# The acceleration of the 2-demension inertial system and the matter wave

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**ABSTRACT**

In the special relativity theory, the acceleration $a$ about the accelerated matter that has the initial velocity $v_0$ in 2-Dimension inertial coordinate system $S(t,x)$ and the other acceleration $a'$ about the accelerated matter that has not the initial velocity $v_0$ in 2-Dimension inertial coordinate system $S'(t',x')$ are same. Therefore using it, derive the moving formula and the transformation about the matter wave

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## I.Introduction

Use following the formula about the constant accelerated matter.

$$x+\frac{c^2}{a_0}=\frac{c^2}{a_0}\cosh(\frac{a_0\tau}{c}), t=\frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \quad (1)$$

$x$ and $t$ is the coordinate and the time in the inertial system about the constant accelerated matter. $a_0$ is the constant acceleration, $\tau$ is invariable time about the constant accelerated matter, $c$ is light speed in the inertial system in the free space-time.

In the special relativity, the formula about 2-Dimension inertial coordinate system $S(t,x)$ and $S'(t',x')$ is

$$V=\frac{u+v_0}{1+\frac{u}{c^2}v_0} \ , \ V=\frac{dx}{dt}, u=\frac{dx'}{dt'} \ , \ dx=\frac{dx'+v_0dt'}{\sqrt{1-\frac{v_0^2}{c^2}}}, \ dt=\frac{dt'+\frac{v_0}{c^2}dx'}{\sqrt{1-\frac{v_0^2}{c^2}}}$$

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}), \ a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (2)$$

The velocity $V$ has the initial velocity $v_0$ and the velocity $u$ is the velocity by the pure acceleration $a'$.

$$a=\frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}})=\frac{\sqrt{1-\frac{v_0^2}{c^2}}}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{v_0^2}{c^2}}\sqrt{1-\frac{u^2}{c^2}}})=\frac{1}{1+\frac{v_0}{c^2}u}\frac{d}{dt'}(\frac{u+v_0}{\sqrt{1-\frac{u^2}{c^2}}})$$

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}}) \quad (3)$$

In this time , if the pure acceleration $a'$ of the velocity $u$ is

$$a'=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) \ , \ u=\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \quad (4)$$

Eq(3) is

$$a(1+\frac{v_0}{c^2}u)=\frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}})+\frac{d}{dt'}(\frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}})=a'+v_0\frac{d}{dt'}(\sqrt{1+\frac{1}{c^2}[\int a'dt']^2})$$

$$= a'+v_0 \frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}} \frac{a'}{c^2} = a'(1+\frac{v_0}{c^2}\frac{\int a'dt'}{\sqrt{1+\frac{1}{c^2}[\int a'dt']^2}})$$

$$= a'(1+\frac{v_0}{c^2}u) \qquad (5)$$

Therefore, the acceleration $a$ about the accelerated matter that has the initial velocity $v_0$ in 2-Dimension inertial coordinate system $S(t,x)$ and the other acceleration $a'$ about the accelerated matter that has not the initial velocity $v_0$ in 2-Dimension inertial coordinate system $S'(t',x')$ are same. In this time, if the acceleration $a'$ is the constant acceleration $a_0$, the inertial acceleration in 2-Dimension inertial coordinate system $S(t,x)$ and in 2-Dimension inertial coordinate system $S'(t',x')$ is the constant acceleration $a_0$.

$$a_0 = a' = \frac{d}{dt'}(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}) = a = \frac{d}{dt}(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}) \quad (6)$$

## II. Additional chapter-I

Therefore,

$$V = \frac{dx}{dt} = \frac{a_0 t + C}{\sqrt{1+\frac{1}{c^2}(a_0 t + C)^2}}, \quad u = \frac{dx'}{dt'} = \frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}, x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)$$

$$= \frac{\gamma a_0 (t'+\frac{v_0}{c^2}x') + C}{\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}x') + C)^2}} \quad , \quad C \text{ is the constant number}$$

$$= \frac{\gamma a_0 (t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)) + C}{\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1)) + C)^2}}$$

$$= \frac{\gamma a_0 t' + \gamma v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C}{\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - \gamma v_0 + C)^2}}$$

$$= \frac{u+v_0}{1+\frac{u}{c^2}v_0} = \frac{\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}+v_0}{1+\frac{v_0}{c^2}\frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}} = \frac{a_0 t' + v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}+\frac{v_0}{c^2}a_0 t'} \quad (7)$$

In this time,

$$\sqrt{1+\frac{1}{c^2}(\gamma a_0 t' + \gamma v_0\sqrt{1+\frac{1}{c^2}(a_0 t')^2})^2} = \gamma(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}+\frac{v_0}{c^2}a_0 t') \quad (8)$$

Therefore,

$$C = \gamma v_0 \qquad (9)$$

Hence,

$$x = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2} - \sqrt{1+\frac{1}{c^2}(\gamma v_0)^2})$$

$$= \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2} - \gamma) = \frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}} - \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}), V = \frac{a_0 t+\gamma v_0}{\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}}$$

$$x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2} - 1) = \frac{c^2}{a_0}(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1) \ , \ u = \frac{a_0 t'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}},$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (10)$$

And

$$d\tau = \sqrt{1-V^2/c^2}\,dt = \frac{dt}{\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}}, \ d\tau = \sqrt{1-u^2/c^2}\,dt' = \frac{dt'}{\sqrt{1+\frac{1}{c^2}(a_0 t')^2}}$$

$$\tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c}) - \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}) = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c}) - \tau_0$$

$$\tau + \tau_0 = \frac{c}{a_0}\sinh^{-1}(\frac{a_0}{c}t+\gamma\frac{v_0}{c}), \qquad \tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0 t'}{c})$$

$$\tau_0 = \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}), \quad \gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (11)$$

Therefore,

$$t+\gamma\frac{v_0}{a_0} = \frac{c}{a_0}\sinh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)$$

$$= \frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c})+\cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (12)$$

In this time,

$$\tau = \frac{c}{a_0}\sinh^{-1}(\frac{a_0 t'}{c}) \rightarrow \sinh(\frac{a_0\tau}{c}) = \frac{a_0 t'}{c},$$

$$x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1) \rightarrow \cosh(\frac{a_0\tau}{c}) = \sqrt{1+\frac{a_0^2 t'^2}{c^2}} = 1+\frac{a_0}{c^2}x'$$

$$\tau_0 = \frac{c}{a_0}\sinh^{-1}(\gamma\frac{v_0}{c}) \rightarrow \sinh(\frac{a_0\tau_0}{c}) = \frac{\gamma v_0}{c}, \quad \cosh(\frac{a_0\tau_0}{c}) = \sqrt{1+\frac{\gamma^2 v_0^2}{c^2}} = \gamma,$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (13)$$

Therefore, Eq(12) is

$$t+\gamma\frac{v_0}{a_0} = \frac{c}{a_0}\sinh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)$$

$$= \frac{c}{a_0}[\sinh(\frac{a_0\tau}{c})\cosh(\frac{a_0\tau_0}{c})+\cosh(\frac{a_0\tau}{c})\sinh(\frac{a_0\tau_0}{c})]$$

$$= \frac{c}{a_0}[\gamma\sinh(\frac{a_0\tau}{c})+\cosh(\frac{a_0\tau}{c})\frac{\gamma v_0}{c}]$$

$$= \frac{c}{a_0}[\frac{a_0 t'}{c}\cdot\gamma+(1+\frac{a_0}{c^2}x')\cdot\frac{\gamma v_0}{c}] = \gamma(t'+\frac{v_0}{c^2}x')+\gamma\frac{v_0}{a_0}, \gamma = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (14)$$

Therefore, Eq(10) is

$$x = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t+\gamma v_0)^2}-\gamma), \quad x' = \frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0 t')^2}-1)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}x')+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0\gamma(t'+\frac{v_0}{c^2}\cdot\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1))+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(\gamma a_0t'+\gamma v_0\sqrt{1+\frac{1}{c^2}(a_0t')^2})^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{(\gamma\sqrt{1+\frac{1}{c^2}(a_0t')^2}+\gamma a_0\frac{v_0}{c^2}t')^2}-\gamma)$$

$$=\gamma\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t')^2}-1)+\gamma v_0t'=\gamma(x'+v_0t')\quad,\quad\gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}}\qquad(15)$$

or by Eq(13),Eq(14)

$$x=\frac{c^2}{a_0}(\sqrt{1+\frac{1}{c^2}(a_0t+\gamma v_0)^2}-\gamma)$$

$$=\frac{c^2}{a_0}(\sqrt{1+\sinh^2(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)}-\gamma)=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau+\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\cosh(\frac{a_0}{c}\tau_0)+\sinh(\frac{a_0}{c}\tau)\sinh(\frac{a_0}{c}\tau_0)-\gamma)$$

$$=\frac{c^2}{a_0}(\cosh(\frac{a_0}{c}\tau)\gamma+\sinh(\frac{a_0}{c}\tau)\frac{\gamma v_0}{c}-\gamma)$$

$$=\frac{c^2}{a_0}\gamma\sqrt{1+\frac{1}{c^2}(a_0t')^2}+\gamma v_0t'-\frac{c^2}{a_0}\gamma,\quad\gamma=\frac{1}{\sqrt{1-\frac{v_0{}^2}{c^2}}}\qquad(16)$$

## V. Conclusion

Hence, Eq(1) is in the 2-Dimension inertial coordinate system $S'(t',x')$

$$x'=\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c})-1)$$

$$t' = \frac{c}{a_0}\sinh(\frac{a_0\tau}{c}) \qquad (17)$$

Therefore, in the 2-Dimension inertial coordinate system $S(t,x)$

$$t = \gamma(t' + \frac{v_0}{c^2}x') = \gamma(\frac{c}{a_0}\sinh(\frac{a_0}{c}\tau) + \frac{v_0}{a_0}(\cosh(\frac{a_0}{c}\tau) - 1))$$

$$x = \gamma(x' + v_0 t') = \gamma(\frac{c^2}{a_0}(\cosh(\frac{a_0\tau}{c}) - 1) + \frac{v_0 c}{a_0}\sinh(\frac{a_0\tau}{c})), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^{\ 2}}{c^2}}} \qquad (18)$$

$$dt = \gamma(\cosh(\frac{a_0}{c}\tau) + \frac{v_0}{c}\sinh(\frac{a_0}{c}\tau))d\tau,$$

$$dx = \gamma(c\sinh(\frac{a_0}{c}\tau) + v_0\cosh(\frac{a_0}{c}\tau))d\tau,$$

$$V = \frac{dx}{dt} = (c\tanh(\frac{a_0}{c}\tau) + v_0)/(1 + \frac{v_0}{c}\tanh(\frac{a_0}{c}\tau)), \gamma = \frac{1}{\sqrt{1 - \frac{v_0^{\ 2}}{c^2}}} \quad (19)$$

In this time, treat about the matter wave.

The energy $E'$ in 2-Dimension inertial coordinate system $S'(t',x')$ and the energy $E$ in 2-Dimension inertial coordinate system $S(t,x)$ is

$$a' = \frac{d}{dt'}(\frac{u}{\sqrt{1 - \frac{u^2}{c^2}}}), u = \frac{\int a'dt'}{\sqrt{1 + \frac{1}{c^2}[\int a'dt']^2}}, a = \frac{d}{dt}(\frac{V}{\sqrt{1 - \frac{V^2}{c^2}}}), V = \frac{\int adt}{\sqrt{1 + \frac{1}{c^2}[\int adt]^2}} \quad (20)$$

$$E' = \frac{m_0c^2}{\sqrt{1 - \frac{u^2}{c^2}}} = m_0c^2\sqrt{1 + \frac{1}{c^2}[\int a'dt']^2}, \quad E = \frac{m_0c^2}{\sqrt{1 - \frac{V^2}{c^2}}} = m_0c^2\sqrt{1 + \frac{1}{c^2}[\int adt]^2} \quad (21)$$

$$\frac{dE'}{dt'} = m_0c^2\frac{\int a'dt'}{\sqrt{1 + \frac{1}{c^2}[\int a'dt']^2}}\frac{a'}{c^2} = m_0a'u \quad (22)$$

$$\frac{dE}{dt} = m_0c^2\frac{\int adt}{\sqrt{1 + \frac{1}{c^2}[\int adt]^2}}\frac{a}{c^2} = m_0aV = m_0a(\frac{u + v_0}{1 + \frac{v_0}{c^2}u}) \quad (23)$$

In Eq(6),

$$a' = \frac{d}{dt'}\left(\frac{u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = a = \frac{d}{dt}\left(\frac{V}{\sqrt{1-\frac{V^2}{c^2}}}\right)$$

$$F' = m_0 a' = \frac{d}{dt'}\left(\frac{m_0 u}{\sqrt{1-\frac{u^2}{c^2}}}\right) = m_0 a = \frac{d}{dt}\left(\frac{m_0 V}{\sqrt{1-\frac{V^2}{c^2}}}\right) = F \quad (24)$$

Therefore, the transformation is

$$\frac{dE}{dt} = m_0 a\left(\frac{u+v_0}{1+\frac{v_0}{c^2}u}\right) = m_0 a'\left(\frac{u+v_0}{1+\frac{v_0}{c^2}u}\right) = \frac{m_0 a' u + m_0 a' v_0}{1+\frac{v_0}{c^2}u}$$

$$= \frac{\frac{dE'}{dt'} + F' v_0}{1+\frac{v_0}{c^2}u}, \quad F' = m_0 a', \quad \frac{dE'}{dt'} = m_0 a' u \quad (25)$$

The inverse-transformation is

$$\frac{dE'}{dt'} = m_0 a' u = m_0 a u = m_0 a\left(\frac{V-v_0}{1-\frac{v_0}{c^2}V}\right) = \frac{m_0 a V - m_0 a v_0}{1-\frac{v_0}{c^2}V}$$

$$= \frac{\frac{dE}{dt} - F v_0}{1-\frac{v_0}{c^2}V}, \quad F = m_0 a, \quad \frac{dE}{dt} = m_0 a V \quad (26)$$

In this time, the frequency $\upsilon$ and the wavelength $\lambda$ about the matter wave is

$$E = h\upsilon, \quad E' = h\upsilon', \quad (27)$$

$$\lambda = \frac{h}{p}, \quad \lambda' = \frac{h}{p'},$$

$$\frac{1}{\lambda} = \frac{p}{h}, \quad \frac{1}{\lambda'} = \frac{p'}{h} \quad (28)$$

Therefore, if Eq(25) and Eq(26) is inserted by Eq(27)

$$\frac{dE}{dt} = h\frac{d\upsilon}{dt} = \frac{h\frac{d\upsilon'}{dt'} + F' v_0}{1+\frac{v_0}{c^2}u}, \quad F' = m_0 a'$$

$$\frac{d\upsilon}{dt} = \frac{\frac{d\upsilon'}{dt'} + \frac{F' v_0}{h}}{1 + \frac{v_0}{c^2} u}, \quad F' = m_0 a' \qquad (29)$$

$$\frac{dE'}{dt'} = h\frac{d\upsilon'}{dt'} = \frac{h\frac{d\upsilon}{dt} - F v_0}{1 - \frac{v_0}{c^2} V}, \quad F = m_0 a$$

$$\frac{d\upsilon'}{dt'} = \frac{\frac{d\upsilon}{dt} - \frac{F v_0}{h}}{1 - \frac{v_0}{c^2} V}, \quad F = m_0 a \qquad (30)$$

In Eq(28),

$$\frac{d}{dt}(\frac{1}{\lambda}) = \frac{d}{dt}(\frac{p}{h}) = \frac{1}{h}\frac{dp}{dt} = \frac{F}{h} = \frac{F'}{h} = \frac{1}{h}\frac{dp'}{dt'} = \frac{d}{dt'}(\frac{p'}{h}) = \frac{d}{dt'}(\frac{1}{\lambda'})$$
$$F' = m_0 a' = m_0 a = F$$
$$(31)$$

Hence,

$$\frac{d}{dt}(\frac{1}{\lambda}) = -\frac{1}{\lambda^2}\frac{d\lambda}{dt} = \frac{d}{dt'}(\frac{1}{\lambda'}) = -\frac{1}{\lambda'^2}\frac{d\lambda'}{dt'} \qquad (32)$$

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