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Static Response of Transversely Isotropic Elastic Medium with Irregularity Present in the Medium

DINESH KUMAR MADAN[1], ANITA DAHIYA[2] AND SHAMTA CHUGH[3]

1 Department of Mathematics, Technological Institute of Textile & Sciences, Bhiwani-127021(India)
Email: dineshmadan@titsbhiwani.ac.in, dk_madaan@rediffmail.com
2 Department of Mathematics, Manav Bharti University, Solan,Himachal Pardesh -173229(India)
Email: anitamalik76@gmail.com
3 Department of Mathematics, Shanti Devi Institute of Technology & Management,Panipat(India)
Email: shamta.chugh@gmail.com

Abstract. In the present paper, the closed form expressions for the displacements at any point of the transversely isotropic elastic medium with irregularity present in the medium have been obtained. A model is considered in which the irregularity is expressed by a rectangle shape and the medium is taken in a state of free from initial stress. To study the effect of irregularity present in the medium , the variation of displacements with horizontal distance have been drawn for different values of irregularity size. Also the comparison between the displacements for isotropic and transversely isotropic elastic medium is shown graphically. It is found that the irregularity have a notable effect on this deformation.

Keywords: Static deformation; Irregularity, transversely isotropic medium, Eigenvalues

1 Introduction

For convenience elastic problems are formulated under the assumptions of homogeneity, perfect elasticity, plane parallel boundaries and isotropy. Under these assumptions certain ambiguities exist between theory and observation that point out the need for reformulating some problems under less restrictive assumptions and possible more realistic. In the present problem we drop the assumption of isotropy and considered to be medium as transversely isotropic elastic.

Transverse isotropy results in the same set of elastic constants as that for hexagonal symmetry and hence is exhibited in all metals or minerals crystallizing in the hexagonal system. This symmetry is also expected to be displayed by sediments, planner igneous bodies, floating ice sheets and rolled or extruded metal and plastic sheets. This later material is often used in two-dimensional model experiments. The elastic problems with transverse symmetry has been discussed by Love[1], Stonley[2],Singh[3], Pan[4] , etc.

The problem of static deformation with irregularity present in the elastic medium which is due to continental margin, mountain roots, etc., is very important to study. The problem with irregular thickness has been discussed by De Noyer [5]; Sato [6];Mal [7],Kar et al. [8],Chhattopadhyay and Pal [9], Chhattopadhyay et al [10-12], Selim [13], and others. In the present paper a model is considered in which the irregularity on the interface has been expressed by a rectangle shape and the medium was taken in a state of free from initial stress. The closed form expressions for the displacements at any point of the transversely isotropic medium are obtained. To study the effect of irregularity in the medium, the variation of displacements with horizontal distance have been drawn. Also the comparison of

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displacements for isotropic and transversely isotropic medium is shown graphically for different values of irregularity size.

2 Generalised Hook's Law and Equations of Equilibrium.

The stress strain relations in matrix form for a medium with hexagonal or transverse isotropic elastic symmetry is Love [1]

$$\begin{bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{bmatrix}=\begin{bmatrix}c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{c_{11}-c_{12}}{2}\end{bmatrix}\begin{bmatrix}e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{13}\\ 2e_{12}\end{bmatrix} \quad (1)$$

where the two-suffix quantity c_{ij} denotes the elastic constants of the medium.
So in transversely isotropic elastic medium we have five elastic constants. For an elastic isotropic medium, these constants reduce to just two as given below.

$$c_{11}=c_{33}=\lambda+2\mu$$
$$c_{12}=c_{13}=\lambda \quad (2)$$
$$c_{44}=\frac{c_{11}-c_{12}}{2}=\mu$$

where λ and μ are the Lame's constants.
In the absence of body forces, the equilibrium equations in the Cartesian co-ordinate system(x, y, z) are

$$\frac{\partial\sigma_{11}}{\partial x}+\frac{\partial\sigma_{12}}{\partial y}+\frac{\partial\sigma_{13}}{\partial z}=0$$
$$\frac{\partial\sigma_{21}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y}+\frac{\partial\sigma_{23}}{\partial z}=0 \quad (3)$$
$$\frac{\partial\sigma_{31}}{\partial x}+\frac{\partial\sigma_{32}}{\partial y}+\frac{\partial\sigma_{33}}{\partial z}=0$$

where $\tau_{ij}(i,j=1,2,3)$are the stress components for transversely isotropic medium.The strain displacement relation are

$$e_{ij}=e_{ji}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right),\quad 1\le i,j\le 3, \quad (4)$$

where $(u_1,u_2,u_3)=(u,v,w)$and$(x_1,x_2,x_3)=(x,y,z)$.
The equilibrium equations in terms of displacement components can be obtained from equations
$(1)-(4)$.We obtain

$$c_{11}\frac{\partial^2 u}{\partial x^2}+\left(\frac{c_{11}-c_{12}}{2}\right)\frac{\partial^2 u}{\partial y^2}+c_{44}\frac{\partial^2 u}{\partial z^2}+\left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 v}{\partial x\,\partial y}+(c_{13}+c_{44})\frac{\partial^2 w}{\partial x\,\partial z}=0$$
$$\left(\frac{c_{11}-c_{12}}{2}\right)\frac{\partial^2 v}{\partial x^2}+c_{11}\frac{\partial^2 v}{\partial y^2}+c_{44}\frac{\partial^2 v}{\partial z^2}+\left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 u}{\partial x\,\partial y}+(c_{13}+c_{44})\frac{\partial^2 w}{\partial y\,\partial z}=0 \quad (5)$$
$$c_{44}\frac{\partial^2 w}{\partial x^2}+c_{44}\frac{\partial^2 w}{\partial y^2}+c_{33}\frac{\partial^2 w}{\partial z^2}+(c_{13}+c_{44})\frac{\partial^2 u}{\partial x\,\partial z}+(c_{13}+c_{44})\frac{\partial^2 v}{\partial y\,\partial z}=0$$

3 Formulation and solution of the problem

Consider an unbounded transversely isotropic elastic half space with x-axis vertically downwards. The origin of Cartesian coordinates is situated at x=0.

Suppose that a normal line load R_0 per unit length is acting vertically downwards on a line parallel to the z-axis and passing through the point $(H, 0)$.Assume that the

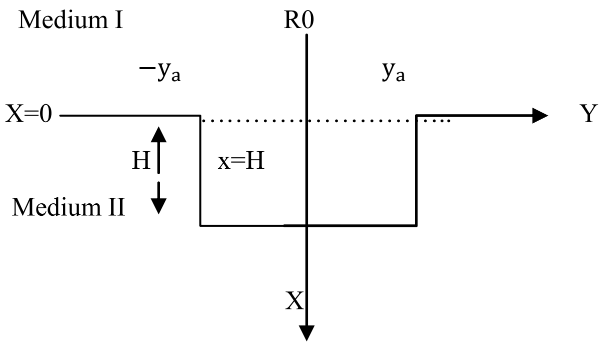
irregularity is of the form of a rectangle with length $2y_a$ and depth H (Fig.1). Let the equation of irregularity is

$$x = \epsilon f(y) \tag{6}$$

where

$$\epsilon f(y) = \begin{Bmatrix} H & \text{for} |y| \le y_a \\ 0 & \text{for } |y| > y_a \end{Bmatrix} \tag{7}$$

where ϵ is a small parameter $\epsilon = \frac{H}{2y_a} \ll 1.$



(fig 1)

Let the elastic medium under consideration be under the condition of plain strain deformation, parallel to the xy-plane,in which the displacement components are independent of z and are of the type

$$u = u(x, y), \quad v = v(x, y), \quad w = 0 \tag{8}$$

The non zero stresses for the plane strain problem are

$$\tau_{11} = c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y}$$

$$\tau_{22} = c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} \tag{9}$$

$$\tau_{12} = \left(\frac{c_{11}-c_{12}}{2}\right)\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

The equilibrium equations reduced to

$$c_{11}\frac{\partial^2 u}{\partial x^2} + \left(\frac{c_{11}-c_{12}}{2}\right)\frac{\partial^2 u}{\partial y^2} + \left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 v}{\partial x\,\partial y} = 0$$

$$\left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 u}{\partial x\,\partial y} + \left(\frac{c_{11}-c_{12}}{2}\right)\frac{\partial^2 v}{\partial x^2} + c_{11}\frac{\partial^2 v}{\partial y^2} = 0 \tag{10}$$

On taking fourier transformation, equation (10) becomes

$$c_{11}\frac{d^2\bar{u}}{dx^2} + \left(\frac{c_{11}-c_{12}}{2}\right)(-k^2)\bar{u} - ik\left(\frac{c_{11}+c_{12}}{2}\right)\frac{d\bar{v}}{dx} = 0$$

$$-ik\left(\frac{c_{11}+c_{12}}{2}\right)\frac{d\bar{u}}{dx} + \left(\frac{c_{11}-c_{12}}{2}\right)\frac{d^2\bar{v}}{dx^2} - c_{11}k^2\bar{v} = 0 \tag{11}$$

where bar stands for Fourier Transform with transformed fourier parameter k
In the vector matrix differential equation form ,the equation (11)

$$P_1 \frac{d^2U}{dx^2} - i\eta P_2 \frac{dU}{dx} - k^2 P_3 U = 0 \tag{12}$$

where

$$P_1 = \begin{pmatrix} c_{11} & 0 \\ 0 & \frac{c_{11}-c_{12}}{2} \end{pmatrix} \qquad P_2 = \begin{pmatrix} 0 & \frac{c_{11}+c_{12}}{2} \\ \frac{c_{11}+c_{12}}{2} & 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} \frac{c_{11}-c_{12}}{2} & 0 \\ 0 & c_{11} \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \tag{13}$$

Let the solution of the matrix equation (12) is of the form

$$U(x,k) = E(k)e^{mx}, \tag{14}$$

where m is a parameter and E(k) is a matrix of the type 2×1.
Substitution of the value of U from Eqs.(14) − (13), we get the following characteristic equation

$$m^4 - 2k^2m^2 + k^4 = 0, \tag{15}$$

The solution of the characteristic equation (15) are

$$m = m_1 = m_2 = -m_3 = -m_4 = |k| \tag{16}$$

Which are repeated eigenvalues
Ross [14] has given a procedure to tackle with repeated Eigen values, provided the governing vector differential equation is of the first-order. The equilibrium equations (11) are equivalent to the first-order vector differential equation. By applying the process as given by Ross[14] the equilibrium equation becomes

$$\frac{dU_1}{dx} = A_1 U_1 \tag{17}$$

$$\text{where } U_1 = \begin{bmatrix} \bar{u} \\ \bar{v} \\ \frac{d\bar{u}}{dx} \\ \frac{d\bar{v}}{dx} \end{bmatrix}, A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{c_{11}-c_{12}}{2c_{11}}k^2 & 0 & 0 & \frac{c_{11}+c_{12}}{2c_{11}}ik \\ 0 & \frac{2c_{11}}{c_{11}-c_{12}}k^2 & \frac{c_{11}+c_{12}}{c_{11}-c_{12}}ik & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k^2}{\beta} & 0 & 0 & \frac{ik(\beta-1)}{\beta} \\ 0 & k^2\beta & ik(\beta-1) & 0 \end{bmatrix}$$

where $\beta = \frac{2c_{11}}{c_{11}-c_{12}}$

Then,the independent eigenvectors are found to be

$$E_1 = \begin{bmatrix} i|k| \\ k \\ ik^2 \\ k|k| \end{bmatrix}, E_2 = \begin{bmatrix} i\left\{x|k| - \left(\frac{2\beta}{\beta-1}\right)\right\} \\ k\left(x - \frac{1}{|k|}\right) \\ i|k|\left\{x|k| - \left(\frac{\beta+1}{\beta-1}\right)\right\} \\ xk|k| \end{bmatrix},$$

$$E_3 = \begin{bmatrix} -i|k| \\ k \\ ik^2 \\ -k|k| \end{bmatrix}, \quad E_4 = \begin{bmatrix} -i\left\{x|k| + \left(\frac{2\beta}{\beta-1}\right)\right\} \\ k\left(x + \frac{1}{|k|}\right) \\ i|k|\left\{x|k| + \left(\frac{\beta+1}{\beta-1}\right)\right\} \\ -xk|k| \end{bmatrix} \tag{18}$$

Thus ,the general solution of equation (17) for transversely isotropic medium is

$$U_1 = (BE_1 + CE_2)e^{|k|x} + (DE_3 + GE_4)e^{-|k|x} \tag{19}$$

where B, C, D, G are constants to be determined from boundary conditions, which may be dependent upon k and elastic constants

Solving the matrix equation (14) and using Eqs. (18) − (19), the displacements and stresses in the transformed domain are found to be

$$\bar{u}(x,k) = i\left[\{B|k| + C(x|k| - M_1)\}e^{|k|x} - \{D|k| + G(x|k| + M_1)\}e^{-|k|x}\right],$$

where $M_1 = \left(\frac{2\beta}{\beta-1}\right)$

$$\bar{v}(x,k) = k\left[\left\{B + C\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x} + \left\{D + G\left(x + \frac{1}{|k|}\right)\right\}e^{-|k|x}\right],$$

$$\frac{d\bar{u}}{dx} = i\left[\left\{Bk^2 + C|k|\left(x|k| - \left(\frac{\beta+1}{\beta-1}\right)\right)\right\}e^{|k|x} - \left\{Dk^2 + G|k|\left(x|k| + \left(\frac{\beta+1}{\beta-1}\right)\right)\right\}e^{-|k|x}\right]$$

$$\frac{d\bar{v}}{dx} = k|k|\left[\{B + Cx\}e^{|k|x} - \{D + Gx\}e^{-|k|x}\right] \tag{20}$$

$$\bar{\sigma}_{11} = i\begin{bmatrix} \{Bk^2(c_{11} - c_{12}) + C(xk^2(c_{11} - c_{12}) + |k|(c_{12} - M_2c_{11}))\}e^{|k|x} + \\ \{Dk^2(c_{11} - c_{12}) + G(xk^2(c_{11} - c_{12}) - |k|(c_{12} - M_2c_{11}))\}e^{-|k|x} \end{bmatrix}$$

where $M_2 = \left(\frac{2\beta-1}{\beta-1}\right)$

$$\bar{\sigma}_{12} = \left(\frac{c_{11}-c_{12}}{2}\right)k[\{2B|k| + C(2|k|x - M_1)\}e^{|k|x} - \{2D|k| + G(2|k|x + M_1)\}e^{-|k|x}]$$

By using inverse fourier transform the displacements and stresses are obtained as

$$u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\{B|k| + C(x|k| - M_1)\}e^{|k|x} - \{D|k| + G(x|k| + M_1)\}e^{-|k|x}\right]e^{-iky}dk,$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} k\left[\left\{B + C\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x} + \left\{D + Gk\left(x + \frac{1}{|k|}\right)\right\}e^{-|k|x}\right]e^{-i\eta y}dk,$$

$$\sigma_{11}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i[\{(c_{11} - c_{12})(B + Cx)k^2 + |k|(c_{12} - M_2c_{11})\}e^{|k|x}\{(c_{11} - c_{12})(D + Gx)k^2 - |k|(c_{12} - M_2c_{11})\}e^{-|k|x}]e^{-iky}dk$$

$$\sigma_{12}(x,y) = \frac{1}{2\pi}\left(\frac{c_{11}-c_{12}}{2}\right)\int_{-\infty}^{\infty} k[\{B|k| + C(2|k|x - M_1)\}e^{|k|x} - \{2D|k| + G(2|k|x + M_1)\}e^{-|k|x}]e^{-iky}dk. \tag{21}$$

3.1 Normal Line-Load in an Irregular Transversely isotropic Elastic Half-Space

To find the elastic displacements and stresses at any point of an irregular transversely isotropic elastic medium due to normal line load R_0per unit length acting on z-axis,we consider the irregular medium consisting of region $x < \epsilon f(y)$ (Medium I)and region$x > \epsilon f(y)$ (Medium II).of identical elastic properties.From equation(21) The displacement and stress component for the Medium I are

$$u^{I}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\{B|k| + C(x|k| - M_1^1)\}e^{|k|x}\right]e^{-iky}dk,$$

$$v^{I}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} k\left[\left\{B + C\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x}\right]e^{-iky}dk,$$

$$\sigma_{11}^{I}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\left\{Bk^2(c_{11} - c_{12}) + C\begin{pmatrix}xk^2(c_{11} - c_{12}) + \\ |k|(c_{12} - M_2^1 c_{11})\end{pmatrix}\right\}e^{|k|x}e^{-iky}dk\right]$$

$$\sigma_{12}^{I}(x,y) = \frac{1}{2\pi}\left(\frac{c_{11}-c_{12}}{2}\right)\int_{-\infty}^{\infty} k\left[\{2B|k| + C(2|k|x - M_1^1)\}e^{|k|x}\right]e^{-iky}dk. \quad (22)$$

And for medium II

$$u^{II}(x,y) = \frac{1}{2\pi} i\int_{-\infty}^{\infty}\left[-\{D|k| + G(x|k| + M_1^{11})\}e^{-|k|x}\right]e^{-iky}dk.$$

$$v^{II}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} k\left[\left\{D + Gk\left(x + \frac{1}{|k|}\right)\right\}e^{-|k|x}\right]e^{-iky}dk.$$

$$\sigma_{11}^{II}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\left\{Dk^2(c_{11} - c_{12}) + G\begin{pmatrix}xk^2(c_{11} - c_{12}) \\ -|k|(c_{12} - M_2^{11}c_{11})\end{pmatrix}\right\}e^{-|k|x}\right]e^{-iky}dk$$

$$\sigma_{12}^{II}(x,y) = \frac{1}{2\pi}\left(\frac{c_{11}-c_{12}}{2}\right)\int_{-\infty}^{\infty} k\left[-\{2D|k| + G(2|k|x + M_1^{11})\}e^{-|k|x}\right]e^{-iky}dk. \quad (23)$$

3.2. Boundary conditions.

Consider a normal line-load R_0 per unit length, is acting vertically downwards on the interface irregularity $x = H$ along z-axis (Fig. 1). Then the boundary conditions at $x = H$ are

$$u^{I}(H,y) = u^{II}(H,y),$$

$$v^{I}(H,y) = v^{II}(H,y).$$

$$\sigma_{12}^{I}(x = H,y) = \sigma_{12}^{II}(x = H,y)$$

$$\sigma_{11}^{II}(x = H,y) - \sigma_{11}^{I}(x = H,y) = -R_0\delta(y), \quad (24)$$

where $H = \epsilon f(y)$ and $\delta(y)$ is the Dirac-delta satisfying the following properties:

$$\int_{-\infty}^{\infty}\delta(y)f(y)\,dy = 1, \quad \delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iky}dk = 1 \quad (25)$$

Following the method of Erigen and Suhubi [15] ,the arbitrary functions B,C,D and G can be evaluated by expanding these in terms of ϵ and retaining only the linear terms, we may write

$B = B_0 + \epsilon B_1$, $C = C_0 + \epsilon C_1$

$D = D_0 + \epsilon D_1 , \qquad G = G_0 + \epsilon G_1$ (26)

And since $|\epsilon f(y)| \ll 1$, then

$\exp[\pm|k|\epsilon f(y)] \approx 1 \pm |k|\epsilon f(y).$ (27)

Applying the boundary conditions (24) and using equations (25) – (27),we obtain the following equations:

CASE 1

The zero order terms:

$$B_0 - \frac{M_1^I}{|k|} C_0 + D_0 + \frac{M_1^{II}}{|k|} G_0 = 0$$

$$B_0 - \frac{C_0}{|k|} - D_0 - \frac{G_0}{|k|} = 0 \tag{28}$$

$$B_0 - \frac{M_1^I}{2|k|} C_0 + D_0 + \frac{M_1^{II}}{2|k|} G_0 = 0$$

$$B_0 + \frac{M_3^I}{2|k|} C_0 - D_0 + \frac{M_3^{II}}{2|k|} G_0 = \frac{-iR_0}{2k^2}$$

where

$$M_3^I = \frac{2(M_2^1 c_{11} - c_{12})}{c_{11} - c_{12}} \quad \text{and} \quad M_3^{II} = \frac{2(M_2^{11} c_{11} - c_{12})}{c_{11} - c_{12}}$$

Thus we have the zero order solution as

$$B_0 = \frac{-iR_0}{k^2} k_1, \quad C_0 = \frac{-iR_0}{|k|} k_2, D_0 = \frac{iR_0}{k^2} k_1, C_0 = \frac{-iR_0}{|k|} k_3 \tag{29}$$

where

$$k_1 = \frac{M_1^{II} + M_1^I}{\delta} \quad k_2 = \frac{M_1^{II}}{\delta} \quad k_3 = \frac{M_1^I}{\delta}$$

$$\delta = 2M_1^{II} - M_3^I M_1^{II} + 2M_1^I - M_3^{II} M_1^I$$

CASE 2

The first order terms are evaluated following Erigen and Suhubi[15] The final results for the first order solutions are

$$B_1 = \frac{-iR_0 f(y)}{2|k|} \delta_1$$

$$C_1 = -iR_0 f(y) \delta_2$$

$$D_1 = \frac{-iR_0 f(y)}{2|k|} \delta_3$$

$$G_1 = -iR_0 f(y)\, \delta_4 \tag{30}$$

where

$$\delta_1 = \left[\frac{4M_1^I M_1^{II} M_3^I + {M_1^I}^2 M_4^{II} + 4M_3^I {M_1^{II}}^2 - M_4^I {M_1^{II}}^2 - 2{M_1^I}^2 - 4M_1^I M_1^{II} - 2{M_1^I}^2 M_1^{II} - 2{M_1^{II}}^2 - 2M_1^I {M_1^{II}}^2}{M_1^I + M_1^{II}}\right]$$

$$\delta_2 = \frac{M_1^{II}[-M_1^{II} M_4^I + 4M_1^{II} M_3^I + M_1^I M_4^{II} - 4M_1^I]}{M_1^I + M_1^{II}}$$

$$\delta_3 = \left[\frac{-2{M_1^I}^2 - 2{M_1^I}^2 M_1^{II} - 4M_1^I M_1^{II} - 2{M_1^{II}}^2 M_1^I - 2{M_1^{II}}^2 - 4M_1^I M_1^{II} M_3^I - {M_1^I}^2 M_4^{II} - 4M_3^I {M_1^{II}}^2 + M_4^I {M_1^{II}}^2}{M_1^I + M_1^{II}}\right]$$

$\delta_4 = \frac{M_1^I[4M_1^{II} - M_1^{II}M_4^I + 4M_1^{II}M_3^I + M_1^I M_4^{II}]}{M_1^I + M_1^{II}}$

Inserting the values of the various coefficients from equations.(30), (29)and(26)into Eqs. (22), (23) and using Eqs. (6) and values of the integral given in appendix A, we get the following closed form expressions for displacements at any point of an irregular transversely isotropic elastic half space due to normal line force acting at (H, 0).

$$u(x,y) = \frac{R_0}{2\pi}\left[\left\{(k_2M_1^1 + k_3M_1^{11} - 2k_1)\log(x^2+y^2) - (k_2+k_3)\frac{2x^2}{x^2+y^2}\right\} + \gamma\left\{\left(\frac{\delta_1-\delta_3}{2} + (\delta_4 M_1^{11} - M_1^1)\right)\frac{2x^2}{x^2+y^2} - 2(\delta_2+\delta_4)\frac{x^2(x^2-y^2)}{(x^2+y^2)^2}\right\}\right] \quad (31)$$

$$v(x,y) = \frac{-R_0}{2\pi}\left[\left\{2(k_2 - 2k_1 + k_3)\tan^{-1}\frac{y}{x} + (k_2+k_3)\frac{2xy}{x^2+y^2}\right\} + \gamma\left\{\left(\frac{\delta_1+\delta_3}{2} + (\delta_4 - \delta_2)\right)\frac{2xy}{x^2+y^2} + (\delta_4-\delta_2)\frac{4x^3y}{(x^2+y^2)^2}\right\}\right] \quad (32)$$

4. Particular Case : For isotropic elastic half space

In the previous expressions for transversely isotropic medium if we put $c_{11} = c_{33} = \lambda + 2\mu$, $c_{12} = c_{13} = \lambda$, , $c_{44} = \frac{c_{11}-c_{12}}{2} = \mu$ Where λ and μ are the Lame's constants.,the corresponding expressions for isotropic half space can be obtained.

5. Numerical results and discussion.

In this section we intend to study the effect of irregularity present in the transversely isotropic elastic medium and to compare the results numerically between the displacements for transversely isotropic and isotropic elastic medium. For transversely isotropic elastic medium , we used the values of elastic constants given by Haojiang Ding , Weigiu Chen, Liangchi[16] for medium I and II. For medium I we consider Graphite and for medium II, we consider Aluminium Oxide for which the values of elastic constants are given as(in units of GPa)
For medium I, $c_{11} = 460.2$ $c_{12=}174.7$ $c_{13} = 127.4$ $c_{33} = 509.5$, $c_{44} = 126.9$, and for medium II, the elastic constants are $c_{11} = 1060$, $c_{12=}180$, $c_{13} = 15$, $c_{33} = 37$, $c_{44} = 0.35$.
And for isotropic elastic medium, we used the values of elastic constants given by Sokolnikoff[17]. For medium I we consider Carbon and for Medium II we consider Glass for which the values of elastic constants are given as: For medium I $c_{11} = .00006619$, $c_{12=}.000022063$, $c_{44} = .000022063$ and for Medium II $c_{11} = .000261656$, $c_{12=}.000103077$ $c_{44} = .00007929$.
From Fig. 2.1 to Fig. 2.3 and from Figure 3.1 to Fig.3.3, it has been observed that the distance in magnitude between the corresponding normal displacements and corresponding tangential displacement for transversely isotropic and isotropic medium increases as the size of irregularity increases..This problem is useful in many civil and mechanical engineering problems. In Fig. 4.1 and Fig. 4.2, the variation of normal (UT) and tangential (VT) dimensionless displacement corresponding to horizontal distance y for a dimensionless value of x=1, for different sizes of irregularity $\gamma = 0.0,\ 0.25,\ 0.50,\ 0.75$ have been shown. In

these figures it has been observed that the distance between the displacements increases as the size of irregularity increase.

Appendix. A $(\xi) > 0$

$$\int_{-\infty}^{\infty} \exp(-|\eta|\xi)\exp(-\iota\eta y)d\eta = \frac{2\xi}{y^2+\xi^2}$$

$$\int_{-\infty}^{\infty} \frac{\eta}{|\eta|}\exp(-|\eta|\xi)\exp(-\iota\eta y)d\eta = \frac{-2iy}{y^2+\xi^2}$$

$$\int_{-\infty}^{\infty} |\eta|^{-1}\exp(-|\eta|\xi)\exp(-\iota\eta y)d\eta = -\log(y^2+\xi^2)$$

$$\int_{-\infty}^{\infty} \eta\exp(-|\eta|\xi)\exp(-\iota\eta y)d\eta = \frac{-4iy\xi}{(y^2+\xi^2)^2}$$

$$\int_{-\infty}^{\infty} |\eta|\exp(-|\eta|\xi)\exp(-\iota\eta y)d\eta = \frac{2(\xi^2-y^2)}{(y^2+\xi^2)^2}$$

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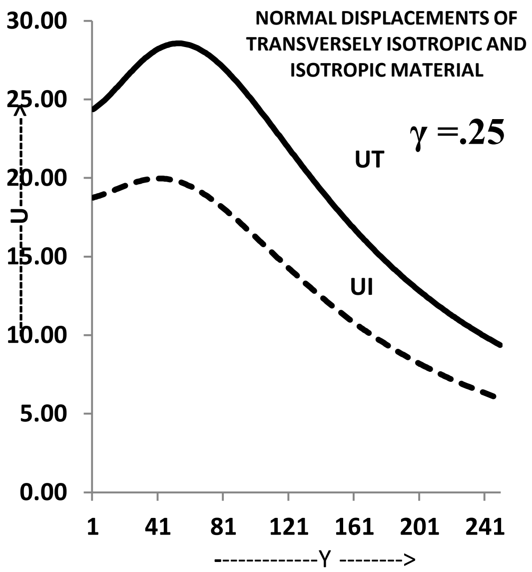
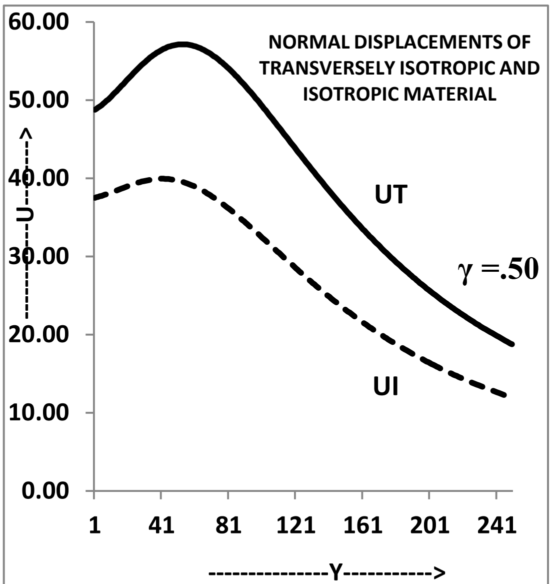


Fig. 2.1



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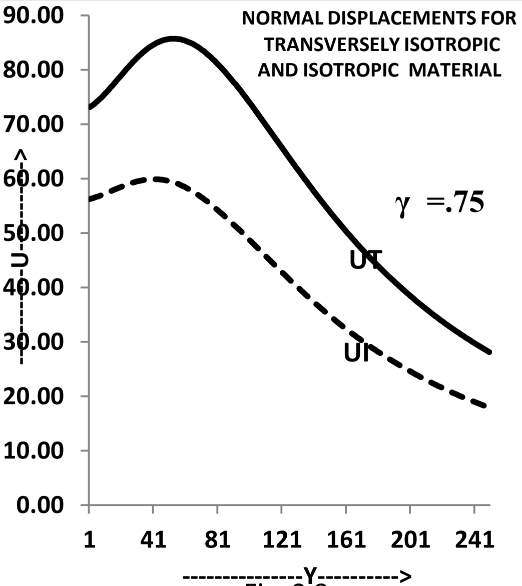

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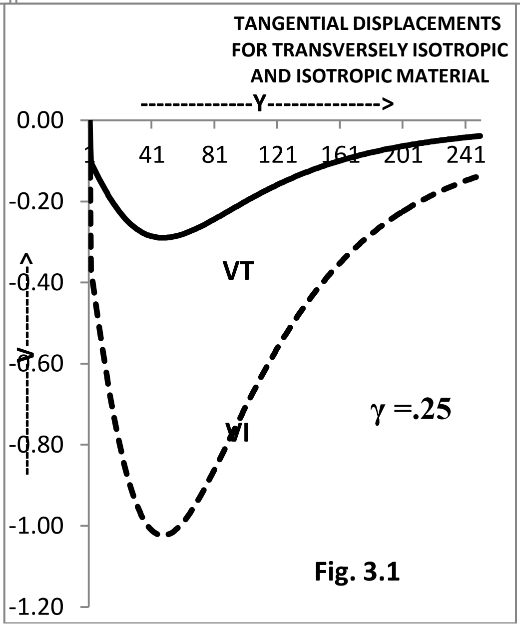

Fig. 3.1

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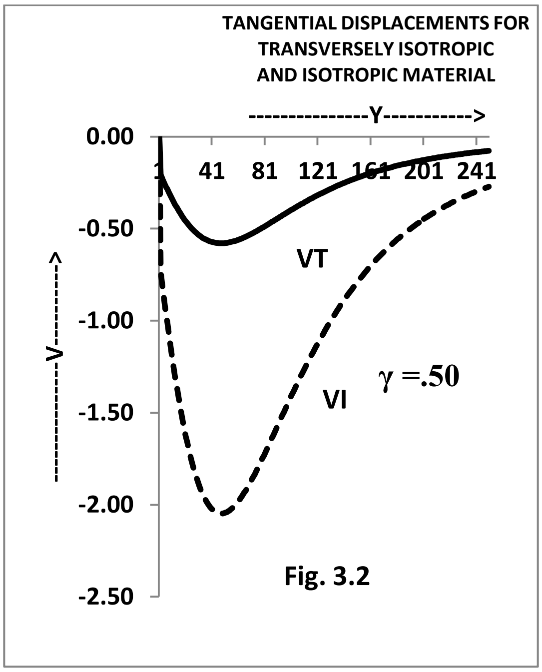

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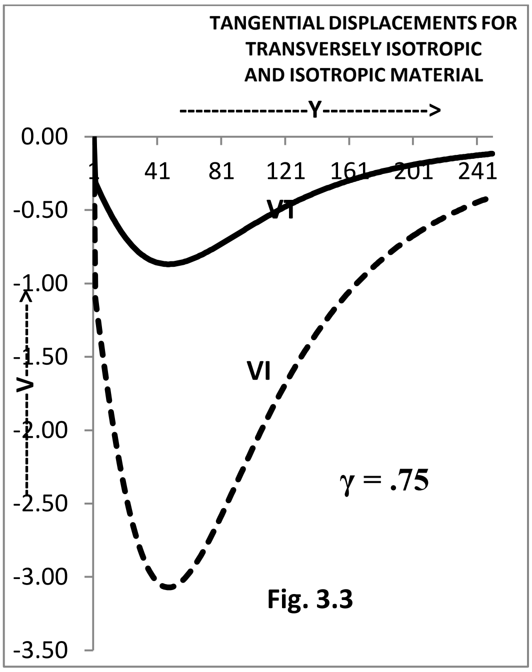

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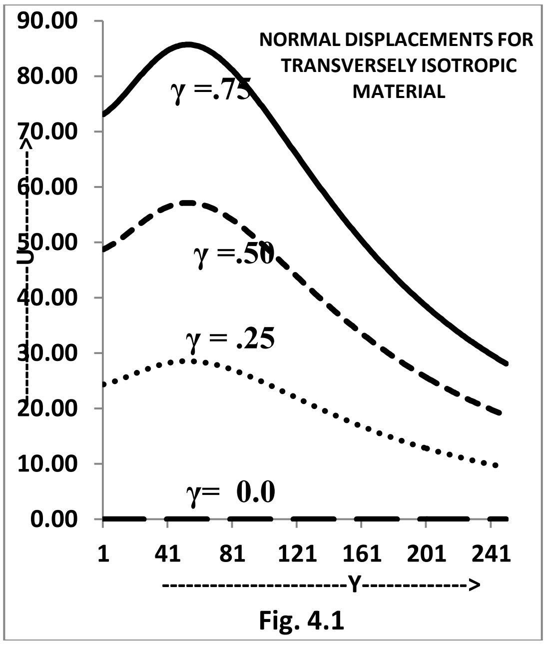

Fig. 4.1

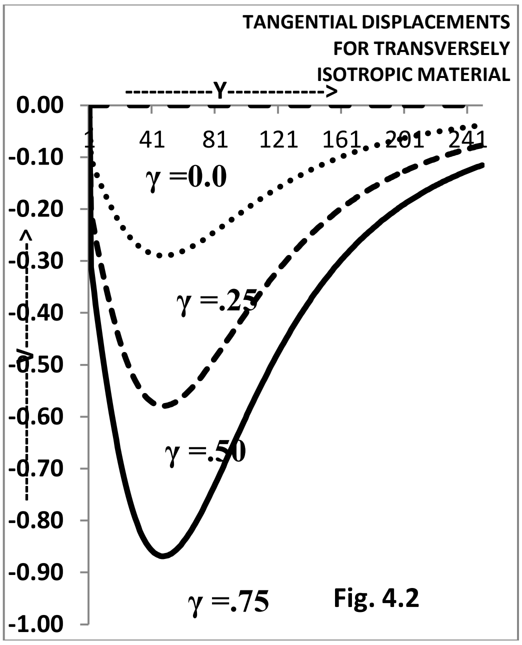

Fig. 4.2