Article 20:
Two experimental consequences of the theory of gravitational relativity

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Two experimentally verifiable implications of the expressions for mass of a test particle and gravitational potential on flat spacetime in a gravitational field of arbitrary strength, in the context of the theory of gravitational relativity (TGR) are derived. The first is the existing expression for the shift in perihelion per revolution of a planet round an assumed spherical Sun, derived on a proposed curved spacetime in a gravitational field in the context of the general theory of relativity (GR), with excellent observational support in the case of the planet mercury, which is re-derived in a more compact and more straight forward manner on flat spacetime in the context of TGR. The second is a set of new (hybrid) forces of nature namely, gravi-electric, gravi-magnetic and laser-anti-gravitational forces, which arise when a capacitor (with electric field of extremely large strength between its plates), an electromagnet (with magnetic field of extremely large strength between its pole pieces) and a sphere containing very high energy radial laser beams respectively, each of which is safely enclosed in a box, falls freely towards a gravitational field source on flat spacetime in the context of TGR. Each of these forces is repulsive and opposes the gravitational attraction on a body towards a gravitational field source. They can therefore be used to control the weight (or effectively the inertia) of a body (without engine power or any other aid) in the gravitational field of the earth or a host planet.

1 Introduction

This article is devoted to two experimentally verifiable results of the theory of gravitational relativity (TGR), while all other experimental tests of the general theory of relativity (GR) and more shall be considered in the context of TGR in later articles. The first is the recalculation of the shift in perihelion per revolution of the planets on the flat relativistic spacetime of TGR, done in section two of this article. This serves as the first test of the effective gravitational acceleration on a test particle towards a gravitational field source on flat spacetime in the context of TGR presented as Eq. (86) or (87) of [1]. The shift in perihelion per revolution of the planets derived in the context of TGR in this article had been derived in the context of GR and confirmed experimentally already.

The second experimentally verifiable result of TGR are a set of repulsive hybrid forces that arise from the coupling of electrical and magnetic energy to gravity on the flat relativistic spacetime in the context of TGR, referred to as gravi-electric force and gravi-magnetic force and a third force that arises from the coupling of laser energy to gravity in the context of TGR, referred to as laser-anti-gravitational force. The gravi-electric and gravi-magnetic forces are isolated and severe odds against their experimental tests and applications shown in section three of this article.

The laser-anti-gravitational force is isolated and better prospects for its experimental test and application shown in section four of this article. Various super-aerodynamical feat in the gravitational field of the earth, which can be achieved with a craft equipped with the laser-anti-gravitational force are derived. These include hanging freely in mid-air; falling freely vertically towards the earth at a reduced free-fall acceleration that can be as low as desired; rising vertically freely against earth's gravity at any desired low or large acceleration; hovering at constant (or non-constant) velocity at low elevation; attaining large acceleration rapidly from an initial low velocity motion; breaking rapidly from high velocity motion; and changing direction of motion rapidly at high velocity. Possible future applications of laser-anti-gravitational force in aviation, mass-transit and space research are also mentioned.

2 Calculating the shift in perihelion per revolution of a planet in the context of the theory of gravitational relativity

The effective gravitational force on a test particle of rest mass m_0, which is located at radial distance r' from the center of the rest mass M_0 of an assumed spherically symmetric gravitational field source on the flat proper spacetime (Σ', ct') of classical gravitation, which corresponds to the inertial mass m of the test particle at radial distance r from the center of the inertial mass M of the gravitational field source on the flat relativistic spacetime (Σ, ct) of the theory of gravitational relativity (TGR), which follows from the effective gravitational acceleration suffered by a test particle towards a gravitational field source on the flat spacetime (Σ, ct) of TGR, derived and presented as equation (88) or (89) of [1] shall be reproduced here as follows

$$\begin{aligned}\vec{F}_{\text{eff}}(r') &= m_0\vec{g}_{\text{eff}}(r') \\ &= -\frac{GM_{0\text{a}}m_0}{r'^2}\left(1-\frac{2GM_{0\text{a}}}{r'c_g^2}\right)^{3/2}\hat{r} + \frac{3G^2M_{0\text{a}}^2m_0}{r'^3c_g^2}\left(1-\frac{2GM_{0\text{a}}}{r'c_g^2}\right)^{1/2}\hat{r} \qquad (1)\end{aligned}$$

This effective gravitational force is exact in a gravitational field of arbitrary strength. It operates on the flat relativistic spacetime (Σ, ct) of TGR in a gravitational field. However we shall gain better insight into the implications of Eq. (1) by writing its weak gravitational field limit (or post-Newtonian) approximation as follows

$$\vec{F}_{\text{eff}}(r') \approx -\frac{GM_{0\text{a}}m_0}{r'^2}\hat{r} + \frac{6G^2M_{0\text{a}}^2m_0}{r'^3c_g^2}\hat{r} \qquad (2)$$

It is to be recalled that the post-Newtonian gravitation (PNG) limit in weak gravitational fields and the Newtonian gravitation (NG) limit in the extremely weak gravitational field to the exact Eq. (1) have been derived in sub-section 3.1 of [1].

The attractive Newtonian force of gravity namely, the first term at the right-hand side of Eq. (2) is the only fundamental force. The second repulsive force is a second-order gravitational-relativistic correction term. There are more gravitational-relativistic correction force terms of higher orders in the full expansion of the exact relation (1). All the gravitational-relativistic correction force terms disappear in Newtonian gravitation limit for which $2GM_{0\mathrm{a}}/r'c_g^2 = 0$.

Now let m be the inertial mass of a planet moving in orbit of radius r from the center of the inertial mass, $M \equiv M_\odot$, of the assumed spherical sun on the flat relativistic spacetime (Σ, ct) of TGR. Eq. (1) expresses the net gravitational force on the planet due to the sun without approximation, while the post-Newtonian relation (2) gives its approximate weak field limit. The extra repulsive force term in Eq. (2) gives rise to a perturbation of the elliptical orbit of the planet calculated using the Newtonian force only, thereby causing a shift in perihelion of the planet as it revolves round the sun.

Now the motion on the equatorial plane of a particle of rest mass m_0 under a central force $\vec{F}(r')$ is described by the following well known equation in classical mechanics,

$$u'' + u = -\frac{F(u)}{m_0 H^2 u^2} \tag{3}$$

where $u = 1/r'(\phi)$, H = twice the constant areal velocity, $F(u) = F(1/r')$, $u'' = d^2u/d\phi^2$ and r', ϕ are polar coordinates. In the case of a planet moving under the effective gravitational force $\vec{F}_{\mathrm{eff}}(r')$ of Eq. (1) we have the following

$$-\frac{F(r')}{m_0} = -g_{\mathrm{eff}}(r') \tag{4}$$

If we substitute for $g_{\mathrm{eff}}(r')$ in Eq. (4) from the exact relation (1) we have

$$-\frac{F(u)}{m_0} = GM_{0\mathrm{a}}u^2\left(1 - \frac{2GM_{0\mathrm{a}}u}{c_g^2}\right)^{3/2} - \frac{3G^2M_{0\mathrm{a}}^2u^3}{c_g^2}\left(1 - \frac{2GM_{0\mathrm{a}}u}{c_g^2}\right)^{1/2} \tag{5}$$

Then by substituting Eq. (5) into Eq. (3) we have

$$u'' + u = \frac{GM_{0\mathrm{a}}}{H^2}\left(1 - \frac{2GM_{0\mathrm{a}}u}{c_g^2}\right)^{3/2} - \frac{3G^2M_{0\mathrm{a}}^2u}{H^2c_g^2}\left(1 - \frac{2GM_{0\mathrm{a}}u}{c_g^2}\right)^{1/2} \tag{6}$$

Eq. (6) is highly nonlinear. Its solution can be sought numerically. However we shall consider its weak gravitational field limit (or post-Newtonian approximation) in this paper, which is given as follows

$$u'' + u = \frac{GM_{0\mathrm{a}}}{H^2} - \frac{6G^2M_{0\mathrm{a}}^2 u}{H^2c_g^2}$$

or

$$u'' + (1 + \frac{6G^2M_{0\mathrm{a}}^2}{H^2c_g^2})u = \frac{GM_{0\mathrm{a}}}{H^2} \tag{7}$$

Equation (6) is the polar coordinate form of the modified Newton's second law of motion on flat relativistic spacetime in a gravitational field in the context of the theory of gravitational relativity (TGR), presented in rectangular coordinates as Eq. (90) of [1], which must be used to describe the motion of a test particle on the flat relativistic spacetime (Σ, ct) in a gravitational field of arbitrary strength in the context of TGR, while Eq. (7) is its weak gravitational field limit approximation. The calculation being done in this section is a test of Eq. (88) or (90) of [1] and, indeed, a test of the modified form of Newton's (or classical) law of gravity in the context of TGR.

If we let

$$\epsilon = \frac{6G^2M_{0\mathrm{a}}^2}{H^2c_g^2} \tag{8}$$

Then Eq. (7) becomes the following

$$u'' + (1 + \epsilon)u = \frac{GM_{0\mathrm{a}}}{H^2} \tag{9}$$

In the classical Kepler problem, only the Newtonian gravitational potential is present, the extra relativistic correction potential term in the context of TGR is absent. Hence $\epsilon = 0$, and Eq. (7) or (9) reduces as the classical Kepler problem

$$u'' + u = \frac{GM_{0\mathrm{a}}}{H^2} \tag{10}$$

The suitable form of the general solution to Eq. (10) is the following

$$u = A + B\cos\phi \tag{11}$$

where $A = GM_{0\mathrm{a}}/H^2$ and B is an integration constant. The solution (11) is purely periodic, hence, as well known, the classical Kepler problem does not predict a shift in perihelion of a planet. The suitable general solution to Eq. (9), on the other hand, is the following

$$u = A' + B' \cos(1+\epsilon)^{1/2}\phi \tag{12}$$

where $A' = A/(1+\epsilon) = GM_{0\mathrm{a}}/H^2(1+\epsilon)$ and B' is an integration constant related to B, see pages 207 - 209 of [2].

The fact that $\epsilon \neq 0$ in the cosine argument in Eq. (12) implies aperiodicity in the planetary motion causing a shift in perihelion per revolution of the planet. Now the perihelion of a planet occurs when r is a minimum or when, $u = 1/r$, is a maximum. From Eq. (12), we see that u is a minimum when

$$(1+\epsilon)^{1/2}\phi = 2\pi n \tag{13}$$

or when

$$\phi = 2\pi n(1+\epsilon)^{-1/2} \tag{14}$$

where n is a positive integer. Therefore successive perihelia will occur at an interval $\Delta\phi$ where

$$\Delta\phi = 2\pi(1+\epsilon)^{-1/2} \tag{15}$$

The shift in perihelion per revolution is therefore given as follows

$$\delta\phi = 2\pi(1-(1+\epsilon)^{-1/2}) \tag{16}$$

or

$$\delta\phi = 2\pi(\frac{\epsilon}{2} - \frac{3}{8}\epsilon^2 + \frac{5}{16}\epsilon^3 - \cdots) \tag{17}$$

Now in the weak gravitational field limit being considered, $\epsilon << 1$, we can neglect terms in second and higher orders of ϵ in Eq. (17). Therefore for the weak gravitational field of the sun,

$$\delta\phi \approx 2\pi(\frac{\epsilon}{2}) \tag{18}$$

Substitution of the expression for ϵ in Eq. (8) into Eq. (18) yields expression for the shift in perihelion per revolution of a planet in the weak field limit as follows

$$\delta\phi \approx 2\pi(\frac{3G^2M_{0\mathrm{a}}^2}{H^2c_g^2}) \tag{19}$$

Equation (19) gives the well known expression for the shift in perihelion per revolution of a planet in the weak gravitational field limit, which can be found in the standard texts on the general theory of relativity (GR). The only difference is that the inertial mass M that appears in Eq. (19) in GR is replaced by the active gravitational mass M_{0a} in the present theory. The equivalence in magnitude of the rest mass M_0 and the active gravitational mass (see derivation of Eq. (113) in section 4 of [1]), allows M_{0a} to be replaced by M_0 in Eq. (19) in numerical calculations. Then the valid approximation $M \approx M_0$ in the weak gravitational field of the sun further allows M_{0a} to be replaced by M in Eq. (19), thereby converting Eq. (19) to its form in GR. As expected, the calculation on the flat spacetime of TGR in this section is more compact and more straight forward than the geodesic approach on curved spacetime in GR. A comparison of the above calculation on the flat spacetime of TGR with the calculation on pages 199-209 of [2] on curved spacetime in GR will confirm this.

The purpose of re-derivation of the perihelion shifts of the planets, in the context of the present theory in this section, is to show that the relativistic form of Newton's second law of motion on flat spacetime in a gravitational field in the context of TGR, presented as Eq. (90) of [1], which translates into Eq. (6), (or Eq. (7) in the weak field limit) above in polar coordinates, which must be used to describe the motion of a test particle on flat spacetime in the context of TGR, leads to the same expression (19) for the shift in perihelion per revolution of a planet round an assumed spherical sun, usually calculated by considering the motion of a planet along the geodesic on the proposed curved spacetime in a gravitational field in GR. See for example, chapter six of [2] and also [3].

The expression (19) first derived by Albert Einstein [4], is known to be in fair agreement with observational data for the planets, and indeed, in excellent agreement in the case of the planet mercury, for which it predicts a total shift of 43.03 seconds of an arc per century, while the observational result is 43.11 ± 0.45 seconds of an arc per century, as quoted on page 209 of [2]. On the other hand, the prediction of Eq. (19) differs appreciably from observation in the case of the other planets, as Table I taken from page 213 of [2] shows.

Table I: Calculated and observed perihelic shifts of some planets.

	Distance r from sun $\times 10^9$ m	Shift S, sec. of arc/century Calculated	Observed
Mercury	58	43.03	43.11 ± 0.45
Venus	108	8.6	8.4 ± 4.8
Earth	149	3.8	5.0± 1.2
Icarus	161	10.3	9.8 ± 0.8

Effort has been made to reconcile the calculated and observed shifts in perihelia of the planets by including the effect of the quadrupole moment of the sun in general relativity, pages 209-213 of [2]. However it has been remarked that much more precise measurements of the flattening of the sun is needed for the separation of relativistic and quadrupole effects to be feasible. Correction to relation (19) due to the imperfect spherical shape of the sun shall be investigated in the context of the Maxwellian theory of gravity to be developed on the flat relativistic spacetime of the theory of gravitational relativity elsewhere with further development.

The theory of gravitational relativity (TGR) and the relativistic form of Newtonian theory of gravity in the context of TGR have passed the first test. They must also be tested with the other 'traditional' problems namely, gravitational red shift, bending of light by a gravitational field source and radar time delay in the gravitational field of the sun, as benchmark, as shall be done elsewhere with further development.

The entire theories of gravity and cosmology must ultimately be reformulated in the contexts of the co-existing pair of theories of gravity isolated in the present evolving fundamental theory namely, the metric theory of absolute intrinsic gravity (ϕMAG) on curved 'two-dimensional' absolute intrinsic spacetime ($\phi\hat{\rho}, \phi\hat{c}\phi\hat{t}$) and the theory of gravitational relativity/intrinsic theory of gravitational relativity (TGR/ϕTGR) on flat four-dimensional relativistic spacetime (Σ, ct) and its underlying flat two-dimensional relativistic intrinsic spacetime ($\phi\rho, \phi c\phi t$) in a gravitational field, as alternative to their formulation on postulated curved four-dimensional spacetime in the context of the general theory of relativity. Expectedly new results shall be found and new insights into the phenomena shall be gained from such effort. Indeed a new result in gravitation namely, the gravi-electric force, gravi-magnetic force and laser-anti-gravitational force, shall be derived in the context of TGR in the next two sections to support the fact that new results shall definitely be found in reformulating gravity and cosmology in the context of the duo ϕMAG and TGR/ϕTGR.

3 Isolating the gravi-electric and gravi-magnetic forces in the context of the theory of gravitational relativity

As a quick reward of the theory of gravitational relativity (TGR), I shall derive, in this section, a repulsive gravi-electric force on a shielded charged test particle or a box containing a capacitor with electric field within it, which is falling freely towards a gravitational field source with zero net electric charge and a repulsive gravi-magnetic force on a box containing an electromagnet with magnetic field within it, which is falling freely towards a gravitational field source, as other implications of the mass and gravitational potential relations in the context of TGR presented as Eqs. (26) and (54) of [5]. The shifts in perihelia of the planets have been recalculated on flat spacetime in the context of TGR, as an implication of those relations in the preceding section.

Let us consider a spherical metallic ball of rest mass m_0 and classical radius r'_p (of m_0), on which is uniformly distributed net electric charge Q, which is located at radial distance r' from the center of the rest mass M_0 of a spherical gravitational field source on the flat proper spacetime (Σ', ct') of classical gravitation. The rest mass m_0 evolves into the gravitational-relativistic (or inertial) mass m of the charged ball at radial distance r from the center of the gravitational-relativistic (or inertial) mass M of the gravitational field source on the flat relativistic spacetime (Σ, ct) in the context of TGR. The charged ball possesses rest mass m_0 and equivalent rest mass $m_{0\text{eq}}$ in the proper Euclidean 3-space Σ' due to the electric charge Q on it where, $m_{0\text{eq}} = Q^2/4\pi\epsilon'_0 r'_p c^2_\gamma$. Hence the "net rest mass" of the ball is

$$m_{0\text{net}} = m_0 + m_{0\text{eq}} = m_0 + \frac{Q^2}{4\pi\epsilon'_0 r'_p c^2_\gamma}, \tag{20}$$

where ϵ'_0 is the proper (or primed) electric permittivity of the proper Euclidean 3-space Σ'.

However only the rest mass m_0 can be observed or measured as actual rest mass of the ball. The gravitational-relativistic equivalent mass (or equivalent inertial mass) m_{eq} on the flat relativistic spacetime of TGR is related to the equivalent rest mass $m_{0\text{eq}}$ the same way the inertial mass m is related to the rest mass m_0 by Eq. (26) of [1]. The gravitational-relativistic "net mass" (or net inertial mass) m_{net} in the context of TGR of the charged ball is then given at radial distance r from the center of the inertial mass M of the assumed spherical gravitational field source on the flat relativistic spacetime (Σ, ct) of TGR as follows

$$m_{\text{net}} = m + m_{\text{eq}} = (m_0 + \frac{Q^2}{4\pi\epsilon'_0 r'_p c^2_\gamma})(1 - \frac{2GM_{0\text{a}}}{r' c^2_g}) \tag{21}$$

The gravitational-relativistic gravitational potential function in the context of TGR at the location of the charged ball given by equation (54) of [1] shall be rewritten as follows

$$\Phi(r) = -\frac{GM_{0\text{a}}}{r} = -\frac{GM_{0\text{a}}}{r'}(1 - \frac{2GM_{0\text{a}}}{r' c^2_g})^{1/2} \tag{22}$$

Hence the net gravitational-relativistic potential energy, (in the relativistic Euclidean 3-space Σ in context of TGR), of the charged ball is the following

$$\begin{aligned} U_{\text{net}} &= m\Phi(r) \\ &= -\frac{GM_{0\text{a}}}{r'}(m_0 + \frac{Q^2}{4\pi\epsilon'_0 r'_p c^2_\gamma})(1 - \frac{2GM_{0\text{a}}}{r' c^2_g})^{3/2} \end{aligned} \tag{23}$$

This equation is exact in all gravitational fields. However we shall consider its weak gravitational field limit (or its post-Newtonian approximation) in order to gain better insight into its implications, in which case the equation reduces approximately as follows

$$\begin{aligned} U_{\rm net} &\approx -\frac{GM_{0\rm a}}{r'}(m_0 + \frac{Q^2}{4\pi\epsilon_0' r_p' c_\gamma^2})(1 - \frac{3GM_{0\rm a}}{r' c_g^2}) \\ &\approx -\frac{GM_{0\rm a} m_0}{r'} + \frac{3G^2 M_{0\rm a}^2 m_0}{r'^2 c_g^2} - \frac{GM_{0\rm a} Q^2}{4\pi\epsilon_0' r_0 c_\gamma^2 r'} + \frac{3G^2 M_{0\rm a}^2 Q^2}{4\pi\epsilon_0 r_p' r'^2 c_g^2 c_\gamma^2} \end{aligned} \tag{24}$$

The first term at the right-hand side of Eq. (24) is the fundamental (Newtonian) gravitational potential energy, the second term is the gravitational-relativistic correction term, which gives rise to the second-order relativistic correction force term that causes shifts of perihelia of the planets in the preceding section. The third term, which can be written as $-GM_{0\rm a} m_{0\rm eq}/r'$, arises by virtue of the interaction of the equivalent rest mass $m_{0\rm eq}$ with the Newtonian gravitational potential. However there is no interaction between the Newtonian gravitational field and electrostatic potential energy, (or with mass equivalent $m_{0\rm eq}$ of electrostatic potential energy), as explained below.

The third term at the right-hand side of Eq. (24), which represents the interaction of the equivalent mass of electrostatic potential energy with the Newtonian gravitational field does not exist, or is not an actual gravitational potential energy, and must hence be removed. This fact is made more explicit by rewriting Eq. (24), while factorizing $-GM_{0\rm a}/r'$ out, as follows

$$U_{\rm net} \approx -\frac{GM_{0\rm a}}{r'}\left(m_0 - \frac{2GM_{0\rm a} m_0}{r' c_g^2} + \frac{Q^2}{4\pi\epsilon_0' r_p' c_\gamma^2} - \frac{3GM_{0\rm a} Q^2}{4\pi\epsilon_0' r_p' r' c_g^2 c_\gamma^2}\right) \tag{25}$$

The only mass or equivalent mass term inside the parentheses with zero inertial or zero passive gravitational attribute, which can hence not interact with the Newtonian gravitational potential outside the parentheses is $Q^2/4\pi\epsilon_0' r_p' c_\gamma^2$. This term is certainly not a passive gravitational mass. The second-order geometry-induced mass term $-3GM_{0\rm a}Q^2/4\pi\epsilon_0' r_p' r' c_g^2 c_\gamma^2$ possesses both gravitational and electromagnetic attributes, being a product of square of gravitational speed and equivalent mass of electrostatic potential energy, $-(\frac{3}{2})(2GM_{0\rm a}/r' c_g^2)(Q^2/4\pi\epsilon_0' r_p' c_\gamma^2)$, (or being a coupling already of electrostatic potential energy to gravity). It therefore possesses passive gravitational attribute and can interact with the Newtonian gravitational potential, yielding the fourth term at the right-hand side of Eq. (24).

As further justification of the fact that the third term at the right-hand side of Eq. (24) must be allowed to vanish, let us consider the classical theory of gravity.

Let the test particle of rest mass m_0 above, on which net electric charge Q is uniformly distributed, be located at radial distance r' from the center of the rest mass M_0 of a gravitational field source on the flat proper spacetime (Σ', ct') of classical gravitation. Then by allowing the proper (or Newtonian) gravitational potential, $\Phi' = -GM_{0a}/r'$, to interact with the 'net rest mass', $m_{0\mathrm{net}} = m_0 + Q^2/4\pi\epsilon_o' r_p' c_\gamma^2$, of the charged test particle, we obtain the proper gravitational potential energy U'_{net} in Σ' of the test particle as follows

$$\begin{aligned} U'_{\mathrm{net}} &= -\frac{GM_{0a}}{r'}(m_0 + \frac{Q^2}{4\pi\epsilon_o' r_p' c_\gamma^2}) \\ &= -\frac{GM_{0a}m_0}{r'} - \frac{GM_{0a}Q^2}{4\pi\epsilon_o' r_p' r' c_\gamma^2} \end{aligned} \tag{26}$$

Clearly the second term at the right-hand side of Eq. (26) is unknown and, indeed, does not exist in Newtonian gravitation. It is meaningless (or invalid) and must be allowed to vanish. The second term at the right-hand side of Eq. (26) is the third term at the right-hand side of Eq. (24), which must be allowed to vanish.

As a rule, a classical non-gravitational energy E' has no interaction with the Newtonian gravitational field on the flat proper spacetime (Σ', ct') of classical gravitation. For instance there is no concept of bending of light by a gravitational field source in the context of the Newtonian theory of gravity. However when a classical non-gravitational energy E' interacts with a relativistic gravitational field, it becomes the gravitational-relativistic non-gravitational energy E on the flat relativistic spacetime in the context of TGR, which is given in terms of the factor $\gamma_g(r')^{-2}$, that appears in the expressions for gravitational-relativistic parameters in the context of TGR, derived in [5] and [1], like all other forms of energy, (safe massless radiation energy), as follows

$$\begin{aligned} E = \gamma_g(r')^{-2}E' &= E'(1 - \frac{2GM_{0a}}{r' c_g^2}) \\ &= E' - \frac{2GM_{0a}E'}{r' c_g^2} \end{aligned} \tag{27}$$

The first term at the right-hand side of Eq. (27) is still the classical non-gravitational energy that has no interaction with the Newtonian gravitational potential. On the other hand, the second term at the right-hand side of Eq. (27) is an intrinsic geometry-induced (or intrinsic metric-induced) energy with geometry-induced passive gravitational attribute. It has interaction with the Newtonian gravitational potential or field (in the context of TGR).

Electrostatic potential energy or energy stored in electromagnetic field is an example of non-gravitational energy. The summary of the above is that electrostatic potential energy or energy stored in electromagnetic field, has no first-order or fundamental interaction with the (Newtonian) gravitational field, but a second-order intrinsic geometry- (or intrinsic metric-) induced interaction in a gravitational field in the context of TGR.

Relation (27) is given in the context of the theory of absolute intrinsic gravity (ϕAG) (as derived in [6]) as follows

$$\begin{aligned}\phi E' &= \phi\hat{g}_{00}\phi\hat{E} = (1 - \phi\hat{k}_g(\phi\hat{r})^2)\phi\hat{E} \\ &= \phi\hat{E} - \phi\hat{k}_g(\phi\hat{r})^2\phi\hat{E}\end{aligned} \tag{28}$$

where the absolute intrinsic curvature parameter $\phi\hat{k}(\phi\hat{r})$ of the curved absolute absolute intrinsic space $\phi\hat{\rho}$ is related to the square of the absolute intrinsic gravitational speed as

$$\phi\hat{k}_g(\phi\hat{r})^2 = \phi\hat{V}_g(\phi\hat{r})^2/\phi\hat{c}_g^2 = 2G\phi\hat{M}_{0\mathrm{a}}/\phi\hat{r}\phi\hat{c}_g^2,$$

as derived in [7]. Hence Eq. (28) can be written in terms of the absolute intrinsic gravitational speed as follows

$$\begin{aligned}\phi E' &= \phi\hat{g}_{00}\phi\hat{E} = (1 - \frac{\phi\hat{V}_g^2}{\phi\hat{c}_g^2})\phi\hat{E} = (1 - \frac{2G\phi\hat{M}_{0\mathrm{a}}}{\phi\hat{r}\phi\hat{c}_g^2})\phi\hat{E} \\ &= \phi\hat{E} - \frac{2G\phi\hat{M}_{0\mathrm{a}}}{\phi\hat{r}\phi\hat{c}_g^2}\phi\hat{E}\end{aligned} \tag{29}$$

The second term at the right-hand side of Eq. (28) exists because the absolute intrinsic curvature parameter $\phi\hat{k}_g(\phi\hat{r})$ is non-zero. In other words, the second term at the right-hand side of Eq. (28) or (29) exists because the 'two-dimensional' absolute intrinsic metric spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ is curved, thereby possessing absolute intrinsic sub-Riemannian metric tensor $\phi\hat{g}_{ik}$ in every gravitational field in the context of ϕAG. It is clearly appropriate to describe the second term at the right-hand side of Eq. (28) or (29) as absolute intrinsic spacetime-geometry-induced (or absolute intrinsic metric-induced).

Now Eq. (28) or (29) in ϕAG translates into Eq. (27) in TGR. The component $\phi\hat{g}_{00} = (1 - 2G\phi\hat{M}_{0\mathrm{a}}/\phi\hat{r}\phi\hat{c}_g^2)$ of the absolute intrinsic Riemannian metric tensor of the metric theory of absolute intrinsic gravity (ϕMAG) on the curved 'two-dimensional absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$, which is a component of the theory of absolute intrinsic gravity (ϕAG), translates into the factor $\gamma_g(r')^{-2} = (1 - 2GM_{0\mathrm{a}}/r'c_g^2)$ on flat spacetime (Σ, ct) in the context of TGR. The second term at the right-hand side of Eq. (28) or (29) in ϕAG translates into the second terms at the right-hand side of Eq. (27) in TGR. The second term at the right-hand side of Eq. (27) in TGR would vanish should the second term at the right-hand side of Eq. (28) or (29) in ϕAG vanish, which would be the case if there was no curvature of the absolute intrinsic spacetime $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$, (or if there was no absolute intrinsic Riemannian spacetime geometry). However $(\phi\hat{\rho}, \phi\hat{c}\phi\hat{t})$ is curved in a gravitational field of arbitrary strength. The reference to the second term at the right-hand side of Eq. (27), (and the fourth term at the right-hand side of Eq. (24) or (25) in the context of TGR previously) as intrinsic (Riemann) geometry-induced or intrinsic metric-induced terms, despite the fact that TGR operates on flat spacetime with constant Lorentzian metric tensor, is due to their background in ϕMAG with absolute intrinsic sub-Riemannian metric tensor.

Further justification for allowing the third term at the right-hand side of Eq. (24) or (25) to vanish shall be provided when the Reisner-Nordstrom line element, (written without deriving it as Eq. (18) of [8], shall be derived in the context of the 'two-dimensional' metric theory of combined absolute intrinsic gravity and absolute intrinsic electromagnetism (ϕMAG + ϕMEM) elsewhere with further development.

The hybrid intrinsic geometry-induced fourth term of Eq. (24) shall be named the gravi-electric potential energy. By removing the meaningless third term from the right-hand side of that equation we obtain the following

$$U_{\text{net}}(r') \approx -\frac{GM_{0\text{a}}m_0}{r'} + \frac{3}{2}\frac{GM_{0\text{a}}m_0}{r'}\left(\frac{2GM_{0\text{a}}}{r'c_g^2}\right) + \frac{3}{2}\frac{GM_{0\text{a}}}{r'}\left(\frac{2GM_{0\text{a}}}{r'c_g^2}\right)\frac{Q^2}{4\pi\epsilon_0' r_p' c_\gamma^2} \quad (30)$$

This is the gravitational-relativistic gravitational potential energy of the charged ball on flat relativistic spacetime (Σ, ct) in the post-Newtonian approximation.

By dividing through Eq. (30) by the rest mass m_0 of the charged ball we obtain the effective gravitational-relativistic gravitational potential $\Phi_{\text{eff}}(r')$ ('seen' by the rest mass m_0) on the flat relativistic spacetime (Σ, ct)) in the post-Newtonian approximation as follows

$$\Phi_{\text{eff}}(r') \approx -\frac{GM_{0\text{a}}}{r'} + \frac{3}{2}\frac{GM_{0\text{a}}}{r'}\left(\frac{2GM_{0\text{a}}}{r'c_g^2}\right) + \frac{3}{2}\frac{GM_{0\text{a}}}{r'}\left(\frac{2GM_{0\text{a}}}{r'c_g^2}\right)\frac{Q^2}{4\pi\epsilon_0' m_0 r_p' c_\gamma^2} \quad (31)$$

The effective gravitational-relativistic gravitational acceleration suffered by (the rest mass m_0 of) the charged test particle on the flat relativistic spacetime (Σ, ct), is then given in the post-Newtonian approximation from definition as follows

$$\begin{aligned}\vec{g}_{\text{eff}}(r') &= -\frac{\mathrm{d}\Phi_{\text{eff}}(r')}{\mathrm{d}r'}\hat{r} \\ &\approx -\frac{GM_{0\text{a}}}{r'^2}\hat{r} + \frac{6}{2}\frac{GM_{0\text{a}}}{r'^2}\left(\frac{2GM_{0\text{a}}}{r'c_g^2}\right)\hat{r}\end{aligned}$$

$$+\frac{6}{2}\frac{GM_{0a}}{r'^2}(\frac{2GM_{0a}}{r'c_g^2})\frac{Q^2}{4\pi\epsilon_o' m_0 r_p' c_\gamma^2}\hat{r} \tag{32}$$

where $\hat{r}$ is the unit vector along the radial direction away from the center of the inertial mass M of the assumed spherical gravitational field source to the charged ball in the relativistic Euclidean 3-space Σ of TGR.

Equation (32) gives the effective gravitational acceleration suffered by the charged ball on the flat relativistic spacetime (Σ, ct) of TGR in the weak gravitational field limit (or in the post-Newtonian approximation). The effective gravitational force suffered by the charged ball on the flat relativistic spacetime (Σ, ct) of TGR is therefore given as follows

$$\begin{aligned} \vec{F}_{\text{eff}}(r') &= m_0\vec{g}_{\text{eff}}(r') \\ &= -\frac{GM_{0a}m_0}{r'^2}\hat{r} + \frac{6G^2M_{0a}^2m_0}{r'^3c_g^2}\hat{r} + \frac{6G^2M_{0a}^2Q^2}{4\pi\epsilon_o' r_p' r'^3 c_g^2 c_\gamma^2}\hat{r} \end{aligned} \tag{33}$$

It must be noted that $\vec{g}_{\text{eff}}(r')$ has been multiplied by the rest mass m_0 and not by its inertial mass m of the charged ball. This arises because the effective gravitational potential $\Phi_{\text{eff}}(r')$ has been obtained by dividing the net gravitational potential energy $U_{\text{net}}(r')$ by m_0 between Eqs. (30) and (31).

The second intrinsic geometry-induced force at the right-hand side of Eq. (33), which is the force that gives rise to shift in perihelion of a planet, as found in the preceding section, is negligible compared to the first fundamental (or Newtonian) force in a weak gravitational field, and it is uncontrollable by man. On the other hand, the third intrinsic geometry-induced term, to be referred to as the gravi-electric force, can be made significantly large by increasing the charge Q. Thus by neglecting the second term, equation (33) simplifies further as follows

$$\vec{F}_{\text{eff}}(r') \approx -\frac{GM_{0a}m_0}{r'^2}\hat{r} + \frac{6G^2M_{0a}^2Q^2}{4\pi\epsilon_o' r_p' r'^3 c_g^2 c_\gamma^2}\hat{r} \tag{34}$$

Equation (34) shows that the (attractive) Newtonian gravitational force on the charged ball radially towards the center of the assumed spherical gravitational field source, is reduced by the (repulsive) gravi-electric force on the ball radially away from the center of the gravitational field source. By dividing through equation (34) by the rest mass m_0 of the charged ball, we rewrite the effective gravitational acceleration on it as follows

$$\begin{aligned} \vec{g}_{\text{eff}}(r') &\approx -\frac{GM_{0a}}{r'^2}\hat{r} + \frac{6G^2M_{0a}^2 r' Q^2}{4\pi\epsilon_o' m_0 r_p' r'^4 c_g^2 c_\gamma^2}\hat{r} \\ &\approx -\,|\vec{g}'|\,\hat{r} + \frac{6\,|\vec{g}'|^2\, r' Q^2}{4\pi\epsilon_o' m_0 r_p' c_g^2 c_\gamma^2}\hat{r} \end{aligned} \tag{35}$$

Now let us consider what appears as the reciprocal of the free fall of a charged test particle towards a gravitational field source of zero net electric charge namely, the free fall of a spherical test particle of rest mass m_0 and classical radius r'_p (of m_0) with zero net electric charge towards a spherical gravitational field source of rest mass M_0 on which net electric charge Q is uniformly distributed. The gravitational-relativistic gravitational potential in the relativistic Euclidean 3-space Σ of TGR is given in this case as follows

$$\begin{aligned} \Phi(r') &= -\frac{GM_{0a}}{r'}\gamma_g(r')^{-1} \\ &= -\frac{GM_{0a}}{r'}(1-\frac{2GM_{0a}}{r'c_g^2}+\frac{2GQ^2}{4\pi\epsilon'_o r'^2 c_g^2 c_\gamma^2})^{1/2} \end{aligned} \tag{36}$$

where $\gamma_g(r')^{-2} = \mid \ \phi\hat{g}_{00} \ \mid$, and

$$\phi\hat{g}_{00} = 1 - 2G\phi\hat{M}_{0a}/\phi\hat{r}\phi\hat{c}_g^2 + 2GQ^2/4\pi\phi\hat{\epsilon}_o\phi\hat{r}^2\phi\hat{c}_g^2\hat{c}_\gamma^2,$$

in the context of the metric theory of combined absolute intrinsic gravity and absolute intrinsic electromagnetism (ϕMAG+ϕMEM), (written as Eq. (18) of [8] without deriving it), which shall be derived in a later paper. Hence

$$\gamma_g(r')^{-2} = 1 - 2GM_{0a}/r'c_g^2 + 2GQ^2/4\pi\epsilon'_o r'^2 c_g^2 c_\gamma^2,$$

in the present problem.

Thus the gravitational-relativistic gravitational potential energy U possessed by the test particle in the relativistic Euclidean 3-space Σ in the context of TGR is the following

$$\begin{aligned} U(r') &= m\Phi(r') = \gamma_g(r')^{-2}m_0\gamma_g(r')^{-1}\Phi'(r') \\ &= \gamma_g(r')^{-3}m_0\Phi'(r') \\ &= -\frac{GM_{0a}m_0}{r'}(1-\frac{2GM_{0a}}{r'c_g^2}+\frac{2GQ^2}{4\pi\epsilon'_o r'^2 c_g^2 c_\gamma^2})^{3/2} \end{aligned} \tag{37}$$

Equation (37) is exact for all gravitational field sources and for all quantities of net electric charge Q put on a gravitational field source. However we shall gain insight into the implication of this equation by considering its post-Newtonian approximation, in which case it simplifies as follows

$$\begin{aligned} U(r') &= -\frac{GM_{0a}m_0}{r'}(1-\frac{3GM_{0a}}{r'c_g^2}+\frac{3GQ^2}{4\pi\epsilon'_o r'^2 c_g^2 c_\gamma^2}) \\ &= -\frac{GM_{0a}m_0}{r'}+\frac{3G^2M_{0a}^2 m_0}{r'^2 c_g^2}-\frac{3G^2M_{0a}Q^2 m_0}{4\pi\epsilon'_o r'^3 c_g^2 c_\gamma^2} \end{aligned} \tag{38}$$

Every term in this equation possesses geometry-induced gravitational attribute and must therefore be retained.

By dividing through Eq. (38) by the rest mass m_0 of the test particle, we obtain the effective gravitational potential ('seen' by the rest mass m_0) in the relativistic Euclidean 3-space Σ of TGR as follows

$$\Phi_{\text{eff}}(r') = -\frac{GM_{0\text{a}}}{r'} + \frac{3G^2M_{0\text{a}}^2}{r'^2c_g^2} - \frac{3G^2M_{0\text{a}}Q^2}{4\pi\epsilon_0'r'^3c_g^2c_\gamma^2} \tag{39}$$

Hence the effective gravitational acceleration suffered by the test particle in Σ in the context of TGR is the following

$$\vec{g}_{\text{eff}}(r') = -\frac{GM_{0\text{a}}}{r'^2}\hat{r} + \frac{6G^2M_{0\text{a}}^2}{r'^3c_g^2}\hat{r} - \frac{9G^2M_{0\text{a}}Q^2}{4\pi\epsilon_0'r'^4c_g^2c_\gamma^2}\hat{r} \tag{40}$$

And the effective gravitational force on the test particle towards the center of the charged spherical gravitational field source is

$$\vec{F}_{\text{eff}}(r') = -\frac{GM_{0\text{a}}m_0}{r'^2}\hat{r} + \frac{6G^2M_{0\text{a}}^2m_0}{r'^3c_g^2}\hat{r} - \frac{9G^2M_{0\text{a}}m_0Q^2}{4\pi\epsilon_0'r'^4c_g^2c_\gamma^2}\hat{r} \tag{41}$$

The third term at the right-hand side of Eq.(41) is a gravi-electric force on the test particle, which is attractive towards the field source in this case.

The present problem in which a particle with zero net electric charge falls freely towards a gravitational field source with net electric charge, is really not the reciprocal of the problem we started with, in which a test particle with net electric charge falls freely towards a gravitational field source with zero net electric charge. For while gravi-electric force on the test particle towards the gravitational field source is attractive in the present case, it is repulsive in Eq. (33) in the former problem.

It shall be remarked that the second situation of the combined electromagnetism and gravity in the context of TGR above has been considered for theoretical interest and completeness only. There are perhaps massive bodies with large quantities of net electric charge in the universe. However the situation that shall be of interest to us henceforth, because of prospects for application, is the interaction of a charged test particle with the gravitational field of the earth with zero net charge. We shall therefore return to the problem we started with in this section and apply Eq. (34) or (35) to a test particle with net electric charge falling freely towards the earth.

Now the proper (or primed) parameters that appear in Eq. (34) or (35) are parameters on flat proper spacetime (Σ', ct') of classical gravitation, which are elusive to 3-observers in the relativistic Euclidean 3-space Σ of TGR. Ideally they must be calculated from the respective observed (or measurable) gravitational-relativistic (or unprimed) parameters on the flat relativistic spacetime (Σ, ct) of TGR, by following the procedure described in sub-section 4.1 of [1]. Essentially the factors, $\gamma_g(r')^{-2}$, $\gamma_g(r')^{-1}$ and $\gamma_g(r')$ that appear in parameter transformations in the context of TGR must be evaluated as described in that sub-section.

However while the proper (or primed) parameters must be calculated in strong gravitational fields, the following approximations of the parameter relations derived in [5], [1] and [9] and summarized in Table 1 of [9], can be applied in numerical calculations without significant loss of accuracy in the weak gravitational field of the earth.

$$\left.\begin{array}{rcl} m &=& m_0(1 - \dfrac{2GM_{0\mathrm{a}}}{r'c_g^2}) \approx m_0 \\ M &=& M_0(1 - \dfrac{2GM_{0\mathrm{a}}}{R_0 c_g^2}) \approx M_0 \\ r_p &=& r_p'(1 - \dfrac{2GM_{0\mathrm{a}}}{r'c_g^2})^{1/2} \approx r_p' \\ r &=& \int_0^{R_0}(1 - \dfrac{2GM_{0\mathrm{a}}}{R_0 c_g^2}\dfrac{r'^2}{R_0^2})^{1/2} dr' \approx r' \\ \epsilon_{\mathrm{o}} &=& \epsilon_{\mathrm{o}}'(1 - \dfrac{2GM_{0\mathrm{a}}}{r'c_g^2})^{-1/2} \approx \epsilon_{\mathrm{o}}' \\ \vec{g} &=& \vec{g}' \end{array}\right\} \qquad (42)$$

We shall by virtue of the approximations in system (42) (which are good to be applied in numerical calculations in the weak gravitational field of the earth), replace the proper (or primed) parameters on the flat proper spacetime (Σ', ct') of classical gravitation by their observed gravitational-relativistic (or unprimed) values on the flat relativistic spacetime (Σ, ct) of TGR in Eq. (35). In other words, we shall let $M_{0\mathrm{a}} \to M_{\mathrm{a}} \equiv M$; $m_0 \to m$; $r_p' \to r_p$; $r' \to r$; $\epsilon_{\mathrm{o}}' \to \epsilon_{\mathrm{o}}$ and $\vec{g}' \to \vec{g}$ in Eq. (33).

Thus for our charged ball of inertial mass m of radius r_p with net electric charge Q uniformly distributed over it, which is located at an elevation z above the earth's surface in the relativistic Euclidean 3-space Σ of TGR, we shall approximately replace $M_{0\mathrm{a}}$ by $M_{e\mathrm{e}} \equiv M_{\mathrm{e}}$; m_0 by m; r_p' by r_p; r' by $r_{\mathrm{e}} + z$; ϵ_{o}' by ϵ_{o} and $\vec{g}'$ by $\vec{g}$ in Eq. (35) for the purpose of numerical calculation to have

$$\vec{g}_{\mathrm{eff}} \approx -\frac{GM_{\mathrm{e}}}{(r_{\mathrm{e}} + z)^2}\hat{r} + \frac{6G^2 M_{\mathrm{e}}^2 Q^2}{4\pi\epsilon_{\mathrm{o}} m r_p (r_{\mathrm{e}} + z)^3 c_g^2 c_\gamma^2}\hat{r} \qquad (43)$$

or

$$\vec{g}_{\mathrm{eff}} \approx -|\vec{g}|\hat{r} + \frac{6|\vec{g}|^2 (r_{\mathrm{e}} + z) Q^2}{4\pi\epsilon_{\mathrm{o}} m r_p c_g^2 c_\gamma^2}\hat{r}$$

$$\approx \quad \vec{g}(1 - \frac{6.67 \times 10^{-24}|\vec{g}|(r_{\mathrm{e}} + z)Q^2}{mr_p}) \tag{44}$$

The parameters that appear in Eq. (43) or (44) are the observed (or measurable) parameters in the relativistic Euclidean 3-space Σ. Eq. (43) or (44) approximates Eq. (35) numerically without significant loss of accuracy. It must be noted however that Eq. (35) in terms of the proper (or primed) parameters is the theoretically correct expression.

Thus by increasing the charge Q in Eq. (44), the free-fall acceleration of the charged ball towards the center of the earth can be reduced significantly. Indeed there exists a value Q_{eqm} of charge on the ball that that will cause it to hang motionless freely, (without engine power or any other aid), in mid-air at the elevation z above the earth's surface, which is given as follows

$$\begin{aligned} Q_{\mathrm{eqm}} &= \sqrt{\frac{mr_p}{6.67 \times 10^{-24}(r_{\mathrm{e}} + z)|\vec{g}|}} \\ &= 4.8 \times 10^7 \sqrt{mr_p} \ \text{ Coulombbs} \end{aligned} \tag{45}$$

where z has been neglected compared to r_{e} in the numerical substitution. Thus, for example, a spherical ball of mass 100 kg and radius 20 cm will hang freely in mid-air, (without engine power or any other aid), if a total net electric charge of 3.79×10^8 Coulombs is put on it.

Finally the charged ball can be made to rise freely, (without engine power or any other aid), with a small or large acceleration, as may be desired, from an initial resting position on the ground or at an elevation z, against earth's gravitational pull on it, by simply increasing the net electric charge on it to an appropriate value larger than Q_{eqm}, as follows from equations (44) and (45).

By building a sufficiently thick wall of insulating materials around the charged ball, (i.e. by encapsulating it), the electrostatic field emanating from it can be effectively shielded from the environment; (and other ways around this problem might be possible). On the other hand, gravitational field cannot be shielded, hence no thickness of wall around a charged ball will prevent the interaction of the electrostatic field from the charged ball with the gravitational field of a nearby gravitational field source.

An alternative configuration to the shielded charged ball above is obtained by producing electric field between two parallel metal plates or between concentric spherical metallic shells of a spherical capacitor (by an appropriate procedure). It is easy to show by a derivation similar to the one above, that if a uniform electric field $\vec{E}$ is produced within the gap between two parallel metal plates of a capacitor, each of area A and the gap between them of distance d in the relativistic Euclidean

3-space Σ of SG, then the effective gravitational acceleration suffered towards the center of an assumed spherical gravitational field source of inertial mass M with zero net electric charge, by a box containing the capacitor, which is located at radial distance r from the center of the gravitational field source in Σ, is given as follows in the weak gravitational field limit (or in the post-Newtonian approximation)

$$\vec{g}_{\text{eff}}(r') = -\frac{GM_{0\text{a}}}{r'^2}\hat{r} + \frac{6G^2M_{0\text{a}}^2\epsilon_{\text{o}}'|\vec{E}'|^2A'd'}{m_0r'^3c_g^2c_\gamma^2}\hat{r} \tag{46}$$

$$= \vec{g}'(1 - \frac{6\epsilon_{\text{o}}'r'|\vec{g}'||\vec{E}'|^2A'd'}{m_0c_g^2c_\gamma^2}) \tag{47}$$

It must be noted that it is the proper (or primed) parameters in the flat proper spacetime (Σ', ct') that appear in these equations. Again the second repulsive term at the right-hand side of Eq. (46) is a gravi-electric acceleration and the associated force when Eq. (46) is multiplied through by the rest mass m_0 of the capacitor and the box containing it is a gravi-electric force.

The theoretically correct expression (46) or (47) in the post-Newtonian approximation can be replaced by the following for the purpose of numerical calculation without significant loss of accuracy in a weak gravitational field, such as that of the earth

$$\vec{g}_{\text{eff}}(r) = -\frac{GM}{r^2}\hat{r} + \frac{6G^2M^2\epsilon_{\text{o}}|\vec{E}|^2Ad}{mr^3c_g^2c_\gamma^2}\hat{r} \tag{48}$$

$$= \vec{g}(1 - \frac{6\epsilon_{\text{o}}r|\vec{g}||\vec{E}|^2Ad}{mc_g^2c_\gamma^2}) \tag{49}$$

where m is the inertial mass in Σ of the box and the parallel metal plates contained within it, ϵ_{o} is the electric permittivity of assumed vacuum between the parallel metal plates and $\hat{r}$ is the unit vector radially away from the center of the inertial mass M of the gravitational field source to the box in Σ. The electrostatic potential energy $Q^2/4\pi\epsilon_{\text{o}}r_p$ stored in the charged ball in the second term at the right-hand side of Eq. (43) or (44) has effectively been replaced by the electrical energy $\epsilon_{\text{o}}|\vec{E}|^2Ad$ stored in electric field within the capacitor in Eq. (48) or (49).

Yet an alternative configuration to the shielded charged ball in an external gravitational field is obtained by producing magnetic field within an electromagnet in an external gravitational field. If a uniform magnetic field $\vec{B}$ is produced within a volume V of the cylindrical volume of empty space between the pole pieces (of cast steel) of the electromagnet, which is contained in a box located at radial distance r from the center of the inertial mass M of an assumed spherical gravitational field source in the relativistic Euclidean 3-space Σ of TGR, then we must simply replace the proper electrostatic energy $Q^2/4\pi\epsilon'_o r'_p$ stored within the charged ball by the proper magnetic energy $|\vec{B}'|^2 V'/\mu'_o$ stored in magnetic field within the volume V' of empty space within the electromagnet in the proper Euclidean 3-space Σ' in the second term at the right-hand side of Eq. (35), to obtain the effective gravitational acceleration suffered by the box containing the electromagnet radially towards the center of the gravitational field source in the weak gravitational field limit (or within the post-Newtonian approximation) as follows

$$\vec{g}_{\rm eff}(r') = -\frac{GM_{0\rm a}}{r'^2}\hat{r} + \frac{6G^2M_{0\rm a}^2|\vec{B}'|^2V'}{m_0\mu'_o r'^3 c_g^2 c_\gamma^2}\hat{r} \tag{50}$$

$$= \vec{g}'(1 - \frac{6r'|\vec{g}'||\vec{B}'|^2V'}{m_0\mu'_o c_g^2 c_\gamma^2}) \tag{51}$$

The second repulsive term at the right-hand side of Eq. (50) shall be referred to as gravi-magnetic acceleration and the associated force when Eq. (50) is multiplied through by the rest mass m_0 of the electromagnet and the box containing it as gravi-magnetic force.

Again the theoretically correct expression (50) or (51) in the post-Newtonian approximation can be replaced by the following for the purpose of numerical calculations without significant loss of accuracy in a weak gravitational field, such as that of the earth

$$\vec{g}_{\rm eff}(r) = -\frac{GM}{r^2}\hat{r} + \frac{6G^2M^2|\vec{B}|^2V}{m\mu_o r^3 c_g^2 c_\gamma^2}\hat{r} \tag{52}$$

$$= \vec{g}(1 - \frac{6r|\vec{g}||\vec{B}|^2V}{m\mu_o c_g^2 c_\gamma^2}) \tag{53}$$

where m is the inertial mass in Σ of the electromagnet and the box containing it and μ_o is the magnetic permeability of assumed vacuum within the electromagnet.

Again by increasing the strength of the electric field between the parallel metal plates of the capacitor (in Eq. (48) or (49)) or by increasing the strength of the magnetic field within the electromagnet (in Eq. (52) or (53)), the free-fall acceleration towards the gravitational field source of the box containing the capacitor or electromagnet can be significantly reduced or made to vanish. By sufficiently increasing the strength of the electric field between the plates of the capacitor or of the magnetic field within the electromagnet, the box can be made to rise freely against earth's gravity at any desired acceleration, at least in principle.

Like the testable prediction of every theory, the existence of the gravi-electric force and gravi-magnetic force must be confirmed by experiment ultimately, as shall be discussed hereunder.

3.1 Odds against experimental tests and application of the gravi-electric and gravi-magnetic forces

Let us replace the charged spherical ball surrounded by a shielding material, which is interacting with an external gravitational field, to which Eq. (35) (replaced by Eq. (43) or (44) for numerical calculation) applies, by a spherical capacitor. Let a spherical metallic shell of outer radius r_1 be enclosed by a larger spherical metallic shell of outer radius r_2, such that the two shells have a common center. The walls of the spherical shells are sufficiently thick and the space between them is maintained as vacuum. The two spherical shells are connected by thick rods of an insulator and the exterior wall of the outer shell is earthed. The capacitor enclosed within a box of an insulator is supported in earth's gravitational field by a sensitive mass-balance as illustrated in sketchy form in Fig. 1.

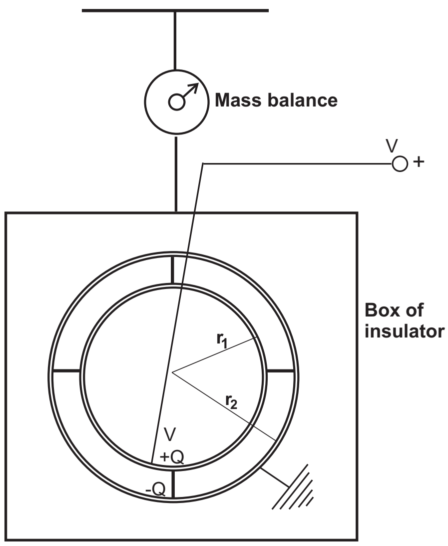

Figure 1: A concentric spherical capacitor enclosed within a box of insulator supported by a mass-balance in earth's gravitational field.

In adapting Eq. (43) (derived for a charged ball of radius r_p and inertial mass m) for the spherical capacitor of Fig. 1, we must simply replace the factor $Q^2/4\pi\epsilon_0 r_p$

in the second term at the right-hand side of that equation by $Q^2(r_2 - r_1)/4\pi\epsilon_o r_1 r_2$ to have as follows

$$\vec{g}_{\text{eff}} \approx -\frac{GM}{r^2}\hat{r} + \frac{6G^2M^2Q^2(r_2 - r_1)}{4\pi\epsilon_o m r_1 r_2 c_g^2 c_\gamma^2} \tag{54}$$

or

$$\vec{F}_{\text{eff}} \approx -\frac{GMm}{r^2}\hat{r} + \frac{6G^2M^2Q^2}{Cc_g^2 c_\gamma^2} \tag{55}$$

where

$$C = 4\pi\epsilon_o r_1 r_2/(r_2 - r_1) \tag{56}$$

is the capacitance of the concentric spherical shell capacitor, assuming the gap between its spherical shells is maintained as vacuum.

The apparatus is suspended on earth's gravitational field close to the surface of the earth. Hence M is the inertial mass of the earth; m is the inertial mass of the apparatus (read by the mass-balance before charging the walls of the shells); Q is the magnitude of net charge on the outer surface of the inner shell or on the inner surface of the outer shell and r is the radius of the observed inertial mass of the earth in the relativistic Euclidean 3-space Σ of TGR, which has been denoted by r_{e} earlier.

Since, $GM/r_{\text{e}}^2 = |\vec{g}|$, is the magnitude of the acceleration due to gravity on the surface of the earth at the location of the experiment, Eq. (55) shall be re-written as follows

$$\begin{aligned} \vec{F}_{\text{eff}} &\approx -m\,|\vec{g}|\,\hat{k} + \frac{6\,|\vec{g}|^2\, r_{\text{e}} Q^2}{Cc_g^2 c_\gamma^2}\hat{k} \\ &\approx \left(\frac{6\,|\vec{g}|\, r_{\text{e}} Q^2}{Cmc_g^2 c_\gamma^2} - 1\right) m\,|\vec{g}|\,\hat{k} \end{aligned} \tag{57}$$

where $\hat{k}$ is the unit vector along the vertical normal to the surface of the earth.

One finds from Eq. (57) that the weight of the apparatus will decrease as the electric charge Q increases, and this will be indicated by the sensitive mass-balance supporting the apparatus. If it is desired to reduce the weight of the apparatus by 1%, for instance, then we must let $\vec{F}_{\text{eff}} = -0.99m\,|\vec{g}|\,\hat{k}$ in Eq. (57) to have as follows

$$\begin{aligned} \left(\frac{6\,|\vec{g}|\, r_{\text{e}} Q^2}{Cmc^4} - 1\right) m\,|\vec{g}|\,\hat{k} &= -0.99m\,|\vec{g}|\,\hat{k} \\ \frac{6\,|\vec{g}|\, r_{\text{e}} Q^2}{Cmc^4} &= 0.01 \end{aligned}$$

$$\begin{aligned} Q &= \sqrt{\frac{0.01Cmc^4}{6\,|\vec{g}|\,r_{\mathrm{e}}}} \\ &= 4.593 \times 10^{11}\sqrt{Cm}\ \ \mathrm{C} \end{aligned}$$

If we consider a moderately large concentric spherical capacitor for which $r_1 = 0.7\,\mathrm{m}$, $r_2 = 1\,\mathrm{m}$ and $m = 50\,\mathrm{kg}$, say, then Eq. (56) gives,

$$C = 4\pi \times 8.85 \times 10^{-12} \times 0.7/0.3 = 2.595 \times 10^{-10}\ \mathrm{F}$$

Hence

$$Q = 4.593 \times 10^{11}\sqrt{50 \times 2.595 \times 10^{-10}} = 5.232 \times 10^{7}\mathrm{C}$$

The required voltage is,

$$V = Q/C = 5.232 \times 10^{7}/2.595 \times 10^{-10} = 2.016 \times 10^{17}\ \mathrm{V}$$

These predicted numerical values of the quantity of electric charge and the required voltage to reduce the weight of the apparatus by mere 1% are daunting, especially when judged against the scope of the existing capacitor and voltage generation technologies. However this does not foreclose experimental test of the gravi-electric force, it only means that some pertinent material and technological problems must be resolved first.

Alternatively let the concentric spherical capacitor be replaced by a parallel plate capacitor with uniform electric field within it inside the box of insulator in Fig. 1. Then the effective gravitational acceleration suffered towards the center of the earth by the apparatus is given by Eq. (48) or (49). The effective gravitational force suffered towards the center of the earth can then be expressed as follows

$$\begin{aligned} \vec{F}_{\mathrm{eff}} &= -m|\vec{g}|\hat{k} + \frac{6|\vec{g}|^2 r_{\mathrm{e}}\epsilon_{\mathrm{o}}|\vec{E}|^2 Ad}{c_g^2 c_\gamma^2}\hat{k} \\ &= \left(\frac{6|\vec{g}|r_{\mathrm{e}}\epsilon_{\mathrm{o}}|\vec{E}|^2 Ad}{mc_g^2 c_\gamma^2} - 1\right)m|\vec{g}|\hat{k} \end{aligned} \tag{58}$$

where $\hat{k}$ is the unit vector along the vertical direction normal to the surface of the earth, as defined under Eq. (57).

The weight of the apparatus will decrease as the intensity of the electric field increases. If it is again desired to reduce the weight of the apparatus by 1%, for instance, then we must write as follows

$$\left(\frac{6|\vec{g}|r_{\mathrm{e}}\epsilon_{\mathrm{o}}|\vec{E}|^2 Ad}{mc_g^2 c_\gamma^2} - 1\right)m|\vec{g}|\hat{k} = -0.99m|\vec{g}|\hat{k}$$

$$\begin{aligned}\frac{6|\vec{g}|r_{\mathrm{e}}\epsilon_{\mathrm{o}}|\vec{E}|^2 Ad}{mc_g^2 c_\gamma^2} &= 0.01\\ |\vec{E}| &= \sqrt{\frac{0.01mc^4}{6|\vec{g}|r_{\mathrm{e}}\epsilon_{\mathrm{o}}Ad}}\\ &= 1.54\times 10^{17}\sqrt{m/Ad}\ \ \mathrm{V/m}\end{aligned}$$

If we let $m = 25\,\mathrm{kg}$, $A = 1\,\mathrm{m}^2$ and $d = 0.3\,\mathrm{m}$, say, then

$$|\vec{E}| = 1.41\times 10^{18}\quad \mathrm{V/m}$$

Finally if the concentric spherical shell capacitor inside the box in Fig. 1 is replaced by an electromagnet containing uniform magnetic field within it, then the effective gravitational acceleration suffered towards the center of the earth by the apparatus is given by Eq. (52) or (53) and the effective gravitational force on it can be expressed as follows

$$\begin{aligned}\vec{F}_{\mathrm{eff}} &= -m|\vec{g}|\hat{k} + \frac{6|\vec{g}|^2 r_{\mathrm{e}}|\vec{B}|^2 V}{\mu_{\mathrm{o}}c_g^2 c_\gamma^2}\hat{k}\\ &= (\frac{6|\vec{g}|r_{\mathrm{e}}|\vec{B}|^2 V}{\mu_{\mathrm{o}}mc^4} - 1)m|\vec{g}|\hat{k} \end{aligned}\tag{59}$$

The strength of the magnetic field within the electromagnet that will reduce the weight of the apparatus by 1% is then given from the following

$$\begin{aligned}(\frac{6|\vec{g}|r_{\mathrm{e}}|\vec{B}|^2 V}{\mu_{\mathrm{o}}mc^4} - 1)m|\vec{g}|\hat{k} &= -0.99m|\vec{g}|\hat{k}\\ \frac{6|\vec{g}|r_{\mathrm{e}}|\vec{B}|^2 V}{\mu_{\mathrm{o}}mc^4} &= 0.01\\ |\vec{B}| &= \sqrt{\frac{0.01\mu_{\mathrm{o}}mc^4}{6|\vec{g}|r_{\mathrm{e}}V}}\\ &= 5.15\times 10^8\sqrt{m/V}\quad \mathrm{Tesla}\end{aligned}$$

By letting $m = 25\,\mathrm{kg}$ and $V = 1\,\mathrm{m}^3$, say, then

$$|\vec{B}| = 5.89\times 10^9\quad \mathrm{Teslas}$$

The super-high strengths of the electric and magnetic fields needed to reduce the weight of a 25-kilogram apparatus by mere 1% calculated above are not only daunting but foreboding. The initial enthusiasm that attended the discovery of the gravi-electric and gravi-magnetic forces must have been seriously dampened by what appears an insurmountable material and technological barriers towards their experimental test and application.

4 Isolating the laser-anti-gravitational force

The general concept within which gravi-electric force (GEF) and gravi-magnetic force (GMF) have been isolated in the preceding sub-section shall be referred to as gravitation of non-gravitational energy. This concept exists on the flat relativistic spacetime (Σ, ct) in a gravitational field in the context of the theory of gravitational relativity (TGR).

For if we consider the expression for the gravitational-relativistic value E in the relativistic Euclidean 3-space Σ of TGR of a general non-gravitational energy of proper value E' in the proper Euclidean 3-space Σ' of classical gravitation, given by Eq. (27) in the weak gravitational field limit (or within the post-Newtonian approximation), then as mentioned earlier, the first term at the right-hand side of that equation namely, the proper non-gravitational energy E' possesses no passive gravitational attribute and hence does not interact with the Newtonian gravitational potential or field, whereas the second term at the right-hand side possesses intrinsic geometry-induced (or intrinsic metric induced) negative passive gravitational attribute and consequently anti-gravitates in an external gravitational field. This is the origin of the concept of gravitation of non-gravitational energy in the present theory.

We have simply let the proper non-gravitational energy E' in Eq. (27) to be the proper electrostatic energy $Q^2/4\pi\epsilon_0' r_p'$ stored within the charged ball of rest mass m_0 and radius r_p' with net electric charge Q uniformly distributed over it to be equal; let E' in Eq. (27) to the proper electrical energy $\epsilon_0' |\vec{E}'|^2 A' d'$ stored in electric field within the parallel plate capacitor and let E' in Eq. (27) to be equal to the proper magnetic energy $|\vec{B}'|^2 V'/\mu_0'$ stored in magnetic field within an electromagnet, in isolating the gravi-electric and gravi-magnetic forces in the weak gravitational field limit in the preceding section.

However the non-gravitational energy E' can be replaced by other forms of non-gravitational energy, such as proper electromagnetic radiation (or light) energy $h\nu_0$. In this wise, let us consider a spherical enclosure (of the appropriate material), at the center of which is located a source of very high energy laser beams, which are produced continuously along radial directions from the center and terminate at the inner wall of the spherical enclosure of the beams and their source at the center. We shall assume that there is a large number of such high energy radial laser beams within the spherical enclosure at every instant, so that the total proper laser energy within it at any given instant is E_l'.

Let the rest mass of the material of the spherical shell containing the radial laser beams and all other material objects within the spherical shell be m_0. The net rest mass of the apparatus (i.e. of the spherical shell and all other material objects and laser beams within it) is given as

$$m_{\text{net}} = m_0 + E'_l/c_\gamma^2 \tag{60}$$

However only the rest mass m_0 of the spherical shell and material objects within it can be observed and measured as rest mass in the Newtonian gravitational field on the flat proper spacetime (Σ', ct') of classical gravitation.

Now let us obtain the transformation of the net rest mass (60) in the context of TGR. In doing this we recall that the laser energy E'_l, being a radiation energy $h\nu_0$, transforms in the context of TGR as

$$E_l = \gamma_g(r')^{-1}E'_l = E'_l(1 - \frac{2GM_{0\text{a}}}{r'c_g^2})^{1/2} \tag{61}$$

(Radiation energy transforms as $h\nu = \gamma_g(r')^{-1}h\nu_0$ from Table I of [9]). On the other hand, the mass relation in the context of TGR is

$$m = \gamma_g(r')^{-2}m_0 = m_0(1 - \frac{2GM_{0\text{a}}}{r'c_g^2}) \tag{62}$$

Thus the gravitational-relativistic net mass (or net inertial mass) of the spherical shell and all the material objects and all radial laser beams contained within it on the flat relativistic spacetime (Σ, ct) of TGR is

$$m_{\text{net}} = m_0(1 - \frac{2GM_{0\text{a}}}{r'c_g^2}) + \frac{E'_l}{c_\gamma^2}(1 - \frac{2GM_{0\text{a}}}{r'c_g^2})^{1/2} \tag{63}$$

The gravitational-relativistic gravitational potential $\Phi(r')$ in the relativistic Euclidean 3-space Σ of TGR at the location of the spherical enclosure (or apparatus) of Fig. 2, located at radial distance r from the center of the inertial mass M of the gravitational field source in Σ, derived in [9] is

$$\begin{aligned}\Phi(r') &= \gamma_g(r')^{-1}\Phi'(r') = \Phi'(r')(1 - \frac{2GM_{0\text{a}}}{r'c_g^2})^{1/2} \\ &= -\frac{GM_{0\text{a}}}{r'}(1 - \frac{2GM_{0\text{a}}}{r'c_g^2})^{1/2}\end{aligned} \tag{64}$$

The net gravitational-relativistic gravitational potential energy $U_{\text{net}}(r')$ that the spherical enclosure of laser beams (or apparatus) possesses in the relativistic Euclidean 3-space Σ of TGR is then given from Eqs. (63) and (64) as follows

$$U_{\text{net}}(r') = -\frac{GM_{0\text{a}}m_0}{r'}(1 - \frac{2GM_{0\text{a}}}{r'c_g^2})^{3/2} - \frac{GM_{0\text{a}}E'_l}{r'c_\gamma^2}(1 - \frac{2GM_{0\text{a}}}{r'c_g^2}) \tag{65}$$

The post-Newtonian approximation to Eq. (65) in a weak gravitational field is the following

$$\begin{aligned} U_{\mathrm{net}}(r') &\approx -\frac{GM_{0\mathrm{a}}m_0}{r'}(1-\frac{3GM_{0\mathrm{a}}}{r'c_g^2})-\frac{GM_{0\mathrm{a}}E_l'}{r'c_\gamma^2}(1-\frac{2GM_{0\mathrm{a}}}{r'c_g^2}) \\ &\approx -\frac{GM_{0\mathrm{a}}m_0}{r'}+\frac{3G^2M_{0\mathrm{a}}^2m_0}{r'^2c_g^2}-\frac{GM_{0\mathrm{a}}E_l'}{r'c_\gamma^2}+\frac{2G^2M_{0\mathrm{a}}^2E_l'}{r'^2c_g^2c_\gamma^2} \end{aligned} \tag{66}$$

Let us factor out the Newtonian gravitational potential and re-write Eq. (66) as follows

$$U_{\mathrm{net}}(r') = -\frac{GM_{0\mathrm{a}}}{r'}(m_0-\frac{3GM_{0\mathrm{a}}m_0}{r'c_g^2}+\frac{E_l'}{c_\gamma^2}-\frac{2GM_{0\mathrm{a}}E_l'}{r'c_g^2c_\gamma^2}) \tag{67}$$

Of all the mass and equivalent mass terms inside the parentheses in Eq. (67), only E_l'/c_γ^2 — the equivalent 'rest mass' of the proper radiation (or light) energy E_l', possesses no passive gravitational attribute. Hence it has no interaction with the Newtonian gravitational potential potential outside the parentheses. The fourth term inside the parentheses in Eq. (67), being a product of the square of gravitational speed $V_g'(r') = 2GM_{0\mathrm{a}}/r'$ and proper radiation energy E_l' (or being a local coupling of radiation energy to gravity already) possesses intrinsic geometry-induced (or intrinsic metric induced) passive gravitational attribute. Consequently it can interact with the Newtonian gravitational potential outside the parentheses.

By removing the third term inside the parentheses in Eq. (67), which corresponds to discarding the meaningless (or non-existing) third term at the right-hand side of Eq. (66) we have

$$U_{\mathrm{eff}}(r') \approx -\frac{GM_{0\mathrm{a}}m_0}{r'}+\frac{3G^2M_{0\mathrm{a}}^2m_0}{r'^2c_g^2}+\frac{2G^2M_{0\mathrm{a}}^2E_l'}{r'^2c_g^2c_\gamma^2} \tag{68}$$

This is the effective gravitational potential energy possessed by the spherical enclosure of radial laser beams and their source (or sources) in the relativistic Euclidean 3-space Σ of TGR.

Division of Eq. (68) by the rest mass m_0 gives the effective gravitational potential ('seen' by the rest mass m_0) in the relativistic Euclidean 3-space Σ in the context of TGR as follows

$$\Phi_{\mathrm{eff}}(r') \approx -\frac{GM_{0\mathrm{a}}}{r'}+\frac{3G^2M_{0\mathrm{a}}^2}{r'^2c_g^2}+\frac{2G^2M_{0\mathrm{a}}^2E_l'}{m_0r'^2c_g^2c_\gamma^2} \tag{69}$$

The implied effective gravitational acceleration (suffered by the rest mass m_0) in Σ

in the context of TGR is

$$\vec{g}_{\text{eff}}(r') \approx -\frac{GM_{0\text{a}}}{r'^2}\hat{r} + \frac{6G^2M_{0\text{a}}^2}{r'^3c_g^2}\hat{r} + \frac{4G^2M_{0\text{a}}^2E_l'}{m_0r'^3c_g^2c_\gamma^2}\hat{r} \tag{70}$$

The second term at the right-hand side of Eq. (70) is negligibly small compared to the Newtonian first term and cannot be controlled by man. On the other hand, the third term can be made significantly large by increasing the laser energy E_l'. Consequently let us approximate Eq. (70) further by removing the second term at the right-hand side to have

$$\vec{g}_{\text{eff}}(r') \approx -\frac{GM_{0\text{a}}}{r'^2}\hat{r} + \frac{4G^2M_{0\text{a}}^2E_l'}{m_0r'^3c_g^2c_\gamma^2}\hat{r} \tag{71}$$

$$\approx \vec{g}'(1 - \frac{4|\vec{g}'|r'E_l'}{m_0c_g^2c_\gamma^2}) \tag{72}$$

And the effective gravitational force on the spherical enclosure of laser beams and their source (or sources) is

$$\vec{F}_{\text{eff}}(r') = m_0\vec{g}_{\text{eff}}(r')$$

$$\approx -\frac{GM_{0\text{a}}m_0}{r'^2}\hat{r} + \frac{4G^2M_{0\text{a}}^2E_l'}{r'^3c_g^2c_\gamma^2}\hat{r} \tag{73}$$

$$\approx m_0\vec{g}'(1 - \frac{4|\vec{g}'|\,r'E_l'}{m_0c_g^2c_\gamma^2}) \tag{74}$$

We shall by virtue of the approximations in system (42) in addition to $E_l = (1 - 2GM_{0\text{a}}/r'c_g^2)^{1/2} \approx E_l'$, which are good for numerical calculation purpose in the gravitational field of the earth, adapt Eq. (71) or (72) and (73) or (74) for a spherical enclosure of high energy radial laser beams and their source (or sources), which is located at an elevation z above the earth's surface as follows

$$\vec{g}_{\text{eff}} \approx -\frac{GM_{\text{e}}}{(r_{\text{e}}+z)^2}\hat{r} + \frac{4G^2M_{\text{e}}^2E_l}{m(r_{\text{e}}+z)^3c_g^2c_\gamma^2}\hat{r} \tag{75}$$

$$\approx \vec{g}(1 - \frac{4|\vec{g}|(r_{\text{e}}+z)E_l}{mc_g^2c_\gamma^2}) \tag{76}$$

and

$$\vec{F}_{\text{eff}} = m\vec{g}_{\text{eff}}$$

$$\approx \quad -\frac{GM_{\mathrm{e}}m}{(r_{\mathrm{e}}+z)^2}\hat{r} + \frac{4G^2M_{\mathrm{e}}^2E_l}{(r_{\mathrm{e}}+z)^3c_g^2c_\gamma^2}\hat{r} \tag{77}$$

$$\approx \quad m\vec{g}\left(1 - \frac{4|\vec{g}|\,(r_{\mathrm{e}}+z)E_l}{mc_g^2c_\gamma^2}\right) \tag{78}$$

The parameters that appear in Eqs. (75) – (78) are the observed and measurable parameters in the relativistic Euclidean 3-space Σ of TGR. Although Eqs. (71) – (74) are the theoretically correct expressions, they can be replaced by Eqs.(75) – (78) for the purpose of numerical calculation without significant loss in accuracy.

On finds from Eq. (75) that by increasing the total laser energy E_l contained within the spherical enclosure, the free-fall acceleration of the spherical enclosure towards the center of the earth can be reduced significantly. Indeed there exists a value $E_{l\,\mathrm{eqm}}$ of total laser energy within the spherical enclosure that will cause the enclosure to hang motionless freely (without engine power or any other aid) in mid-air at the elevation z above the earth's surface, which is given from Eq. (75) as follows

$$E_{l\,\mathrm{eqm}} = \frac{mc^4}{4|\vec{g}|(r_{\mathrm{e}}+z)} = 3.24\times10^{25}m \tag{79}$$

where z has been neglected compared to the radius of the earth r_{e} in the numerical substitution. Thus a spherical shell enclosure of radial laser beams and all other material objects contained within it of mass 50 kg, say, will hang motionless freely in mid-air (without engine power or any other aid), if a total laser energy of 1.62×10^{26} Joules is contained within it.

The spherical enclosure of laser beams (or a box containing it) can be made to rise freely, (without engine power or any other aid), with a small or large acceleration, as may be desired, from an initial resting position on the ground or at an elevation z, against earth's gravitational pull on it, by simply increasing the total laser energy within it to an appropriate value larger than $E_{l\,\mathrm{eqm}}$, as follows from equations (78) and (79).

4.1 On a possible experimental test of laser-anti-gravitational force

Let the spherical enclosure of radial laser beams of high energy and their source (or sources) at the center be supported by a sensitive mass-balance as illustrated in Fig. 2, where only a few radial beams are shown.

The weight of the apparatus in Fig. 2 will decrease as the total laser energy E_l contained within it increases, according to Eq. (74), and this will be indicated by the mass-balance. If it is desired to reduce the weight of the apparatus by 1%, for instance, then we must write as follows from Eq. (74)

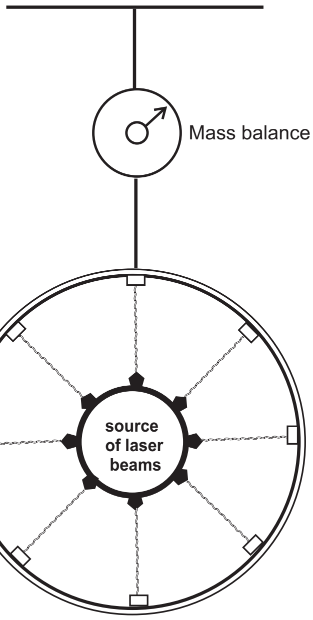

Figure 2: A spherical shell containing high energy radial laser beams and their source at its center supported by a mass-balance in earth's gravitational field.

$$\begin{aligned}
\left(\frac{4|\vec{g}|(r_{\mathrm{e}}+z)E_l}{mc^4}-1\right)m|\vec{g}|\hat{k} &= -0.99m|\vec{g}|\hat{k} \\
\frac{4|\vec{g}|(r_{\mathrm{e}}+z)E_l}{mc^4} &= 0.01 \\
E_l &= \frac{0.01mc^4}{4|\vec{g}|(r_{\mathrm{e}}+z)} = 1.62\times 10^{24} \text{ Joules}
\end{aligned}$$

where the elevation z has been neglected compared to the radius r_{e} of the earth in the numerical substitution.

Although the total energy of 1.62×10^{24} Joules required to reduce the weight of the 50 kg apparatus in Fig. 2 by 1% on earth's gravitational field calculated above is very large, there is much better prospect for producing and applying the anti-gravitational force (LAGF) than the gravi-electric force (GEF) and gravi-magnetic force (GMF) for the following reasons.

First of all, once the laser beams are contained within the spherical enclosure, as illustrated in Fig. 2, the LAGF can be safely produced and utilized. It poses no danger to people and environment. This is not so with dealing with electric charge of the order of 10^7 Coulombs on a metallic ball or electric field intensity of the other of 10^{18} V/m within a capacitor, required to produce gravi-electric force (GEF) that will reduce the weight of 100 kg apparatus n Fig. 1 by 1% in earth's gravitational field or a magnetic field strength of the order of 10^9 Teslas within an electromagnet required to produce gravi-magnetic force (GMF) that will reduce the weight of 100 kg apparatus by 1%. There is little or no hope for achieving anti-gravitational thrust with the aid of GEF or GMF, because of the extreme hazard and disaster that they could cause to people and environment.

Secondly laser production technology has advanced tremendously after fifty years of continuous effort, since the discovery of laser in 1960. From the account of the past and present advances in laser technology and the near-future direction of efforts, reported in [10], it appears that producing laser energy to achieve laser-anti-gravitational thrust is not far from reach.

4.2 Prospects for future applications of the laser-anti-gravitational force

The effective gravitational force $\vec{F}_{\text{eff}}$ of Eq. (78) is the effective weight of the spherical shell enclosure of radial beams of high energy laser and their source (or sources) in Fig. 2. Thus the effect of the presence within the gravitational field of the earth of high energy laser beams within the spherical shell enclosure is to reduce its weight.

Thus by equipping a craft with one or more units of spherical shell enclosures of high energy radial laser beams, to be referred to as laser-anti-gravitational force units, the effective gravitational force on the craft towards the center of the earth (or its weight), can be reduced by increasing the total laser energy E_l within each spherical enclosure. In other words, by varying E_l, the weight (or effectively the inertia in earth's gravitational field) of the craft can be controlled. It shall be assumed in what follows that all the associated material and technological problems shall be resolved with time.

A craft equipped with one or more laser-anti-gravitational force units shall be referred to as laser-antigravity craft and given the acronym LAGC. A laser-antigravity craft (LAGC) must be equipped with a variable super-high energy laser machine to increase or decrease the total laser energy E_l within its laser-anti-gravitational force (LAGF) units at the desired fast rates.

4.2.1 Performing super-aerodynamical feat with a smart variety of laser-anti-gravity craft

Hanging motionless freely in mid-air

The condition for vertical equilibrium of a laser-antigravity craft (LAGC), that is, for it to hang freely motionless vertically in mid-air without engine power or any other aid, is the vanishing of the effective gravitational force on it towards the earth. Thus for a LAGC at elevation z_0 above the earth's surface, which is equipped with a pair of identical laser-anti-gravitational force units in order to avoid turning moment on it, the condition for hanging motionless in mid-air is given from Eq. (78) as follows

$$(1 - \frac{4|\vec{g}|(r_{\mathrm{e}} + z)E_l}{mc^4})m\vec{g} = 0 \tag{80}$$

where m is the inertial mass of the craft, that is, the inertial mass of its body and all objects including persons and the pair of laser-anti-gravitational force units contained within the craft. The total laser energy E_l that satisfies Eq. (80) − the condition for the craft to hang freely at elevation z_0 − shall be referred to as the equilibrium total laser energy and denoted by $E_{l\ \mathrm{eqm}}$ as done previously. It is given by Eq. (79) in the case of a single laser-anti-gravitational force unit. That value must be divided by a factor of 2 in order to determine the total laser energy within each of the pair of spherical shell enclosures of laser beams, in the case of a pair of laser-anti-gravitational force units.

Thus a laser-antigravity craft of mass 300 kg, say, which is equipped with a pair of identical laser-anti-gravitational force units, will hang motionless freely in mid-air on earth, (without engine power or any other aid), barring other external forces, at any elevation, $z_0 << r_{\mathrm{e}}$, when a total laser energy of 4.86×10^{27} Joules is contained within each of it pair of laser-anti-gravitational force units (i.e. within each of its two spherical shell enclosures of high energy radial laser beams).

Hovering at constant elevation

Having suspended the laser-antigravity craft (LAGC) at elevation z_0 by producing total laser energy of $E_{l\ \mathrm{eqm}} = 4.86 \times 10^{27}$ Joules within each of its pair of laser-anti-gravity force units, as calculated above, the 300 kg craft can be made to sail at any velocity at constant elevation, by ejecting gas backward from the nozzles at its back, (like a hover-craft or rocket). The craft has to be equipped with a burner and an appropriate solid fuel for this purpose. Ejecting gas forward through front nozzles will decelerate the craft and eventually bring it to a halt. The back nozzles and nozzles at other locations on the craft can be used to steer it and to maneuver.

Rising vertically slowly or rapidly freely from the earth and dropping vertically slowly freely to the earth from an elevation

The effective gravitational acceleration on a laser-antigravity craft (LAGC) at elevation z_0, given by Eq. (76) shall be re-written as follows

$$\begin{aligned}\vec{g}_{\text{eff}} &\approx \left(\frac{4|\vec{g}|(r_{\text{e}}+z)E_l}{mc_g^2c_\gamma^2}-1\right)|\vec{g}|\hat{k} \\ &= \left(3.088\times10^{-26}\frac{E_l}{m}-1\right)|\vec{g}|\hat{k} \qquad (81)\end{aligned}$$

where $\hat{k}$ is the unit vector pointing upward along the vertical normal to the surface of the earth.

Thus by suddenly making the total laser energy E_l within each of the pair of laser-anti-gravitational force units of the craft to be far larger than $E_{l\ \text{eqm}} = 4.8\times10^{27}$ Joules, calculated above, the effective acceleration on the craft becomes large suddenly, pointing in the positive $z-$direction, (vertically away from the earth's surface). In this case, a LAGC landed on the ground, ($z_0 = 0$), or hanging motionless in mid-air or hovering at constant elevation in mid-air initially, will begin to ascend rapidly at an acceleration that may be as large as desired. This way, vertical rise (or take-off) acceleration of $10|\vec{g}|$ or larger can be achieved without engine power or any other aid.

On the other hand, the total laser energy E_l can be regulated to a value that is only larger than $E_{l\ \text{eqm}}$ by a very small amount. Then the effective acceleration on the craft still points upward in the positive $z-$direction, but with a small magnitude of $0.1|\vec{g}|$ or $0.01|\vec{g}|$, say, or even smaller. In this case the craft begins to rise freely slowly against earth's gravity from its initial resting position on the ground or at elevation z_0 or from its initial hovering at a constant elevation.

If the craft was initially hovering at a constant elevation z_0 above the earth at constant velocity v along the $x-$axis, when the effective upward acceleration was given to it by regulating the total laser energy E_l contained within each of its pair of laser-anti-gravitational force units, then it will move along an upward parabolic trajectory, as illustrated in Fig. 4(a) for fast ascent and Fig. 4(b) for slow ascent, while Fig. 4(c) illustrates vertical rise from the ground of a craft initially at rest on the ground.

If it is desired for the craft to fall slowly to the earth from an elevation z_0 at which it was hanging motionless or hovering at constant velocity initially, then the total laser energy E_l should be made slightly smaller than $E_{l\ \text{eqm}}$. The effective acceleration on the craft will point vertically downward, (towards the earth's surface), but with a small magnitude of $0.1|\vec{g}|$ or $0.01|\vec{g}|$, say, or even smaller, as may be

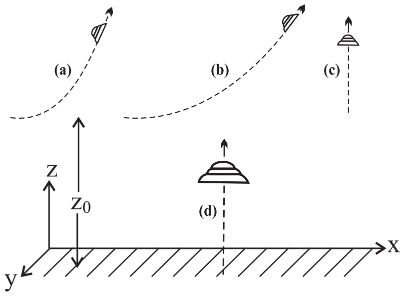

Figure 3: Possible trajectories of a laser-antigravity craft, which is ascending freely (without engine power or any other aid), against earth's gravitational pull on it, due to operating total laser energy E_l larger than the total laser energy $E_{l\,\mathrm{eqm}}$ that nullifies the weight of the craft, for the craft which was, (a) or (b) hovering at a constant elevation initially and (c) at rest on the ground initially.

desired. If the craft was initially hovering at a constant velocity along the $x-$axis at elevation z_0 when the effective downward acceleration of $0.1|\vec{g}|$ or $0.01|\vec{g}|$ was applied to it by regulating the total laser energy E_l, then the craft will follow the downward parabolic trajectory of Fig. 5(a) on the xz-plane, while Fig. 5(b) illustrates the vertical descent from elevation z_0 of the craft in a situation where it was initially stationary at elevation z_0.

Breaking rapidly from high velocity and changing direction of motion rapidly at high velocity

Let us consider a laser-antigravity craft operating at the equilibrium total laser energy $E_{l\,\mathrm{eqm}}$ within each of its pair of laser-anti-gravitational force units and hovering along the x-axis at elevation z_0. The craft is literally walking in the air. Consequently making sharp turns, such as right-angle turn, performing run-stops and abruptly coming to a halt in mid-air should not be problems for the craft. Likewise a laser-antigravity craft, which is operating at the equilibrium total laser energy $E_{l\,\mathrm{eqm}}$ and moving at a high velocity, such as of a super-sonic jet, at constant elevation, can easily make U-turn round a circular path of five mile radius or smaller − a feat which will be suicidal for a super-sonic jet.

On the other hand, let the total laser energy E_l be suddenly increased to a value much larger than $E_{l\,\mathrm{eqm}}$, for a craft which is hovering at velocity v along the $x-$axis initially, so that the craft is given a large effective upward acceleration, thereby following the trajectory of Fig. 4(a) or 4(b) as a consequence. The velocity of the craft

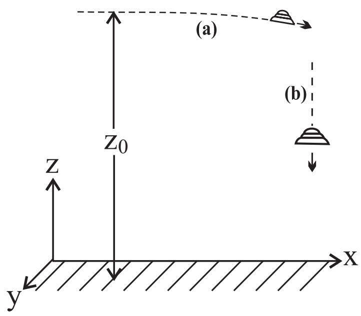

Figure 4: Possible trajectories of a laser-antigravity craft, which is descending freely under reduced gravitational acceleration towards the earth, due to operating total laser energy E_l that is slightly smaller than the total laser energy $E_{l\,\mathrm{eqm}}$ that nullifies the weight of the craft, for the craft which was (a) hovering at an elevation initially and (b) stationary at an elevation initially.

as it moves along this trajectory on the xz-plane is given in terms of its components as follows

$$v_z(t) = \left(3.088 \times 10^{-26} \frac{E_l}{m} - 1\right) |\vec{g}| t; \quad v_x = v_0 \tag{82}$$

For a large E_l giving rise to a large upward acceleration of $5|\vec{g}|$, say, along the trajectory, the velocity of the craft is large and is predominantly along the vertical. If, as the craft is in this motion, the total laser energy E_l is suddenly reduced to a value lower than $E_{l\,\mathrm{eqm}}$ again, then the craft will suddenly receive a downward acceleration of $3 \mid \vec{g} \mid$, say, and will, in a matter of a few minutes, begin to move along a downward parabolic trajectory. Thus the craft will be observed to break rapidly from a high velocity motion and to change its direction of motion rapidly simultaneously.

On the other hand, if the craft initially hovering at velocity v_0 along the x-direction at elevation z_0, while operating at total laser energy $E_{l\,\mathrm{eqm}}$, is given net downward acceleration by making the operating total laser energy E_l smaller than $E_{l\,\mathrm{eqm}}$, then the craft will follow the trajectory of Fig. 5(a). Its velocity is still given by Eq. (82) along this trajectory. Now if as it is moving along this trajectory, the E_l is suddenly increased to a value much larger than $E_{l\,\mathrm{eqm}}$, so that the craft suddenly receives a large upward acceleration of $5|\vec{g}|$, say, then the vertical component of its velocity pointing downward during descent will rapidly reduce to zero and begin to increase rapidly while pointing upward, as the craft rapidly changes its downward motion to an upward motion along a trajectory on the vertical xz-plane. The velocity at which the craft shoots off in its upward motion can be made far larger than the velocity at which it was descending.

We have discussed in this sub-section some of the strange feats which a smart variety of laser-antigravity crafts (LAGC) can be made to perform in mid-air, within the gravitational field of the earth, by simply regulating the total laser energy within each of its pair of laser-anti-gravitational force units. Since engine power is not required, the feats can be accomplished noiselessly. The feats have been described as strange because they cannot be explained within the knowledge of physical science until now, and consequently no earthly craft has been observed to perform them hitherto (as far as I know).

4.3 The condition of persons inside a laser-antigravity craft which is performing super-aerodynamical feat in mid-air

The laser-anti-gravitational force acts vertically upwards from the earth's surface on the pair of spherical shell enclosures of high energy radial laser beams (the laser-anti-gravitational force units) within a laser-antigravity craft and not on any other object or person inside the craft. On the other hand, the Newtonian gravitational force acts vertically downwards towards the earth's surface on the pair of laser-anti-gravitational force units and on every other object (and every person) inside the craft.

Thus a person of mass m who is standing on the floor of a laser-antigravity craft exerts a weight on the floor, while the floor exerts a reaction vertically on the person. For a craft which is given an effective gravitational acceleration $\vec{g}_{\text{eff}}$ by regulating the total laser energy E_l on each of its pair of laser-anti-gravitational force units, the weight $m\vec{g}$ of the person and the reaction $\vec{R}$ of the floor of the craft on him are related thus

$$\vec{R} - mg\hat{k} = m\vec{g}_{\text{eff}}$$

or

$$\vec{R} = m(g\hat{k} + \vec{g}_{\text{eff}}) \tag{83}$$

where $\hat{k}$ is unit vector pointing vertically upwards from the earth's surface.

In a situation where $\vec{g}_{\text{eff}} = 0$, such as when the craft is landed on the ground, is hanging motionless in mid-air or is hovering at a constant elevation, Eq. (83) simplifies as follows

$$\vec{R} = mg\hat{k} \tag{84}$$

Thus persons inside the craft will feel their normal weights acting on the floor of the craft under any of these conditions. They will therefore feel no discomfort whatsoever under any of these conditions.

In a situation where the craft is given a downward effective gravitational acceleration $-g_{\text{eff}}\hat{k}$, Eq. (83) becomes the following

$$\vec{R} = m(g - g_{\text{eff}})\hat{k} \tag{85}$$

For the desired low downward effective acceleration of $0.01\vec{g}$ or $0.001\vec{g}$, say, needed to bring the craft gently to the ground from an elevation, Eq. (85) gives, $\vec{R} = 0.99m\vec{g}$ or $\vec{R} = 0.999m\vec{g}$. Hence persons inside the craft will feel their weights reduced by mere 1% or 0.1%. They will feel comfortable when the craft is made to drop vertically very slowly to the ground with the aid of the laser-anti-gravitational force.

Finally, in a situation where the craft is given an upward effective acceleration $g_{\text{eff}}\hat{k}$, Eq.(83) becomes the following

$$\vec{R} = m(g + g_{\text{eff}})\hat{k} \tag{86}$$

Again if the craft is made to rise vertically freely from the earth against earth's gravity at a low effective acceleration of $\vec{g}_{\text{eff}} = 0.01g\ \hat{k}$ or $0.1g\ \hat{k}$, say, then $\vec{R} = 1.01mg\ \hat{k}$ or $\vec{R} = 1.1mg\hat{k}$. Persons inside the craft will feel their weights increased by 1% or 10% in this case.

If, on the other hand, the craft is given a large upward effective acceleration of, $\vec{g}_{\text{eff}} = 5g\,\hat{k}$ or $\vec{g}_{\text{eff}} = 10g\hat{k}$, say, then Eq. (86) gives $\vec{R} = 6mg\hat{k}$ or $\vec{R} = 11mg\hat{k}$. Persons inside the craft will feel six times or eleven times heavier in this case. They must be dressed in the appropriate suit to withstand the crushing effect of their weights.

One finds from equations (84), (85) and (86) that the reactions from the floor of a laser-antigravity craft on a person inside it for the situations of zero effective vertical acceleration, vertical downward accelerated motion and vertical upward accelerated motion of the craft respectively, are the same as the reactions of the floor of an elevator on a person inside it, when the elevator is hanging motionless mid-way, (such as when off-loading at a floor), in downward accelerated motion and in upward accelerated motion respectively. There is no feeling of discomfort due to large reduction in weight or due to weightlessness of persons inside a laser-antigravity craft, except when the craft falls vertically to the earth at large effective acceleration close to or equal to $\vec{g}$, due to failure of its laser-anti-gravitational force unit. These facts further make practical application of the laser-anti-gravitational force in laser-antigravity crafts feasible and promising.

4.4 Laser-anti-gravitational navigation: Possible future applications of the laser-anti-gravitational force in aviation and space travels

All the possible future applications of the laser-anti-gravitational force can hardly be imagined at present. Nevertheless some readily come to focus. A commercial air-craft equipped with laser-anti-gravitational force units could make use of the laser-anti-gravitational force to aid its take-offs and landings, thereby cutting down spending tremendously on aviation fuels. Such air-craft could be made to fall slowly to the ground during mid-air emergency, such as in a situation of loss of engine, or to hang freely in mid-air, (if above an ocean), until rescue action is taken, by regulating the laser-anti-gravitational force on it.

Laser-anti-gravitational force has prospects for revolutionizing the present aviation technology. Future air-crafts would not have to be equipped with sophisticated engines, rather they would be laser-antigravity crafts (LAGC), which would be made to rise freely from the ground vertically against earth's gravity and be suspended at a desired elevation in mid-air with the aid of the laser-anti-gravitational force. Then by ejecting gas backward by burning a solid fuel, (like a rocket), they would be made to move forward at constant elevations, even at much lower noise levels and at larger speeds than attainable by super-sonic jets at present, if so desired. Since future air-crafts would be laser-antigravity crafts suspended at constant elevations in the sky by laser-anti-gravitational force, without the need for jet engines that use inflammable aviation fuels, the present 'primitive' form of air travels in which lives are helplessly lost in mid-air crashes due to loss of engines and mid-air fires, would be a thing of the past.

Laser-antigravity air-trains hovering at low elevations within and between cities would be used in mass-transit. Such air-trains would fall vertically gently to the ground from elevation in order to off-load and to pick passengers, and rise vertically gently back to elevation again with the aid of the laser-anti-gravitational force. This application has prospect for replacing metropolitan sub-ways and hanging trains, and indeed the present transportation systems in the future. A laser-antigravity car would be able to scale a barrier, such as a wall or mountain, and to fly over a river, in a golden future time that is now around the corner.

The laser-anti-gravitational force would also be used to aid space-ships at take-offs. In particular, rockets and space shuttles would be made to safely attain larger accelerations at take-offs with the aid of the laser-anti-gravitational force than attainable hitherto by the conventional methods. Space shuttles would apply the laser-anti-gravitational force to slow down when re-entering earth's atmosphere from outer space. The landing module of a manned voyage to the moon, Mars and other planets would also apply laser-anti-gravitational force to achieve soft landing and for hovering at low elevation above the host planet. It must be remarked however that foreseeable sizable technological and material problems must be resolved in order to harness the laser-anti-gravitational force.

Finally, on a humorous note, one of the most persistent and most enigmatic features of the unidentified flying objects (UFOs) (or 'flying saucers'), even if we restrict to the residue of the unexplained cases, [11], is their ability to perform the feat described in sub-section 4.2 above in mid-air in our earth environment without producing audible sound. It thus appears that UFOs (or 'flying saucers') make use of the laser-anti-gravitational force to perform the mid-air "gymnastics" for which they are well known.

Indeed exact and experimentally testable explanations of the origin, the extreme maneuverability and all other essential features of UFOs have been derived within the many-world background of the present evolving fundamental theory, and reported in a volume of this monograph series devoted to the theory. Comprehensive summaries of the explanations are reported in a mini book [14]. The book targeted towards all people who are interested in seeing UFO unraveled.

5 Non-validity of the weak equivalence principle with the presence of non-gravitational energy

A theoretical issue, apart from prospects for applications, has arisen from the derivations of the gravi-electric force, gravi-magnetic force and laser-anti-gravitational force in sections 3 and 4, which must be pointed out. One observes from equation (34) or (35) and equation (46) or (47) that the effective gravitational acceleration on a charged spherical ball or on a box housing two parallel metal plates containing uniform electric field between them, (the test particle), towards a gravitational field source, depends on properties of the charged ball or box, (i.e. of the test particle). It depends on the net electric charge Q, the rest mass m_0 and radius $r_p{}'$ of the charged ball in Eq. (35), and the electric field strength $|\vec{E}'|$ within the parallel metal plates of the capacitor, the volume $A'd'$ of the gap between the parallel metal plates of the capacitor and the rest mass m_0 of the capacitor and the box containing containing it in Eq. (47).

One also observes from Eq. (71) or (72) that the effective gravitational acceleration on a spherical shell enclosure of radial beams of high energy laser and their source (or sources) (the test particle), towards a gravitational field source, depends on the rest mass m_0 of the spherical shell enclosure and all material objects within it, as well as the total laser energy E_l' within the enclosure. This and the foregoing paragraph represent a violation of the weak equivalence principle (WEP), which states that all objects fall at the same rate in a given gravitational field.

What has been achieved essentially in the derivations of gravi-electric force, gravi-magnetic force and laser-anti-gravitational force is local coupling to gravity of electrostatic potential energy stored in a test particle with net electric charge, energy stored in electric field within a test particle, energy stored in magnetic field in a test particle and laser energy contained in a test particle, on the flat relativistic spacetime (Σ, ct) in a gravitational field in the context of the theory of gravitational relativity (TGR). It can also be interpreted as gravitation of non-gravitational energy stored within a test particle, (in addition to gravitation of the mass of the test particle), in the context of TGR. This yields an additional term in the gravitational acceleration suffered by the test particle on flat spacetime of TGR, which contains parameters (or properties) of the test particle. The new term could not arise in the context of the classical theory of gravity since there is no coupling of non-gravitational energy to gravity in the classical context, neither could it show up explicitly in the context of the general theory of relativity (GR), since the concept of gravitational acceleration does not appear in GR.

Indeed there is a local coupling to gravity of every non-gravitational energy located in free space within a gravitational field or contained within a test particle falling towards a gravitational field source in the context of TGR, as explained with Eq. (27). This coupling yields a new term containing the parameters of the test particle in the resultant gravitational acceleration on it. The weak equivalence principle (WEP) is therefore not valid whenever the test particle contains non-gravitational energy (in a large quantity). The test particles namely, pieces of wood, platinum, aluminum and gold, in the Eötvös-Dicke experiment, see, for example, page 251 of [12] and page 15 of [13], generally remarked to confirm WEP, indeed did not contain (large quantity of) non-gravitational energy. In other words, that experiment had tested the situation where the test particle contains no non-gravitational energy in the gravitation field of the earth. No one had tested the situation where the test particle (in the experiment) contained large quantity of non-gravitational energy, as far as I can find. Experimental tests of the gravi-electric force, gravi-magnetic force and laser-anti-gravitational force are therefore highly desirable and recommended.

The validity of Einstein's principle of equivalence (EEP), (composed of LLI, WEP and SEP), has been shown in the context of TGR in [9]. It was remarked there however that the non-validity of WEP when the test particle contains large quantity of non-gravitational energy shall be discussed in this paper, as has now been done. The non-validity of WEP when the test particle contains large quantity of non-gravitational energy does not detract from the validity of EEP.

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