force, $F_{\mathrm{MAX}} = c^4/4G$, equal to the maximum force expounded upon by Schiller. Squaring both sides, we get

$$V_{\mathrm{S}}^2 = \frac{2GM}{r(1+\frac{2GM}{rc^2})} = \frac{2GM}{(r+\frac{2GM}{c^2})}\;. \tag{4}$$

The length in the denominator on the right side is the sum of the *coordinate radius*, r and the *gravitational radius*, $2GM/c^2$. Let's call this sum, $r_\gamma = r + 2GM/c^2$. This suggests that, whatever the coordinate radius may be, by virtue of its mass, a body possesses an additional spatial extent. This idea is consistent with general relativity. Spacetime curvature—or at least the spatial part of the curvature—can be described in similar terms. Motivated by the suggestiveness of (3), we diverge from standard general relativity, however, by treating $2GM/rc^2$ as a quantity to be added to rather than subtracted from unity. Thus we assume that the quantity $(1+2GM/rc^2)$ appearing in (4) may play a role similar to $(1-2GM/rc^2)^{-1}$ appearing in the Schwarzschild solution—applying to both space and (its inverse) to time. Since this is clearly a mathematical possibility, perhaps it is also a physical possibility. (See Appendix.) The likelihood that we are within the limits set by empirical observations follows from the smallness of the difference, for most cases, between the quantities:

$$\left[1-\frac{2GM}{rc^2}\right]^{-1} - \left[1+\frac{2GM}{rc^2}\right] = \frac{4G^2M^2}{r^2c^4(1-2GM/rc^2)}\,. \tag{5}$$

5 Maximum force

Since $(r+2GM/c^2)$ is the radial length whose inverse square root gives V_{S}, we assume that its inverse square gives the surface acceleration, g_{S}. Recalling the kinematic origins of this derivation, we expect g_{S} to be the acceleration given

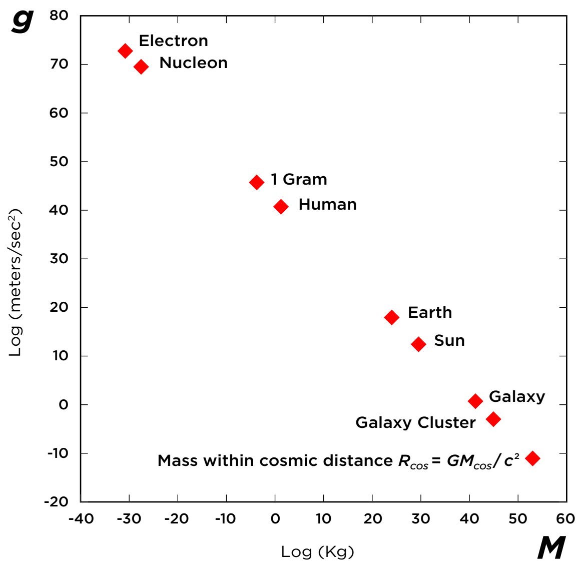

Fig. 2 The maximum acceleration, $c^4/4GM$, is given as the limit when $r \to 0$. The maximum force, $c^4/4G$, is gotten by multiplying this acceleration by the corresponding mass. Since massive bodies always have finite radii, these maxima are never attained in nature.

by an accelerometer at the body's surface. When we expand the square of the sum r_γ, we get

$$g_{\mathrm{S}} = \frac{GM}{r_\gamma^2} = \frac{GM}{(r + \frac{2GM}{c^2})^2} = \frac{GM}{r^2 + \frac{4rGM}{c^2} + \frac{4G^2M^2}{c^4}} \ . \tag{6}$$

In the limit, $r \to 0$, this leads to

$$g_{\mathrm{MAX}} = g_{\mathrm{S}(r\to 0)} = \frac{c^4}{4GM} \ . \tag{7}$$

In Figure 2 this acceleration is plotted against the full range of known masses in the universe. Multiplying (6) by any mass, M', will result in a force less than the maximum, $F_{\mathrm{MAX}} = c^4/4G$, because multiplication in the numerator also entails adding (at least) the distance $2GM'/c^2$ within the parentheses in the denominator. Thus, the maximum force is the product of the mass of any body, such as those in Figure 2, times the corresponding acceleration (7):

$$F_{\mathrm{MAX}} = \frac{c^4}{4G} = 3.0256 \times 10^{43} \text{ N} \ . \tag{8}$$

6 Singularity-free geometry

Let's now consider a few of the geometrical consequences. To reiterate what was said above in connection with (3), any M/r ratio is permissible. Since there can be no mass within zero volume, if $r = 0$, M is also zero, so we simply get zero velocity. But any other M/r leaves V_S, the "stationary surface velocity," finite and less than c. This implies that a gravitational horizon can never form. We can see this graphically by using the quantity $(1+2GM/rc^2)$ [from (4)] to make an embedding diagram and a plot which compares it to the Schwarzschild metric coefficient, $(1-2GM/rc^2)^{-1}$. These are shown in Figure 3. It is a curious fact that, though our initial equation involving kinematic acceleration gives rise to a horizon, our gravitational adaptation of this equation does not.

Since the form of the equations is the same, we naturally expect the new one to also exhibit a hyperbola for some physical circumstance. This comes about when we increase the M/r ratio by adding ever more shells of matter of the same density. In this case the slope of the asymptote is 2, as shown in Figure 4. In the figure the increasing size of the embedding parabolas represents mass increases in increments of $\sqrt{8}$. Astronomical sized spheres of constant density are unlikely or impossible in nature. But this idealization is useful for illustrating some interesting geometrical relationships.

Progression up the figure can be understood as follows. By adding ever more matter, both M and R increase. As the surface grows, so does the size of the embedding parabola. But the relation between M and R is such that, with each increase, R grows *proportionally* closer to the vertex of the parabola. Points on the *hyper*bola are the distances, $R_{\rm PT}$, gotten by multiplying the circumference, C, (measured with unshortened rods) by $\sqrt{1+2GM/Rc^2}/2\pi$. Since $C/2\pi = R$, we have

$$R_{\rm PT} = R\sqrt{1+2GM/Rc^2}\ . \tag{9}$$

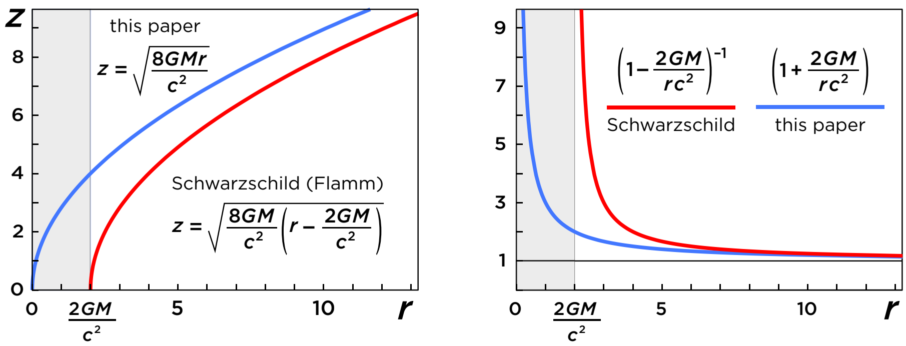

Fig. 3 In the strong field regime the curvature implied by the present approach deviates markedly from general relativity. Left — Profile of the usual Flamm paraboloid compared with the profile of the present model. Right — The Schwarzschild coefficient can become infinite at the horizon distance, $r = 2GM/c^2$. Whereas in the present approach, since it is impossible for a body's mass to be contained within zero volume ($r = 0$), spacetime is well-behaved from the body's surface to ∞. The interior is similarly well-behaved, as we will see later. When $2GM/c^2$ is small compared to r the curves in both graphs nearly coincide.

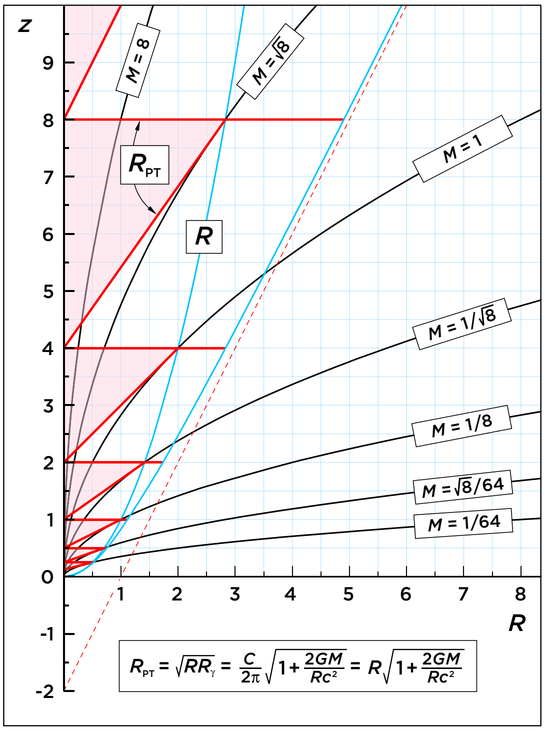

Fig. 4 Series of embedding parabolas corresponding to spheres of constant density, in steps of increasing mass, M ($\times\sqrt{8}$). The surfaces of these masses correspond to coordinate radii R (in steps $\times\sqrt{2}$). The latter points lie on the upwardly opening parabola as shown. The tangents from these points to the z-axis have lengths, $R_{\rm PT}$, that are equal to the horizontal lengths whose end points lie on the upwardly opening hyperbola. The relationship between $R_{\rm PT}$, the coordinate radius R, R_γ, and the circumference, C, are given by the equation. Note that the case ($R = 2$, $M = 1$, $z = 4$) corresponds to that of a Schwarzschild black hole. In the present model, it is just one unexceptional case in a continuous series.

Thus as $M/R \to \infty$, $z/R_{\rm PT} \to 2$. This may therefore be called *hyperbolic stationary motion*, which does not increase with time, but with increasing M/R.

7 Physical implications, Tangherlini's shell, and experimental test

7.1 Interior questions

If the only difference between general relativity and the present approach were that represented by (5), it would be extremely difficult to decide between them from observations. Of the other possible differences one can deduce, we'll address the most important one: What happens for the *interior*? For example, though we can build up a mass, as in connection with Figure 4, so that the *surface* remains well-behaved ($V_{\mathrm{S}} < c$), what happens *inside* the body? This question brings out the curious feature of general relativity that the spatial and temporal parts of the metric are affected in equal magnitude only *outside* massive bodies. In the exterior Schwarzschild solution the inverse of the temporal coefficient is everywhere equal to the spatial coefficient. As exemplified by the Schwarzschild *interior* solution, however, [4] within massive bodies the spatial coefficient goes back to unity at $r = 0$; at the center space is flat. By contrast, from the surface inward, the inverse of the temporal coefficient continues increasing to $r = 0$. A clock located there would be the slowest one in the field. This is shown graphically in Figure 5 for a rather strong field case, $R = 3GM/c^2$. The figure displays these temporal and spatial coefficients in terms of r, R and M from both Schwarzschild solutions.

It is important to emphasize that if we had empirical evidence proving the correctness of Figure 5 or its weak field counterparts, there would be little point in exploring alternatives. But we do not. We certainly have no *direct* evidence. The difference between the rate of a clock at the center and at the surface of any convenient-sized massive body would be much too small to measure. Indirect evidence would be convincing, but this too has not been gathered—although in this case it could be. Specifically, a consequence of the central clock having the slowest rate is that motion through the center—as in the common, idealized "hole through the center of Earth" problem—would yield harmonic oscillation from one end of the hole to the other. Though a laboratory test

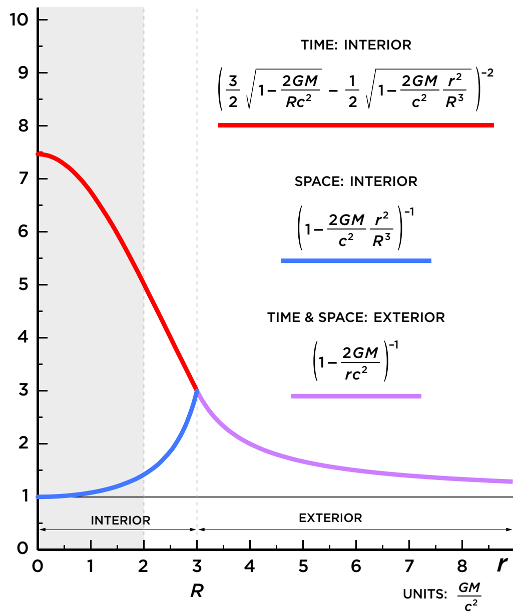

Fig. 5 Schwarzschild interior and exterior time and space coefficients. From the surface inward, clocks get slower and space gets flatter.

of this prediction is possible (using a modified Cavendish balance) it has not yet been carried out. Our trail thus far—which was initiated by modifying the proper acceleration equation—has led to the maximum force in nature, and now to some empirically unexplored territory. Hence, we continue. We'll return to the possibility of a laboratory test in §7.4.

7.2 Tangherlini's solution

This is not the first time that the interior question and alternative answers to it have been discussed. In a paper by Tangherlini titled, 'Postulational approach to Schwarzschild's exterior solution with application to a class of interior solutions,' [5] one of the latter (interior) solutions led to predictions similar to those suggested by the present inquiry. Perhaps not surprisingly, Tangherlini's postulates were similar to our starting point: assumed validity of the equivalence principle and the inverse square law of gravity. Tangherlini also began with a few auxiliary assumptions that differ from ours, so the results differ correspondingly. The case exhibiting the closest similarity is that of a spherical *shell* of matter. According to the usual application of general relativity, the spacetime properties found inside the shell would be essentially an enlarged version of what is found at $r = 0$ for the case of a uniformly dense sphere. That is, space would be flat throughout the interior and the rates of clocks throughout would be a uniform minimum.

What Tangherlini derived on the basis of his postulates, by contrast, is that clocks inside the shell have *maximum* rates, such that "the region inside the shell [may be regarded as] an inversion of the region 'outside matter at infinity'." Therefore, as Tangherlini also explains, an object dropped into the shell from its outer surface would not fall through to the inner cavity. Tangherlini acknowledges the "rather peculiar" nature of these features. Surely it is shocking to one's physical instinct to think Newton's predictions for this problem could be so grossly violated.

The reason for the peculiar behavior in Tangherlini's solution traces back to one of the auxiliary assumptions alluded to above. Within the boundary of

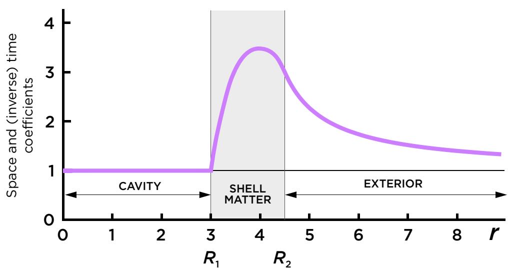

Fig. 6 Approximate relative magnitude of space and inverse time coefficients for Tangherlini's shell solution. Inside the cavity space is flat and clocks have maximum rates. An object dropped into a hole through the shell from the outer surface would never enter the cavity.

the sphere, the space curvature coefficient does not abruptly start going back to unity; rather it changes continuously so as to always remain the inverse of the temporal coefficient. Figure 6 is a graphic approximation of the coefficients (for space and the inverse for time) pertaining to Tangherlini's shell.

Although extremely unlikely to be physically true, this is of interest for the present exploration because it illustrates the possibility that the spatial and temporal coefficients need not diverge as they do in the usual treatment. Furthermore, it is of interest because Tangherlini's "postulational approach" resulted in an *exact* derivation of the exterior Schwarzschild solution. [6] Thus he demonstrated that it is possible to have a solution which matches the Newtonian approximation and general relativity for exterior fields, but which predicts novel, unexpected properties for interior fields.

In light of this, a third possibility presents itself. It is best illustrated not for a material *shell*, but for a uniformly dense sphere. Instead of having the spatial coefficient continue to increase along with the inverse temporal coefficient (as Tangherlini did) suppose it is the other way around; perhaps inside matter the inverse temporal coefficient decreases along with the spatial coefficient. If that were true, it would permit our shifted parabolic profile and metric coefficient, as in Figure 3; and it would permit the horizonless build-up of massive bodies, as in Figure 4. A comparison of these cases is illustrated in Figure 7. Figure 7a is a simplification of Figure 5; in 7b we have added the results of Tangherlini; and 7c represents the implications of the present approach. Justification for Figure 7c is found in an analogy intimated by Tangherlini's remark about the interior being an inversion of the exterior.

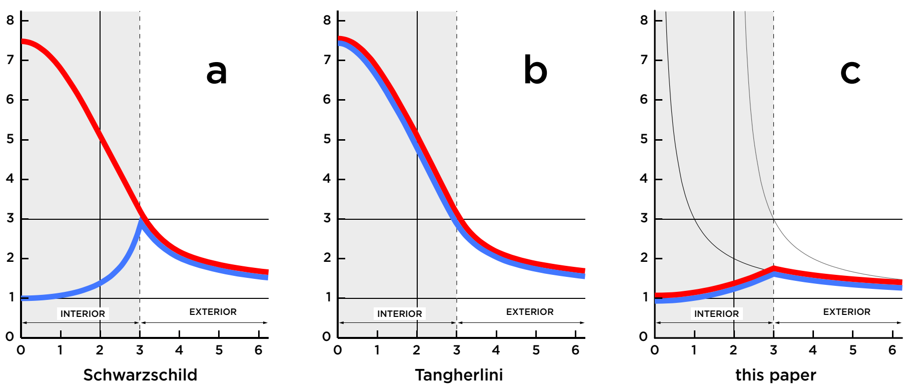

Fig. 7 Schematic comparison of the behavior of space (blue) and time (red) coefficients inside a uniformly dense sphere for three cases. a) Schwarzschild solution exhibits pronounced divergence. b) Unlike the behavior in Tangherlini's shell solution, the inverse of the time coefficient continues increasing to the center and the space coefficient coincides with it. c) Based on the present scheme, we expect the coefficients to coincide, but in the sense of decreasing to unity at the center. Also shown in (c) are the Schwarzschild curve tending to infinity as it approaches the Schwarzschild radius, and the same curve, based on this paper, offset toward the vertical axis by two units ($2GM/c^2$).

7.3 Rotation analogy

Reflecting on Tangherlini's remark, we note that at least one gravitational effect goes to zero at the center of a body, not because it is infinitely far away, but because of *symmetry.* The acceleration due to gravity goes to zero at the center because mass, which produces the effect, is distributed equally in every direction, so the effect is exactly neutralized. This is analogous to the phenomenon of rotation. A rotating body may possess lots of energy due to its motion; but there is none at the axis, which remains motionless.

It is widely known that, because of its properties that are analogous to gravitation, uniform rotation played almost as important a role as the equivalence principle in guiding Einstein to general relativity. On a rotating body there are actually *four* effects that are neutralized to zero at the center and increase with radial distance. First, there is inward acceleration, which is always accompanied by a tangential velocity—both of which vary directly as the distance. The other two effects are more subtle, but their inevitable existence, as deduced by Einstein, led him to conceive of non-Euclidean spacetime. These effects are the shortening of measuring rods and the slowing of clocks—both of which are caused by the velocity and both of which occur in equal magnitude.

At that time, Einstein was motivated by the idea that all motion should be relative, so he reasoned as follows: Since a non-moving gravitating body (and its field) can be described in terms of non-Euclidean geometry, a rotating body, which also exhibits properties of non-Euclidean geometry, invites the conception that it too can be regarded as being "at rest." The effects of *motion* were to be subsumed under the more fundamental idea (to Einstein) of spacetime curvature, i.e., gravitational field. [7]

I have summarized the story here to provide the context for taking the opposite approach. It is equally (if not more) logical, I propose, to reason as follows. First, acknowledge the absoluteness of rotational motion. Acknowledge all the resulting effects suggesting non-Euclidean geometry, especially, non-zero accelerometer readings, shortened rods and slow clocks. Then, upon finding or deducing these same physical effects on or near a gravitating body, hypothesize that they are due to the same cause: motion.

Based on this reasoning and intimated throughout this paper is the following set of propositions that we now make explicit: 1) *Gravitational spacetime curvature is caused by stationary motion.* 2) *Accelerometer readings and the variation of clock rates establish the existence of this motion.* And 3) *If* (1) *and* (2) *are correct, then gravitating bodies do not induce geodesic motion through their centers.* Though these propositions are clearly motivated by the rotation analogy, it is important to point out some key distinctions. Rotation is stationary motion *through* space; whereas gravitational stationary motion is motion *of* space. (A spherical array of accelerometers surrounding a body give a volumetric measurement of this motion; i.e., the product, $4\pi GM$.) A corollary of this distinction is a comparison of the respective symmetry properties. Rotation may be characterized as having essentially planar, or cylindrical symmetry. Whereas gravitational stationary motion is clearly of a volumetric, omnidirectional character, which implies a higher dimension of space.

The latter implication can be understood by comparing it to a more popular conception of higher dimensional space. Space dimensions beyond the third are often imagined as being "compactified" to an imperceptibly small size. By contrast, according to the present idea, we and other familiar bodies of matter are in the relatively "compactifed," seemingly three-dimensional state, and the higher dimension is an "expandification" thereof. The fourth dimension of space subsumes the first three; and the whole manifold is in a state of perpetual outward motion. Since the accelerations and velocities produced by gravitation are locally quite inhomogeneous, it is obvious that this kind of stationary motion cannot be conceived as motion through pre-existing three-dimensional space. Material bodies would rapidly disintegrate. To be consistent, the idea therefore requires a fourth space dimension to accommodate the inhomogeneous motion and to insure the integrity of material bodies. Though this conception stretches the imagination, it stems from a straightforward interpretation of accelerometer readings. And a simple experiment can reveal whether or not it is correct.

7.4 Laboratory test

The scope of this paper does not allow going into more detail about its higher dimensional implications. Rather, it should suffice to elucidate the basis for future work, to show the logical consistency by way of analogy and mathematical connection to well established foundations. But future work in this direction would clearly be pointless if we could prove with empirical evidence that the idea is contrary to fact. Therefore, a brief description of an apparatus for acquiring the needed fact is in order.

First, however, let's clarify our prediction. The above reasoning implies that, not just acceleration, but all four of the effects of spacetime curvature (including now velocity, rod shortening and clock slowing) are due only to the mass *within* a given radial distance. The gravitational effect of concentrically distributed matter beyond this distance is canceled by symmetry. By this reasoning, or by analogy with rotation, we predict a maximum clock rate at

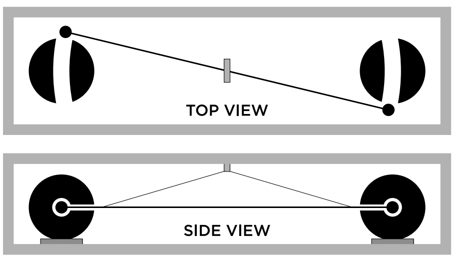

Fig. 8 Schematic of modified Cavendish balance for testing interior field motion predictions.

the center, which corresponds to the prediction that a test object dropped into an antipodal hole through a massive body will not pass the center. This can be tested by modifying a Cavendish balance so as to allow motion of the balance arm through the center of the large source masses. The basic idea is shown in Figure 8.

Though simple in principle, the experiment is not easy because of the stringent requirements of the arm's suspension system. Almost every previous Cavendish-like balance has involved a suspension system with a restoring force. The arm is allowed to move through only a short range of motion. Clearly, this will not work for our purpose. We need to allow a wide range of free motion. This becomes possible with either a fluid or magnetic suspension. In 1976 a measurement of Newton's constant was conducted by Faller and Koldewyn with a balance using a magnetic suspension. [8,9] Especially since electronic and magnetic technology have vastly improved since then, it is reasonable to expect that a similar apparatus could be adapted to the present purpose.

8 Interior acceleration, velocity and embedding diagram

8.1 Stationary acceleration

Our route to the maximum force has illuminated a new interpretation of the meaning of spacetime curvature, and a way to test whether or not this new interpretation is correct. Since this test involves the *interiors* of massive bodies, we now give the interior a fuller (though certainly far from complete) mathematical and graphical expression. Recalling that the acceleration due to gravity outside a spherical mass is given by $GM/(r + \frac{2GM}{c^2})^2$, adapting this equation for the simplest case of uniform density yields:

$$g_{\mathrm{SINT}} = \frac{4\pi}{3} \frac{G\rho\, r}{\left[1 + \frac{8\pi}{3}\frac{G\rho\, r^2}{c^2}\right]^2} \ . \tag{10}$$

For weak fields, g_{SINT} varies directly as the distance. But for densities and/or distances so large that $8\pi G\rho\, r^2/3c^2$ approaches or exceeds unity, a maximum acceleration is reached inside the body, as shown in Figure 9. The rise and fall of acceleration within a uniformly dense body only happens for systems with large m/r ratios, and is a manifestation of remaining below the maximum force, which is equivalent to the stationary velocity remaining less than c. No matter how large the density, the product of density, volume, and acceleration never reaches $c^4/4G$.

8.2 Stationary velocity

The interior stationary velocity equation follows from a similar adaptation of the exterior equation:

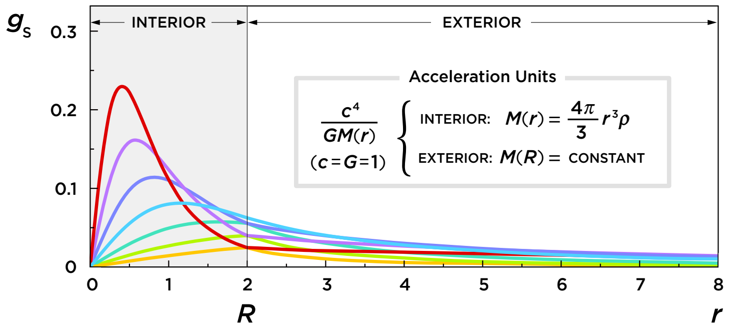

Fig. 9 Stationary acceleration inside and outside of a uniformly dense sphere. In this very strong field regime, the same acceleration at the surface of a body could be due to at least two different density distributions in the interior. The curves are color coded to facilitate comparison with Figures 10 and 11. In the units of the graph, the cyan curve corresponds to a density $\rho = 3/32\pi$, which makes $M(R) = 1$.

$$V_{\text{SINT}} = \frac{r\sqrt{\frac{8\pi}{3}G\rho}}{\sqrt{1 + \frac{8\pi}{3}\frac{G\rho\, r^2}{c^2}}} \,. \tag{11}$$

This has the same form as (1), of course. For weak fields the velocity varies directly as the distance, and as $8\pi G\rho\, r^2/3c^2$ approaches or exceeds unity, V_{SINT} flattens out as it approaches c. When the density changes abruptly, so does the stationary velocity. This is evident in Figure 10 at the surface radius, $r = 2$.

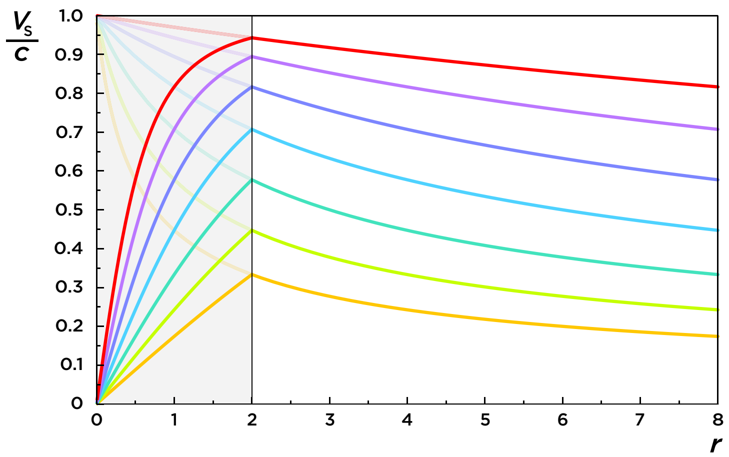

Fig. 10 Stationary velocity inside and outside of a uniformly dense sphere. For the highly idealized case of uniform density, the velocity varies directly as the radius for weak fields; but for very strong fields (as shown here) the variation is non-linear.

8.3 Embedding diagram

It is well known that the spatial part of the Schwarzschild exterior solution, as represented by Flamm's paraboloid, joins up with the interior solution as a "spherical cap." [10] By contrast, our interior field "cap" is a paraboloid of revolution. The cross-section is an upwardly opening parabola that joins smoothly to the exterior, given by

$$z = \frac{1}{4}r^2\sqrt{\frac{32\pi}{3}\frac{G\rho}{c^2}} + \frac{3}{4}R^2\sqrt{\frac{32\pi}{3}\frac{G\rho}{c^2}}\ . \tag{12}$$

The right hand term is a constant which defines the surface radius, R, and the vertex height on the z-axis. Figure 11 shows a series of different interior profiles all joined to one exterior profile. The colored curves correspond to the densities from Figures 9 and 10. In the latter figures each spherical body has a different coordinate mass and has the same coordinate surface radius, equal to that given by $R = 2GM/c^2$ for the cyan colored curve. Surface radii in Figure 11, on the other hand, vary so that the coordinate mass (active gravitational mass) of each sphere is the same. Note that this means the *proper* masses would have to be greater as they get smaller and denser. This is due to the greater spatial curvature in such compact fields. It bears repeating that, for the present model, this embedding diagram indicates both spatial and temporal curvature.

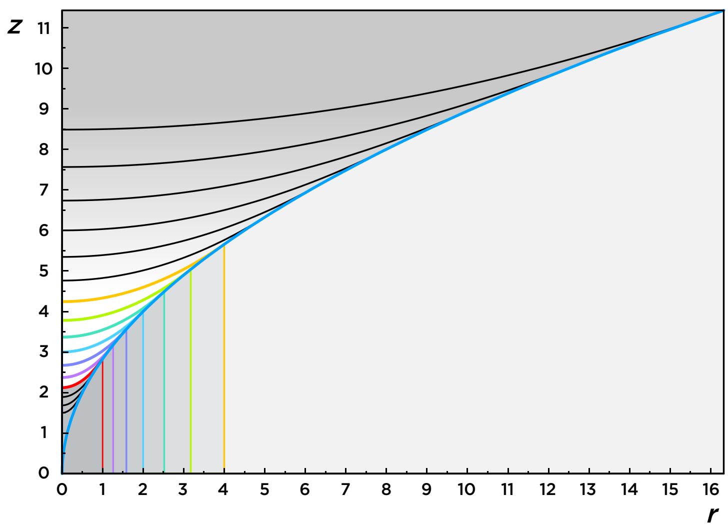

Fig. 11 Nested interior parabolas. Projected length segments of the parabolas onto corresponding length segments on the r-axis represent both rod length and clock rate ratios. (Coordinate lengths are shorter and coordinate clocks tick faster.) The colored curves correspond to density variations as in Figures 9 and 10. In this figure the active gravitational mass is the same for each case, as represented by the solitary exterior parabola.

9 Rethinking motion

Having no horizons or singularities, the geometry of the present scheme is, in at least this respect, simpler than general relativity. Also the conceptual basis is simpler. In general relativity, a positive accelerometer reading is equivocal as to whether it indicates motion or not. Of course, it indicates "acceleration with respect to a local geodesic." But the body on which it rests is typically deemed to be *static*. The prevailing understanding of motion thus involves scrambling up the terms so that it is not unusual to find oxymoronic expressions as "acceleration of a particle at rest." [11, 12] This is all due to our heritage of having evolved on the surface of a huge spherical mass. In spite of the readings on co-moving accelerometers, most things around us *appear* not to move, so we think we too are at rest. Our visual impressions dominate our thinking, even as our tactile experience (flattened undersides) indicates that we accelerate, as though matter were an inexhaustible source of perpetual propulsion. Contrary to this experience, the laws of physics have evolved to reflect our visual impression of staticness. Of course these laws have proven to be remarkably successful for an impressively wide range of circumstances.

But there is a huge gap—not because it is inaccessible, but because we just haven't thought about looking there. We don't know how test objects fall near the centers of gravitating bodies. The laws give clear predictions. But these particular predictions have not been tested. If in fact gravity is a force of attraction, if spacetime curvature causes falling bodies to move inwardly, then the predictions will be verified when they are finally tested. But if accelerometer readings are actually not equivocal, if they really indicate the state of motion of matter and space, then how are we to conceive that a falling test object doesn't pass the center?

We again come to the distinction between motion *through* pre-existing space and the motion *of* space. This corresponds to the distinction between thinking spacetime curvature causes inward motion versus the present idea that outward motion is the cause of spacetime curvature. Attractive forces cause motion through space. If true for gravity, then the test object would oscillate through the antipodal hole. By the present view, what happens instead is that the space that once separated the test object from the center—when the object begins to fall—moves outwardly past it. At first this results in an increasing relative speed. But as the amount of intervening space diminishes and as the amount of matter responsible for the separation also diminishes (because the falling body is increasingly below the surface of the larger body) so does the rate at which it moves past the falling body.

It must be borne in mind that this description rests on the idea that differences in accelerometer readings and differences in clock rates correspond to physically real differences in acceleration and velocity. An object rigidly attached to the gravitating body (beyond $r = 0$) is thus initially endowed with both a stationary outward acceleration and a stationary outward velocity. Accordingly, the speed of the dropped object immediately after release does not fall from zero to increasingly negative values. Rather, its initially *positive* value *remains positive* and decreases to zero as it gets closer to the center. The standard of "rest" is thus not the seemingly static body, but the trajectory of a test object falling radially from infinity ("maximal geodesic"). If this view is correct then any test object whose apparent motion is due only to the gravitating mass and which falls radially inside the gravitating mass, will not quite reach the center.

10 Deeper implications: inertial mass and the direction of time

This conception of motion conflicts with standard physics in many ways. To make sense it would require the existence of a fourth dimension of space, as mentioned in §7.3. If the laboratory test described in §7.4 should nevertheless support our prediction, then at least two persistent enigmas in standard physics could begin to be understood. If gravity is correctly conceived as a process of stationary outward motion, then the resistance posed to linear acceleration (inertia) could be understood as being due to this same process. The greater the magnitude of *omnidirectional* motion (of space) the more difficult it is to change the state of *linear* motion (through space).

Finally, we have the potential to shed light on the time asymmetry problem. This is easily understood in terms of the proposed experiment. If the Newtonian oscillation prediction were to be confirmed, then an idealized video of the motion would look exactly the same whether it was played forward or backward. Whereas, if the non-oscillation prediction were to be confirmed, then one direction could be clearly distinguished from the other. If the test object appears to move upward and reach the surface, the video is being played backward because this cannot happen in nature (without an extraneous source of propulsion). If the non-oscillation prediction were confirmed then time asymmetry could be succinctly characterized as follows. Time only increases because space and matter also only increase. The failure to solve the problem of time's arrow has been due to the failure to discover space's arrow and matter's arrow.

11 Conclusion

Schiller has argued that the maximum force principle and general relativity are equivalent, that they can each be derived from the other. In the present paper we have shown that exactly the same maximum force follows from a simple application of the equivalence principle, the limiting speed of light and the inverse square law of gravity. Our first equation, representing the speed acquired from constant proper acceleration, involves a horizon, a communication barrier with respect to the accelerating observer and an observer remaining at rest in the original inertial system. Motivated by the equivalence principle, we have exchanged the time variable and the acceleration in this equation with stationary gravitational quantities (3).

By the reasoning elucidated in the later sections we have come to see that the key difference in the meaning of these equations is that (1) represents motion through space, whereas (3) represents motion of space. In both cases the speed of light is an unreachable limit. But in the latter, gravitational case, this does not lead to a horizon. There is no communication barrier. Also there are no singularities. These features are all conducive to simple geometrical expression.

Of great importance for the new approach is that the magnitude of spacetime curvature for exterior fields is nearly the same as that for general relativity, except in the strong field regime. Even more important is that it would be relatively easy to test the emerging model with a laboratory experiment. If the results of the modified Cavendish experiment should confirm the standard prediction, then our derivation of the maximum force would be proven to be an inconsequential coincidence. The novel conceptions of matter, space, time, and gravitation presented in this paper should then all be discarded. But perhaps the experiment will support these conceptions. The highest priority is to find out, one way or the other.

12 Appendix

A reviewer has suggested that the step from Eq (1) to Eq (3) may be mistaken, that it fails to account for discussions such as that of Hamilton [13] or Desloge and Philpott,[14,15] and therefore the implications concerning horizons and deviations from general relativity may also be mistaken. The cited literature concerns uniformly accelerated reference frames. It concerns the question of exactly how such frames should be defined, consequences for observers therein and some discussion about the connection to general relativity. The reviewer pointed out the standard result that, "the existence of horizons for accelerated observers in flat spacetime is known to be equivalent to the existence of a horizon for an observer of a black hole located at spatial infinity." This view is thus similar to that of Schiller, which may be correct. I admit that the approach taken in the present paper might not be physically true.

But the work of Hamilton, Desloge and Philpott (et al) does not invalidate the logic of the present approach. Eq (3)—which simply replaces a kinematic quantity with a gravitational quantity—is a logically possible step from Eq (1). The step is non-standard, but it does not conflict with any known physical facts. The equivalence principle, being a heuristic device for relating the effects of uniform acceleration with spacetime curvature, motivated the step. But even independent of the equivalence principle, Eq (3) is clearly mathematically correct. Only Nature can tell us whether or not it is also physically correct. The fact that the equation so transparently leads to the same maximum force discussed by Schiller suggests that it is worthy of further exploration.

Following the trail where it leads, we directly come to the question of spacetime curvature *inside* massive bodies—a domain for which empirical evidence is clearly lacking. The degree of spacetime curvature implied by the present approach, as per Eq (5), is nearly the same as that of general relativity for (weak) *exterior* fields. But it deviates markedly for *interior* fields. Which approach holds up better for the interior is an empirical question whose answer remains to be discovered. Therefore, I have emphasized the importance of testing the validity of the derivation of maximum force and the other analogies I've proposed with the simple laboratory experiment discussed in §7.4.

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