Article 26:

Hierarchy of Theories of Unified Gravity and Dynamics at the Neighborhood of Several Gravitational Field Sources. Part II.

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Corresponding to the special theory of relativity/intrinsic special theory of relativity (SR/ϕSR) and the theory of gravitational relativity/intrinsic intrinsic theory of gravitational relativity (TGR/ϕTGR), on flat four-dimensional spacetime/flat two-dimensional intrinsic spacetime, and the metric theory of absolute intrinsic motion (ϕMAM) and metric theory of absolute intrinsic gravity (ϕMAG), on curved 'two-dimensional' absolute intrinsic spacetime, at the second stage of evolutions of spacetime/intrinsic spacetime and parameters/intrinsic parameters in every gravitational field, there are unified SR/ϕSR and TGR/ϕTGR on flat four-dimensional spacetime/flat two-dimensional intrinsic spacetime, denoted by SR/ϕSR $\cup$ TGR/ϕTGR, and unified ϕMAM and ϕMAG on curved 'two-dimensional' absolute intrinsic spacetime, denoted by ϕMAM $\cup$ ϕMAG. These unified theories are accomplished in this article for two cases of, (i) a test particle is in motion at a large velocity relative to an observer at the neighborhood of one, two and several gravitational field sources, and (ii) a gravitational field source, such as a massive star or a neutron star, is in motion at a large velocity relative to an observer in a region of space that is devoid of the gravitational field of any other source. These entail essentially the incorporation of the velocity v of relative motion into the results of TGR and of absolute intrinsic dynamical speed $\phi\hat{V}_d$ of absolute intrinsic motion into the absolute intrinsic metric tensors of ϕMAG, developed in the previous articles for the two situations. It is shown that the existing special theory of relativity, referred to as Lorentz-Einstein-Minkowski special relativity (LEM), is valid strictly for the relative motion of the electron or its anti-particle.

1 Introduction

The incompatibility of Newton's laws of gravity and the special theory of relativity is well discussed in the literature [1]. Newton's laws of gravity is given in differential equation form as follows

$$\mathrm{d}^2x^i/\mathrm{d}t^2 = -\partial\Phi/\partial x^i; \quad i = 1, 2, 3 \tag{1}$$

$$\nabla^2\Phi = 4\pi G\varrho \tag{2}$$

As usually remarked, these equations cannot be incorporated into special relativity because Eq. (1) is in three-dimensional rather than four-dimensional form. It must be modified into a four-dimensional form for it to be compatible with special relativity. It is also usually remarked that the appearance of the three-dimensional

Laplacian operator instead of the D'Alembertian operator in Eq. (2), means that the potential Φ responds instantaneously to changes in ϱ at arbitrarily large distance. This means that the Newtonian gravitational field propagates with infinite velocity, which is outside the scope of special relativity.

All initial attempts to resolve the problem outside the framework of the general theory of relativity (GR) by considering Φ as a scalar, then as a vector and finally as a symmetric tensor field in flat space, (pages 1070-1071 and 181-186 of [1], all failed on the ground of consistency, completeness or agreement with experiment (pages 1066-1068 of [1]. Various later attempts to overcome the inconsistency in the symmetric tensor field theory based on the modification of the flat space tensor theory in the context of relativistic field theory, led uniquely to the 1915 general relativity on curved four-dimensional spacetime (page 186 of [1].

The incompatibility of the special theory of relativity (SR) and gravity is usually considered to have been resolved on curved spacetime in the general theory of relativity (GR), as demonstrated on pages 121 – 132 of [2]. The conclusion of that demonstration is that the metric tensor $g_{\mu\nu}$ of GR unifies the speed of the test particle and gravitational potential Φ of the field source on curved spacetime. However this conclusion is faulty for at least one reason when considered from the perspective of the present theory. This is the fact that the parameter $2\Phi = 2GM/R$ in $g_{\mu\nu}$, which we would call the square of gravitational speed and denote by $V_g(R)^2$ in the present theory, is an absolute parameter in the context of the special theory of relativity (SR). That is, $2\Phi = V_g(R)^2$ is invariant with the observer or frame of reference. Consequently it cannot appear at equal footing (or cannot be mixed) with the square of relative dynamical speed v^2 of SR in a unification theory of GR and SR. The only square of dynamical speed that can appear at equal footing with 2Φ is the square of absolute dynamical speed $\hat{V}_d^2$, which is invariant with the observer like 2Φ (or $V_g(R)^2$). It can thus be said that, from the perspective of the present theory, there is no valid unified theory of gravity and relative motion in the context of the general theory of relativity.

On the other hand, there is a hierarchy of unified theories of gravity and dynamics in the context of the present theory, which is composed of the unified theory of gravitational relativity an the special theory of relativity (TGR $\cup$ SR) on the flat four-dimensional relativistic spacetime (Σ, ct); unified intrinsic theory of gravitational relativity and intrinsic special theory of relativity (ϕTGR $\cup$ ϕSR) on the flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ and unified metric theory of absolute intrinsic gravity and metric theory of absolute intrinsic motion (ϕMAG $\cup$ ϕMAM) on curved 'two-dimensional' absolute intrinsic spacetime ($\phi\hat{\rho}, \phi\hat{c}\phi\hat{t}$), in every gravitational field. Since (TGR $\cup$ SR) on the flat spacetime (Σ, ct) is mere outward manifestation of (ϕTGR $\cup$ ϕSR) on flat ($\phi\rho, \phi c\phi t$), the two theories are counted as one and denoted by (TGR $\cup$ SR)/(ϕTGR $\cup$ ϕSR) and added to (ϕMAG $\cup$ ϕMAM) to have a two-theory approach to the unification of gravity and dynamics in the present theory.

The two-theory approach to the unification of gravity and dynamics namely, (TGR $\cup$ SR)/(ϕTGR $\cup$ ϕSR) and (ϕMAG $\cup$ ϕMAM) have been well accomplished at the exterior of one gravitational field source in [3–5]. They shall be extended to the exterior neighborhood of several isolated gravitational field sources in this second part of this paper, upon the corresponding two-theory approach to the theories of gravity at the neighborhood of several isolated gravitational field sources developed in the first part [6].

The unification of the special theory of relativity/intrinsic special theory of relativity (SR/ϕSR) and the theory of gravitational relativity/intrinsic theory of gravitational relativity (TGR/ϕTGR) at the neighborhood of several isolated gravitational field sources shall be derived in section 2, while the unification of the metric theory of absolute intrinsic gravity and absolute intrinsic motion (ϕMAG $\cup$ ϕMAM) at the neighborhood of several isolated gravitational field sources shall be derived in section 3.

The effect of the gravitational field of the test particle (assumed to be an extended object), shall be incorporated into the results of sections 2 and 3 in section 4, and in section 5, the combined special theory of relativity and the theory of gravitational relativity (TGR $\cup$ SR) and combined metric theory of absolute intrinsic motion and metric theory of absolute intrinsic gravity (ϕMAG $\cup$ ϕMAM) for a gravitational field source in motion relative to the observer, in the absence of external gravitational field of any other source, shall be inferred from the results of section 4.

2 Unification of the special theory of relativity/intrinsic special theory of relativity and theory of gravitational relativity/intrinsic theory of gravitational relativity at the neighborhood of several isolated gravitational filed sources

The flat four-dimensional relativistic spacetime (Σ, ct) (with constant Lorentzian metric tensor), established in the context of the theory of gravitational relativity (TGR), in all finite neighborhood of any number N of gravitational field sources scattered arbitrarily in the Euclidean 3-space Σ, derived in the first part of this article [6], constitute a platform for the special theory of relativity (SR) at the neighborhood of the N isolated gravitational field sources. The inertial masses m of test particles that evolve on the flat (Σ, ct) in the context of TGR, are involved in relative motions on (Σ, ct) in the context of SR, at the neighborhood of the N gravitational field sources.

Likewise the flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ that underlies the flat relativistic spacetime (Σ, ct), established in the context of the intrinsic theory of gravitational relativity (ϕTGR) in all finite neighborhood of N isolated gravitational field sources, derived in [6], constitutes a platform for the intrinsic special theory of relativity (ϕSR) at the neighborhood of the N isolated gravitational field sources.

2.1 Validating local Lorentz invariance at the neighborhood of N isolated gravitational field sources

Although spacetime is flat, gravitational potential varies with position in space at the neighborhood of N isolated gravitational field sources. A local Lorentz frame is a neighborhood about every given point in spacetime within which gravitational potential is constant or within which gravitational potential can be taken to be constant. Lorentz transformation/intrinsic Lorentz transformation must be restricted to local Lorentz frames, thereby being local Lorentz transformation/intrinsic local Lorentz transformation (LLT/ϕLLT), at the neighborhood of N isolated gravitational field sources, as done at the neighborhood of one gravitational field source in [3] and [4].

Intrinsic local Lorentz transformation and its inverse take on their usual forms in terms of intrinsic affine coordinates within local Lorentz frames on the flat relativistic intrinsic metric spacetime $(\phi\rho, \phi c\phi t)$ of ϕTGR, at the neighborhood of N isolated gravitational field sources as derived formally in [3] as follows

$$\left.\begin{array}{lcl} \phi\tilde{t} &=& \phi\gamma(\phi v)(\phi\tilde{\tilde{t}} - \dfrac{\phi v}{\phi c_\gamma^2}\phi\tilde{\tilde{x}}) \ ; \\ && (\text{w.r.t. } 1 - \text{observer } \tilde{\text{P}}\text{eter in } ct) \ ; \\ \phi\tilde{x} &=& \phi\gamma(\phi v)(\phi\tilde{\tilde{x}} - \phi v\phi\tilde{\tilde{t}}) \ ; \\ && (\text{w.r.t. } 3 - \text{observer Peter in } \Sigma) \end{array}\right\} \qquad (3)$$

and

$$\left.\begin{aligned} \phi\tilde{\tilde{t}} &= \phi\gamma(\phi v)(\phi\tilde{t} + \frac{\phi v}{\phi c_\gamma^2}\phi\tilde{x})\ ; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma)\ ; \\ \phi\tilde{\tilde{x}} &= \phi\gamma(\phi v)(\phi\tilde{x} + \phi v\phi\tilde{t})\ ; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{4}$$

where

$$\phi\gamma(\phi v) = (1 - \phi v^2/\phi c_\gamma^2)^{-1/2} \tag{5}$$

The ϕLLT (3) or its inverse (4) yields the following invariance,

$$\phi c_\gamma^2 \phi\tilde{\tilde{t}}^2 - \phi\tilde{\tilde{x}}^2 = \phi c_\gamma^2 \phi\tilde{t}^2 - \phi\tilde{x}^2 \tag{6}$$

This intrinsic Lorentz invariance obtains within every local Lorentz frame and it is hence intrinsic local Lorentz invariance (ϕLLI), at the neighborhood of N isolated gravitational field sources. We have thus validated ϕLLI at the neighborhood of any number N of isolated gravitational field sources.

Now the intrinsic local Lorentz transformation (3) and its inverse (4), in the context of ϕSR on flat two-dimensional intrinsic spacetime $(\phi\rho, \phi c\phi t)$, are made manifest in local Lorentz transformation and its inverse on the flat four-dimensional relativistic spacetime (Σ, ct) respectively as follows

$$\left.\begin{aligned} \tilde{t} &= \gamma(v)(\tilde{\tilde{t}} - \frac{v}{c_\gamma^2}\tilde{\tilde{x}})\ ; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct)\ ; \\ \tilde{x} &= \gamma(v)(\tilde{\tilde{x}} - v\tilde{\tilde{t}})\ ;\ \tilde{y} = \tilde{\tilde{y}}\ ;\ \tilde{z} = \tilde{\tilde{z}}\ ; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma) \end{aligned}\right\} \tag{7}$$

and

$$\left.\begin{aligned} \tilde{\tilde{t}} &= \gamma(v)(\tilde{t} + \frac{v}{c_\gamma^2}\tilde{x})\ ; \\ &\quad (\text{w.r.t. } 3-\text{observer Peter in } \Sigma)\ ; \\ \tilde{\tilde{x}} &= \gamma(v)(\tilde{x} + v\tilde{t})\ ;\ \tilde{\tilde{y}} = \tilde{y}\ ;\ b\tilde{a}rz = \tilde{z}\ ; \\ &\quad (\text{w.r.t. } 1-\text{observer } \tilde{\text{P}}\text{eter in } ct) \end{aligned}\right\} \tag{8}$$

where

$$\gamma(v) = (1 - v^2/c_\gamma^2)^{-1/2} \tag{9}$$

The LLT (7) or its inverse (8) yields the following invariance,

$$c_\gamma^2\tilde{\tilde{t}}^2 - \tilde{\tilde{x}}^2 = c_\gamma^2\tilde{t}^2 - \tilde{x}^2 \tag{10}$$

This Lorentz invariance obtains within every local Lorentz frame and it is hence local Lorentz invariance (LLI), at the neighborhood of N isolated gravitational field sources. The LLI has thus been validated at the neighborhood of any number N of isolated gravitational field sources.

2.2 Intrinsic time dilation and intrinsic length contraction formulae and some intrinsic parameter relations in the context of combined ϕTGR and ϕSR at the neighborhood of N isolated gravitational field sources

The special-relativistic intrinsic time dilation and special-relativistic intrinsic length contraction formulae implied by systems (3) and (4), derived formally in [7] and [3] are the following

$$\phi\tilde{\tilde{t}} = \phi\gamma(\phi v)\phi\tilde{t} = (1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}\phi\tilde{t} \tag{11}$$

$$\phi\tilde{\tilde{x}} = \phi\gamma(\phi v)^{-1}\phi\tilde{x} = (1 - \frac{\phi v^2}{\phi c_\gamma^2})^{1/2}\phi\tilde{x} \tag{12}$$

Intrinsic gravitational time dilation and intrinsic gravitational length contraction must then be incorporated into Eqs. (11) and (12) respectively. This is possible because the intrinsic affine space and intrinsic affine time coordinates $\phi\tilde{x}$ and $\phi\tilde{t}$ have suffered intrinsic gravitational contraction and intrinsic gravitational dilation from the primed intrinsic affine coordinates $\phi\tilde{x}'$ and $\phi\tilde{t}'$ respectively, in the context of ϕTGR, at the neighborhood of N isolated gravitational field sources, as has been developed in the first part of this paper [6] and expressed as Eqs. (18) and (20) of that paper. Let us replace $d\phi t$ and $d\phi t'$ by $\phi\tilde{t}$ and $\phi\tilde{t}'$ respectively in Eq. (18) of [6] and $d\phi\rho$ and $d\phi\rho'$ by $\phi\tilde{x}$ and $\phi\tilde{x}'$ respectively in Eq. (20) of that paper to have

$$\begin{aligned} \phi\tilde{t} &= \phi\bar{\gamma}_g^t\phi\tilde{t}' \\ &= \prod_{i=1}^{N}(1 - \frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i'\phi c_g^2})^{-1/2}\phi\tilde{t}' \end{aligned} \tag{13}$$

$$\begin{aligned} \phi\tilde{x} &= (\phi\bar{\gamma}_g^t)^{-1}\phi\tilde{x}' \\ &= \prod_{i=1}^{N}(1 - \frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i'\phi c_g^2})^{1/2}\phi\tilde{x}' \end{aligned} \tag{14}$$

Equations (13) and (14) in the context of ϕTGR must then be substituted into

Eqs. (11) and (12) in the context of ϕSR to have

$$\begin{aligned}\phi\tilde{\tilde{t}} &= \phi\bar{\gamma}_g^t\phi\gamma(v)\phi\tilde{t}' \\ &= \prod_{i=1}^{N}(1-\frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i'\phi c_g^2})^{-1/2}(1-\frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}\phi\tilde{t}' \qquad (15)\\ \phi\tilde{\tilde{x}} &= (\phi\bar{\gamma}_g^t)^{-1}\phi\gamma(v)^{-1}\phi\tilde{x}' \\ &= \prod_{i=1}^{N}(1-\frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i'\phi c_g^2})^{1/2}(1-\frac{\phi v^2}{\phi c_\gamma^2})^{1/2}\phi\tilde{x}' \qquad (16)\end{aligned}$$

Equations (15) and (16) are the gravitational-relativistic cum special-relativistic intrinsic time dilation and gravitational-relativistic cum special-relativistic intrinsic length contraction formulae at the neighborhood of N isolated gravitational field sources.

If the test particle is in motion at constant acceleration $\vec{a}$ within a local Lorentz frame relative to the observer, then the constant intrinsic speed ϕv relative to the observer must be replaced with $2\phi a\phi\tilde{x}'$ in Eqs. (15) and (16), as follows from the intrinsic theory of relativity of uniformly accelerated systems in [8], to have

$$\begin{aligned}\phi\tilde{\tilde{t}} &= \phi\bar{\gamma}_g^t\phi\gamma(\tilde{x}')\phi\tilde{t}' \\ &= \prod_{i=1}^{N}(1-\frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i'\phi c_g^2})^{-1/2}(1-\frac{2\phi a\phi\tilde{x}'}{\phi c_\gamma^2})^{-1/2}\phi\tilde{t}' \qquad (17)\\ \phi\tilde{\tilde{x}} &= (\phi\bar{\gamma}_g^t)^{-1}\phi\gamma(\tilde{x}')^{-1}\phi\tilde{x}' \\ &= \prod_{i=1}^{N}(1-\frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i'\phi c_g^2})^{1/2}(1-\frac{2\phi a\phi\tilde{x}'}{\phi c_\gamma^2})^{1/2}\phi\tilde{x}' \qquad (18)\end{aligned}$$

The uniformly accelerated intrinsic motion of the test particle within a local Lorentz frame relative to the observer is assumed to start from zero intrinsic speed ($\phi v_0 = 0$) from an origin $\phi\tilde{x}' = 0$ of the intrinsic coordinate $\phi\tilde{x}'$ within the local Lorentz frame, so that $\phi v^2 = 2\phi a\phi\tilde{x}'$.

The special-relativistic intrinsic mass and special-relativistic intrinsic kinetic energy relations are given respectively as

$$\begin{aligned}\phi\overline{m} &= \phi\gamma(v)\phi m = \phi m(1\phi v^2\phi c_\gamma^2)^{-1/2} \qquad (19)\\ \phi\overline{T} &= \phi m\phi c_\gamma^2(\phi\gamma(\phi v)-1) \\ &= \phi m\phi c_\gamma^2[(1-\frac{\phi v^2}{\phi c_\gamma^2})^{-1/2}-1] \qquad (20)\end{aligned}$$

where $\overline{m}$ is the special-relativistic inertial mass that evolves in $\phi\rho$ from ϕm in the context of ϕSR and ϕm is the intrinsic inertial mass that evolves in $\phi\rho$ from the intrinsic rest mass ϕm_0 in the context of ϕTGR.

Now the intrinsic inertial mass ϕm is related to the intrinsic rest mass ϕm_0 in the context of ϕTGR at the neighborhood of N isolated gravitational field sources by Eq. (24) of the first part of this paper [6], which shall be reproduced here as follows

$$\phi\overline{m} = \phi m_0(\phi\overline{\gamma}_g^t)^{-2} = \phi m_0 \prod_{i=1}^{N}(1 - \frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i' \phi c_g^2}) \tag{21}$$

By substituting Eq. (21) in ϕTGR into Eqs. (19) and (20) in ϕSR we have respectively as follows

$$\begin{aligned} \phi\overline{m} &= \phi m_0(\phi\overline{\gamma}_g^t)^{-2}\phi\gamma(\phi v) \\ &= \phi m_0 \prod_{i=1}^{N}(1 - \frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i' \phi c_g^2})(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2} \end{aligned} \tag{22}$$

$$\begin{aligned} \phi\overline{T} &= \phi m \phi c_\gamma^2(\phi\overline{\gamma}_g^t)^{-2}(\phi\gamma(\phi v) - 1) \\ &= \phi m \phi c_\gamma^2 \prod_{i=1}^{N}(1 - \frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i' \phi c_g^2})[(1 - \frac{\phi v^2}{\phi c_\gamma^2})^{-1/2} - 1] \end{aligned} \tag{23}$$

Equations (22) and (23) express intrinsic mass relation and intrinsic kinetic energy relation in the context of combined ϕSR and ϕTGR at the neighborhood of N gravitational field sources that are scattered arbitrarily in space about the moving test particle. Again Eqs. (22) and (23) must be modified as follows if the test particle is in uniformly accelerated motion within the local Lorentz frame relative to the observer

$$\begin{aligned} \phi\overline{m} &= \phi m_0(\phi\overline{\gamma}_g^t)^{-2}\phi\gamma(\phi\tilde{x}') \\ &= \phi m_0 \prod_{i=1}^{N}(1 - \frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i' \phi c_g^2})(1 - \frac{2\phi a\,\phi\tilde{x}'}{\phi c_\gamma^2})^{-1/2} \end{aligned} \tag{24}$$

$$\begin{aligned} \phi\overline{T} &= \phi m \phi c_\gamma^2(\phi\overline{\gamma}_g^t)^{-2}(\phi\gamma(\phi\tilde{x}') - 1) \\ &= \phi m \phi c_\gamma^2 \prod_{i=1}^{N}(1 - \frac{2G\phi M_{0\mathrm{a}\,i}}{\phi r_i' \phi c_g^2})[(1 - \frac{2\phi a\,\phi\tilde{x}'}{\phi c_\gamma^2})^{-1/2} - 1] \end{aligned} \tag{25}$$

Extension to other intrinsic parameter relations in the context of combined ϕTGR and ϕSR at the neighborhood of N isolated gravitational field sources is straight forward.

2.3 Time dilation and length contraction formulae and some parameter relations in the context of combined TGR and SR at the neighborhood of N isolated gravitational field sources

We must simply obtain the outward manifestations on the flat relativistic four-dimensional spacetime (Σ, ct) of the results of the preceding sub-section on the flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$. That is, we must obtain the outward manifestations in (Σ, ct) of Eqs. (9) and (10), Eqs. (11) and (12), Eqs. (14) and (17) and Eqs. (18) and (19).

For the outward manifestation of the intrinsic time dilation formulae (9) and (11), we must simply drop the symbol ϕ in those equations to have the corresponding time dilation formulae in the context of combined TGR and SR at the neighborhood of N isolated gravitational field sources as follows

$$\begin{aligned} \tilde{\tilde{t}} &= \overline{\gamma}_g^t \gamma(v)\tilde{t}' \;; \\ &= \prod_{i=1}^{N}(1 - \frac{2GM_{0\mathrm{a}\,i}}{r_i' c_g^2})^{-1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2}\phi\tilde{t}' \;; \end{aligned} \tag{26}$$

in the case of uniform velocity motion of the test particle, and

$$\begin{aligned} \tilde{\tilde{t}} &= \overline{\gamma}_g^t \gamma(\tilde{x}')\tilde{t}' \;; \\ &= \prod_{i=1}^{N}(1 - \frac{2GM_{0\mathrm{a}\,i}}{r_i' c_g^2})^{-1/2}(1 - \frac{2a\tilde{x}'}{c_\gamma^2})^{-1/2}\tilde{t}' \;; \end{aligned} \tag{27}$$

in the case of uniformly accelerated motion of the test particle.

In the case of the length contraction formula in the context of combined TGR and SR at the neighborhood of N isolated gravitational field sources, on the other hand, no unique outward manifestations in (Σ, ct) of the intrinsic length contraction formulae (10) and (12) can be written, unlike the time dilation formulae (26) and (27) are the unique outward manifestations of Eqs. (15) and (17). This is so because the outward manifestations of Eq. (16) and (18) depend on the arrangement of the N isolated gravitational field sources and the direction of motion of the test particle in the Euclidean 3-space Σ.

Each arrangement of the N isolated gravitational field sources and direction of motion of the test particle in Σ, has its associated length contraction formula, as the outward manifestation in Σ of Eq. (16) or (18) in $\phi\rho$. For instance, let us consider the rarely possible arrangement in which the centers of all N gravitational field sources lie along the same direction in Σ, along which the test particle is moving. That is, along the $\tilde{x}-$axis, say, of the Cartesian coordinate system $(\tilde{x}, \tilde{y}, \tilde{z})$ of the Euclidean 3-space attached to the test particle, along which it is moving. The length contraction formula in this rare situation is,

$$\begin{aligned}\tilde{x} &= (\overline{\gamma}^t_g)^{-1}\gamma(v)^{-1}\tilde{x}'\ ;\ \tilde{\tilde{y}} = \tilde{y}'\ ;\ \tilde{\tilde{z}} = \tilde{z}'\ ;\\ &= \prod_{i=1}^{N}(1 - \frac{2GM_{0\mathrm{a}i}}{r'_i c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{1/2}\tilde{x}'\ ;\ \tilde{\tilde{y}} = \tilde{y}'\ ;\ \tilde{\tilde{z}} = \tilde{z}'\ ; \end{aligned} \tag{28}$$

in the case of uniform velocity motion of the test particle, and

$$\begin{aligned}\tilde{\tilde{x}} &= (\overline{\gamma}^t_g)^{-1}\gamma(\tilde{x}')^{-1}\tilde{x}'\ ;\ \tilde{\tilde{y}} = \tilde{y}'\ ;\ \tilde{\tilde{z}} = \tilde{z}'\ ;\\ &= \prod_{i=1}^{N}(1 - \frac{2GM_{0\mathrm{a}i}}{r'_i c_g^2})^{1/2}(1 - \frac{2a\tilde{x}'}{c_\gamma^2})^{1/2}\tilde{x}'\ ;\ \tilde{\tilde{y}} = \tilde{y}'\ ;\ \tilde{\tilde{z}} = \tilde{z}'\ ; \end{aligned} \tag{29}$$

in the case of uniformly accelerated motion of the test particle.

The mass and kinetic energy relations in the context of combined TGR and SR at the neighborhood of N isolated gravitational field sources, must be obtained as the outward manifestations on flat spacetime (Σ, ct) of Eqs. (22) and (23) in the case of uniform velocity motion of the test particle and Eqs. (24) and (25) in the case of uniformly accelerated motion of the test particle, to be obtained by simply dropping the symbol ϕ in those equations. They are given as follows

$$\begin{aligned}\overline{m} &= (\gamma^t_g)^{-2}\gamma(v)m_0\\ &= m_0\prod_{i=1}^{N}(1 - \frac{2GM_{0\mathrm{a}i}}{r'_i c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2} \end{aligned} \tag{30}$$

$$\begin{aligned}\overline{T} &= m_0 c_\gamma^2[\gamma(v) - 1]\\ &= m_0 c_\gamma^2\prod_{i=1}^{N}(1 - \frac{2GM_{0\mathrm{a}i}}{r'_i c_g^2})[(1 - \frac{v^2}{c_\gamma^2})^{-1/2} - 1] \end{aligned} \tag{31}$$

and

$$\overline{m} = (\gamma^t_g)^{-2}\gamma(\tilde{x}')m_0$$

$$= \quad m_0 \prod_{i=1}^{N} (1 - \frac{2GM_{0\mathrm{a}\,i}}{r_i' c_g^2})(1 - \frac{2a\tilde{x}'}{c_\gamma^2})^{-1/2} \qquad (32)$$

$$\overline{T} \quad = \quad m_0 c_\gamma^2 [\gamma(2\dot{v}\tilde{x}') - 1]$$

$$= \quad m_0 c_\gamma^2 \prod_{i=1}^{N} (1 - \frac{2GM_{0\mathrm{a}\,i}}{r_i' c_g^2})[(1 - \frac{2a\tilde{x}'}{c_\gamma^2})^{-1/2} - 1] \qquad (33)$$

Extension to other parameter relations in the context of combined TGR and SR at the neighborhood of N isolated gravitational field sources is straight forward.

We have not only achieved the unification of gravity and dynamics as unification of the theory of gravitational relativity and the special theory of relativity (TGR $\cup$ SR) on flat spacetime at the exterior neighborhood of one gravitational field source in [3, 4], but have extended it to the neighborhood of any number N of gravitational field sources that are scattered arbitrarily in the Euclidean 3-space Σ in this section, along with the results of the first part of this paper [6]. It is safe to say that this is the resolution of another outstanding problem in the general theory of relativity namely unification of gravity and dynamics, as follows from the discussion under the introduction to this article.

3 Unification of the metric theory of absolute intrinsic gravity and metric theory of absolute intrinsic motion at the neighborhood of N isolated gravitational field sources

Just as N isolated gravitational field sources prescribe flat four-dimensional relativistic spacetime (Σ, ct) in the context of TGR and its underlying flat two-dimensional relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ in the context of ϕTGR, in all their finite neighborhood, as platforms for SR and ϕSR respectively, they prescribe a resultant curved 'two-dimensional' absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ with resultant absolute intrinsic metric tensor, in the context of ϕMAG, in all their finite neighborhood, as platform for the metric theory of absolute intrinsic motion (ϕMAM).

Let the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ in the 'two-dimensional' absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ of a test particle be in absolute intrinsic motion at absolute intrinsic dynamical speed $\phi\hat{V}_d$ within a local Lorentz frame, at the neighborhood of the two isolated gravitational field sources in Fig.a or 1b of the first part of this paper [6]. Then $\phi\hat{m}_0$ and $\phi\hat{\varepsilon}/\phi\hat{c}^2$ will be in absolute intrinsic translation along the absolute intrinsic affine spacetime coordinates $\phi\hat{\tilde{x}}$ and $\phi\hat{c}_\gamma\phi\hat{\tilde{t}}$ respectively, which are inclined to the upper curved absolute intrinsic spacetime 'dimensions' $\phi\hat{\rho}$ and

$\phi\hat{c}\phi\hat{t}$ respectively in Fig. 7 of [6]. Fig. 7 of [6] is the resultant spacetime/intrinsic spacetime geometry in all finite neighborhood of the two isolated gravitational field sources in Fig. 1a or 1b of that article.

The resultant spacetime/intrinsic spacetime geometry of the absolute intrinsic motion of the absolute intrinsic rest mass of the test particle at the neighborhood of the two isolated gravitational field sources described above is depicted in Fig. 1. The special-relativistic intrinsic inertial mass $(\phi\gamma\,(\phi v)\phi m, \phi\gamma\,(\phi v)\phi\varepsilon/\phi c_\gamma^2)$ in intrinsic motion on flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ and its special-relativistic inertial mass $(\gamma\,(v)m, \gamma\,(v)\varepsilon/c_\gamma^2)$ in motion on flat relativistic spacetime (Σ, ct) relative to an observer are also shown in Fig. 1.

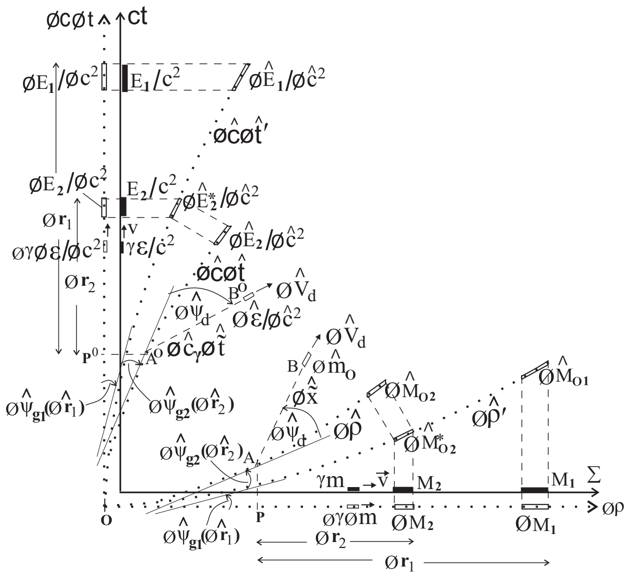

Fig. 1: The spacetime/intrinsic spacetime geometry of the absolute intrinsic motion of the absolute intrinsic rest mass of a test particle at the neighborhood of two isolated gravitational field sources.

The resultant inclination of the path AB along the absolute intrinsic affine coordinate $\phi\hat{\hat{x}}$ of the absolute intrinsic translation of $\phi\hat{m}_0$ to $\phi\rho$ along the horizontal and the equal inclination of of the path A^0B^0 along the absolute intrinsic affine coordinate $\phi\hat{c}_\gamma\phi\hat{\hat{t}}$ of the absolute intrinsic translation of $\phi\hat{\varepsilon}/\phi\hat{c}^2$ to $\phi c\phi t$ along the vertical in Fig. 1, is $\phi\hat{\psi}_{\text{res}}$ where

$$\phi\hat{\psi}_{\text{res}} = \phi\hat{\psi}_{g1}(\phi\hat{r}_1) + \phi\hat{\psi}_{g2}(\phi\hat{r}_2) + \phi\hat{\psi}_d \quad (34a)$$

$$= \phi\hat{\psi}_{g\,\text{res}} + \phi\hat{\psi}_d \quad (34b)$$

and

$$\phi\hat{\psi}_{g\,\text{res}} = \phi\hat{\psi}_{g1}(\phi\hat{r}_1) + \phi\hat{\psi}_{g2}(\phi\hat{r}_2) \quad (34c)$$

is the resultant inclination of the upper curved absolute intrinsic metric spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ to the flat relativistic intrinsic spacetime $(\phi\rho, \phi c\phi t)$ at point P in Σ of radial distances r_1 and r_2 from the center of the inertial masses M_1 and M_2 of the gravitational field sources in Σ, which correspond to 'distances' $\phi\hat{r}_1$ from the base of $\phi\hat{M}_{01}$ along the curved $\phi\hat{\rho}'$ and $\phi\hat{r}_2$ from the base of $\phi\hat{M}_{02}$ along the curved $\phi\hat{\rho}$ in Fig. 1.

Now the absolute intrinsic line element of combined metric theory of absolute intrinsic gravity and absolute intrinsic motion (ϕMAG $\cup$ ϕMAM) is valid within the elementary interval of absolute intrinsic metric spacetime $(d\hat{\rho}, \phi\hat{c}d\phi\hat{t})$ occupied by the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ of the test particle. It is given in terms of the resultant absolute intrinsic angle $\phi\hat{\psi}_{\text{res}}$ of Eq. (34b) as follows, as derived originally in [9, 10] and applied in [5, 11],

$$d\phi\hat{s}^2 = \cos^2\phi\hat{\psi}_{\text{res}}\phi\hat{c}^2 d\phi\hat{t}^2 - \sec^2\phi\hat{\psi}_{\text{res}} d\phi\hat{\rho}^2 \quad (35a)$$

$$= (1 - \sin^2\phi\hat{\psi}_{\text{res}})\phi\hat{c}^2 d\phi\hat{t}^2 - (1 - \sin^2\phi\hat{\psi}_{\text{res}})^{-1} d\phi\hat{\rho}^2 \quad (35b)$$

Now

$$\sin^2\phi\hat{\psi}_{\text{res}} = \sin^2[\phi\hat{\psi}_{g1}(\phi\hat{r}_1) + \phi\hat{\psi}_{g2}(\phi\hat{r}_2) + \phi\hat{\psi}_d] \quad (36a)$$

$$= \sin^2[\phi\hat{\psi}_{g\,\text{res}} + \phi\hat{\psi}_d] \quad (36b)$$

$$= \sin^2\phi\hat{\psi}_{g\,\text{res}} + \sin^2\phi\hat{\psi}_d \quad (36c)$$

$$= \sin^2[\phi\hat{\psi}_{g1}(\phi\hat{r}_1) + \phi\hat{\psi}_{g2}(\phi\hat{r}_2)] + \sin^2\phi\hat{\psi}_d \quad (36d)$$

$$= \sin^2\phi\hat{\psi}_{g1}(\phi\hat{r}_1) + \sin^2\phi\hat{\psi}_{g2}(\phi\hat{r}_2)) + \sin^2\phi\hat{\psi}_d \quad (36e)$$

where the rule for the composition of two absolute intrinsic angles $\phi\hat{\psi}_1$ and $\phi\hat{\psi}_2$ for the purpose of writing the absolute intrinsic line element namely,

$$\sin^2[\phi\hat{\psi}_1 + \phi\hat{\psi}_2] = \sin^2\phi\hat{\psi}_1 + \sin^2\phi\hat{\psi}_2,$$

derived originally in [9, 12] and re-derived as Eq. (117) of part one of this article [6], has been used in arriving at Eq. (36e).

By using Eq. (36e) in Eq. (35b we have

$$d\phi\hat{s}^2 = \left(1 - \sin^2\phi\hat{\psi}_{g1}(\phi\hat{r}_1) - \sin^2\phi\hat{\psi}_{g2}(\phi\hat{r}_2)) - \sin^2\phi\hat{\psi}_d\right)\phi\hat{c}^2 d\phi\hat{t}^2 - \left(1 - \sin^2\phi\hat{\psi}_{g1}(\phi\hat{r}_1) - \sin^2\phi\hat{\psi}_{g2}(\phi\hat{r}_2)) - \sin^2\phi\hat{\psi}_d\right)^{-1} d\phi\hat{\rho}^2 \quad (37)$$

Then by using the following definitions,

$$\sin^2\phi\hat{\psi}_{g1}(\phi\hat{r}_1) = \phi\hat{k}_{g1}(\phi\hat{r}_1)^2 = \frac{2G\phi\hat{M}_{0a1}}{\phi\hat{r}_1\phi\hat{c}_g^2} \; ;$$

$$\sin^2\phi\hat{\psi}_{g2}(\phi\hat{r}_2) = \phi\hat{k}_{g2}(\phi\hat{r}_2)^2 = \frac{2G\phi\hat{M}_{0a2}}{\phi\hat{r}_2\phi\hat{c}_g^2} \; ;$$

$$\sin^2\phi\hat{\psi}_d = \phi\hat{k}_d^2 = \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2} \; ;$$

which have been derived in the previous papers and made use of in Eqs. (121) and (122) of the first part of this paper [6], the absolute intrinsic line element (37) can be re-written in the following alternative forms,

$$d\phi\hat{s}^2 = \left(1 - \phi\hat{k}_{g1}(\phi\hat{r}_1)^2 - \phi\hat{k}_{g2}(\phi\hat{r}_2)^2 - \phi\hat{k}_d^2\right)\phi\hat{c}^2 d\phi\hat{t}^2 - \left(1 - \phi\hat{k}_{g1}(\phi\hat{r}_1)^2 - \phi\hat{k}_{g2}(\phi\hat{r}_2)^2 - \phi\hat{\psi}_d^2\right)^{-1} d\phi\hat{\rho}^2 \quad (38)$$

or

$$d\phi\hat{s}^2 = \left(1 - \frac{2G\phi\hat{M}_{0a1}}{\phi\hat{r}_1\phi\hat{c}_g^2} - \frac{2G\phi\hat{M}_{0a2}}{\phi\hat{r}_2\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2}\right)\phi\hat{c}^2 d\phi\hat{t}^2 - \left(1 - \frac{2G\phi\hat{M}_{0a1}}{\phi\hat{r}_1\phi\hat{c}_g^2} - \frac{2G\phi\hat{M}_{0a2}}{\phi\hat{r}_2\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2}\right)^{-1} d\phi\hat{\rho}^2 \quad (39)$$

The absolute intrinsic line element (37), (38) and (39) admit of generalizations to the case of a test particle in motion at the neighborhood of any number N of gravitational field sources that are scattered arbitrarily in space about the test particle respectively as follows

$$d\phi\hat{s}^2 = \left(1 - \sum_i^N \sin^2\phi\hat{\psi}_{gi}(\phi\hat{r}_i) - \sin^2\phi\hat{\psi}_d\right)\phi\hat{c}^2 d\phi\hat{t}^2 - \left(1 - \sum_i^N \sin^2\phi\hat{\psi}_{gi}(\phi\hat{r}_i) - \sin^2\phi\hat{\psi}_d\right)^{-1} d\phi\hat{\rho}^2, \quad (40)$$

$$d\phi\hat{s}^2 = \left(1 - \sum_i^N \phi\hat{k}_{gi}(\phi\hat{r}_i)^2 - \phi\hat{k}_d^2\right)\phi\hat{c}^2 d\phi\hat{t}^2 - \left(1 - \sum_i^N \phi\hat{k}_{gi}(\phi\hat{r}_i)^2 - \phi\hat{k}_d^2\right)^{-1} d\phi\hat{\rho}^2 \quad (41)$$

or

$$d\phi\hat{s}^2 = \left(1 - \sum_i^N \frac{2G\phi\hat{M}_{0\mathrm{a}\,i}}{\phi\hat{r}_i\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2}\right)\phi\hat{c}^2 d\phi\hat{t}^2 - \left(1 - \sum_i^N \frac{2G\phi\hat{M}_{0\mathrm{a}\,i}}{\phi\hat{r}_i\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2}\right)^{-1} d\phi\hat{\rho}^2 \quad (42)$$

Equation (40), (41) or (42) gives the absolute intrinsic lone element of combined ϕMAG and ϕMAM for a test particle whose absolute intrinsic rest mass $\phi\hat{m}_0$ is in absolute intrinsic motion at constant absolute intrinsic dynamical speed $\phi\hat{V}_d$ within a local Lorentz frame at the neighborhood of N isolated gravitational field sources. If $\phi\hat{m}_0$ is in a uniformly accelerated absolute intrinsic motion at absolute intrinsic acceleration $\dot{\hat{V}}_d$ within the local Lorentz frame, then the term $\hat{V}_d^2/\hat{c}^2$ must be replaced with $2\dot{\hat{V}}_d\hat{\hat{x}}/\hat{c}^2$ in Eq. (42) to have

$$d\phi\hat{s}^2 = \left(1 - \sum_i^N \frac{2G\phi\hat{M}_{0\mathrm{a}\,i}}{\phi\hat{r}_i\phi\hat{c}_g^2} - \frac{2\phi\dot{\hat{V}}_d\phi\hat{\hat{x}}}{\phi\hat{c}_\gamma^2}\right)\phi\hat{c}^2 d\phi\hat{t}^2 - \left(1 - \sum_i^N \frac{2G\phi\hat{M}_{0\mathrm{a}\,i}}{\phi\hat{r}_i\phi\hat{c}_g^2} - \frac{2\phi\dot{\hat{V}}_d\phi\hat{\hat{x}}}{\phi\hat{c}_\gamma^2}\right)^{-1} d\phi\hat{\rho}^2 \quad (43)$$

It shall be reiterated for emphasis that the absolute intrinsic line element (40), (41) or (42) in the case of uniform velocity absolute intrinsic motion and Eq. (43) in the case of uniformly accelerated absolute intrinsic motion, are valid within the elementary interval of absolute intrinsic spacetime $(d\phi\hat{\rho}, \phi\hat{c}d\phi\hat{t})$ occupied by the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ of the test particle. For positions outside the test particle at any instant during its motion, the absolute intrinsic speed $\phi\hat{V}_d$ or 2ϕ
$hat\dot{V}_d\phi\hat{\hat{x}}$ must be set to zero, since there is nothing in absolute intrinsic dynamics to transmit the absolute intrinsic speed of a particle to regions of space exterior to the particle. The absolute intrinsic line elements (40) – (42) and (43) reduce to that of ϕMAG purely for positions outside the test particle at any instant during its motion.

We have again extended the unified metric theory of absolute intrinsic gravity and metric theory of absolute intrinsic motion (ϕMAG $\cup$ ϕMAM) at the neighborhood of one gravitational field source in [5, 11] to the neighborhood of N isolated gravitational field sources in this section, along with the results of the first part of this paper [4]. The two-theory approach to combined gravity and dynamics achieved at the neighborhood of one gravitational field source in [3–5] has thus been extended to the neighborhood of any number N of gravitational field sources that are scattered arbitrarily in the Euclidean 3-space σ in the preceding section and this, along with results of the first part of this paper [6].

4 Incorporating the effect of the gravitational field of the test particle into the hierarchy of theories of unification of gravity and dynamics

The effect of the gravitational field of the test particle has not been considered in the unification of TGR/ϕTGR and SR/ϕSR in section 2 and the unification of ϕMAG and ϕMAM in section 3 of this paper. It has inherently been assumed that the test particle is a non-extended point particle without gravitational field or an extended particle with negligible gravitational field in those sections. The test particle shall now be considered to be a gravitational field source and the effect of its field shall be incorporated into the unified theories in this section. In effect the moving test particle shall become the (N+1)th gravitational field source (or the (N+1)th body) in this section.

4.1 Incorporating the effect of the gravitational field of the test particle into TGR/ϕTGR and SR/ϕSR

In the general theory of relativity (GR), the test particle is assumed to be a mass-point (of zero gravitational field). Thus the problem of incorporating the effect of the gravitational potential of the test particle into GR does not arise. However in reality the test particle is an extended object with non-zero mass and non-zero dimensions, and hence with non-zero gravitational field. It is pertinent for us to consider all ramifications in the present theory, since the ultimate goal is a complete theory of the whole of physics. Consequently the effect of the gravitational field (or potential) of the test particle in the separate and combined theories cannot be neglected at the onset.

So far we have not considered the effect of the gravitational field of the moving test particle or body in the special theory of relativity (SR) and in the combined theory of gravitational relativity and special theory of relativity (TGR $\cup$ SR) in [3,4] and in section 2 of this paper; nor in the combined metric theories of absolute intrinsic motion and absolute intrinsic gravity (ϕMAG $\cup$ ϕMAM) in [5, 11] and in section 3 of this article. By incorporating the effect of the gravitational field of the macroscopic test particle into the combined theories of sections 2 and 3 in this section, the effect of the gravitational field of the test particle on TGR and SR as separate theories shall be deduced by allowing the velocity of the test particle to vanish and by allowing external gravitational field to vanish separately in the resultant combined theories, (containing the effect of the gravitational field of the test particle), to be derived here.

Indeed a body in motion with the presence or absence of external gravitational field can be quite large both in size and mass, to make the effect of its gravitational field important in the theories of motion namely, ϕMAM and SR/ϕSR for the body. For example, stars in distant galaxies are in motion at large velocities relative to us on earth. Also it can be imagined that a medium star or planet is captured in orbit round a super-massive star (or a giant). There is therefore the need to incorporate the gravitational velocity (or potential) due to the mass of a test particle in motion in an external gravitational field into the combined theories of dynamics and gravity as done hereunder.

Let the moving test particle or body be spherical in shape of rest mass m_0 and classical radius r'_p (of m_0). The gravitational speed at the surface of the test particle or body is $V'_g(r'_p) = -(2Gm_{0\mathrm{a}}/r'_p)^{1/2}$. In incorporating the gravitational speed due to the mass of the test particle or body into combined TGR and SR of section 2, the resultant factor $\overline{\gamma}_g^{\,t}$ in the case of N isolated sources of the external gravitational field at the location of the moving test particle at any instant must be modified as follows

$$\overline{\gamma}_g^{\,t} = \gamma_{g1}(r'_1)\gamma_{g2}(r'_2)\gamma_{g3}(r'_3)\cdots\gamma_{gN}(r'_N)\gamma_g(r'_p)\,\gamma \tag{44a}$$

or

$$\sec\overline{\psi}^{\,t} = \sec\psi_{g1}(r'_1)\sec\psi_{g2}(r'_2)\cdots\sec\psi_{gN}(r'_N)\sec\psi_g(r'_p)\sec\psi \tag{44b}$$

or

$$\overline{\gamma}_g^{\,t} = \left(1 - \frac{2GM_{0\mathrm{a}1}}{r'_1 c_g^2}\right)^{-1/2}\left(1 - \frac{2GM_{0\mathrm{a}2}}{r'_2 c_g^2}\right)^{-1/2}\cdots\left(1 - \frac{2GM_{0\mathrm{a}N}}{r'_N c_g^2}\right)^{-1/2}$$
$$\times\left(1 - \frac{2Gm_{0\mathrm{a}}}{r'_p c_g^2}\right)^{-1/2}\left(1 - \frac{v^2}{c_\gamma^2}\right)^{-1/2} \tag{44c}$$

Equations (44a-c) simplify for the case of one source of external gravitational field as follows

$$\overline{\gamma}_g^{\,t} = \gamma_g(r')\gamma_g(r'_p)\,\gamma \tag{45a}$$

or

$$\sec \overline{\psi}^{\,t} = \sec \psi_g(r') \sec \psi_g(r'_p) \sec \psi \tag{45b}$$

or

$$\overline{\gamma}_g^{\,t} = (1 - \frac{2GM_{0a}}{r' c_g^2})^{-1/2} (1 - \frac{2Gm_{0a}}{r'_p c_g^2})^{-1/2} (1 - \frac{v^2}{c^2})^{-1/2} \tag{45c}$$

Even with the consideration of the gravitational speed due to the moving test particle, gravitational local Lorentz invariance (GLLI) of TGR holds still. One must write a tandem of gravitational local Lorentz transformations as done in the preceding article. Gravitational local Lorentz transformation and its inverse must be written for the moving test particle, (as a moving gravitational field source), while assuming that any stationary massive field source is absent, and gravitational local Lorentz invariance validated. It must be remembered that the gravitational speed $V'_g(r')$ due to the test particle at radial distance r' from the location of the moving test particle at any given instant, which appears in the gravitational local Lorentz transformation due to the test particle, is invariant with the motion of the test particle. Consequently the gravitational local Lorentz transformation derived with respect to the gravitational field of the test particle, is invariant with the motion of the test particle. Then by bringing a stationary massive field source into the flat spacetime established by the moving gravitational field source (the test particle) in all its finite neighborhood, gravitational local Lorentz transformation and its inverse must again be written, and gravitational local Lorentz invariance validated. This confirms gravitational local Lorentz invariance in the gravitational field of the moving test particle and one stationary massive gravitational field source.

Thus the combination of the gravitational field of a moving test particle or body with the gravitational field of one external field source, does not alter the Lorentzian metric tensor of the flat four-dimensional spacetime in all finite neighborhood of the moving test particle (or body) and the stationary gravitational field source. Then by obtaining gravitational local Lorentz transformation and its inverse at the location of the moving test particle or body at an instant, when a second external field source is brought in place, upon the flat spacetime prescribed by the first external field source, gravitational local Lorentz invariance holds again in the gravitational field of two external sources in combination with the gravitational field of the moving test particle. By bringing a third external field source in place, then a fourth, and so on, until all N external field sources have been brought in place, we find that gravitational local Lorentz invariance holds in an external gravitational field of N isolated sources plus the gravitational field of the moving test particle, no matter how the N field sources are scattered in 3-space about the moving test particle or body.

The validity of gravitational local Lorentz invariance in an external gravitational field implies that the four-dimensional spacetime (Σ, ct) is flat with constant Lorentzian metric tensor within the external gravitational field. It is upon the flat spacetime established by gravity (in the context of TGR) that the special theory of relativity due to the motion of the test particle or body operates. This then implies that local Lorentz transformation (LLT) and its inverse in the context of SR can be derived, and local Lorentz invariance (LLI) validated (within local Lorentz frames) in an external gravitational field of any number of isolated sources, even with the inclusion of the effect of the gravitational field of the moving test particle or body.

The LLT and its inverse take on their usual forms within the local Lorentz frame located at a point P in the external gravitational field of N isolated sources, even with the inclusion of the effect of the gravitational field of the moving test particle or body as follows

$$\tilde{x} = \gamma(v)(\tilde{\tilde{x}} - v\tilde{\tilde{t}});\ \tilde{y} = \tilde{\tilde{y}};\ \ \tilde{z} = \tilde{\tilde{z}};\ \tilde{t} = \gamma(v)(\tilde{\tilde{t}} - (v/c_\gamma^2)\tilde{\tilde{x}}) \tag{46}$$

and

$$\tilde{\tilde{x}} = \gamma(v)(\tilde{x} + v\tilde{t});\ \tilde{\tilde{y}} = \tilde{y};\ \tilde{\tilde{z}} = \tilde{z};\ \tilde{\tilde{t}} = \gamma(v)(\tilde{t} + (v/c_\gamma^2)\tilde{x}) \tag{47}$$

where $\gamma(v) = (1 - v^2/c_\gamma^2)^{-1/2}$.

The extended affine coordinates in Eqs. (46) and (47) are limited within local Lorentz in which the test particle is moving. Consequently the transformations (46) and (47) are local Lorentz transformations (LLT). Each yields local Lorentz invariance,

$$c_\gamma^2\tilde{\tilde{t}}^2 - \tilde{\tilde{x}}^2 - \tilde{\tilde{y}}^2 - \tilde{\tilde{z}}^2 = c_\gamma^2\tilde{t}^2 - \tilde{x}^2 - \tilde{y}^2 - \tilde{z}^2 \tag{48}$$

Although the effect of gravity does not appear explicitly in LLT and its inverse of systems (46) and (47), the time $\tilde{t}$ has suffered gravitational dilation and the spatial coordinates $\tilde{x}, \tilde{y}$ and $\tilde{z}$ have suffered gravitational contractions in the context of TGR from the original proper coordinates $\tilde{t}'$ and $\tilde{x}', \tilde{y}', \tilde{z}'$ in the flat proper spacetime (Σ', ct'). Hence the time $\tilde{\tilde{t}}$ is gravitational-relativistic cum special-relativistic time and the spatial coordinates $\tilde{\tilde{x}}, \tilde{\tilde{y}}$ and $\tilde{\tilde{z}}$ are gravitational-relativistic cum special-relativistic spatial coordinates. The resultant time dilation formula at the surface of the test particle, in the case of test particle moving in the gravitational field of N external sources is given as follows

$$\begin{aligned} d\tilde{\tilde{t}} &= \sec\psi_{g1}(r'_1)\sec\psi_{g2}(r'_2)\cdots\sec\psi_{gN}(r'_N)\sec\psi_g(r'_p)\sec\psi\, d\tilde{t}' \\ &= (1-\frac{2GM_{0\mathrm{a}1}}{r'_1c_g^2})^{-1/2}(1-\frac{2GM_{0\mathrm{a}2}}{r'_2c_g^2})^{-1/2}\cdots(1-\frac{2GM_{0\mathrm{a}N}}{r'_Nc_g^2})^{-1/2} \\ &\times(1-\frac{2Gm_{0\mathrm{a}}}{r'_pc_g^2})^{-1/2}(1-\frac{v^2}{c_\gamma^2})^{-1/2}d\tilde{t}' \end{aligned} \tag{49}$$

The time dilation formula (49) is true irrespective of how the N gravitational field sources are scattered in 3-space about the moving test particle or body. On the other hand, no straight forward formula can be written for the resultant length contraction of the coordinates $\tilde{x}', \tilde{y}'$ and $\tilde{z}'$ attached to the rest mass m_0 of the moving test particle originally in the flat proper spacetime (Σ', ct'), since the orientations of these coordinates with respect to the lines joining the centers of the gravitational field sources to the moving test particle at any given instant, as well as the direction of motion of the test particle, count in determining their resultant contractions.

Let us consider an elementary 4-box of proper dimensions cdt', dx', dy' and dz' and rest mass $m_{0\mathrm{b}}$, which is tied to the surface of the spherical test particle (or body) of rest mass m_0 and classical radius r'_p that is in motion in an external gravitational field. Let us assume the presence of only one gravitational field source, and that the spherical test particle with the elementary box at its surface is moving radially towards the gravitational field source at a large velocity $\vec{v}$ relative to the observer, while it is momentarily passing through radial distance r from the center of the field source. It shall also be assumed that the dimension dx of the elementary box is along the direction of motion of the spherical test particle transporting it, as illustrated in Fig. 2.

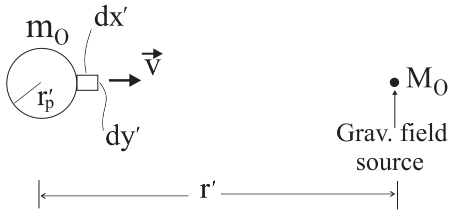

Fig. 2: A little box at rest with respect to a spherical test particle (or body) moving radially towards a gravitational field source.

The resultant time dilation formula at any point at the surface of the moving spherical test particle relative to the observer in this case is given as follows

$$d\tilde{\tilde{t}} = (1 - \frac{2GM_{0\mathrm{a}}}{r' c_g^2})^{-1/2}(1 - \frac{2Gm_{0\mathrm{a}}}{r'_p c_g^2})^{-1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2} d\tilde{t}' \qquad (50)$$

The dimension dx' of the elementary box suffers both gravitational and special-relativistic contractions, but its dimensions dy' and dz' are not contracted. The resultant length contraction formulae of the spatial dimensions of the box are then given as follows

$$d\overline{x} = (1 - \frac{2GM_{0\mathrm{a}}}{r' c_g^2})^{1/2}(1 - \frac{2Gm_{0\mathrm{a}}}{r'_p c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{1/2} dx'; \; d\overline{y} = dy'; \; d\overline{z} = dz' \qquad (51)$$

The gravitational velocity due to the external field source(s) at the point where the test particle is momentarily passing through, as well as the energy stored in the external gravitational field within the test particle, must be incorporated into the expressions for the relativistic mass, relativistic total energy and relativistic kinetic energy in the situation of combined special theory of relativity (SR) and theory of gravitational relativity (TGR). If we let the rest mass of the elementary box be $m_{0\mathrm{b}}$ as said earlier, then its relativistic mass $\tilde{m}_\mathrm{b}$, and its relativistic kinetic energy $\overline{T}_\mathrm{b}$ in the situation depicted in Fig. 2 are given relative to the observer as follows

$$\overline{m}_\mathrm{b} = m_{0\mathrm{b}}(1 - \frac{2GM_{0\mathrm{a}}}{r' c_g^2})(1 - \frac{2Gm_{0\mathrm{a}}}{r'_p c_g^2})(1 - \frac{v^2}{c_\gamma^2})^{-1/2} \qquad (52)$$

and

$$\overline{T}_\mathrm{b} = m_{0\mathrm{b}} c_\gamma^2 (1 - \frac{2GM_{0\mathrm{a}}}{r' c_g^2})(1 - \frac{2Gm_{0\mathrm{a}}}{r'_p c_g^2})[(1 - \frac{v^2}{c_\gamma^2})^{-1/2} - 1] \qquad (53)$$

Equations (52) and (53) can be generalized to a situation where N gravitational field sources are present nearby by simply replacing $(1 - 2GM_{0\mathrm{a}}/r' c_g^2)$ by $\prod_{i=1}^{N}(1 - 2GM_{0\mathrm{a}\,i}/r'_i c_g^2)$.

More importantly, the relativistic mass $\overline{m}$ and the relativistic kinetic energy $\overline{T}$ relative to the observer, in the context of combined TGR and SR of the spherical test particle in motion in Fig. 2, with the elementary box removed, are given in terms of its rest mass m_0 as follows

$$\overline{m} = m_0(1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})(1 - \frac{2Gm_{0\mathrm{a}}}{r'_p c_g^2})(1 - \frac{v^2}{c_\gamma^2})^{-1/2} \quad (54)$$

$$\overline{T} = m_0 c_\gamma^2 (1 - \frac{2GM_{0\mathrm{a}}}{r'c_g^2})(1 - \frac{2Gm_{0\mathrm{a}}}{r'_p c_g^2})[(1 - \frac{v^2}{c_\gamma^2})^{-1/2} - 1] \quad (55)$$

Again Eq. (54) and (55) can be generalized to a situation where the spherical test particle (or body) is in motion at the common neighborhood of N gravitational field sources that are scattered arbitrarily in the Euclidean 3-space Σ about it, by simply replacing the factor $(1 - 2GM_{0\mathrm{a}}/r'c_g^2)$ by $\prod_{i=1}^{N}(1 - 2GM_{0\mathrm{a}\,i}/r'_i c_g^2)$. The fact that the mass and kinetic energy relations (54) and (55) are valid, despite the fact that the factor $2Gm_{0\mathrm{a}}/r'_p c_g^2$ is evaluated at the surface of the particle only shall be justified in the next section.

4.2 Incorporating the gravitational field of the test particle into combined metric theories of absolute intrinsic gravity and absolute intrinsic motion

In incorporating the effect of the gravitational field of the moving test particle (or body) into combined 'two-dimensional' metric theories of absolute intrinsic motion and absolute intrinsic gravity (ϕMAM $\cup$ ϕMAG) of section 3, the absolute intrinsic line element still takes the general form of Eq. (40), (41) or (42). However in expanding the resultant absolute intrinsic curvature parameter $\phi\hat{k}_{\mathrm{res}}$ at the neighborhood of N isolated gravitational field sources, we must add an extra term $\phi\hat{k}_g(\phi\hat{r}_p)^2$ due to the gravitational field of the moving test particle (or body) at its surface to the right-hand side of Eq. (41), which corresponds to adding an extra term $\sin^2\phi\hat{\psi}_g(\phi\hat{r}_p)$ to the right-hand side of Eq. (40). Thus $\phi\hat{k}_{\mathrm{res}}{}^2$ and $\sin^2\phi\hat{\psi}_{\mathrm{res}}$ in Eq. (40) and (41) must be expressed respectively as follows in putting the effect of the gravitational field of the moving test particle (or body) into consideration,

$$\phi\hat{k}_{\mathrm{res}}^2 = \sum_{i=1}^{N} \phi\hat{k}_{gi}(\phi\hat{r}_i)^2 + \phi\hat{k}_g(\phi\hat{r}_p)^2 + \phi\hat{k}_d^2 \quad (56)$$

$$\sin^2\phi\hat{\psi}_{\ \mathrm{res}} = \sum_{i=1}^{N} \sin^2\phi\hat{\psi}_{gi}(\phi\hat{r}_i) + \sin^2\phi\hat{\psi}_g(\phi\hat{r}_p) + \sin^2\phi\hat{\psi}_d \quad (57)$$

where

$$\phi\hat{k}_{gi}(\phi\hat{r}_i)^2 = \sin^2\phi\hat{\psi}_{gi}(\phi\hat{r}_i) = 2G\phi\hat{M}_{0\mathrm{a}\,i}/\phi\hat{r}_i\phi\hat{c}_g^2;$$

$\phi\hat{k}_g(\phi\hat{r}_p)^2 = \sin^2\phi\hat{\psi}_g(\phi\hat{r}_p) = 2G\phi\hat{m}_{0\mathrm{a}}/\phi\hat{r}_p\phi\hat{c}_g^2$ and $\phi\hat{k}_d^2 = \sin^2\phi\hat{\psi}_d = \phi\hat{V}_d^2/\phi\hat{c}_\gamma^2$.

Thus the absolute intrinsic line element of combined ϕMAG and ϕMAM at the neighborhood of N isolated gravitational field sources, with the effect of the gravitational field of the moving test particle (or body) put into consideration is the following

$$\begin{aligned} d\phi\hat{s}^2 \;=\; & (1 - \sum_{i=1}^{N}\frac{2G\phi\hat{M}_{0\mathrm{a}\,i}}{\phi\hat{r}_i\,\phi\hat{c}_g^2} - \frac{2G\phi\hat{m}_{0\mathrm{a}}}{\phi\hat{r}_p\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2})\phi\hat{c}^2 d\phi\hat{t}^2 \\ & -(1 - \sum_{i=1}^{N}\frac{2G\phi\hat{M}_{0\mathrm{a}\,i}}{\phi\hat{r}_i\,\phi\hat{c}_g^2} - \frac{2G\phi\hat{m}_{0\mathrm{a}}}{\phi\hat{r}_p\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2})^{-1} d\phi\hat{\rho}^2 \end{aligned} \tag{58}$$

The components of the absolute intrinsic metric tensor in this case are then given as follows

$$\phi\hat{g}_{00} = -\phi\hat{g}_{11}^{-1} = (1 - \sum_{i=1}^{N}\frac{2G\phi\hat{M}_{0\mathrm{a}\,i}}{\phi\hat{r}_i\,\phi\hat{c}_g^2} - \frac{2G\phi\hat{m}_{0\mathrm{a}}}{\phi\hat{r}_p\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2});\; \phi\hat{g}_{12} = \phi\hat{g}_{21} = 0 \tag{59}$$

It shall be noted in concluding this section, since the factor $(1 - 2Gm_{0\mathrm{a}}/r'_p\,c_g^2)$, which appears in combined TGR and SR, and the term $-2G\phi\hat{m}_{0\mathrm{a}}/\phi\hat{r}_p\,\phi\hat{c}_g^2$, which appears in the components of the absolute intrinsic metric tensor of unified metric theories of absolute intrinsic motion and absolute intrinsic gravity (ϕMAM $\cup$ ϕMAG), have been evaluated at the surface of the test particle, the validity of the results of this section, (except the relation for relativistic mass and relativistic kinetic energy of Eq. (54) and (55)), are restricted to the surface of the test particle. In modifying the results for other positions at the interior of the test particle, which is necessary only for massive and large moving test particles, such as a planet moving round a star or a moderate star in motion in the gravitational field of a giant, one must first of all derive the special theory of gravity and metric theory of absolute intrinsic gravity at the interior of the test particle, as shall be done in an article later in this volume.

5 The special theory of relativity and the metric theory of absolute intrinsic motion of a moving gravitational field source in the absence of external gravitational field due to any other source

The results of this section are already contained in the results of the preceding section. It is only of interest to show explicitly how the existing (Lorentz-Einstein-Minkowski) special theory of relativity (LEM) becomes modified by the effect of the gravitational field of the moving particle or body and, consequently, to identify the particle for which LEM is valid in a strict sense.

Just as there is no combined 'two-dimensional' metric theory of absolute intrinsic gravity and absolute intrinsic motion (ϕMAM $\cup$ ϕMAG) in the extended spacetime outside a test particle or body in motion in an external gravitational field at any instant during its motion, as discussed in the preceding section, there is no combined 'two-dimensional' metric theory of absolute intrinsic motion and absolute intrinsic gravity in the extended spacetime outside a gravitational field source in motion in the absence of external gravitational field due to any other source. This is so since, as discussed earlier in this article, there is nothing in dynamics to transmit the velocity of a particle or body to region of spacetime outside the particle or body.

Graphically, the superposition of the inclined affine intrinsic spacetime frame $(\phi\hat{\tilde{x}}, \phi\hat{c}_\gamma\phi\hat{\tilde{t}})$ of the absolute intrinsic motion at absolute intrinsic dynamical speed $\phi\hat{V}_d$ of the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ of the test particle (or body) and the extended curved absolute intrinsic metric spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ prescribed by the moving test particle (or body) solely, in the context of the metric theory of absolute intrinsic gravity (ϕMAG), at positions outside its body in Fig. 3, does not alter the curved absolute intrinsic metric spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ and the absolute intrinsic metric tensor prescribed by gravity at such positions at any instant during the motion of the test particle.

It is only the little interval of absolute intrinsic metric spacetime $(d\phi\hat{\rho}, \phi\hat{c}d\phi\hat{t})$ containing the absolute intrinsic rest mass $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ of the test particle (or body), on top of the inclined absolute intrinsic affine spacetime $(\phi\hat{\tilde{x}}, \phi\hat{c}_\gamma\phi\hat{\tilde{t}})$, which when superposed on the curved absolute intrinsic metric spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$, causes a change in the absolute intrinsic metric tensor due to gravity solely. It is within the little interval of absolute intrinsic metric spacetime $(d\phi\hat{\rho}, \phi\hat{c}d\phi\hat{t})$ containing $(\phi\hat{m}_0, \phi\hat{\varepsilon}/\phi\hat{c}^2)$ that there is unified metric theory of absolute intrinsic motion and metric theory of absolute intrinsic gravity.

Now system (59) simplifies as follows in the absence of external gravitational field,

$$\phi\hat{g}_{00} = -\phi\hat{g}_{11}^{-1} = 1 - \frac{2G\phi\hat{m}_{0\mathrm{a}}}{\phi\hat{r}_p\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi c_\gamma^{\,2}};\ \phi\hat{g}_{12} = \phi\hat{g}_{21} = 0;\ r' \le r'_p \qquad (60)$$

Equation (60) is correct strictly at the surface of the moving gravitational field source (or test particle), because the absolute intrinsic gravitational speed, $\phi\hat{V}_g(\phi\hat{r}_p)$ $= (2G\phi\hat{m}_{0\mathrm{a}}/\phi\hat{r}_p\,\phi\hat{c}_g^2)^{1/2}$, has been evaluated there. However it shall be considered to be valid at the interior of the test particle for now, until the interior absolute intrinsic metric tensor shall be derived in an article later in this volume.

Thus the components of the absolute intrinsic metric tensor naturally contains both the absolute intrinsic gravitational speed term and the absolute intrinsic dynamical speed of absolute intrinsic motion term within the moving gravitational field source. If an external gravitational field source of rest mass M_0 is located at radial distance r' from the moving gravitational field source of rest mass m_0 and radius r'_p (of m_0) at a given instant, then Eq. (60) must be replaced by Eq. (59) for $\mathrm{N} = 1$ at that instant.

On the other hand, no effect of motion of the moving gravitational field source can be felt in the extended spacetime exterior to it at any given instant, since there is nothing in dynamics, (no action-at-a-distance), which could make the motion of a body through a given point in space, at a given instant in time, to induce the absolute intrinsic dynamical speed $\phi\hat{V}_d$ of the body in space outside it at that instant, as mentioned above. The absolute intrinsic dynamical speed $\phi\hat{V}_d$ of the absolute intrinsic rest mass of a moving gravitational field source (or test particle) cannot appear directly in the components of the absolute intrinsic metric tensor in regions exterior to the moving body at any instant. On the other hand, gravitational speed $V'_g(r')$ and absolute intrinsic gravitational speed $\phi\hat{V}_g(\phi\hat{r})$ are established at every point in space of radial distance r' up to infinity from the center of a gravitational field source. Hence only the term $2G\phi\hat{m}_{0\mathrm{a}}/\phi\hat{r}_p\phi\hat{c}_g^2$ should appear in $\phi\hat{g}_{00}$ and $\phi\hat{g}_{11}$ for regions outside the moving gravitational field source at any instant. This reduces Eq.(60) as follows

$$\phi\hat{g}_{00} = -\phi\hat{g}_{11}^{-1} = 1 - \frac{2G\phi\hat{m}_{0\mathrm{a}}}{\phi\hat{r}_p\phi\hat{c}_g^2};\ \phi\hat{g}_{12} = \phi\hat{g}_{21} = 0;\ r' \leq r'_p \qquad (61)$$

There is yet another argument in support of the union of the theories of motion and gravity within a moving gravitational field source. A particle or body with nonzero gravitational mass, which is stationary relative to an observer in the absence of external gravitational field, gives rise to curvature of the 'two-dimensional' absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\phi\hat{t})$ with respect to observers in in spacetime (Σ, ct), (no matter how microscopic the curvature), both at its interior and in all finite exterior neighborhood of it. Therefore the boundary condition geometry of the metric theory of absolute intrinsic motion (ϕMAM) of a particle or body with nonzero gravitational field, (while at rest relative to an observer in a region of space devoid of external gravitational field), is not a Euclidean geometry purely. This non-

Euclidean absolute intrinsic spacetime geometry boundary condition for the metric theory of absolute intrinsic motion of a particle or body, due to the gravitational field of the particle or body, has been discussed in [13] and illustrated in Fig. 1(b) of that article.

By incorporating the non-Euclidean absolute intrinsic spacetime geometry boundary condition due to the gravitational field of a moving gravitational field source into the metric theory of absolute intrinsic motion of the moving gravitational field source, in the absence of gravitational field of other sources, one has effectively derived a unified metric theory of absolute intrinsic motion and metric theory of absolute intrinsic gravity with the absolute intrinsic metric tensor of Eq. (60) within the moving gravitational field source.

Having gone through the above preamble, the results of the preceding section shall now be reduced to the case of a gravitational field source in motion in a space devoid of external gravitational field due to any other source of this section, by simply letting the terms due to external gravitational field sources to vanish in those results.

Foremost, there is Lorentz invariance on the global flat four-dimensional spacetime of the theory of gravitational relativity (TGR) within and outside the moving gravitational field source. Consequently local Lorentz transformation and its inverse take on their usual forms, which for a gravitational field source moving along the coordinate x attached to it, are given as follows

$$\tilde{\tilde{x}} = \gamma(v)(\tilde{x} - v\tilde{t}) \ ; \ \tilde{\tilde{y}} = \tilde{y} \ ; \ \tilde{\tilde{z}} = \tilde{z} \ ; \ \tilde{\tilde{t}} = \gamma(v)(\tilde{t} - \frac{v}{c_\gamma^2}\tilde{x}) \tag{62}$$

and

$$\tilde{x} = \gamma(v)(\tilde{\tilde{x}} + v\tilde{\tilde{t}}) \ ; \ \tilde{y} = \tilde{\tilde{y}} \ ; \ \tilde{z} = \tilde{\tilde{z}} \ ; \ \tilde{t} = \gamma(v)(\tilde{\tilde{t}} + \frac{v}{c_\gamma^2}\tilde{\tilde{x}}) \tag{63}$$

where, $\gamma(v) = (1 - v^2/c_\gamma^2)^{-1/2}$, and the affine coordinates are local coordinates, limited within the local Lorentz frame in which LLT and its inverse are written at a given instant.

If the coordinates $\tilde{x}, \tilde{y}$ and $\tilde{z}$ are chosen so that the coordinate $\tilde{x}$ is along the direction of motion of the gravitational field source, which is also along a radial direction from the center of the moving field source, then the following time dilation and length contraction formulae obtain at the surface of the moving field source at any instant

$$d\bar{t} \quad = \quad \gamma_g(r')\,\gamma(v)\,dt' = \sec\psi_g(r')\sec\psi_d\,dt'$$

$$= \quad (1 - \frac{2Gm_{0a}}{r' c_g^2})^{-1/2}(1 - \frac{v^2}{c_\gamma^2})^{-1/2} dt' \tag{64}$$

$$d\overline{x} \quad = \quad \gamma_g(r')^{-1}\,\gamma(v)^{-1}\,dx' = \cos\psi_g(r')\cos\psi_d\,dx';\; d\overline{y} = dy';\; \text{and } d\overline{z} = dz'$$

$$= \quad (1 - \frac{2Gm_{0a}}{r' c_g^2})^{1/2}(1 - \frac{v^2}{c_\gamma^2})^{1/2} dx';\; d\overline{y} = dy';\; \text{and } d\overline{z} = dz' \tag{65}$$

The relations (54) and (55) for relativistic mass and relativistic kinetic energy of the moving gravitational field source in the context of combined TGR and SR, reduce as follows in the present case of no other gravitational field source apart from the moving gravitational field source (or test particle)

$$\overline{m} \quad = \quad m_0(1 - \frac{2Gm_{0a}}{r'_p c_g^2})(1 - \frac{v^2}{c_\gamma^2})^{-1/2} \tag{66}$$

$$\overline{T} \quad = \quad m_0 c_\gamma^2(1 - \frac{2Gm_{0a}}{r'_p c_g^2})[(1 - \frac{v^2}{c_\gamma^2})^{-1/2} - 1] \tag{67}$$

Finally the absolute intrinsic line element implied by the components of the absolute intrinsic metric tensor of combined metric theory of absolute intrinsic gravity and absolute intrinsic motion of system (60), in the case of the presence of no other gravitational field source apart from the one moving, is the following at the surface of the moving field source

$$d\phi\hat{s}^2 = (1 - \frac{2G\phi\hat{m}_{0a}}{\phi\hat{r}_p\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2})\phi\hat{c}^2 d\phi\hat{t}^2 - (1 - \frac{2G\phi\hat{m}_{0a}}{\phi\hat{r}_p\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2})^{-1} d\phi\hat{\rho}^2 \tag{68}$$

where m_0 is the rest mass and r'_p is the classical radius (i.e. the radius of m_0) of the moving gravitational field source (or test particle), as defined earlier. The absolute intrinsic line element (68) is valid strictly at the surface of the moving field source. It shall become modified as follows at radial distances, $r' < r'_p$, within the moving field source upon deriving interior absolute intrinsic metric tensor of ϕMAG $\cup$ ϕMAM of a solid spherical body in an article later in this volume,

$$d\phi\hat{s}^2 = (1 - \frac{2G\phi\hat{m}_{0a}\phi\hat{r}^2}{\phi\hat{r}_p^3\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2})\phi\hat{c}^2 d\phi\hat{t}^2 - (1 - \frac{2G\phi\hat{m}_{0a}\phi\hat{r}^2}{\phi\hat{r}_p^3\,\phi\hat{c}_g^2} - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2}) d\phi\hat{\rho}^2;\; r' < r'_p \tag{69}$$

At a point P in space, which is momentarily located at radial distance, $r'(t') > r'_p$, from the center of the moving gravitational field source at any given instant t', we must allow $\phi\hat{V}_d$ to vanish and replace $\phi\hat{r}_p$ by $\phi\hat{r}(\phi\hat{t})$ in Eq. (68) to have as follows

$$d\phi\hat{s}^2 = (1 - \frac{2G\phi\hat{m}_{0\mathrm{a}}}{\phi\hat{r}(\phi\hat{t})\phi\hat{c}_g^2})\phi c^2 d\phi\hat{t}^2 - (1 - \frac{2G\phi\hat{m}_{0\mathrm{a}}}{\phi\hat{r}(\phi\hat{t})\phi\hat{c}_g^2})^{-1} d\phi\hat{\rho}^2;\ r'(t') \geq r'_p \qquad (70)$$

We have thus incorporated the effect of the gravitational field of a moving particle or body into the Lorentz-Einstein-Minkowski special relativity (LEM) and the metric theory of absolute intrinsic motion (ϕMAM), (in the absence of gravitational field of any other source). One finds, by letting $2Gm_{0\mathrm{a}}/r'_p c_g^2 = 0$, that the results from Eq. (64) through Eq. (67) become their usual forms in LEM, while the components of the absolute intrinsic metric tensor in the line element (68) and (69) become the following in the context of ϕMAM purely,

$$\phi\hat{g}_{00} = -\phi\hat{g}_{11}^{-1} = 1 - \frac{\phi\hat{V}_d^2}{\phi\hat{c}_\gamma^2};\ \phi\hat{g}_{12} = \phi\hat{g}_{21} = 0 \qquad (71)$$

Now $2Gm_{0\mathrm{a}}/r'_p c_g^2$ is approximately equal to zero for particles and bodies with relatively small masses, such as encountered in classical mechanics and classical gravitation. Hence LEM is approximately a valid special theory of relativity for such particles and bodies. Ideally, however, $2Gm_{0\mathrm{a}}/r'_p c_g^2$ vanishes for a particle or body with zero active gravitational rest mass ($m_{0\mathrm{a}} = 0$), which can hence not give rise to Newtonian gravitational field, that is, for which $-Gm_{0\mathrm{a}}/r' = 0$ for all r'. One particle with zero gravitational rest mass ($m_{0\,g} = 0$), but with non-zero dynamical rest mass ($m_{0\,\mathrm{em}} \neq 0$), is the electron. This fact that has been mentioned at different points in the previous articles shall be adequately justified with further development of the present theory. Thus LEM is correct in a strict sense for the electron and its anti-particle.

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