The arithmetic of binary representations of even positive integer 2n and its application to the solution of the Goldbach's binary problem

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Abstract

One of causes why Goldbach's binary problem was unsolved over a long period is that binary representations of even integer 2n (BR2n) in the view of a sum of two odd primes(VSTOP) are considered separately from other BR2n. By purpose of this work is research of connections between different types of BR2n. For realization of this purpose by author was developed the "Arithmetic of binary representations of even positive integer 2n" (ABR2n). In ABR2n are defined four types BR2n. As shown in ABR2n all types BR2n are connected with each other by relations which represent distribution of prime and composite positive integers less than 2n between them. On the basis of this relations (axioms ABR2n) are deduced formulas for computation of the number of BR2n (NBR2n) for each types. In ABR2n also is defined and computed Average value of the number of binary sums are formed from odd prime and composite positive integers $< 2n$ (AVNBS). Separately AVNBS for prime and AVNBS for composite positive integers. We also deduced formulas for computation of deviation NBR2n from AVNBS. It was shown that if n go to infinity then NBR2n go to AVNBS that permit to apply formulas for AVNBS to computation of NBR2n. At the end is produced the proof of the Goldbach's binary problem with help of ABR2n. For it apply method of a proof by contradiction in which we make an assumption that for any 2n not exist BR2n in the VSTOP then make computations at this conditions then we come to contradiction. Hence our assumption is false and forall $2n > 2$ exist BR2n in the VSTOP.

1 Introduction

. On 7 June, 1742,the Prussian mathematician Christian Goldbach wrote a letter to Leonhard Euler in which he proposed the following conjecture: Every integer greater than 2 can be written as the sum of three primes. He considered 1 to be a prime number. In mathematics, a prime number is a natural number that has exactly two natural number divisors, which are 1 and the prime... , a convention subsequently abandoned. A modern version of Goldbach's original conjecture is: Every integer greater than 5 can be written as the sum of three primes. Euler, becoming interested in the problem, replied by noting that this conjecture is equivalent with another version: Every even integer greater than 2 can be written as the sum of two primes. Euler's version is the form in which the conjecture is usually expressed today. It is also known as the strong,even, or binary Goldbach's conjecture. For small values of n,the strong Goldbach conjecture (and hence the weak Goldbach conjecture) can be verified directly. For instance, Nils Pipping in 1938 laboriously verified the conjecture up to $n \leq 10^5$ [1] With the advent of computers, many more values of n have been checked; T. Oliveira e Silva is running a distributed computer search that h verified the conjecture for $n \leq 4 \cdot 10^{18}$ (and double-checked up to $3 \cdot 10^{17}$). [2] Statistical considerations which focus on the probabilistic distribution of prime numbers present informal evidence in favour of the conjecture (in both the weak and strong forms) for sufficiently large integers: the greater the integer, the more ways there are available for that number to be represented as the sum of two or three other numbers, and the more "likely" it becomes that at least one of these representations consists entirely of primes. Number of ways to write an even number n as the sum of two primes (4 - 1,000,000) A very crude version of the heuristic probabilistic argument (for the strong form of the Goldbach conjecture) is as follows. The prime number theorem asserts that an integer m selected at random has roughly a $1/lnm$ chance of being prime. Thus if n is a large even integer and m is a number between 3 and n/2, then one might expect the probability of m and n - m simultaneously being prime to be $1/\big[\ln m \, \ln(n-m)\big]$. If one pursues this heuristic, one might expect the total number of ways to write a large even integer n as the sum of two odd primes to be roughly $\sum_{m=3}^{n/2} \frac{1}{\ln m} \frac{1}{\ln(n-m)} \approx \frac{n}{2\ln^2 n}$. Since this quantity goes to infinity as n increases, we expect that every large even integer has not just one representation as the sum of two primes, but in fact has very many such representations. This heuristic argument is actually somewhat inaccurate, because it assumes that the events of m and n m being prime are statistically independent of each other. For instance, if m is odd then n m is also odd, and if m is even, then n m is even, a non-trivial relation because (besides 2) only odd numbers can be prime. Similarly, if n is divisible by 3, and m was already a prime distinct from 3, then n m would also be coprime to 3 and thus be slightly more likely to be prime than a general number. Pursuing this type of analysis more carefully, Hardy and Littlewood in 1923 conjectured (as part of their famous Hardyand Littlewood prime tuple conjecture) that for any fixed $c2$, the number of representations of a large integer n as the sum of c primes $n = p_1 + \cdots + p_c with p_1 \leq \cdots \leq p_c$ should be asymptotically equal to $\left(\prod_p \frac{p\gamma_{c,p}(n)}{(p-1)^c}\right) \int_{2 \leq x_1 \leq \cdots \leq x_c : x_1 + \cdots + x_c = n} \frac{dx_1 \cdots dx_{c-1}}{\ln x_1 \cdots \ln x_c}$ where the product is over all primes p, and $\gamma_{c,p}(n)$ is the number of solutions to the equation $n = q_1 + \cdots + q_c \mod p$ in modular arithmetic, subject to the constraints $q_1, \ldots, q_c \neq 0 \mod p$. This formula has been rigorously proven to be asymptotically valid for $c > 3$ from the work of Vinogradov, but is still only a conjecture when $c = 2$. In the latter case, the above formula simplifies to 0 when n is odd, and to 2 $\Pi_2 \left(\prod_{p|n;p\geq 3} \frac{p-1}{p-2}\right) \int_2^n \frac{dx}{\ln^2 x} \approx 2\Pi_2 \left(\prod_{p|n;p\geq 3} \frac{p-1}{p-2}\right) \frac{n}{\ln^2 n}$ when n is even, where Π_2 is the twin prime constant $\Pi_2 := \prod_{p\geq 3} \left(1 - \frac{1}{(p-1)^2}\right) = 0.6601618158\ldots$. This is sometimes known as the extended Goldbach conjecture. The strong Goldbach conjecture is in fact very similar to the twin prime conjecture, and the two conjectures are believed to be of roughly comparable difficulty. In 2013, Provatidis et al. reported on a "Rule of Thumb" lower bound for the number of representations. [3] The strong Goldbach conjecture is much more difficult. Using Vinogradov's method, Chudakov, [4] van der Corput, [5] and Estermann [6] showed that almost all even numbers can be written as the sum of two primes (in the sense that the fraction of even numbers which can be so written tends towards 1). In 1930, Lev Schnirelmann proved that every even number $n > 4$ can be written as the sum of at most 20 primes. This result was subsequently enhanced by many authors; currently, the best known result is due to Olivier Ramar, who in 1995 showed that every even number $n > 4$ is in fact the sum of at most six primes. In fact, resolving the weak Goldbach conjecture will also directly imply that every even number $n > 4$ is the sum of at most four primes. [7] Chen Jingrun showed in 1973 using the methods of sieve theory that every sufficiently large even number can be written as the sum of either two primes, or a prime and a semiprime (the product of two primes) [8] e.g., $100 = 23 + 7 \cdot 11$. See Chen's theorem. In 1975, Hugh Montgomery and Robert Charles Vaughan showed that "most" even numbers were expressible as the sum of two primes. More precisely, they

showed that there existed positive constants c and C such that for all sufficiently large numbers N, every even number less than N is the sum of two primes, with at most CN^{1-c} exceptions. In particular, the set of even integers which are not the sum of two primes has density zero. Linnik proved in 1951 the existence of a constant K such that every sufficiently large even number is the sum of two primes and at most K powers of 2. Roger Heath-Brown and Jan-Christoph Schlage-Puchta in 2002 found that K = 13 works. [9] This was improved to K=8 by Pintz and Ruzsa in 2003. [10] One can pose similar questions when primes are replaced by other special sets of numbers, such as the squares. It was proven by Lagrange that every positive integer is the sum of four squares. See Waring's problem and the related Waring Goldbach problem on sums of powers of primes. Hardy and Littlewood listed as their Conjecture I: "Every large odd number ($n > 5$) is the sum of a prime and the double of a prime." Mathematics Magazine, 66.1 (1993): 45-47. This conjecture is known as Lemoine's conjecture (also called Levy's conjecture). The Goldbach conjecture for practical numbers, a prime-like sequence of integers, was stated by Margenstern in 1984, [11] and proved by Melfi in 1996: every even number is a sum of two practical numbers.
But in spite of these large energies Goldbach conjecture was unsolved up to date.

2 Method

Considering great number proposed solutions of this problem it can make conclusion that analytical methods exhausted their resources since none of them don't bring to purpose. And for solution of Goldbach' binary problem it needs to develop the new method . Considered all representations of even integer 2n , and not only binary representations in the view of a sum of two odd primes (BRVSTOP). It can make conclusion that to BRVSOTP no approach but it is for other binary representations of 2n for example in the view of a sum of two odd composite integers (BRVSTOCI) or in the view of a sum of two odd integers one is composite other is prime (BRVSTOICP). So came idea to develop the arithmetic of binary representations of even positive integer 2n (ABR2n). In which it was found the connection between types of binary representation of even integer 2n (BR2n). Which underlie of axioms of ABR2n. In ABR2n the binary representations of even positive integer 2n are defined as bijective mappings : $f : X \to Y$ y = 2n - x; or x+y = 2n; it coincidences with the binary representations of even positive integer 2n : x+y = 2n . It permits to compute all the base quantities of ABR2n with help of the set theory. Thus the New method as follows : To ABR2n is used standard procedure : the proof of contradiction i.e. don't break generality we make assumption that for any valoe of 2n it is not the BVRSTOP . Next we compute all corollaries from it into ABR2n . Specifically from it appears two identities which not were the early. From these identities it follows that to the each number of noncomposite ("1" + primes) $< 2n$ (p) correspond to infinite set of 2n that contradict by proposition proved in ABR2n which assert that each p correspond to finite set of 2n included between neighboring values of p . Next it follows standard conclusion that our assumption that the BVRSTOP are not for some value is false and it is true that the BVRSTOP are forall $2n > 2$. Hence $\forall\ 2n > 2$ exists at least one representation of even positive integer $2n$ in the view of a sum of two odd prime positive integers or "1" and odd prime positive integer.

3 The arithmetic of binary representations of even positive integer 2n

3.1 General conception

Definition 3.1 *The binary representations of even positive integer 2n in ABR2n are defined as bijective mappings :*

$$f : X \to Y \tag{1}$$

$$y = 2n - x \tag{2}$$

Where:

$$X = \{x | x \in N, 1 \leq x < n\} \tag{3}$$

$$Y = \{y | y \in N, n < y < 2n\} \tag{4}$$

n- positive integer .

$$|X| = |Y| \tag{5}$$

Definition 3.2 *The set of binary representations of even positive integer 2n (SBR2n) are defined as follows :*
SBR2n $= \{1 + (2n - 1) = 2n$ *;*
$2 + (2n - 2) = 2n$ *;*
...
$n - 1 + (2n - (n - 1)) = 2n\}$
The last is got by represent (2)
in the view of $x + y = 2n$.

Definition 3.3 *The set XUY consist of: even positive integers , odd positive integers inclusive odd composite positive integers and noncomposite positive integers (primes and "1").*

Remark 3.1 *Since n is not mapped into Y and it is mapped into itself (automorphism) therefore mapping* $n \to n$ *and corresponding binary representation* $n+n = 2n$ *do not enter into SBR2n which is formed only from bijective mappings. By this reason n do not enter into XUY. But it is not denoted that as a result of the exception of binary representations: 2+2=4 and 3+3=6 from SBR2n, 4 and 6 have not binary representations in the view of a sum of two noncomposite positive integers. But it is not the case.In SBR2n is in existence the binary representations 1+3=4 and 1+5 = 6. It is significant "1" with primes are related in ABR2n to noncomposite positive integers. At that special status of "1" in N is ignored in ABR2n.*

Definition 3.4 s_e *- the number of even composite positive integers into SBR2n* $< 2n$ *.*
s_e *- positive integer* $> 0 \ \forall\ 2n > 6$
$s_e = 0 \ \ \forall\ 2 < 2n < 8.$

Definition 3.5 s_o *- the number of odd composite positive integers into SBR2n* $< 2n$ *.*
s_o *- positive integer* $> 0 \ \forall\ 2n > 8$
$s_o = 0 \ \ \forall\ 2 < 2n < 10.$

Definition 3.6 *p- the number of odd noncomposite integers (primes and "1") into SBR2n* $< 2n$ *.*
$p - positive\ integer \ > \ 0 \ \forall\ 2n \ > \ 2.$

Corollary 3.1 *By reason of exclusion n from the set XUY if n is even then n or* s_e *are decremented by two (since it is excluded* $n + n = 2n$*):*
$n* = n - 2; s_e* = s_e - 2.$

Corollary 3.2 *By reason of exclusion n from the set X U Y If n is odd then n or* s_o *are decremented by two:*
$n* = n - 2; s_o* = s_o - 2.$

Corollary 3.3 *By reason of exclusion n from the set X U Y If n is odd prime then n or p are decremented by two:*
$n* = n - 2; p* = p - 2.$

Remark 3.2 *It needs to say that corollaries 3.1 , 3.2 , 3.3 are took into account automatically at forming SBR2n since it makes until appearance of representation in the view of* $n + n = 2n$ *which throw-off . Thus if* n *is odd or odd prime then* n, s_o *automatically are decremented by two at the direct computation them into SBR2n. If* n *is even then* n, s_o, p *are not decremented since* s_o, p *by definitions related to odd composite and noncomposite integers.*

Remark 3.3 *In the equations which include* n *is even* n, s_o, p *are not decremented (see rem 3.2).*
At the computations which related to s_e n *is decremented by two.*

Remark 3.4 *In the equations which include* n *is odd or odd prime then it is decremented by one since* n *enters in the equation with value which is required the correction.*

Proposition 3.1 $\forall\ n > 1$ *always is fulfilled the condition* $|X| = |Y|$.

Proof 3.1 *Taking into account that* $2n$ *do not enter into the set* XUY *and* n *is excluded from the set* XUY *(see remark 3.1)*
then we have :

$$|XUY| = 2n - 2 = 2(n - 1) \tag{6}$$

Hence $|XUY|$ *is even for any* n *and thus* $\forall\ n > 1$ *always is fulfilled the condition* $|X| = |Y|$. □

3.2 The types of binary representations even positive integer 2n

In depend of that are x,y in the view of $x + y = 2n$ prime or composite it can be four types binary representations of even positive integer $2n$:

Definition 3.7 *There is Type "H" if x-odd prime positive integer or"1" and y- odd prime positive integer.*
$|H|$*- the number of binary representations of Type "H".*
$|H|$*- positive integer* $> 0\ \forall\ 2n > 2$ *(see theorem (10.1.)).*

Definition 3.8 *There is Type "Q" if x-odd composite positive integer; and y-odd composite positive integer .*
$|Q|$*- the number of binary representations of Type "Q".*
$|Q|$*- positive integer* $> 0\ \forall\ 2n > 22$
exepting $2n = 26; 28; 32; 38;$ *in which* $|Q| = 0$
(see numerical solution $|Q| = 0$ *subsection (12.2) .*
$|Q| = 0\ \ \forall\ 2 < 2n < 24$.

Definition 3.9 *There is Type "L" if x- odd prime positive integer or"1" and y-odd composite positive integer or x-odd composite positive integer and y- odd prime positive integer.*
$|L|$ *- the number of binary representations of Type "L".*
$|L|$*- positive integer* $> 0 \ \forall \ 2n > 8$.
$|L| = 0 \ \forall \ 2 \ < \ 2n \ < \ 10$.

Definition 3.10 *There is Type "E" if x-even positive integer and y-even positive integer .*
$|E|$*- the number of binary representations of Type"E".*
$|E|$*- positive integer* $> 0 \ \forall \ 2n > 4$.

3.3 The axioms of ABR2n

Axiom 3.1 *The number of the binary representations type H (NBRH) is connected with The number of the binary representations type L (NBRL) as follows:*

$$2|H| + |L| = p \tag{7}$$

$\forall \ 2n > 2$.
The equation (7) asserts that odd noncomposite positive integers less than $2n$ are allotted to types "H"," L" in compliance with balance (7).

Remark 3.5 *From equations (7 and 8) it follows :*
$|H| = (p - |L|)/2; \ |Q| = (s_0 - |L|)/2$.
In these equations is division by '2'. From this it can conclude that equations are correct only for n is even. But it is not the case. Below we show that in ABR2n all equations where is division by '2' It always is possibility for any n's and p's parity.

Proposition 3.2 *The equation (7) is true for any n's and p's parity.*

Proof 3.2 *For n is even p is even*
By equation (7) we get: $|H| = (p - |L|)/2$.
By equation (26) $|L| = n - p - 2|Q|$ *and* $|L|$ *is even for n is even p is even then* $(p - |L|)$ *is even .*
Hence the division by"2" is possibility then $|H|$ *is integer .*
Thus equation (7) is true for n is even p is even.
Next for n is even ; p is odd
By equation (7) we get: $|H| = (p - |L|)/2$.

By equation (26) $|L| = n - p - 2|Q|$ *and* $|L|$ *is odd*
for n is even and p is odd then $(p - |L|)$ *is even .*
Hence the division by"2" is possibility then $|H|$
is integer .
Thus equation (7) is true for n is even p is odd.

Next for n is odd or odd prime p is even
By equation (7) we get: $|H| = (p - |L|)/2$ *.*
By equation (27) $|L| = n - p - 1 - 2|Q|$ *and* $|L|$ *is even*
for n is odd or odd prime and p is even then $(p - |L|)$*is even .*
Hence the division by"2" is possibility then $|H|$
is integer .
Thus equation (7) is true for n is odd or odd prime p is even.

Next for n is odd or odd prime p is odd
By equation (7) we get: $|H| = (p - |L|)/2$ *.*
By equation (27) $|L| = n - p - 1 - 2|Q|$ *and* $|L|$ *is odd*
for n is odd or odd prime and p is odd then $(p - |L|)$*is even .*
Hence the division by"2" is possibility then $|H|$
is integer .
Thus equation (7) is true for n is odd or odd prime p is odd.

Finally equation (7) is true for any n's and p's parity. □

Axiom 3.2 *The number of the binary representations Type Q (NBRQ) is connected with the number of the binary representations Type "L" (NBRL) as follows:*

$$2|Q| + |L| = s_0 \tag{8}$$

$\forall\ 2n > 2$.
The expression(8) asserts that odd composite positive integers less than $2n$ *are allotted to types "Q","L" in compliance with balance (8).*

Proposition 3.3 *The equation (8) is true for any n's and p's parity.*

Proof 3.3 *For n is even p is even*
By equation (8) we get: $|Q| = (s_0 - |L|)/2$ *.*
By equation (7) $|L| = p - 2|H|$ *and* $|L|$ *is even*
for n is even and p is even .
By equation (17) $s_o = n - p$ *is even then* $(s_0 - |L|)$ *is even .*

Hence the division by"2" is possibility then $|Q|$
is integer .
Thus equation (8) is true for n is even ; p is even.

Next for n is even p is odd
By equation (8) we get: $|Q| = (s_0 - |L|)/2$ *.*
By equation (7) $|L| = p - 2|H|$ *and*$|L|$ *is odd*
for n is even p is odd .
By equation (17) $s_0 = n - p$ *is odd then* $(s_0 - |L|)$ *is even .*
Hence the division by"2" is possibility then $|Q|$
is integer
Thus equation (8) is true for n is even p is odd

Next for n is odd or odd prime p is even
By equation (8) we get: $|Q| = (s_0 - |L|)/2$ *.*
By equation (7) $|L| = p - 2|H|$ *and* $|L|$ *is even*
for p is even .
By equation (18) $s_0 = n - p - 1$ *is even then* $(s_0 - |L|)$ *is even .*
Hence the division by"2" is possibility then $|Q|$
is integer .
Thus equation (8) is true for n is odd or odd prime p is even.

Next for n is odd or odd prime p is odd
By equation (8) we get: $|Q| = (s_0 - |L|)/2$ *.*
By equation (7) $|L| = p - 2|H|$ *and* $|L|$ *is odd*
for p is odd .
By equation (18) $s_0 = n - p - 1$ *is odd then* $(s_0 - |L|)$ *is even .*
Hence the division by"2" is possibility then $|Q|$
is integer .
Thus equation (8) is true for n is odd or odd prime p is odd.
Finally equation (8) is true for any n's and p's parity. □

Definition 3.11 *G - the general number of binary representations in ABR2n.*
G - positive integer $\forall\ 2n > 2$ *.*

Axiom 3.3

$$|Q| + |L| + |H| + |E| = G \tag{9}$$

Definition 3.12 *F-the general number of binary representations with odd positive integers .*
$F -$ *positive integer* $\forall\ 2n > 2$ *.*

Axiom 3.4

$$|Q| + |L| + |H| = F \tag{10}$$

3.4 The computation of $G, F, S_o, p, |Q|, |L|, |H|, |E|$

3.4.1 The computation of G

Proposition 3.4

$$G = n - 1 \tag{11}$$

$\forall\ n\ >\ 1$

Proof 3.4 *The general number of elements in the set* XUY *by equation (6) equals* $2(n-1)$. *Taking into account that in forming of each binary representation participate with two elements from the set* XUY *then we have:* $G = 1/2(2(n-1) = n-1\ \forall\ n\ >\ 1$. □

3.4.2 The computation of $|E|$

Proposition 3.5

$$|E| = [(n-1)/2] \tag{12}$$

$\forall\ n\ >\ 1$

Proof 3.5 *By definition (3.3) by elements of the set* XUY
are even integer which consist of half of all integers
Then the number of even integers in the set XUY *equals:*
$1/2|XUY| = 1/2(2(n-1)) = n-1$.
Taking into account that in forming of each binary representation
participate by two elements from the set XUY *we have:* $|E| = (n-1)/2$.
Taking into account that for n *- even* $|E|$ *is not integer*
that breaks the status of $|E|$ *(see definition (3.10)*
then $|E| = [(n-1)/2]$ *is aliquot of* $(n-1)/2$
then we get : $|E| = [(n-1)/2]\ \forall\ n\ >\ 1$ □

Proposition 3.6

$$|E| = n/2 - 1 \tag{13}$$

$for\ n\ is\ even\ \ n\ >\ 1$

Proof 3.6 *For* $n = 2i$ *where* $i \in N$ *by equation (12) we get:* $|E| = [(2i-1)/2] = [i-1/2] = [(i-1)+1/2] = i-1$
returning to n *finally we get:* $|E| = n/2 - 1\ \ n\ >\ 1$. □

Proposition 3.7

$$|E| = (n-1)/2 \quad (14)$$

for n is odd or odd prime $n > 2$

Proof 3.7 *For n = 2i + 1 where* $i \in N$ *by equation (12) we get:* $|E| = [(2i+1-1)/2] = [i] = i$
returning to n finally we get: $|E| = (n-1)/2$ $n > 2$. □

3.4.3 The computation of F

Proposition 3.8

$$F = n/2 \quad (15)$$

for n is even $n > 1$

Proof 3.8 *Subtracting equation (10) from equation (9) we get:* $|E| = G - F$ *whence* $F = G - |E|$ *. Taking into account equations (11), (12) we get:*
$F = (n-1) - [(n-1)/2]$ *for n = 2i we get:*
$F = 2i - 1 - [(2i-1)/2] = 2i - 1 - [i - 1/2] =$
$= 2i - 1 - [i - 1 + 1/2] = 2i - 1 - (i-1) = i$
returning to n finally we get: $F = n/2$ $n > 1$

Remark 3.6 *Since n is even then the division by"2"*
is possibility. Thus F is integer and equation (15) is true for n is even.
□

Proposition 3.9

$$F = (n-1)/2 \quad (16)$$

for n is odd or odd prime $n > 2$

Proof 3.9 *Subtracting equation (10) from equation (9) we get:* $|E| = G - F$ *whence* $F = G - |E|$ *. Taking into account equations (11), (12) we get:*
$F = (n-1) - [(n-1)/2]$ *for n = 2i + 1 we get:*
F = 2i - i = i returning to n finally we get: F = (n - 1)/2 $n > 2$. □

Remark 3.7 *Since n is odd or odd prime then (n - 1) is even . Hence the division by"2"*
is possibility.Thus F is integer and equation (16) is true for n is odd or odd prime.

3.4.4 The computation of s_o

Proposition 3.10
$$s_o = n - p \tag{17}$$
.

$for\ n\ is\ even,\ n\ >\ 1$

Proof 3.10 *By definition (3.3) for computation s_o it needs to subtract from $|XUY|$ the number of even composite positive integers $(n-2)$ here(-2) takes into account corollary 3.1 $n* = n - 2$ and also subtract the number of odd noncomposite positive integers p then we get:*
$s_o = 2(n-1) - (n-2) - p = n - p\ for\ n\ is\ even\ n\ >\ 1$. □

Proposition 3.11
$$s_o = n - p - 1 \tag{18}$$
.

$for\ n\ is\ odd\ or\ odd\ prime\ n > 1$

Proof 3.11 *Making the change equation (17) by rem 3.4 we get:* $s_o = n - p - 1\ for\ n\ is\ odd\ n\ >\ 1$. □

3.4.5 The computation of p

Definition 3.13 $p_1 - the\ first\ approximation\ of\ p.$
$p_2 - the\ second\ approximation\ of\ p.$
$p_3 - the\ third\ approximation\ of\ p.$

Proposition 3.12
$$p_1 = round(2n/ln2n) \tag{19}$$

Proposition 3.13

$$p_2 = round(2n/ln2n + 2n/ln^2 2n) \tag{20}$$

Proposition 3.14

$$p_3 = round(2n/ln2n + 2n/ln^2 2n + 4n/ln^3 2n) \tag{21}$$

Proof 3.12 *As everybody knows* [*12*] *that the number of the primes less than* $2n$ *is expressed as follows:*
$\pi(2n) = (2n/ln2n)\int_0^1(1-(lny/ln2n)+(ln^2y/ln^22n)+\ldots)dy$
We are limited to three of the first terms of the series.
Integrating in parts then we get:
$\pi(2n) = (2n/ln2n)(1+1/ln2n+2/ln^22n)$
Whence taking into account definition (3.6) that p is (primes + 1) then we get :
$p = round(2n/ln2n + 2n/ln^22n + 4n/ln^32n + 1)$
But taking into account that "2" is not odd prime and that by definition (3.6) p is the number of odd prime < 2n then
$p = round(2n/ln2n + 2n/ln^22n + 4n/ln^32n + 1 - 1)$
Finally we get:
$p = round(2n/ln2n + 2n/ln^22n + 4n/ln^32n)$
Whence we get:
$p_1 = round(2n/ln2n)$
$p_2 = round(2n/ln2n + 2n/ln^22n)$
$p_3 = round(2n/ln2 + 2n/ln^22n + 4n/ln^32n)$ □

3.4.6 The computation of $|Q|$

Proposition 3.15

$$|Q| = (n - p - |L|)/2 \tag{22}$$

for n is even $n > 1$

Proof 3.13 *By equation (8) we get:* $|Q| = (s_o - |L|)/2$. *Taking into account equation (17) we get for n is even*
$|Q| = (n-p-|L|)/2$. *Now we show that for n is even* $(n-p-|L|) = (n-(p+|L|))$ *is even:*
By equation (28) $|L| = p-2|H|$ *whence* $|L|$ *has parity of p hance* $(p+|L|)$ *is even*
for any p's and $|L|'s$ *parity and for n is even then* $(n-p-|L|)$ *is even then the division by"2" is possibility*
thus $|Q|$ *is integer and equation (22) is true for n is even* $n > 1$. □

Proposition 3.16

$$|Q| = (n - p - 1 - |L|)/2 \tag{23}$$

.

for n is odd or odd prime $n > 1$

Proof 3.14 *Changing equation (22) by rem (3.4) we get:* $|Q| = (n - p - 1 - |L|)/2 = (n - 1 - (p + |L|)$ $n > 1$.
Now we show that $(n - p - 1 - |L|)$ *is even:*
By equation (28) $|L| = p - 2|H|$ *whence* $|L|$ *has parity of* p *hance* $(p + |L|)$ *is even for any p's and* $|L|'s$ *parity*
then $(n - 1 - (p + |L|)$ *is even then the division by"2" is possibility thus* $|Q|$ *is integer and equation (23) is true for n is odd or odd prime* $n > 1$.
□

Proposition 3.17

$$|Q| = (n - 2p + 2|H|)/2 \tag{24}$$

.

for n is even $n > 1$

Proof 3.15 *subtracting equation (7) from equation (8) we get:* $2|Q| - 2|H| = s_0 - p$ *Whence we get:* $|Q| = (2|H| + s_0 - p)/2$. *Taking into account equation (17)*
$|Q| = (2|H| + n - p - p)/2 = (n - 2p + 2|H|)/2$. *and for n is even* $(n - 2p + 2|H|)$ *is even then the division by"2"*
is possibility thus $|Q|$ *is integer and equation (24) is true for n is even* $n > 1$. □

Proposition 3.18

$$|Q| = (n - 2p + 2|H| - 1)/2 \tag{25}$$

for n is odd or odd prime $n > 1$

Proof 3.16 *Changing equation (24) by rem (3.4) we get:* $|Q| = (n - 2p + 2|H| - 1)/2$ $n > 1$.
For n is odd or odd prime $(n - 2p + 2|H| - 1)$ *is even then the division by"2" is possibility thus* $|Q|$ *is integer and equation (25) is true for n is odd or odd prime* $n > 1$. □

3.4.7 The computation of $|L|$

Proposition 3.19

$$|L| = n - p - 2|Q| \tag{26}$$

for n is even $n > 1$

Proof 3.17 *By equation (8) we get:* $|L| = (s_o - 2|Q|)$ *Taking into account (17) we get for n is even*
$|L| = (n - p - 2|Q|)$ *For n is even* $n > 1$. □

Proposition 3.20

$$|L| = n - p - 1 - 2|Q| \tag{27}$$

for n is odd or odd prime $n > 1$

Proof 3.18 *Changing equation (26) by rem (3.4) we get:* $|L| = (n - p - 1 - 2|Q|)/2$ *for n is odd or odd prime* $n > 1$. □

Proposition 3.21

$$|L| = p - 2|H| \tag{28}$$

$\forall\ n\ >\ 1$

Proof 3.19 *By equation (7) and rem (3.3) we get:* $|L| = p - 2|H|$
$for\ n\ is\ even\ n\ >\ 1$
For n is odd or odd prime by rem (3.4) we also get:$|L| = p - 2|H|\ n\ >\ 1$
Thus $|L| = p - 2|H|\ \forall\ n\ >\ 1$. □

3.4.8 The computation of $|H|$

Proposition 3.22

$$|H| = (p - |L|)/2 \tag{29}$$

$\forall\ n\ >\ 1$

Proof 3.20 *By equation(7) and rem (3.3) we get:* $|H| = (p - |L|)/2$ *for n is even* $n\ >\ 1$.
For n is odd or odd prime by rem (3.4) we also get: $|H| = (p - |L|)/2\ n\ >\ 1$.
Now we show that $\forall\ n\ (p - |L|)$ *is even*
By equation (26) $|L| = n - p - 2|Q|$ *whence* $|L| + p$ *is even for n is even .*
By equation (27) $|L| = n - p - 1 - 2|Q|$ *whence* $|L| + p$ *is even for n is odd or odd prime.*
It is possibility if $|L|$ *and p have equal parity then* $(p - |L|)$ *is even for any p's and* $|L|'s$ *parity.*
Thus $(p - |L|)$ *is even* $\forall\ n$.
Hence the division by"2" is possibility then $|H|$ *is integer and equation (29) is true* $\forall\ n\ >\ 1$. □

Proposition 3.23

$$|H| = (2p + 2|Q| - n)/2 \tag{30}$$

$for\ n\ is\ even\ n\ >\ 1$

Proof 3.21 *Subtracting equation (7) from equation (8) we get:* $2|Q| - 2|H| = s_0 - p$ *Whence we get:* $|H| = (2|Q| - s_0 + p)/2$. *Taking into account equation (17) we have for n is even:*
$|H| = (2|Q| - (n - p) + p)/2 = (2p + 2|Q| - n)/2$. *Now we can see that for n is even* $(2p + 2|Q| - n)$ *is even*
then the division by"2" is possibility thus $|H|$ *is integer and equation (30) is true for n is even* $n > 1$ □

Proposition 3.24

$$|H| = (2p + 2|Q| - n + 1)/2 \tag{31}$$

for n is odd or odd prime $n > 1$

Proof 3.22 *Subtracting equation (7) from equation (8) we get:* $2|Q| - 2|H| = s_o - p$ *Whence we get:* $|H| = (2|Q| - s_0 + p)/2$. *Taking into account (18) we have for:*
$|H| = (2|Q| - (n - p - 1) + p)/2 = (2p + 2|Q| - n + 1)/2$. *Now we can see that for n is odd or odd prime*
$(2p + 2|Q| - n + 1)$ *is even then the division by"2" is possibility thus* $|H|$ *is integer and equation (31) is true*
for n is odd or odd prime $n > 1$. □

Definition 3.14 $|Q| - |H|$ *- lower limit of possible range of* $|Q| - |H|$.
$|Q| - |H|$*- positive integer* $> 0 \ \ \forall\ 2n > 120$.
$|Q| - |H|$ *- negative integer* $< 0 \ \ \forall\ 2 < 2n < 120$.
excepting $2n = 94; 96; 100; 106; 118;$ *for which* $|Q| - |H| = 0($ *see subsection* $12.1)$
.

Proposition 3.25

$$|Q| - |H| = (n - 2p)/2 \tag{32}$$

for n is even $n > 1$

Proof 3.23 *Subtracting equation (7) from equation (8) we get:* $2|Q| - 2|H| = s_0 - p$ *Whence we get:* $|Q| - |H| = (s_0 - p)/2 =$. *Taking into account equation (17) we :*
$|Q| - |H| = (n - p - p)/2 = (n - 2p)/2$. *Since for n is even* $(n - 2p)$ *is even then the division by"2"*
is possibility thus $|Q| - |H|$ *is integer and equation (32) is true for n is even* $n > 1$ □

Proposition 3.26

$$|Q| - |H| = (n - 2p - 1)/2 \quad (33)$$

for n is odd or odd prime $n > 1$

Proof 3.24 *Subtracting equation (7) from equation (8) we get:*
$2|Q| - 2|H| = s_0 - p$ *Whence we get:* $|Q| - |H| = (s_0 - p)/2 =$. *Taking into account equation (18)*
we have :
$|Q| - |H| = (n-1-p-p)/2 = (n-2p-1)/2$. *For n is odd or odd prime* $(n - 2p - 1)$ *is even then*
the division by"2" is possibility thus $|Q| - |H|$ *is integer*
and equation (33) is true for n is odd or odd prime $n \ > \ 1$. □

Remark 3.8 *It was controlled ABR2n for some values of2n. For each value of 2n it was computed by direct computation the following:* $p, s_o, |Q|, |L|, |H|, F$ *for values* $< 2n$. *Next we compare data direct computations with the same parameters which are computed by equations of ABR2n and we get full coincidence. The full text of the control ABR2n see subsection (12.3) of appendices.*

4 The limited values of possible range of $|Q|, |L|, |H|$

Definition 4.1 $|H|_b$ *-lower limit of possible range of* $|H|$.

Axiom 4.1

$$|H|_b = 0 \quad (34)$$

Definition 4.2 *p(n) is the number of odd noncomposite integers (primes and "1")* $< \ n$
p(n) is positive integer $\ > \ 0 \ \forall \ n \ > \ 1$

Definition 4.3 $p - p(n)$ *- the number of odd noncomposite integers in Y* .

Definition 4.4 $|H|_c$*-upper limit of possible range of* $|H|$.

Proposition 4.1

$$|H|_c = p - p(n) \quad (35)$$

Proof 4.1 *By Low of distribution of primes the number of odd noncomposite integers in X is greater than the number of odd noncomposite integers in Y :* $p(n) > p - p(n)$ *.*
Since the number of primes decreases with increase of n. Hence maximal number of pair of odd noncomposite integers in the set XUY equals the number of odd noncomposite integers in Y : p - p(n)
then $|H|_c = p - p(n)$ *.* □

Corollary 4.1 *The number of unpaired odd noncomposite positive integers in X equals* $2p(n) - p$ *and are allotted to type "L". Then* $|L| > 0 \; \forall 2n > 8$

Proof 4.2 *The number of unpaired odd noncomposite positive integers in X by definitions 4.2, 4.3 equals :*
(p(n) - (p - p(n)) = 2p(n) - p
and are allotted to type "L". □

Definition 4.5 $|L|_b$ *- lower limit of possible range of* $|L|$*.*

Proposition 4.2

$$|L|_b = 2p(n) - p \tag{36}$$

Proof 4.3 *Substituting upper limit of* $|H|$ *by (35) to (7) then we get: lower limit for* $|L|$ *:* $|L|_b = 2p(n) - p$ □

Definition 4.6 $|L|_c$ *-upper limit of possible range of* $|L|$*.*

Proposition 4.3

$$|L|_c = p \tag{37}$$

Proof 4.4 *Substituting lower limit of* $|H|$ *by (34) to (7) then we get upper limit for* $|L|$ *:* $|L|_c = p$ □

Definition 4.7 $|Q|_b$ *- lower limit of possible range of* $|Q|$*.*
$|Q|_b$*- positive integer* $> 0 \;\; \forall \; 2n > 120$ *see proposition (9.2).*
$|Q|_b$ *- negative integer* $< 0 \;\; \forall \; 2 < 2n < 120$ *see proposition (9.1).*
excepting $2n = 94; 96; 100; 106; 118$ *for which* $|Q|b = 0$ *see subsection (12.1).*

At that $2n = 120; (|Q|_b = 0)$ *is border point.*

Proposition 4.4

$$|Q|_b = (n - 2p)/2 \tag{38}$$

n is even $n > 1$

Proof 4.5 *Substituting upper limit of* $|L|$ *by equation (37) to equation (8) then we get*
lower limit for $|Q|$*:* $|Q|_b = (S_o - p)/2$.
Substituting S_0 *by equation (17), then we get:*
$|Q|_b = (n - 2p)/2 \; n > 1$. □

Proposition 4.5

$$|Q|_b = (n - 2p - 1)/2 \tag{39}$$

n is odd or odd prime $n > 1$

Proof 4.6 *Substituting upper limit of* $|L|$ *by equation (37) to equation (8) then we get*
lower limit for $|Q|$*:* $|Q|_b = (S_0 - p)/2$.
Substituting S_0 *by equation (18) finally we get:* $|Q|_b = (n - 2p - 1)/2$
$for\ n\ is\ odd\ or\ odd\ primen > 1$. □

Definition 4.8 $|Q|_c$ *- upper limit of possible range of* $|Q|$*.*

Proposition 4.6

$$|Q|_c = (n - 2p(n))/2 \tag{40}$$

$for\ is\ even$

Proof 4.7 *Substituting lower limit of* $|L|$ *by equation (36) to equation (8) then we get*
upper limit for $|Q|$*:* $|Q|_c = (S_0 - (2p(n) - p))/2$.
Substituting S_0 *by equation (17) finally we get:* $|Q|_c = (n - 2p(n))/2$;
for n is even □

Proposition 4.7

$$|Q|_c = (n - 2p(n) - 1)/2 \tag{41}$$

for n is odd or odd prime

Proof 4.8 *Substituting lower limit of* $|L|$ *by equation (36) to equation (8) then we get*
upper limit for $|Q|$*:* $|Q|_c = (S_0 - (2p(n) - p))/2$
Substituting S_0 *by equation (18) finally we get:* $|Q|_c = (n - 2p(n) - 1)/2$;
for n is odd or odd prime . □

Proposition 4.8

$$|Q| - |H| = |Q|_b \tag{42}$$

$\forall\ n\ >\ 1$

Proof 4.9 *By equation (32) we have:* $|Q| - |H| = (n-2p)/2$ *for* n *is even By equation (33) we have:* $|Q| - |H| = (n-2p-1)/2$ *for* n *is odd or odd prime.*
By equation (38) we have: $|Q|_b = (n-2p)/2$ *for* n *is even. By equation (39) we have:* $|Q|_b = (n-2p-1)/2$ *for* n *is odd or odd prime.*
Whence we get: $|Q| - |H| = |Q|_b\ \forall\ n\ > 1$. □

5 Average value of the number of binary sums are formed from odd composite positive integers $< 2n$

.

Definition 5.1 $S-ordered\ set\ of\ odd\ composite\ positive\ integers\ < 2n$
$s - element\ of\ S$
$|S| = s_o - power\ of\ S$
$s_i - vary\ over\ all\ s$
$s_j - vary\ over\ all\ s$

Definition 5.2 $V = \{v_k | v_k \in N, v_k = s_i + s_j\}$
is a set by elements of which are every possible binary sums of odd composite
integers $< 2n$*. (each with all the rest)*
Since $s_i < 2n; s_j < 2n$ *then max* $v_k < 4n$*.*
$|V|$ *- the power of set V.*

Definition 5.3 $W = \{w | w \in N, w = 2k, 1 \leq k \leq 2n\}$
is a set of even composite positive integers $< 4n + 2$
$(inf\ W = 2; sup\ W = 4n)$.
$|W| - the\ power\ of\ set\ W |W| = 2n.$

Definition 5.4 $|Q|_m$ - *mean quantity of binary sums* $v_k = s_i + s_j$ *which can be formed of odd composite positive integers* $< 2n$ *and which are mapped into* W *by surjective mapping :*
$f : V \Rightarrow W$

$$|Q|_m = |V|/|W| \tag{43}$$

i.e. uniform mapping regardless of real.
$|Q|_m$ $-$ *positive rational number* > 0

$\forall\ 2n > 4.$

Proposition 5.1

$$|V| = s_0^2 \tag{44}$$

Proof 5.1 *The number of every possible binary sums in the view of* $v_k = s_i + s_j$ *are formed of odd composite positive integers* $< 2n$ *is equal the power of Cartesian product:* $S\ X\ S$ *[13] [14]. Then:*
$|V| = s_0 \cdot s_0 = s_0^2$ □

Proposition 5.2

$$|Q|_m = s_0^2/2n \tag{45}$$

Proof 5.2 *By equations (43 and 44) we have :* $|Q|_m = |V|/|W| = s_0^2/2n$
□

Proposition 5.3

$$|Q|_m = (n-p)^2/2n \tag{46}$$

$For\ n\ is\ even\ n > 1$

Proof 5.3 *Substituting* s_0 *by equation (17) to equation (45) then we get :* $|Q|_m = (n-p)^2/2n;$
For n is even $n > 1$ □

Proposition 5.4

$$|Q|_m = (n-p-1)^2/2n \tag{47}$$

For n is odd or odd prime $n > 1$

Proof 5.4 *Substituting* s_0 *by equation (18) to equation (45) then we get :*
$|Q|_m = (n-p-1)^2/2n;$
For n is odd or odd prime $n > 1$ □

6 Average value of the number of binary sums are formed from odd noncomposite positive integers $< 2n$

Definition 6.1 *P - ordered set of odd noncomposite positive integers* $< 2n$
p - elements of P
$|P| = p$ *- power of P*
p_i*- vary over all p*
p_j *- vary over all p*

Definition 6.2 $T = \{t_k | t_k \in N, t_k = p_i + p_j\}$
is a set by elements of which are every possible binary sums of odd noncomposite integers $< 2n$. *(each with all the rest)*
Since $p_i < 2n; p_j < 2n$ *then max* $t_k < 4n$.
$|T|$ *- the power of set T.*

Definition 6.3 $|H|_m$ *is mean quantity of binary sums* $p_i + p_j = 2k$ *which can be formed of odd noncomposite positive integers* $< 2n$ *and which are mapped into W by surjective mapping :*
$f: T \Rightarrow W$

$$|H|_m = |T|/|W| \tag{48}$$

i.e. uniform mapping regardless of real.
$|H|_m$*-positive rational number* $> 0 \quad \forall 2n > 4$.

Proposition 6.1

$$|T| = p^2 \tag{49}$$

Proof 6.1 *The number of every possible binary sums are formed of odd noncomposite positive integers* $< 2n$ *in the view of* $p_i + p_j = 2k$ *is equal the power of Cartesian product P X P [13] [14] .*
Then $|T| = p \cdot p = p^2 \quad \square$

Proposition 6.2

$$|H|_m = p^2/2n \tag{50}$$

$\forall\ n > 1$

Proof 6.2 *By equations (48 and 49) we have :* $|H|_m = p^2/2n\ \forall\ n > 1$ $\square$

7 The deviation of $|Q|, |H|\ from\ |Q|_m, |H|_m$

Definition 7.1 *The deviation of* $|Q|\ from\ |Q|_m$ *is:*
$\Delta|Q| = |Q|_m - |Q|$
$\Delta|Q| > 0\ if\ |Q|_m > |Q|$
$\Delta|Q| < 0\ if\ |Q|_m < |Q|$

Definition 7.2 *The deviation of* $|H|\ from\ |H|_m$ *is:*
$\Delta|H| = |H|_m - |H|$
$\Delta|H| > 0\ if\ |H|_m > |H|$
$\Delta|H| < 0\ if\ |H|_m < |H|$

7.1 Relationship between deviations of $\Delta|Q|\ and\ \Delta|H|$

Proposition 7.1

$$\Delta|Q| = \Delta|H| + |Q|_m - |H|_m - |Q|_b \tag{51}$$

$\forall\ n > 1$

Proof 7.1 *By equation (42)* $|Q| - |H| = |Q|_b$
By definition (7.1) $\Delta|Q| = |Q|_m - |Q|$
By definition (7.2) $\Delta|H| = |H|_m - |H|$
Whence : $|Q| = |Q|_m - \Delta|Q|$
$|H| = |H|_m - \Delta|H|$
Then by equation (42)
$(|Q|_m - \Delta|Q|) - (|H|_m - \Delta|H|) = |Q|_b$
$|Q|_m - \Delta|Q| - |H|_m + \Delta|H| = |Q|_b$
Whence:
$\Delta|Q| = \Delta|H| + |Q|_m - |H|_m - |Q|_b$. $\square$

Remark 7.1 *It was controlled the equation for at the some values of 2n. For each value of 2n it was took data direct computations of* $p, s_o, |Q|, |l|, |H|$ *from section (12.3) . Next we compute by equation (51) using data direct computations. The compare of* $\Delta|Q|$ *computed by data of direct computations with computed value*
by equation (51) shows full coincidence of results. The full text of the control you can see section (13).

8 The relationship between n and p

8.1 The relationship between n and p for $n < 60$

Proposition 8.1

$$2p > n \tag{52}$$

for n is even $n < 60$

Proof 8.1 *From the numerical solution (see.sabsection 12.1) it follows that:*
$2p > n$ *for n is even* $n < 60$ □

Proposition 8.2

$$2p + 1 > n \tag{53}$$

for n is odd or odd prime $n < 59$

Proof 8.2 *From the numerical solution (see.sabsection 12.1) it follows that:*
$2p + 1 > n$ *for n is odd or odd prime*
$n < 59$. □

8.2 The relationship between n and p for $n > 60$

Proposition 8.3

$$n > 2p \tag{54}$$

for n is even $n > 60$

Proposition 8.4

$$n > 2p + 1 \tag{55}$$

for n is odd or odd prime $n > 59$.

Proof 8.3 *We need to prove that:* $n > 2p$ *for n is even,* $n > 60$.
$n > 2p + 1$ *for n is odd or odd prime,* $n > 59$.
For it we must find a dependence of differences :
$f_1 = n - 2p;$ *and* $f_2 = n - (2p + 1);$ *from n*
Substituting for p its the second order approximation by equation (20) we get:
$f_1 = n - 2(2n/ln2n + 2n/ln^2 2n);\ f_2 = n - (2(2n/ln2n + 2n/ln^2 2n) + 1)$.
$f_1 = n - 4n/ln2n - 4n/ln^2 2n;\ f_2 = n - 4n/ln2n - 4n/ln^2 2n - 1$.
Next we computation the derivatives :

$(f_1)' = (ln^4 2n - 4ln^3 2n + 8ln2n)/ln^4 2n$.
$(f_2)' = (ln^4 2n - 4ln^3 2n + 8ln2n)/ln^4 2n$.
Whence $(f_1)' > 0$; $(f_2)' > 0$; $\forall\ 2n > 2$.
Then f_1, f_2 *increase* $\forall\ 2n > 2$.
Next we compute the points of intersection of f_1, f_2 *with abscissa axis.*
For it we need to test of fulfillment of conditions:
$(f_1(2n) = 0\ f_1(2n+2) > 0)$ *and* $(f_2(2n) = 0 f_2(2n+4) > 0)$.
As follows from numerical solution (see subsubsection 12.1).
The points of intersection are:
2n = 120 for n is even; and 2n = 118 for n is odd or odd prime
Since conditions are fulfilled only for them.
Hence $f_1 > 0\ forn$ *is even,* $n > 60$.
And $f_2 > 0$ *for n is odd or odd prime* $n > 59$
Thus $n > 2p$; *for n is even ,* $n > 60$.
And $n > 2p + 1$ *for n is odd or odd prime* $n > 59$. □

8.2.1 The control of relationship $n > 2p$; $n > 2p + 1$

Remark 8.1 *p∗ is decremental value of p by corollary 3.3*

From data direct computations exmp 12.5; examp 12.6 ; examp 12.7 we have :
n = 66 , p = 32 ; n = 67, p* = 30 n = 69, p = 33
By proposition (8.3) 66 > 64 .
By proposition (8.4) 67 > 60 + 1 69 > 66 + 1 . Hence propositions (8.3 and 8.4) are correct for n stated above.

8.3 The correspondence between n and p $\forall\ n$

Definition 8.1 *The correspondence of p to finite set of 2n it is when for several 2n it takes at the same p at computations by equations ABR2n.*

Proposition 8.5 *To each p is correspond finite set of 2n included between*
neighboring values of p .

Proof 8.4 *Let for any* $2n_1$ *exists* $p < 2n_1$ *and for* $2n_k$ *exists* $p + 1 < 2n_k$ *then to each p is correspond to set of* $2n = 2n_1 ... 2n_k$ *This set is restrictedly.*
Hence to each p is correspond finite set
of 2n included between neighboring values of p. □

Remark 8.2 *It was conrolled the correspondence of finite set of 2n included between neighboring values of p to it. It was shown that each p correspond to finite set of 2n included between neighboring values of p. The full text of the control we can see the subsection (13.1).*

9 The properties of $|Q|_b$

Proposition 9.1 $|Q|_b$ *is negative integer in the range of* $2 < 2n < 120$.
Excepting $2n = 94, 96, 100, 118$.

Proof 9.1 *By proof (4.5)* $|Q|_b$ *is integer for n is even .*
By proof (4.6) $|Q|_b$ *is integer for n is odd or odd prime .*
Then $|Q|_b$ *is integer* $\forall\ n > 1$.
Since by propositions (8.1 and 8.2) $2p > n$ *and* $2p + 1 > n$
then by equation (38) $|Q|_b < 0$ *for n is even* < 60 .
And by equation (39) $|Q|_b < 0$ *for n is odd or odd prime* < 60.
By direct computation with help of numerical solution of $|Q|_b$ *(see subsection 12.1)*
we find 2n for which $|Q|_b = 0$.
Thus In the range of $2 < 2n < 120$ $|Q|_b$ *is negative integer excepting* $2n = 94, 96, 100, 118$. *(see subsetion 12.1)* □

Proposition 9.2 $|Q|_b$ *is positive integer* $\forall 2n > 120$

Proof 9.2 *By proposition (4.4)* $|Q|_b$ *is integer for n is even .*
By proposition (4.5) $|Q|_b$ *is integer for n is odd or odd prime .*
Then $|Q|_b$ *is integer* $\forall\ n > 1$.
Since by propositions (8.3 and 8.4) $2p < n$ *and* $2p + 1 < n$
then by equation (38) $|Q|_b > 0$ *for n is even* > 60 .
And by equation (39) $|Q|_b > 0$ *for n is odd or odd prime* > 60.
Thus $|Q|_b$ *is positive integer* $\forall 2n > 120$ □

Corollary 9.1 *There is no less than* $|Q|_b$ *of representations of the type "Q"* $\forall\ 2n > 120$.

Remark 9.1 *The following proposition explains the cause by which* $|Q| = 0$ *in the range of* $2 < 2n < 120$.

Proposition 9.3 *In the range of* $2 < 2n < 120$ *if fulfilled the condition* $|H| = ||Q|_b|$ *then* $|Q| = 0$.

Proof 9.3 *By equation (42) and proposition (9.1) we have for* $2 < 2n < 120$
$|H| = |Q| + ||Q|_b|$.
Let $|Q| = 0$; *then* $|H| = ||Q|_b|$.
Thus if it is fulfilled the condition $|H| = ||Q|_b|$ *then* $|Q| = 0$
In the range of $2 < 2n < 120$. □

10 The solution of the Goldbach's binary problem

Lemma 10.1 $\forall\ 2 < 2n < 120$ *exists at least one representation of type "H".*

Proof 10.1 *Taking into account equation (42) we get:* $|H| = |Q| - |Q|_b$. *Since in the range of* $2 < 2n < 120$ *by proposition*(9.1) $|Q|_b < 0$ *then* $|H| = |Q| + ||Q|_b|$
Whence $|H| > 0$ *excepting* $2n = 94; 96; 100; 106; 118;$ *in which* $|Q|_b = 0$ *(see subsection 12.1).*
For this $2n$ *the truth of lemma follows from that for* $2n$ *in which* $|Q|_b = 0$ *then* $|Q| > 0$ *in this points since the points of exclusion for* $|Q|$ *2n = 26, 28,32,38 (see subsection 12.2) don't coincidence with points of exclusion* $|Q|_b$ *2n = 94,96, 100,106,118 (see subsection 12.1) .*
Thus $|H| > 0\ \forall\ 2 < 2n < 120$ □

Theorem 10.1 $\forall\ 2n > 2$ *exists at least one representation of even positive integer* $2n$ *in the view of a sum of two odd prime positive integers or "1" and odd prime positive integer.*

Proof 10.2 *Earlier by Lemma we proved that* $|H| > 0\ \forall\ 2 < 2n < 120$.
For the full proof of the theorem we need to prove that
$|H| > 0\ \forall\ 2n > 120$
Let $|H| = 0$ *for some value* $n > 60$ *then by definition (7.2) we get:*

$$\Delta|H| = |H|m \tag{56}$$

Also from definition (7.1) we get:

$$|Q| = |Q|m - \Delta|Q| \tag{57}$$

Taking into account that at $|H| = 0$ *from equation (42) it follows that:*

$$|Q| = |Q|_b \tag{58}$$

we get:

$$|Q|_b = |Q|m - \Delta|Q| \tag{59}$$

Taking into account proposition (7.1) and equation (56) we get:

$$|Q|_b = |Q|m - (|Q|_m - |H|_m - |Q|_b + |H|_m) \tag{60}$$

Whence :

$$|Q|_b = |Q|_b \tag{61}$$

Taking into account equation (38) we get identity for n *is even,* $n > 1$ *:*

$$(n - 2p)/2 = (n - 2p)/2 \tag{62}$$

Taking into account equation (39) we get identity for n *is odd or odd prime,* $n > 1$ *:*

$$(n - 2p - 1)/2 = (n - 2p - 1)/2 \tag{63}$$

Thus in the result of assumption that $|H| = 0$ *for some value* $n > 60$
we come to identities: (62 and 63) next we transform these identities into as follows :
$2n(n - 2p) = 2n(n - 2p); 2n^2 - 4pn = 2n^2 - 4pn;$ *whence :* $4pn = 4pn;$ *or* $2np = 2np$.
$2n(n-2p-1) = 2n(n-2p-1); 2n^2-4pn-2n = 2n^2-4pn-2n;$ *whence :* $4pn = 4pn;$ *or* :

$$2np = 2np \tag{64}$$

We fix value of p *and will be substitute values of infinite series 2n to(64) and identity (64) is correct for all values of 2n*
then it is determine following relationship between n and p : each p correspond to infinite set of 2n .
That contradict by proposition (8.5) which assert that each p correspond to finite set of 2n included between neighboring values of p.
Hence our assumption that $|H| = 0$ *for some value* $n > 60$ *is false and it is true that* $|H| > 0 \ \forall \ 2n > 120$. *Thus we proved that* $|H| > 0 \ \forall \ 2n > 2$.
Hence $\forall \ 2n > 2$ *exists at least one representation of even positive integer 2n in the view of a sum of two odd prime positive integers or "1" and odd prime positive integer.* □

Corollary 10.1

$$|Q| > |Q|_b \tag{65}$$

$\forall\ n > 1$

Proof 10.3 *From equation (42) we get:*

$$|Q| = |Q|_b + |H|$$

Then for $|H| > 0$ *we get:*
$|Q| \ > \ |Q|_b \ \forall\ n \ > \ 1$. $\square$

Corollary 10.2

$$|L| < p \tag{66}$$

$\forall\ n > 1$

Proof 10.4 *From equation (2.7) we get:*

$$|L| = p - 2|H|$$

Then for $|H| > 0$ *we get:*
$|L| < p; \forall n > 1.$ $\square$

11 The computation of the real values of $|Q|, |H|$

11.1 The relative accuracy of computation of $|Q|, |H|$

Definition 11.1 *The relative accuracy of computation of* $|Q|$ *as follows below :*

$$\delta_Q = ((\Delta|Q|/|Q|_m)100)\% \tag{67}$$

Definition 11.2 *The relative accuracy of computation of* $|H|$ *as follows below:*

$$\delta_H = ((\Delta|H|/|H|_m)100)\% \tag{68}$$

11.2 The estimation of δ_Q

Proposition 11.1

$$(100p^2/(n-p)^2) \ > \ \delta_Q \tag{69}$$

for n is even $n > 2$;

Proof 11.1 *By definition (7.1) we have* $|Q| = |Q|_m - \Delta|Q|$.
By definition. (11.1) we have:
$\Delta|Q| = (\delta_Q|Q|_m)/100$
then we get : $|Q| = |Q|_m - (\delta_Q|Q|_m)/100$.
By corollary (10.1) $(|Q|_m - (\delta_Q|Q|_m)/100) > |Q|_b$.
Whence it follows that $(100(|Q|_m - |Q|_b)/|Q|_m)) > \delta_Q$.
Taking into account equations (46 and 38).
Then we get:
$(100((n-p)^2)/2n - (n-2p)/2)/((n-p)^2)/2n) > \delta_Q$.
Hence $(100p^2/(n-p)^2) > \delta_Q$ *for n is even* $n > 2$. □

Proposition 11.2

$$((100(p^2 + 2p - n + 1)/(n-p-1)^2)) > \delta_Q \qquad (70)$$

for n is odd or oddprime $n > 4$

Proof 11.2 *By definition (7.1) we have* $|Q| = |Q|_m - \Delta|Q|$.
By definition. (11.1) we have:
$\Delta|Q| = (\delta_Q|Q|_m)/100$
then we get : $|Q| = |Q|_m - (\delta_Q|Q|_m)/100$.
By corollary (10.1) $|Q|_m - (\delta_Q|Q|_m)/100 > |Q|_b$.
Whence it follows that $100((|Q|_m - |Q|_b)/|Q|_m) > \delta_Q$.
Taking into account equations (47 and 39) then we get:
Then we get:
$(100((n-p-1)^2)/2n - (n-2p-1)/2)/((n-p-1)^2)/2n) > \delta_Q$.
Hence $(100(p^2 + 2p - n + 1)/(n-p-1)^2) > \delta_Q$.
for n is odd or odd prime $n > 4$. □

11.3 The dependence of $\delta_Q(2n)$

Theorem 11.1 *For n is even* > 2 *If* $n \to \infty$ *then* $\delta_Q \to 0$.

Proof 11.3 *We equate* δ_Q *with its estimation by equation(68)*
then for n is even we have:$\delta_Q = 100p^2/(n-p)^2$.
We replace p with its first order approximation by equation (19) we get:
$\delta_Q = (1004n^2/ln^2 2n)/(n^2 ln^2 2n - 4n^2 ln2n + 4n^2)/ln^2 2n = 400/(ln^2 2n - 4ln^2 2n + 4)$
Whence follows that if $n \to \infty$ *then* $\delta_Q \to 0$
for n is even $n > 2$. □

Theorem 11.2 *For n is odd or odd prime* > 4 *If* $n \to \infty$ *then* $\delta_Q \to 0$.

Proof 11.4 *We equate* δ_Q *with its estimation by equation (69)* *then for n is odd or odd prime we have:*$\delta_Q = 100(p^2+2p-n+1)/(n-p-1)^2$.
We replace p with its first order approximation by equation (19) we get:
$\delta_Q = 100(4n^2 + 4nln2n - nln^2 2n + ln^2 2n)/(n^2 ln^2 2n - 4n^2 ln2n + 4n^2 - 2nln^2 2n + 4nln2n + ln^2 2n)$;
$\delta_Q = 100(4n^2 ln^2 2n(1/ln^2 2n + 1/nln2n - 1/4n + 1/4n^2))/4n^2 ln^2 2n(1/4 - 1/ln2n + 1/ln^2 2n - 1/2n + 1/nln2n + 1/4n^2)$
$\delta_Q = 100(1/ln^2 2n + 1/nln2n - 1/4n + 1/4n^2)/(1/4 - 1/ln2n + 1/ln^2 2n - 1/2n + 1/nln2n + 1/4n^2)$
Whence follows that if $n \to \infty$ *then* $\delta_Q \to 0(limn{\to}\infty\ \delta_Q = 0/0,25 = 0)$.
for n is odd or odd prime $n > 4$.
Thus we proved that if $n \to \infty$ *then* $\delta_Q \to 0\ \forall\ n > 4$ □

11.4 The character of dependence of $|Q|(2n)$

Theorem 11.3 *If* $n \to \infty$ *then* $|Q| \to |Q|_m$.

Proof 11.5 *By Theorems (11.1 and 11.2)* $\delta_Q \to 0$ *if* $n \to \infty$
then by Defenition 11.1 $\Delta|Q| \to 0$ *since* $\delta_Q \to 0$.
Whence by Definition 7.1 we have:
If $n \to \infty$*then* $|Q| \to |Q|_m$. □

11.5 The formulas for computation of the real values of $|Q|, |H|$

Remark 11.1 *By theorem (11.3) we can replace the computation of* $|Q|$ *with* $|Q|_m$ *. The mistake of it is decreased with increase of n.*

$$|Q| = round((n-p)^2/2n) \tag{71}$$

for n is even $n > 16$

$$|Q| = round((n-p-1)^2/2n) \tag{72}$$

for n is odd or odd prime $n > 17$
Where: $p = round(2n/ln2n + 2n\ln^2 2n)$. By (20)
The computed value of $|Q|$ it can be used for
computation of $|H|$ by equations(30 and 31):
$|H| = (2|Q| + 2p - n)/2$ *for n is even* $n > 16$
$|H| = (2|Q| + 2p - n + 1)/2$ *for n is odd or odd prime* $n > 17$

12 The appendices

12.1 The numerical solution of $|Q|_b = 0$ *in the range* $2 < 2n < 138$

Remark 12.1 *If* n *is prime then value of* p *is decremented by one .* $p* = p - 1$ *by corollary 3.3 .*

$2n = 4; n = 2; p = 2; |Q|_b = -1$ by (38).
$2n = 6; n = 3; p* = 2; |Q|_b = -1$ by (39).
$2n = 8; n = 4; p = 4; |Q|_b = -2$ by (38).
$2n = 10; n = 5; p* = 3; |Q|_b = -1$ by (39).
$2n = 12; n = 6; p = 5; |Q|_b = -2$ by (38).
$2n = 14; n = 7; p* = 5; |Q|_b = -2$ by (39).
$2n = 16; n = 8; p = 6; |Q|_b = -2$ by (38).
$2n = 18; n = 9; p = 7; |Q|_b = -3$ by (39).
$2n = 20; n = 10; p = 8; |Q|_b = -3$ by (38).
$2n = 22; n = 11; p* = 7; |Q|_b = -2$ by (39).
$2n = 24; n = 12; p = 9; |Q|_b = -3$ by (38).
$2n = 26; n = 13; p* = 8; |Q|_b = -2$ by (39).
$2n = 28; n = 14; p = 9; |Q|_b = -2$ by (38).
$2n = 30; n = 15; p = 10|Q|_b = -3$ by (39).
$2n = 32; n = 16; p = 11; |Q|_b = -3$ by (38).
$2n = 34; n = 17; p* = 10; |Q|_b = -2$ by (39)
$2n = 36; n = 18; p = 11; |Q|_b = -2$ by (38).
$2n = 38; n = 19; p* = 11; |Q|_b = -2$ by (39).
$2n = 40; n = 20; p = 12; |Q|_b = -2$ by (38).
$2n = 42; n = 21; p = 13; |Q|_b = -3$ by (39).
$2n = 44; n = 22; p = 14; |Q|_b = -3$ by (38).
$2n = 46; n = 23; p* = 13; |Q|_b = -2$ by (39).
$2n = 48; n = 24; p = 15; |Q|_b = -3$ by (38).
$2n = 50; n = 25; p = 15; |Q|_b = -3$ by (39).
$2n = 52; n = 26; p = 15; |Q|_b = -2$ by (38)
. $2n = 54; n = 27; p = 16; |Q|_b = -3$ by (39).
$2n = 56; n = 28; p = 16; |Q|_b = -2$ by (38).
$2n = 58; n = 29; p* = 15; |Q|_b = -1$ by (39).
$2n = 60; n = 30; p = 17; |Q|_b = -2$ by (38).
$2n = 62; n = 31; p* = 17; |Q|_b = -2$ by (39).
$2n = 64; n = 32; p = 18; |Q|_b = -2$ by (38).
$2n = 66; n = 33; p = 18; |Q|_b = -2$ by (39).
$2n = 68; n = 34; p = 19; |Q|_b = -2$ by (38).

$2n = 70; n = 35; p = 19; |Q|_b = -2$ by (39).
$2n = 72; n = 36; p = 20; |Q|_b = -2$ by (38).
$2n = 74; n = 37; p* = 20; |Q|_b = -2$ by (39).
$2n = 76; n = 38; p = 21; |Q|_b = -2$ by (38).
$2n = 78; n = 39; p = 21; |Q|_b = -2$ by (39).
$2n = 80; n = 40; p = 22; |Q|_b = -2$ by (38).
$2n = 82; n = 41; p* = 21; |Q|_b = -1$ by (39).
$2n = 84; n = 42; p = 23; |Q|_b = -2$ by (38).
$2n = 86; n = 43; p* = 22; |Q|_b = -1$ by (39).
$2n = 88; n = 44; p = 23; |Q|_b = -1$ by (38).
$2n = 90; n = 45; p = 24; |Q|_b = -2$ by (39).
$2n = 92; n = 46; p = 24; |Q|_b = -1$ by (38).
$2n = 94; n = 47; p* = 23; |Q|_b = 0$ by (39).
$2n = 96; n = 48; p = 24; |Q|_b = 0$ by (38).
$2n = 98; n = 49; p = 25; |Q|_b = -1$ by (39).
$2n = 100; n = 50; p = 25; |Q|_b = 0$ by (38).
$2n = 102; n = 51; p = 26; |Q|_b = -1$ by (39).
$2n = 104; n = 52; p = 27; |Q|_b = -1$ by (38).
$2n = 106; n = 53; p* = 26; |Q|_b = 0$ by (39).
$2n = 108; n = 54; p = 28; |Q|_b = -1$ by (38).
$2n = 110; n = 55; p = 29; |Q|_b = -2$ by (39).
$2n = 112; n = 56; p = 29; |Q|_b = -1$ by (38).
$2n = 114; n = 57; p = 30; |Q|_b = -2$ by (39).
$2n = 116; n = 58; p = 30; |Q|_b = -1$ by (38).
$2n = 118; n = 59; p = 29; |Q|_b = 0$ by (39).
$2n = 120; n = 60; p = 30; |Q|_b = 0$ by (38).
$2n = 122; n = 61; p = 29; |Q|_b = 1$ by (39).
$2n = 124; n = 62; p = 30; |Q|_b = 1$ by (38).
$2n = 126; n = 63; p = 30; |Q|_b = 1$ by (39).
$2n = 128; n = 64; p = 31; |Q|_b = 1$ by (38).
$2n = 130; n = 65; p = 31; |Q|_b = 1$ by (39).
$2n = 132; n = 66; p = 32; |Q|_b = 1$ by (38).
$2n = 134; n = 67; p = 31; |Q|_b = 2$ by (39).
$2n = 136; n = 68; p = 32; |Q|_b = 2$ by (38).
$2n = 138; n = 69; p = 33; |Q|_b = 1$ by (39).

12.2 The numerical solution of $|Q| = 0$ in the range $8 < 2n < 134$

$2n = 10; |Q| = 0; (10 - 9 = 1)$.
$2n = 12; |Q| = 0; (12 - 9 = 3)$.
$2n = 14; |Q| = 0; (14 - 9 = 5)$.
$2n = 16; |Q| = 0; (16 - 9 = 7)$.
$2n = 18; |Q| = 0; (18 - 9 = 9)$.
$2n = 20; |Q| = 0; (20 - 9 = 11)$.
$2n = 22; |Q| = 0; (22 - 9 = 13)$.
$2n = 24; |Q| > 0; (24 - 9 = 15)$.
$2n = 26; |Q| = 0; (26 - 9 = 17)$.
$2n = 28; |Q| = 0; (28 - 9 = 19)$.
$2n = 30; |Q| > 0; (30 - 9 = 21)$.
$2n = 32; |Q| = 0; (32 - 9 = 23)$.
$2n = 34; |Q| > 0; (34 - 9 = 25)$.
$2n = 36; |Q| > 0; (30 - 9 = 21)$.
$2n = 38; |Q| = 0; (38 - 9 = 29)$.
$2n = 40; |Q| > 0; (40 - 15 = 25)$.
$2n = 42; |Q| > 0; (42 - 9 = 33)$.
$2n = 44; |Q| > 0; (44 - 9 = 35)$.
$2n = 46; |Q| > 0; (46 - 21 = 25)$.
$2n = 48; |Q| > 0; (48 - 9 = 39)$.
$2n = 50; |Q| > 0; (50 - 15 = 35)$.
$2n = 52; |Q| > 0; (52 - 25 = 27)$.
$2n = 54; |Q| > 0; (54 - 9 = 45)$.
$2n = 56; |Q| > 0; (56 - 21 = 35)$.
$2n = 58; |Q| > 0; (58 - 9 = 49)$.
$2n = 60; |Q| > 0; (60 - 9 = 51)$.
$2n = 62; |Q| > 0; (62 - 9 = 21)$.
$2n = 64; |Q| > 0; (64 - 9 = 55)$.
$2n = 66; |Q| > 0; (66 - 9 = 57)$.
$2n = 68; |Q| > 0; (68 - 33 = 35)$.
$2n = 70; |Q| > 0; (70 - 15 = 55)$.
$2n = 72; |Q| > 0; (72 - 9 = 63)$.
$2n = 74; |Q| > 0; (74 - 9 = 21)$.
$2n = 76; |Q| > 0; (76 - 21 = 55)$.
$2n = 78; |Q| > 0; (78 - 9 = 69)$.
$2n = 80; |Q| > 0; (80 - 15 = 65)$.
$2n = 82; |Q| > 0; (82 - 25 = 57)$.
$2n = 84; |Q| > 0; (84 - 9 = 75)$.

$2n = 86; |Q| > 0; (86 - 9 = 77)$.
$2n = 88; |Q| > 0; (88 - 25 = 63)$.
$2n = 90; |Q| > 0; (90 - 9 = 81)$.
$2n = 92; |Q| > 0; (92 - 15 = 77)$.
$2n = 94; |Q| > 0; (94 - 9 = 85)$.
$2n = 96; |Q| > 0; (96 - 9 = 87)$.
$2n = 98; |Q| > 0; (98 - 21 = 77)$.
$2n = 100; |Q| > 0; (100 - 15 = 85)$.
$2n = 102; |Q| > 0; (102 - 9 = 93)$.
$2n = 104; |Q| > 0; (104 - 9 = 95)$.
$2n = 106; |Q| > 0; (106 - 15 = 91)$.
$2n = 108; |Q| > 0; (108 - 9 = 99)$.
$2n = 110; |Q| > 0; (110 - 15 = 95)$.
$2n = 112; |Q| > 0; (112 - 21 = 91)$.
$2n = 114; |Q| > 0; (114 - 9 = 105)$.
$2n = 116; |Q| > 0; (116 - 21 = 95)$.
$2n = 118; |Q| > 0; (118 - 25 = 93)$.
$2n = 120; |Q| > 0; (120 - 9 = 111)$.
$2n = 122; |Q| > 0; (122 - 35 = 87)$.
$2n = 124; |Q| > 0; (124 - 9 = 115)$.
$2n = 126; |Q| > 0; (126 - 9 = 117)$.
$2n = 128; |Q| > 0; (128 - 9 = 119)$.
$2n = 130; |Q| > 0; (130 - 9 = 121)$.
$2n = 132; |Q| > 0; (132 - 9 = 123)$.

12.3 The control of ABR2n

Remark 12.2 *We take into account that :* $p, s_o, |Q|, |L|, |H|, |Q|_b, |Q| - |H|$, *are defined for values* $< 2n$.
I.e. the direct computations of them are made $\forall$ *values* $< 2n$, *but the computations by equations are made for n = 2n/2.*

Exemple 12.1 *The computation of* $|Q|$, $|L|$, $|H|$, s_o, G , F *for 2n = 4*
Construction SBR2n:
$2n = 4$; $n = 2$
$\oslash$ - *odd noncomposite;* $\oplus$ - *odd composite ,* $\otimes$ - *even composite .*
$\oslash 1 + 3 \oslash = 4(H)$
Data of direct computations for even n = 2 at that it is took into account remark (3.2)

$p = 2,\ s_o = 0,\ |Q| = 0,\ |L| = 0,\ |H|\ =\ 1,\ G = 1,\ F = 1\ .$
The computations of parameters of binary representations of the positive integer 2n = 4 with help of arithmetic stated above
at that $p, s_o, |Q|, |L|, |H|,$ *are took from the data of direct computations.*
n = 4/2 = 2 ;
$G = n - 1 = 2 - 1 = 1;$ *by equation (11)*
$|E| = n/2 - 1 = 1 - 1 = 0;$ *by equation (13)*
$F = n/2 = 2/2 = 1;$ *by equation (15)*
$s_o = n - p = 2 - 2 = 0;$ *by equation (17)*
$|L| = n - p - 2|Q| = 2 - 2 - 0 = 0;$ *by equation (26)*
$|L| = p - 2|H| = 2 - 2 = 0;$ *by equation (28)*
$|H| = (2|Q| + 2p - n)/2 = (0 + 4 - 2)/2 = 1;$ *by equation (30)*
$|H| = (p - |L|)/2 = (2 - 0)/2 = 1;$ *by equation (29)*
$|Q| = (n - p - |L|)/2 = (2 - 2 - 0)/2 = 01;$ *by equation (22)*
$|Q| = (n - 2p + 2|H|)/2 = (2 - 4 + 2)/2 = 0;$ *by equation (24)*
$|Q| - |H| = 0 - 1 = -1;$
$|Q| - |H| = (n - 2p)/2 = (2 - 4)/2 = -1;$ *by equation (32)*
$p = 2|H| + |L| = 2 + 0 = 2;$ *by equation (7)*
$s_o = 2|Q| + |L| = 0 + 0 = 0;$ *by equation (8)*
$F = |Q| + |L| + |H| = 0 + 0 + 1 = 1;$ *by equation (10)*
The compare of data of direct computations with computed values shows full coincidence of results.

Exemple 12.2 *The computation of* $|Q|,\ |L|,\ |H|,\ s_o,\ G,\ F\ for\ 2n\ =\ 6$
.
$\oslash$ *- odd noncomposite;* $\oplus$ *- odd composite ;* $\otimes$ *- even composite .*

Construction SBR2n:
$2n\ =\ 6\ ;\ n\ =\ 3$
$\oslash 1 + 5 \oslash = 6(H)$
$\otimes 2 + 4 \otimes = 6(E)$
Data of direct computations for odd n = 3 at that it is took into account remark (3.2)
$p* = 2;\ s_o = 0;\ |Q| = 0;\ |L| = 0;\ |H| = 1; G = 2; F = 1$
The computations of parameters of binary representations of the positive integer $2n = 6$ *with help of arithmetic stated above*
at that $p, s_o, |Q|, |L|, |H|,$ *are took from the data of direct computations.*
$n = 6/2 = 3;$
$G = n - 1 = 3 - 1 = 2;$ *by equation (11)*
$|E| = (n - 1)/2 = 1;$ *by equation (12)*

$F = (n-1)/2 = (3-1)/2 = 1$; *by equation (16)*
$s_o = n-p-1 = 3-2-1 = 0$; *by equation (18)*
$|L| = n-p-2|Q|-1 = 3-2-0-1 = 0$; *by equation (27)*
$|L| = p-2|H| = 2-2 = 0$; *by equation (28)*
$|H| = (2|Q|+2p-n+1)/2 = (0+4-3+1)/2 = 1$; *by equation (31)*
$|H| = (p-|L|)/2 = (2-0)/2 = 1$; *by equation (29)*
$|Q| = (n-p-|L|-1)/2 = (3-2-0-1)/2 = 0$; *by equation (23)*
$|Q| = (n-2p+2|H|-1)/2 = (3-4+2-1)/2 = 0$; *by equation (25)*
$|Q|-|H| = 0-1 = -1$; $|Q|-|H| = (n-2p-1)/2 = (3-4-1)/2 = -1$; *by equation (33)*
$p = 2|H|+|L| = 2+0 = 2$; *by equation (7)*
$s_o = 2|Q|+|L| = 0+0 = 0$; *by equation (8)*
$F = |Q|+|L|+|H| = 0+0+1 = 1$; *by equation (10)*
The compare of data of direct computations with computed values shows full coincidence of results.

Exemple 12.3 *The computation of* $|Q|$, $|L|$, $|H|$, s_o, G , F *for* $2n = 12$
Construction SBR2n:
$2n = 12$; $n = 6$
$\oslash$ - *odd noncomposite;* $\oplus$ - *odd composite ;* $\otimes$ - *even composite .*
$\oslash 1 + 11\oslash = 12(H)$
$\otimes 2 + 10\otimes = 12(E)$
$\oslash 3 + 9\oplus = 12(L)$
$\otimes 4 + 8\otimes = 12(E)$
$\oslash 5 + 7\oslash = 12(H)$
Data of direct computations for even n = 2 , at that it is took into account remark (3.2)
$p = 5$; $s_o = 1$; $|Q| = 0$; $|L| = 1$; $|H| = 2$; $G = 5$; $F = 3$
The computations of parameters of binary representations of the positive integer 2n = 12 with help of arithmetic stated above
n = 12/2 = 6 ;
$G = n-1 = 6-1 = 5$; *by equation (11)*
$|E| = n/2-1 = 3-1 = 2$; *by equation (13)*
$F = n/2 = 6/2 = 3$; *by equation (15)*
$s_o = n-p = 6-5 = 1$; *by equation (17)*
$|L| = n-p-2|Q| = 6-5-0 = 1$; *by equation (26)*
$|L| = p-2|H| = 5-4 = 1$; *by equation (28)*
$|H| = (2|Q|+2p-n)/2 = (0+10-6)/2 = 2$; *by equation (30)*
$|H| = (p-|L|)/2 = (5-1)/2 = 2$; *by equation (29)*
$|Q| = (n-p-|L|)/2 = (6-5-1)/2 = 01$; *by equation (22)*

$|Q| = (n - 2p + 2|H|)/2 = (6 - 10 + 4)/2 = 0$; *by equation (24)*
$|Q| - |H| = 0 - 2 = -2$;
$|Q| - |H| = (n - 2p)/2 = (6 - 10)/2 = -2$; *by equation (32)*
$p = 2|H| + |L| = 4 + 1 = 5$; *by equation (7)*
$s_o = 2|Q| + |L| = 0 + 1 = 1$; *by equation (8)*
$F = |Q| + |L| + |H| = 0 + 1 + 2 = 3$; *by equation (10)*
The compare of data of direct computations with computed values shows full coincidence of results.

Exemple 12.4 *The computation of* $|Q|,\ |L|,\ |H|,\ s_o,\ G,\ F$ *for* $2n = 14$.
$\oslash$ *- odd noncomposite;* $\oplus$ *- odd composite ;* $\otimes$ *- even composite .*

Construction SBR2n:
$2n = 14$; $n = 7$
$\oslash 1 + 13 \oslash = 14(H)$
$\otimes 2 + 12 \otimes = 14(E)$
$\oslash 3 + 11 \oslash = 14(H)$
$\otimes 4 + 10 \otimes = 14(E)$
$\oslash 5 + 9 \oplus = 14(L)$
$\otimes 6 + 8 \otimes = 14(E)$
Data of direct computations for odd $n = 7$ *, at that it is took into account remark (3.2).*
$p* = 5;\ s_o = 1;\ |Q| = 0;\ |L| = 1;\ |H| = 2; G = 6; F = 3$
The computations of parameters of binary representations of the positive integer $2n = 14$ *with help of arithmetic stated above*
$n = 14/2 = 7$;
$G = n - 1 = 7 - 1 = 6$; *by equation (11)*
$|E| = (n - 1)/2 = 3$; *by equation (12)*
$F = (n - 1)/2 = (7 - 1)/2 = 3$; *by equation (16)*
$s_o = n - p - 1 = 7 - 5 - 1 = 1$; *by equation (18)*
$|L| = n - p - 2|Q| - 1 = 7 - 5 - 0 - 1 = 1$; *by equation (27)*
$|L| = p - 2|H| = 5 - 4 = 1$; *by equation (28)*
$|H| = (2|Q| + 2p - n + 1)/2 = (0 + 10 - 7 + 1)/2 = 2$; *by equation (31)*
$|H| = (p - |L|)/2 = (5 - 1)/2 = 2$; *by equation (29)*
$|Q| = (n - p - |L| - 1)/2 = (7 - 5 - 1 - 1)/2 = 0$; *by equation (23)*
$|Q| = (n - 2p + 2|H| - 1)/2 = (7 - 10 + 4 - 1)/2 = 0$; *by equation (25)*
$|Q| - |H| = 0 - 2 = -2$; $|Q| - |H| = (n - 2p - 1)/2 = (7 - 10 - 1)/2 = -2$; *by equation (33)*
$p = 2|H| + |L| = 4 + 1 = 5$; *by equation (7)*

$s_o = 2|Q| + |L| = 0 + 1 = 1$; *by equation (8)*
$F = |Q| + |L| + |H| = 0 + 1 + 2 = 3$; *by equation (10)*
The compare of data of direct computations with computed values shows full coincidence of results.

Exemple 12.5 *The computation of* $|Q|,\ |L|,\ |H|,\ s_o,\ G\ ,\ F$ *for* $2n = 132$

Remark 12.3 *The representations with x is even; y is even are excluded since*
$|Q|.\ |L|,\ |H|,\ s_o,\ p$, *by definitions related to odd composite and noncomposite integers.*
$\oslash$ *- odd noncomposite;* $\oplus$ *- odd composite ;*

Construction SBR2n:
$2n = 132\ ;\ n = 66$
$\oslash 1 + 131 \oslash = 132 (H)$
$\oslash 3 + 129 \oplus = 132 (L)$
$\oslash 5 + 127 \oslash = 132 (H)$
$\oslash 7 + 125 \oplus = 132 (L)$
$\oplus 9 + 123 \oplus = 132$ *(Q)*
$\oslash 11 + 121 \oplus = 132$ *(L)*
$\oslash 13 + 119 \oplus = 132$ *(L)*
$\oplus 15 + 117 \oplus = 132$ *(Q)*
$\oslash 17 + 115 \oplus = 132$ *(L)*
$\oslash 19 + 113 \oslash = 132$ *(H)*
$\oplus 21 + 111 \oplus = 132$ *(Q)*
$\oslash 23 + 109 \oslash = 132$ *(H)*
$\oplus 25 + 107 \oslash = 132$ *(L)*
$\oplus 27 + 105 \oplus = 132$ *(Q)*
$\oslash 29 + 103 \oslash = 132$ *(H)*
$\oslash 31 + 101 \oslash = 132$ *(H)*
$\oplus 33 + 99 \oplus = 132$ *(Q)*
$\oplus 35 + 97 \oslash = 132$ *(L)*
$\oslash 37 + 95 \oplus = 132$ *(L)*
$\oplus 39 + 93 \oplus = 132$ *(Q)*
$\oslash 41 + 91 \oplus = 132$ *(L)*
$\oslash 43 + 89 \oslash = 132$ *(H)*
$\oplus 45 + 87 \oplus = 132$ *(Q)*
$\oslash 47 + 85 \oplus = 132$ *(L)*

$\oplus 49 + 83 \oslash = 132$ *(L)*
$\oplus 51 + 81 \oplus = 132$ *(Q)*
$\oslash 3 + 79 \oslash = 132$ *(H)*
$\oplus 55 + 77 \oplus = 132$ *(Q)*
$\oplus 57 + 75 \oplus = 132$ *(Q)*
$\oslash 59 + 73 \oslash = 132$ *(H)*
$\oslash 61 + 71 \oslash = 132$ *(H)*
$\oplus 63 + 69 \oplus = 132$ *(Q)*
$\oplus 65 + 67 \oslash = 132$ *(L)*
Data of direct computations for even n = 66 ,at the compare it is took into account remark (3.2).
$p = 32;\ s_o = 34;\ |Q| = 11;\ |L| = 12;\ |H| = 10; F = 33$
The computations of parameters of binary representations of the positive integer 2n = 132 with help of arithmetic stated above
at that $p, s_o, |Q|, |L|, |H|$, *are took from the data of direct computations.*
n = 132/2 = 66 ;
$G = n - 1 = 66 - 1 = 65;$ *by equation (11)*
$|E| = n/2 - 1 = 66/2 - 1 = 32;$ *by equation (13)*
$F = n/2 = 66/2 = 33;$ *by equation (15)*
$s_o = n - p = 66 - 32 = 34;$ *by equation (17)*
$|L| = n - p - 2|Q| = 66 - 32 - 22 = 12;$ *by equation (26)*
$|L| = p - 2|H| = 32 - 20 = 12;$ *by equation (28)*
$|H| = (2|Q| + 2p - n)/2 = (22 + 64 - 66)/2 = 10;$ *by equation (30)*
$|H| = (p - |L|)/2 = (32 - 12)/2 = 10;$ *by equation (29)*
$|Q| = (n - p - |L|)/2 = (66 - 32 - 12)/2 = 11;$ *by equation (22)*
$|Q| = (n - 2p + 2|H|)/2 = (66 - 64 + 20)/2 = 11;$ *by equation (24)*
$|Q| - |H| = 11 - 10 = 1;$ *by equation ()*
$|Q| - |H| = (n - 2p)/2 = (66 - 64)/2 = 1;$ *by equation (32)*
$p = 2|H| + |L| = 20 + 12 = 32;$ *by equation (7)*
$s_o = 2|Q| + |L| = 22 + 12 = 34;$ *by equation (8)*
$F = |Q| + |L| + |H| = 11 + 12 + 10 = 33;$ *by equation (10)*
The compare of data of direct computations with computed values shows full coincidence of results.

Exemple 12.6 *The computation of* $|Q|,\ |L|,\ |H|,\ s_o,\ G,\ F$ *for* $2n = 138$ *.*

Remark 12.4 *The representations with x is even; y is even are excluded since*

$|Q|.|L|, |H|, s_o, p,$ *by definitions related to odd composite and prime integers.*
$\oslash$ *- odd noncomposite;* $\oplus$ *- odd composite ;*

Construction SBR2n:
$2n \ = \ 138 \ ; \ n \ = \ 69$
$\oslash 1 + 137 \oslash = 138(H)$
$\oslash 3 + 135 \oplus = 138(L)$
$\oslash 5 + 133 \oplus = 138(L)$
$\oslash 7 + 131 \oslash = 138(H)$
$\oplus 9 + 129 \oplus = 138(Q)$
$\oslash 11 + 127 \oslash = 138(H)$
$\oslash 13 + 125 \oplus = 138(L)$
$\oplus 15 + 123 \oplus = 138(Q)$
$\oslash 17 + 121 \oplus = 138(L)$
$\oslash 19 + 119 \oplus = 138(L)$
$\oplus 21 + 117 \oplus = 138(Q)$
$\oslash 23 + 115 \oplus = 138(L)$
$\oplus 25 + 113 \oslash = 138(L)$
$\oplus 27 + 111 \oplus = 138(Q)$
$\oslash 29 + 109 \oslash = 138(H)$
$\oslash 31 + 107 \oslash = 138(H)$
$\oplus 33 + 105 \oplus = 138(Q)$
$\oplus 35 + 103 \oslash = 138(L)$
$\oslash 37 + 101 \oslash = 138(H)$
$\oplus 39 + 99 \oplus = 138(Q)$
$\oslash 41 + 97 \oslash = 138(H)$
$\oslash 43 + 95 \oplus = 138(L)$
$\oplus 45 + 93 \oplus = 138(Q)$
$\oslash 47 + 91 \oplus = 138(L)$
$\oplus 49 + 89 \oslash = 138(L)$
$\oplus 51 + 87 \oplus = 138(Q)$
$\oslash 53 + 85 \oplus = 138(L)$
$\oplus 55 + 83 \oslash = 138(L)$
$\oplus 57 + 81 \oplus = 138(Q)$
$\oslash 59 + 79 \oslash = 138(H)$
$\oslash 61 + 77 \oplus = 138(L)$
$\oplus 63 + 75 \oplus = 138(Q)$
$\oplus 65 + 73 \oslash = 138(L)$
$\oslash 67 + 71 \oslash = 138(H)$

Data of direct computations for odd $n = 69$, at that it is took into account remark (3.2) .
$p = 33;\ s*_o = 35;\ |Q| = 10;\ |L| = 15;\ |H| = 9; F = 34$
The computations of parameters of binary representations of the positive integer $2n = 132$ with help of arithmetic stated above
at that $p, s_o, |Q|, |L|, |H|,$ are took from the data of direct computations.
$n = 138/2 = 69;$
$G = n - 1 = 69 - 1 = 68;$ *by equation (11)*
$|E| = (n - 1)/2 = 34;$ *by equation (12)*
$F = (n - 1)/2 = (69 - 1)/2 = 34;$ *by equation (16)*
$s_o = n - p - 1 = 69 - 33 - 1 = 35;$ *by equation (18)*
$|L| = n - p - 2|Q| - 1 = 69 - 33 - 20 - 1 = 15;$ *by equation (27)*
$|L| = p - 2|H| = 33 - 18 = 15;$ *by equation (28)*
$|H| = (2|Q| + 2p - n + 1)/2 = (20 + 66 - 69 + 1)/2 = 9;$ *by equation (31)*
$|H| = (p - |L|)/2 = (33 - 15)/2 = 9;$ *by equation (29)*
$|Q| = (n - p - |L| - 1)/2 = (69 - 33 - 15 - 1)/2 = 10;$ *by equation (23)*
$|Q| = (n - 2p + 2|H| - 1)/2 = (69 - 66 + 18 - 1)/2 = 10;$ *by equation (25)*
$|Q| - |H| = 10 - 9 = 1;\ |Q| - |H| = (n - 2p - 1)/2 = (69 - 66 - 1)/2 = 1;$ *by equation (33)*
$p = 2|H| + |L| = 18 + 15 = 33;$ *by equation (7)*
$s_o = 2|Q| + |L| = 20 + 15 = 35;$ *by equation (8)*
$F = |Q| + |L| + |H| = 10 + 15 + 9 = 34;$ *by equation (10)*
The compare of data of direct computations with computed values shows full coincidence of results.

Exemple 12.7 *The computation of $|Q|, |L|, |H|, s_o, G, F$ for $2n = 134$*

Remark 12.5 *The representations with x is even; y is even are excluded since*
$|Q|.|L|, |H|, s_o, p,$ by definitions related to odd composite and prime integers.
$\oslash$ - odd noncomposite; $\oplus$ - odd composite ;

Construction SBR2n::
$2n\ =\ 134\ \ n\ =\ 67$
$\oslash 1 + 133 \oplus = 134(L)$
$\oslash 3 + 131 \oslash = 134(H)$
$\oslash 5 + 129 \oplus = 134(L)$
$\oslash 7 + 127 \oslash = 134(H)$
$\oplus 9 + 125 \oplus = 134(Q)$
$\oslash 11 + 123 \oplus = 134(L)$
$\oslash 13 + 121 \oplus = 134(L)$

$\oplus 15 + 119 \oplus = 134(Q)$
$\oslash 17 + 117 \oplus = 134(L)$
$\oslash 19 + 115 \oplus = 134(L)$
$\oplus 21 + 113 \oslash = 134(L)$
$\oslash 23 + 111 \oplus = 134(L)$
$\oplus 25 + 109 \oslash = 134(L)$
$\oplus 27 + 107 \oslash = 134(L)$
$\oslash 29 + 105 \oplus = 134(L)$
$\oslash 31 + 103 \oslash = 134(H)$
$\oplus 33 + 101 \oslash = 134(L)$
$\oplus 35 + 99 \oplus = 134(Q)$
$\oslash 37 + 97 \oslash = 134(H)$
$\oplus 39 + 95 \oplus = 134(Q)$
$\oslash 41 + 93 \oplus = 134(L)$
$\oslash 43 + 91 \oplus = 134(L)$
$\oplus 45 + 89 \oplus = 134(Q)$
$\oslash 47 + 87 \oplus = 134(L)$
$\oplus 49 + 85 \oplus = 134(Q)$
$\oplus 51 + 83 \oslash = 134(L)$
$\oslash 53 + 81 \oplus = 134(L)$
$\oplus 55 + 79 \oslash = 134(L)$
$\oplus 57 + 77 \oplus = 134(Q)$
$\oslash 59 + 75 \oplus = 134(L)$
$\oslash 61 + 73 \oplus = 134(H)$
$\oplus 63 + 71 \oslash = 134(L)$
$\oplus 65 + 69 \oslash = 134(Q)$
Data of direct computations for odd prime n = 67 , at that it is took into account remark (3.2)
$p* = 30;\ s_o = 36;\ |Q| = 8;\ |L| = 20;\ |H| = 5; F = 33$
The computations of parameters of binary representations of the positive integer $2n = 134$
with help of arithmetic stated above
at that $p, s_o, |Q|, |L|, |H|,$ *are took from the data of direct computations.*
$n = 134/2 = 67;$
$G = n - 1 = 67 - 1 = 66;$ *by equation (11)*
$|E| = (n - 1)/2 = 33;$ *by equation (12)*
$F = (n - 1)/2 = (67 - 1)/2 = 33;$ *by equation (16)*
$s_o = n - p - 1 = 67 - 30 - 1 = 36;$ *by equation (18)*
$|L| = n - p - 2|Q| - 1 = 67 - 30 - 16 - 1 = 20;$ *by equation (27)*
$|L| = p - 2|H| = 30 - 10 = 20;$ *by equation (28)*
$|H| = (2|Q| + 2p - n + 1)/2 = (16 + 60 - 67 + 1)/2 = 5;$ *by equation (31)*

$|H| = (p-|L|)/2 = (30-20)/2 = 5$; *by equation (29)* $|Q| = (n-p-|L|-1)/2 = (67-30-20-1)/2 = 8$; *by equation (23)*
$|Q| = (n-2p+2|H|-1)/2 = (67-60+10-1)/2 = 8$; *by equation (25)*
$|Q| - |H| = 8-5 = 3$;
$|Q| - |H| = (n-2p-1)/2 = (67-60-1)/2 = 3$; *by equation (33)*
$p = 2|H| + |L| = 10+20 = 30$; *by equation (7)*
$s_o = 2|Q| + |L| = 16+20 = 36$; *by equation (8)*
$F = |Q| + |L| + |H| = 8+20+5 = 33$; *by equation (10)*
The compare of data of direct computations with computed values shows full coincidence of results.

13 The control of equation for $\Delta|Q|$

Exemple 13.1 *The computation* $\Delta|Q|$ *for n is even:* $n = 2$, $2n = 4$.
Data of direct computations we take from exmp 12.1
$p = 2;\ s_o = 0;\ |Q| = 0;\ |L| = 0;\ |H| = 1;\ G = 1;\ F = 1$
$|Q|_m = ((n-p)^2/2n) = ((2-2)^2/4) = 0.$ *by equation (46)*
$|H|_m = (p^2)/2n = (2^2)/4 = 1$; *by equation (50)*
$\Delta|Q| = |Q|_m - |Q| = 0-0 = 0$; *by definition (7.1)*
$\Delta|H| = |H|_m - |H| = 1-1 = 0$; *by definition (7.2)*
$|Q|_b = (n-2p)/2 = (2-4)/2 = -1$ *; by equation (38)*
The computation of Delta$|Q|$ *by equation (51) .*
$\Delta|Q| = |Q|_m - |H|_m - |Q|_b + \Delta|H| = 0-1+1-0 = 0$.
The compare of $\Delta|Q|$ *computed by data of direct computations with computed value*
by equation (51) shows full coincidence of results.

The computation $\Delta|Q|$ for n is odd prime: $n = 3; 2n = 6$.
Data of direct computations for odd prime n = 3 we take from exmp 12.2
$p* = 2;\ s_o = 0;\ |Q| = 0;\ |L| = 0;\ |H| = 1; G = 2; F = 1$
$|Q|_b = |Q| - |H| = 0-1 = -1$. by equation (42)
$|Q|_m = ((n-p-1)^2)/2n = ((3-2-1)^2/134) = 0.$ by equation (47)
$|H|_m = (p^2)/2n = (2^2)/6 = 2/3$; by equation (50)
$\Delta|Q| = |Q|_m - |Q| = 0-8 = 0$; by definition (7.1)
$\Delta|H| = |H|_m - |H| = 2/3-1 = -1/3$; by definition (7.2)
$|Q|_b = (n-2p-1)/2 = (3-4-1)/2 = -1$; by equation (39)
The computation of $\Delta|Q|$ by equation (51) .
$\Delta|Q| = |Q|_m - |H|_m - |Q|)b + \Delta|H| = 0-2/3+1-1/3 = 0$.
The compare of $\Delta|Q|$ computed by data of direct computations with computed value
by equation (51) shows full coincidence of results.

Exemple 13.2 *The computation* $\Delta|Q|$ *for n is even: n = 6, 2n = 12 .*
Data of direct computations we take from exmp 3.3
$p = 5;\ s_o = 1;\ |Q| = 0;\ |L| = 1;\ |H|\ =\ 2;\ G = 5;\ F = 3$
$|Q|_m = ((n-p)^2/2n) = ((6-5)^2/12) = 1/12.$ *by equation (46)*
$|H|_m = (p^2)/2n = (5^2)/12 = 25/12;$ *by equation (50)*
$\Delta|Q| = |Q|_m - |Q| = 1/12 - 0 = 1/12;$ *by definition (7.1)*
$\Delta|H| = |H|_m - |H| = 25/12 - 2 = 1/12;$ *by definition (7.2)*
$|Q|_b = (n-2p)/2 = (6-10)/2 = -2$ *; by equation (38)*
The computation of Delta|Q| by equation (51) .
$\Delta|Q| = |Q|_m - |H|_m - |Q|_b + \Delta|H| = 1/12 - 25/12 + 2 + 1/12 = 1/12$.
The compare of $\Delta|Q|$ *computed by data of direct computations with computed value*
by equation (51) shows full coincidence of results.

Exemple 13.3 *The computation* $\Delta|Q|$ *for n is odd prime:* $n = 7; 2n = 14$
.
Data of direct computations for odd prime n = 67 we take from exmp 12.4
$p* = 5;\ s_o = 1;\ |Q| = 0;\ |L| = 1;\ |H| = 2; G = 6; F = 3$
$|Q|_b = |Q| - |H| = 0 - 2 = -2$. *by equation (42)*
$|Q|_m = ((n-p-1)^2)/2n = ((7-5-1)^2/14) = 1/14.$ *by equation (47)*
$|H|_m = (p^2)/2n = (5^2)/14 = 25/14;$ *by equation (50)*
$\Delta|Q| = |Q|_m - |Q| = 1/14 - 0 = 1/14;$ *by definition(7.1)*
$\Delta|H| = |H|_m - |H| = 25/14 - 2 = -3/14;$ *by definition (7.2)*
$|Q|_b = (n-2p-1)/2 = (7-10-1)/2 = -2$ *; by equation (39)*
The computation of $\Delta|Q|$ *by equation (51) .*
$\Delta|Q| = |Q|_m - |H|_m - |Q|)b + \Delta|H| = 1/14 - 25/14 + 2 - 3/14 = 1/14$.
The compare of $\Delta|Q|$ *computed by data of direct computations with computed value*
by equation (51) shows full coincidence of results.

Exemple 13.4 *The computation* $\Delta|Q|$ *for n is even: n = 66, 2n = 132 .*
Data of direct computations we take from exmp 12.5
$p = 32;\ s_o = 34;\ |Q| = 11;\ |L| = 12;\ |H| = 10; F = 33$
$|Q|_m = ((n-p)^2/2n) = ((66-32)^2/132) = 8,7575757.$ *by equation (46)*
$|H|_m = (p^2)/2n = (32^2)/132 = 7,7575757;$ *by equation (50)*
$\Delta|Q| = |Q|_m - |Q| = 8,7575757 - 11 = -2,2424243;$ *by definition (7.1)*
$\Delta|H| = |H|_m - |H| = 7,7575757 - 10 = -2,2424243;$ *by definition (7.2)*
$|Q|_b = (n-2p)/2 = (66-64)/2 = 1$ *; by equation (38)*
The computation of Delta|Q| by equation (51) .
$\Delta|Q| = |Q|_m - |H|_m - |Q|_b + \Delta|H| = 8,7575757 - 7,7575757 - 1 - 2,2424243 = -2,2424243$.

The compare of $\Delta|Q|$ *computed by data of direct computations with computed value*
by equation (51) shows full coincidence of results.

Exemple 13.5 *The computation* $\Delta|Q|$ *for n is odd :* $n = 69; 2n = 138$ *.*
Data of direct computations we take from exmp 12.6
$p = 33;\ s_o = 35;\ |Q| = 10;\ |L| = 15;\ |H| = 9;$
$|Q|_m = ((n-p-1)^2/2n) = ((69-33-1)^2/138) = 8,8768115.$ *by equation (47)*
$|H|_m = (p^2)/2n = (33^2)/138 = 7,8913043;$ *by equation (50)*
$\Delta|Q| = |Q|_m - |Q| = 8,8768115 - 10 = -1,123189;$ *by definition (7.1)*
$\Delta|H| = |H|_m - |H| = 7,8913043 - 9 = -1,1086957;$ *by definition (7.2)*
$|Q|_b = (n - 2p - 1)/2 = (69 - 66 - 1)/2 = 1$ *; by equation (39)*
The computation of $\Delta|Q|$ *by equation (51) .*
$\Delta|Q| = |Q|_m - |H|_m - |Q|)b + \Delta|H| = 8,8768115 - 7,8913043 - 1 - 1,1086957 = -1,123189$.
The compare of $\Delta|Q|$ *computed by data of direct computations with computed value*
by equation (51) shows full coincidence of results.

Exemple 13.6 *The computation* $\Delta|Q|$ *for n is odd prime:* $n = 67; 2n = 134$ *.*
Data of direct computations for odd prime n = 67 we take from exmp 12.7
$p = 30;\ s_o = 36;\ |Q| = 8;\ |L| = 20;\ |H| = 5;$
$|Q|_b = |Q| - |H| = 8 - 5 = 3$ *. by equation (42)*
$|Q|_m = ((n-p-1)^2)/2n = ((67-30-1)^2/134) = 9,6716417.$ *by equation (47)*
$|H|_m = (p^2)/2n = (30^2)/134 = 6,7164179;$ *by equation (50)*
$\Delta|Q| = |Q|_m - |Q| = 9,6716417 - 8 = 1,6716417;$ *by definition (7.1)*
$\Delta|H| = |H|_m - |H| = 6,7164179 - 5 = 1.7164179;$ *by definition (7.2)*
$|Q|_b = (n - 2p - 1)/2 = (67 - 60 - 1)/2 = 3$ *; by equation (39)*
The computation of $\Delta|Q|$ *by equation (51) .*
$\Delta|Q| = |Q|_m - |H|_m - |Q|)b + \Delta|H| = 2,9552238 - 3 + 1,7164179 = 1,6716417$.
The compare of $\Delta|Q|$ *computed by data of direct computations with computed value*
by equation (51) shows full coincidence of results.

Hence equation (51) is true $\forall n > 1$.

13.1 The examples of the correspondence of 2n included between neighboring values of p to it

For example we consider the set of XUY for 2n = 32. We can see that the set of XUY consist of several sets. To each set of subsets is corresponded at the same value of p. Next for example of computation s_o we show that use at the same value of p for corresponding set gives corrected result which is controlled by direct computation s_o for each subsets of corresponding set.

Exemple 13.7 *Let we have the sat of numbers*
$X\ U\ Y\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,\}\ U$
$\{17,18,19,20,21,22,23,24,25,26,27,28,29,30,31\}$
at that n = 16 is excluded by remark (3.1).
Corollaries (3.1 and 3.2 and 3.3) are changed following way:
$s_e* = s_e - 1; s_o* = s_o - 1 : p* = p - 1$
since in this case is excluded one n.
We can see that :
p = 2 correspond to subset for 2n = 4
p = 3 correspond to subset for 2n = 6
p = 4 correspond to subsets for 2n = 8,10
p = 5 correspond to subset for 2n = 12
p = 6 correspond to subsets for 2n = 14,16
p = 7 correspond to subset for 2n = 18
p = 8 correspond to subsets for 2n = 20,22
p = 9 correspond to subsets for 2n = 24,26,28
p = 10 correspond to subset for 2n = 30
We control the following correspondences: p = 4 correspond 2n = 8,10 subsets: $X\ U\ Y\{1,2,3\}U\{5,6,7\}$
n =4 is excluded by rem.(3.1).
$X\ U\ Y\{1,2,3,4\}U\{6,7,8,9\}$ *n =5 is excluded (rem3.1).*
p = 6 correspond 2n = 14,16 subsets:$X\ U\ Y\{1,2,3,4,5,6\}U$
$\{8,9,10,11,12,13\}$ *n =7 is excluded (rem3.1).*
$X\ U\ Y\{1,2,3,4,5,6,7\}U\{9,10,11,12,13,14,15\}$ *n =8 is excluded (rem3.1).*
p = 8 correspond 2n = 20,22 subsets:$X\ U\ Y\{1,2,3,4,5,6,7,8,9\}U$
$\{11,12,13,14,15,16,17,18,19\}$ *n =10 is excluded (rem3.1). ;*
$X\ U\ Y\{1,2,3,4,5,6,7,9,10\}U$
$\{12,13,14,15,16,17,18,19,20,21\}$ *n =11 is excluded (rem3.1).*
p = 9 correspond 2n = 24,26,28 subsets:$X\ U\ Y\{1,2,3,4,5,6,7,8,9,10,11\}U$
$\{13,14,15,16,17,18,19,20,21,22,23\}$ *n =12 is excluded (rem3.1).*
$X\ U\ Y\{1,2,3,4,5,6,7,9,10,11,12\}U$

$\{14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25\}$ *n =13 is excluded (rem3.1).*
X U Y $\{1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13\}U$
$\{15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27\}$ *n =14 is excluded (rem3.1).*
By equation (17) we get :
for 2n - 8; n = 4 ; p = 4 .
$s_o = n - p = 4 - 4 = 0$.
By direct computation for $2n = 8,\ s_o = 0.$
for 2n - 10; n = 5 ; p = 3 (corrolary (3.3) with real changes).*
By equation (18) we get :
$s_o = n - p - 1 = 5 - 3 - 1 = 1$
By direct computation for $2n = 10,\ s_o = 1.$
for 2n - 14; n = 7 ; p = 5 (corrolary (3.3) with real changes) .*
By equation (18) we get :
$s_o = n - p - 1 = 7 - 5 - 1 = 1$
By direct computation for $2n = 14,\ s_o = 1.$
for 2n - 16; n = 8 ; p = 6 .
By equation (17) we get :
$s_o = n - p = 8 - 6 = 2$
By direct computation for $2n = 16,\ s_o = 2.$
for 2n - 20; n = 10 ; p = 8 .
By equation (17) we get :
$s_o = n - p = 10 - 8 = 2$
By direct computation for $2n = 20,\ s_o = 2.$
for 2n - 22; n = 11; p = 7 (corrolary (3.3) with real changes) .*
By equation (18) we get :
$s_o = n - p - 1 = 11 - 7 - 1 = 3$
By direct computation for $2n = 22,\ s_o = 3.$
for 2n - 24; n = 12 ; p = 9 .
By equation (17) we get :
$s_o = n - p = 12 - 9 = 3$
By direct computation for $2n = 24,\ s_o = 3.$
for 2n - 26; n = 13; p = 8 (corrolary (3.3) with real changes) .*
By equation (18) we get :
$s_o = n - p - 1 = 13 - 8 - 1 = 4$
By direct computation for $2n = 26,\ s_o = 4.$
for 2n - 28; n = 14 ; p = 9 .
By equation (17) we get :
$s_o = n - p = 14 - 9 = 5$
By direct computation for $2n = 28,\ s_o = 5.$
Thus for several 2n it takes at the same p at computations by equations ABR2n and these computations are correct.

Hence for each p correspond to finite set of 2n included between neighboring values of p

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