The Wave Equation and Spherical Time

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Summary

Of all the fundamental assumptions made by humanity, perhaps the most fundamental concerns time. We perceive events sequentially. Therefore, time is assumed to be linear. This text will challenge that assumption.

The classical wave equation is modified by including time as though it were represented by spherical coordinates. The resulting wave equation is then solved using a spherical formulation for both space and time. Both a scalar and a vector solution are presented. These solutions have several noteworthy features. It is demonstrated that for large values of time, the spherical formulation of time simplifies to become the conventional linear formulation of time. It is also demonstrated that for zero values of time and radius, the wave equation solution reduces to a simple formula. This is interpreted as an explanation for wave-particle duality. It is further demonstrated that the inverse of space-time as denoted by 1/rt is easily converted into c/r^2. This is immediately seen to be similar to the forms of gravity and electrostatics. A solution is presented that satisfies both the Schrödinger Wave Equation and the classical wave equation. As a corollary, a method is presented that allows for the exact integration of sin(x)/x. Lastly, it is shown that the difference between the linear and spherical time formulations of the classical wave equation may be responsible for inertia. A pair of solutions is presented that appear to represent the photon and the electron.

Preface

The reader is encouraged to study the Appendices. The author is neither a mathematician nor a physicist. The mathematical tools used are quaternions, differential equations (specifically, separation of variables), and Euler's equation. These tools are perhaps intimidating to the reader. Hopefully the author has presented them in a manner that is readily understandable.

Discussion

The classical wave equation can be written as follows[1,2]:

Equation 1

$$\nabla^2\psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

The left side of equation 1 is the Laplacian. When equation 1 is written using spherical coordinates[3], it becomes equation 2.

Equation 2

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

The author[4] has previously shown that a vector solution is possible if equation 2 is revised as shown in equation 3.

Equation 3

$$\frac{1}{\beta^2}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}$$

In equation 1 above, space can have several representations. Yet the time portion is always considered to be linear. While attempting to apply the Schrödinger Equation to Ohm's Law, the author wondered if there might be a way to have a 1/t term as part of the solution. The solutions for the r portion of equations 2 and 3 contain a 1/r term. Therefore, it is possible to have a 1/t term. It requires that time be represented spherically.

The author conducted an internet search using the key words "spherical time". There were more than 24 million hits. None of the first ~100 hits were meaningful with respect to the above. There was a technical paper regarding signal processing, a technical paper regarding sonar, a person that uses Spherical Time as a username, and page after page of New Age religion.

Putting aside the obvious doubts associated with such a representation, spherical time can be included in the wave equation as follows:

Equation 4

$$\frac{1}{\beta^2}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{c^2}\frac{1}{t^2}\frac{\partial}{\partial t}\left(t^2\frac{\partial\psi}{\partial t}\right)$$

Expanding the differentials associated with time in the right hand side of equation 4 gives the following:

Equation 5

$$\frac{1}{c^2}\frac{1}{t^2}\frac{\partial}{\partial t}\left(t^2\frac{\partial\psi}{\partial t}\right) = \frac{1}{c^2}\frac{1}{t^2}\left(2t\frac{\partial\psi}{\partial t} + t^2\frac{\partial^2\psi}{\partial t^2}\right) = \frac{1}{c^2}\left(\frac{2}{t}\frac{\partial\psi}{\partial t} + \frac{\partial^2\psi}{\partial t^2}\right)$$

For large values of t, the far right hand side of equation 5 reduces to the standard linear form since the 2/t term tends towards zero. For small values of t, the 1'st derivative is dominant. For large values of t, the 2'nd derivative is dominant.

Wolff[5] presents a solution to the spacial portion of this problem. Appendix A presents a scalar solution to equation 4 using this prior solution.

The scalar solution to equation 4 is as follows:

Equation 6

$$\psi(r,t) = -\frac{4AD}{rt}\sin(\alpha\beta r)\sin(\alpha ct)$$

The symbols A and D are constants whose values have yet to be determined. The negative sign is present as a result of an i^2 term. The limit of equation 6 as both r and t approach zero is:

Equation 7.1

$$\psi(0,0) = -4AD\alpha^2\beta c$$

Taking r and t individually to zero produces equations 7.2 and 7.3.

Equation 7.2

$$\psi(0,t) = -\frac{4AD\alpha\beta}{t}\sin(\alpha ct)$$

Equation 7.3

$$\psi(r,0) = -\frac{4AD\alpha c}{r}\sin(\alpha\beta r)$$

Equations 7.2 and 7.3 will be discussed later.

Since equation 6 has finite values for r = 0 and t = 0, a formulation based upon equation 6 avoids the cosmology problems associated with black-holes and the initial time of the Big Bang. This is also a very simple explanation for wave-particle duality. It is not necessary for the wave function to collapse due to measurement or observation as is so often stated. The formulation of equation 6 might also alter the interpretation of the uncertainty principal. Einstein's concept of space-time is given formal meaning as the term rt, and since t is constantly increasing – at least according to our human perception - it follows that space-time is expanding and that it is curved. Lastly, the relation r = ct can be used to convert 1/rt into c/r^2 as shown in equations 8.1 and 8.2. This hints at a basis for both gravity and electrostatics since they are both known to vary with the inverse of r^2. Essentially, gravity and the electrostatic force are the result of multiplying two waveforms ψ_1 and ψ_2 with one being evaluated at $\psi_1(0, 0)$ and the other at $\psi_2(r, t)$ as one possibility. There are several ways to produce the required $1/r^2$.

Equation 8.1

$$\psi(r,t) = -\frac{4ADc}{r^2}\sin(\alpha\beta r)\sin(\alpha ct)$$

Equation 8.2

$$F = \psi_1(0,0)\psi_2(r,t) = +\frac{16A^2D^2\alpha_1^2\beta_1c^2}{r^2}\sin(\alpha_2\beta_2 r)\sin(\alpha_2 ct)$$

Equation 6 is a scalar solution to equation 4. A vector solution to equation 4 is also possible and is presented in Appendix B. The vector solution is as follows:

Equation 9

$$\psi(r,t) = R(r) \times T(t)$$

Where R(r) and T(t) are as follows:

Equation 9.1

$$R(r) = \pm\frac{2A}{r}\begin{bmatrix}+\sin(\alpha\beta r) & +\sin(\alpha\beta r)\\ +\sin(\alpha\beta r) & -\sin(\alpha\beta r)\end{bmatrix}\begin{bmatrix}\mathbf{j}\\ \mathbf{k}\end{bmatrix}$$

Equation 9.2

$$T(t) = \pm\frac{2D}{t}\begin{bmatrix}+\sin(\alpha ct) & +\sin(\alpha ct)\\ +\sin(\alpha ct) & -\sin(\alpha ct)\end{bmatrix}\begin{bmatrix}\mathbf{j}\\ \mathbf{k}\end{bmatrix}$$

The multiplication associated with equation 9 is set of vector cross products. Performing the various vector cross product multiplications produces the following:

Equation 9.3

$$\psi(r,t) = \pm\frac{8AD\sin(\alpha\beta r)\sin(\alpha ct)}{rt}\begin{bmatrix}-1 & -1\\ -1 & +1\end{bmatrix}\begin{bmatrix}1\\ \mathbf{i}\end{bmatrix}$$

This is presented in Appendix C. Figure 1 at the end of the text presents $\psi(r, t)$ for the solution with both the scalar and the vector values at +1. As with equation 6, r = ct can be substituted into equation 9.3. This produces equation 9.4.

Equation 9.4

$$\psi(r,t) = \pm\frac{8ADc\sin(\alpha\beta r)\sin(\alpha ct)}{r^2}\begin{bmatrix}-1 & -1\\ -1 & +1\end{bmatrix}\begin{bmatrix}1\\ \mathbf{i}\end{bmatrix}$$

Equations 9.3 and 9.4 produce a rotating corkscrew that could easily account for action at a distance. They also describe polarized light.

Next, consider the Schrödinger Equation. It can be written as follows[6]:

Equation 10

$$ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2m}\nabla^2\psi$$

In equation 10 and in this paper in general, h is used to represent ħ due to font limitations. If both equation 10 and equation 4 are solved for the Laplacian (i.e., for the spacial derivative) and then the Laplacians are set equal to each other, the following equation can be obtained:

Equation 11

$$\frac{\partial^2\psi}{\partial t^2} + \left(i\frac{2m}{h}\frac{c^2}{\beta^2} + \frac{2}{t}\right)\frac{\partial\psi}{\partial t} = 0$$

The derivation of this equation and its solution are presented in Appendix D. It is shown that there are two requirements for the wave function to satisfy both the classical wave equation and Schrödinger. These are presented in equations 11.1 and 11.2

Equation 11.1

$$\alpha\beta^2 = \frac{2mc}{h}\ ;\ \alpha c = \frac{2mc^2}{h\beta^2}$$

Equation 11.2

$$\alpha c e^{+i\alpha ct} = \frac{\sin(\alpha ct)}{t}$$

Equation 11.1 was determined previously by the author[4]. Equation 11.2 is something of a puzzle. It is true at t = 0 because both sides are equal to αc. The author believes that there is an additional requirement that αct = πn. This forces the isin(x) portion of the complex exponential to be zero. This also effectively quantizes time since t = πn/αc. This also causes the complex exponential of equation 11.2 to be a series of concentric circles – or spheres - since αc is the radius of the circle associated with complex exponential. Interestingly, the circles could get larger or smaller as time progresses since αc = πn/t. If n and t change at the same rate then the radius will remain constant. If n changes faster than t then the radius will increase. If t changes faster than n then the radius will decrease.

Appendix D also provides for an expression for mass. This is as follows:

Equation 11.3

$$m = \frac{h}{2}\frac{\beta^2}{c^2}\left[e^{-i\alpha ct}\right]\left[\frac{\sin(\alpha ct)}{t}\right]$$

This expression is deceptive. It is not explicit because the αc term in the complex exponential and in the sine is proportional to mass.

Minor revisions to equation 11.2 (expand the complex exponential, multiply by i, and divide by t) and application of L'Hôpital's rule (group the icos(x)/x together after L'Hôpital) produces the following:

Equation 11.4

$$\lim_{t\to 0}\left[i\frac{\cos(\alpha ct)}{t}\right] = 2\alpha c$$

If equation 11.2 is correct, then a corollary to it is a method to determine the anti-derivative of sin(x)/x. Expanding equation 11.2, rearranging, and integrating gives the following:

Equation 12

$$Si(x) = \int \frac{\sin(\alpha ct)}{t} dt = \alpha c \int [\cos(\alpha ct) + i\sin(\alpha ct)]\, dt = \sin(\alpha ct) - i\cos(\alpha ct) + Constant$$

The author conducted several internet searches regarding the above integral. These searches produced little of interest. What can be reported is that sin(x)/x cannot be integrated by any of the conventional methods. It is presented in the literature as a numerical solution and it can be determined from infinite series. A comparison of equation 12 with these methods should provide further insight. This should confirm or reject equation 12 and hence either support or reject the concept of spherical time.

There is also an integral form for equation 11.4.

Now consider two versions of the classical wave equation. The purpose of this exercise is to provide insight regarding the origin of inertia. The author is intentionally choosing to place this material in the body of the text rather than an Appendix. Let space be spherical for both equations. Let time be linear in the 1'st and let time be spherical in the 2'nd. The objective is to determine the difference between the solutions to these equations. If the equations are configured to be solved by the separation of variables method, they are written as follows:

Equation 13.1 (linear time)

$$\psi_1(r,t) = R(r)T_1(t)$$

Equation 13.1.1 (linear time)

$$\frac{1}{\beta^2}\frac{1}{r^2}\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = \frac{1}{c^2}\frac{1}{T_1}\frac{d^2T_1}{dt^2} = -\alpha_1^2$$

Equation 13.2 (spherical time)

$$\psi_2(r,t) = R(r)T_2(t)$$

Equation 13.2.1 (spherical time)

$$\frac{1}{\beta^2}\frac{1}{r^2}\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = \frac{1}{c^2}\frac{1}{t^2}\frac{1}{T_2}\frac{d}{dt}\left(t^2\frac{dT_2}{dt}\right) = -\alpha_2^2$$

Since the left most part of equation 13.1.1 equals the left most part of equation 13.2.1, it follows that the separation of variables constants are equal and can be designated simply as $-\alpha^2$. Setting the time portions of equations 13.1.1 and 13.2.1 equal to each other produces the following (αc is substituted from equation 11.1 above):

Equation 13.3

$$\frac{1}{T_1}\frac{d^2T_1}{dt^2} = \frac{1}{T_2}\left(\frac{2}{t}\frac{dT_2}{dt} + \frac{d^2T_2}{dt^2}\right) = -\alpha^2c^2 = -\left[\frac{4m^2}{h^2}\frac{c^4}{\beta^4}\right]$$

The solutions for T_1 and T_2 are found using complex exponentials. The main difference between the two is that T_2 must be limited to sine values to force the solution to be finite at t = 0. Equation 13.3 is satisfied by having $T_1 = tT_2$. Therefore, $T_1 - T_2 = T_2(t - 1)$ and $\Delta\psi = RT_2(t - 1)$.

Before proceeding with the solution, consider the equation $\cos^2(x) + \sin^2(x) = 1$. This can be factored as follows: [+cos(x) + isin(x)][+cos(x) – isin(x)]. These are the two common forms of Euler's Equation and are the basis for the solution of many differential equations. These two forms of Euler's Equation are also conjugates. A wave and its conjugate can be designated as ψ and ψ^*. It is equally valid to factor the identity as follows: [+icos(x) + sin(x)][-icos(x) + sin(x)]. These two forms can be represented as i multiplied by a complex exponential. All of these forms will be used here.

At this point, some new notation is needed. Please note the following definitions:

$$\psi = e^{+i\alpha ct} = \cos(\alpha ct) + i\sin(\alpha ct)\ ;\ \psi^* = e^{-i\alpha ct} = \cos(\alpha ct) - i\sin(\alpha ct)$$

$$\psi_1 = +\psi + \psi^* = 2\cos(\alpha ct) = \begin{bmatrix}+\psi\\+\psi^*\end{bmatrix}\ ;\ \psi_1^* = +\psi^* + \psi = 2\cos(\alpha ct) = \begin{bmatrix}+\psi^*\\+\psi\end{bmatrix}$$

$$\psi_2 = +\psi - \psi^* = 2i\sin(\alpha ct) = \begin{bmatrix}+\psi\\-\psi^*\end{bmatrix}\ ;\ \psi_2^* = -\psi^* + \psi = 2i\sin(\alpha ct) = \begin{bmatrix}-\psi^*\\+\psi\end{bmatrix}$$

$$\mathbf{\Psi}_1 = \begin{bmatrix} +\psi_1 & -\psi_1^* \\ +\psi_1^* & -\psi_1 \end{bmatrix} ; \ \mathbf{\Psi}_2 = \begin{bmatrix} +\psi_2 & -\psi_2^* \\ +\psi_2^* & -\psi_2 \end{bmatrix}$$

The purpose of the notation above is to more easily write all of the possible combinations of Euler's Equation and thereby simplify writing the solutions to equation 13.3. Each set of solutions can be multiplied by ±1 or by ±i as needed.

Now the solutions are presented. For the simplest case, a scalar solution will be determined. This requires that both T_1 and T_2 must be complex so that when they are multiplied by R (which is complex) a real solution will be produced. In general, any sin(x) or cos(x) type function will satisfy the 2'nd derivative terms in equation 13.3. Therefore, the valid complex solutions for T_1 are $\pm i\mathbf{\Psi}_1$ and $\pm\mathbf{\Psi}_2$. The solutions for T_2 must be limited to sin(x) to force the solution to be finite at t = 0. Therefore, the solutions for T_2 are $\pm\mathbf{\Psi}_2$. The solutions used by T_1 but not used by T_2 are $\pm i\mathbf{\Psi}_1$. They satisfy Schrödinger.

Equation 13.4

$$\Delta\psi = \pm Ri\left[\frac{\mathbf{\Psi}_1}{t}\right](t-1) = \pm R(i\mathbf{\Psi}_1 - 4\alpha c) = \pm R\left(i\mathbf{\Psi}_1 - \frac{8mc^2}{h\beta^2}\right)$$

Equation 11.4 was used to produce the αc term in equation 13.4 and equation 11.1 was used to produce the far right hand side of equation 13.4. Since mass appears in equation 13.4, it is entirely possible that what we perceive as inertia is the result of the difference between linear and spherical time! This theme is developed further in a separate text by the author[8].

Appendix E presents a geometric based derivation of a form of the wave equation. The assumptions that are necessary are as follows:

1. Space is represented by a hyper-sphere.
2. Time is included as a physical dimension as ct.
3. A dimension is included for ψ to have amplitude.
4. The radius of the hyper-sphere is expanding at c.

The equation for the hyper-sphere is as follows:

Equation 14.1

$$c^2t^2 + r^2 + \psi^2 = a^2 \, ; \, r^2\left(1 + \frac{\alpha ct}{r^2}i\right)^2 = a^2\left(1 + \frac{ct}{a}i\right)^2 \, ; \, \psi = +\frac{\alpha ct}{r}i$$

These assumptions produce the following equation:

Equation 14.2

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} - \frac{\partial^2\psi}{\partial r^2} = \frac{1}{\psi}\frac{v^2}{c^2}$$

Equation 14.2 is very similar to the classical wave equation and it includes a term that resembles Special Relativity.

Lastly, let us consider equations 7.2 and 7.3. These are restated below.

Equation 7.2

$$\psi(0,t) = -\frac{4AD\alpha\beta}{t}\sin(\alpha ct)$$

Equation 7.3

$$\psi(r,0) = -\frac{4AD\alpha c}{r}\sin(\alpha\beta r)$$

The author believes that these equations represent some form of the energy and mass respectively. Equation 7.3 is every-where at one time. Equation 7.2 is every-when at one place. The strangeness of this latter thought might be enough to discredit the idea of spherical time. But consider for a moment how it might be possible for it to be true. We know that light moves with respect to us at the measured constant velocity c. If light is actually at one place (ie, stationary) then we must be moving at c rather than the light. If the wave media produces our observable universe and it is moving at c and light is actually stationary, it follows that any experiment such as the Michelson-Morley experiment will yield a null result since the wave media is carrying us with it and our relative velocity with respect to the wave media is zero.

Conclusions

It is shown that the use of spherical time and space in the classical wave equation produces a solution that is finite at r = 0 and t = 0 and that the form of the solution is easily made to conform to the $1/r^2$ form of gravity and electrostatics. It is also shown that an expanding hyper-sphere produces a wave equation similar to that commonly used and that it includes Special Relativity. It is speculated that light is stationary and that an expanding wave media carries us with it at a velocity c and hence there is no relative velocity between us and the wave media. The author will conclude by stating his belief that the biggest error in physics today is the belief that there is no aether.

Acknowledgements

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Appendix A

Begin with the wave equation written in spherical form for both r and t.

Equation A.1

$$\frac{1}{\beta^2}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{c^2}\frac{1}{t^2}\frac{\partial}{\partial t}\left(t^2\frac{\partial\psi}{\partial t}\right)$$

Assume that ψ(r, t) has the following form typically used by the separation of variables method:

Equation A.2

$$\psi(r,t) = R(r)T(t)$$

Also assume the following forms for R and T:

Equation A.3

$$R(r) = \frac{1}{r}\left(Ae^{+i\alpha\beta r} + Be^{-i\alpha\beta r}\right) = \frac{1}{r}[(A+B)\cos(\alpha\beta r) + (A-B)i\sin(\alpha\beta r)]$$

Equation A.4

$$T(t) = \frac{1}{t}\left(De^{+i\alpha ct} + Ee^{-i\alpha ct}\right) = \frac{1}{t}[(D+E)\cos(\alpha ct) + (D-E)i\sin(\alpha ct)]$$

The right hand sides of equations A.3 and A.4 are based on Euler's Equation. Both R and T must be finite at r = 0 and t = 0 respectively. This is achieved by selecting A, B, D, and E such that the cosine terms cancel each other and only the sine terms remain. This makes use of the fact that (1/x)sin(x) has a finite limit at x = 0. For R this requires that B = -A. For T this requires that E = -D. Substituting these values into equations A.3 and A.4 produces the following:

Equation A.3.1

$$R(r) = \frac{A}{r}\left(e^{+i\alpha\beta r} - e^{-i\alpha\beta r}\right) = \frac{2Ai}{r}\sin(\alpha\beta r)$$

Equation A.4.1

$$T(t) = \frac{D}{t}\left(e^{+i\alpha ct} - e^{-i\alpha ct}\right) = \frac{2Di}{t}\sin(\alpha ct)$$

Substitution of equations A.3.1 and A.4.1 into A.2 gives the solution.

Equation A.5

$$\psi(r,t) = \frac{-4AD}{rt}\sin(\alpha\beta r)\sin(\alpha ct)$$

The final task for this exercise is to take the various derivatives of A.5 and substitute them into equation A.1 to confirm that ψ(r, t) as presented in equation A.5 is a solution.

Equation A.6.1

$$\frac{\partial\psi}{\partial r} = -\frac{4AD\alpha\beta}{rt}\cos(\alpha\beta r)\sin(\alpha ct) + \frac{4AD}{r^2 t}\sin(\alpha\beta r)\sin(\alpha ct) = -\frac{4AD\alpha\beta}{rt}\cos(\alpha\beta r)\sin(\alpha ct) - \frac{1}{r}\psi$$

Equation A.6.2

$$\frac{\partial^2\psi}{\partial r^2} = +\frac{4AD\alpha^2\beta^2}{rt}\sin(\alpha\beta r)\sin(\alpha ct) + \frac{4AD\alpha\beta}{r^2 t}\cos(\alpha\beta r)\sin(\alpha ct) - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\psi$$

$$\frac{\partial^2\psi}{\partial r^2} = -\alpha^2\beta^2 F - \frac{1}{r}\left[-\frac{4AD\alpha\beta}{rt}\cos(\alpha\beta r)\sin(\alpha ct) - \frac{1}{r}\psi\right] - \frac{1}{r}\frac{\partial\psi}{\partial r}$$

$$\frac{\partial^2\psi}{\partial r^2} = -\alpha^2\beta^2\psi - \frac{2}{r}\frac{\partial\psi}{\partial r}$$

Equation A.7.1

$$\frac{\partial\psi}{\partial t} = -\frac{4AD\alpha c}{rt}\sin(\alpha\beta r)\cos(\alpha ct) + \frac{4AD}{rt^2}\sin(\alpha\beta r)\sin(\alpha ct) = -\frac{4AD\alpha\beta}{rt}\sin(\alpha\beta r)\cos(\alpha ct) - \frac{1}{t}\psi$$

Equation A.7.2

$$\frac{\partial^2\psi}{\partial t^2} = +\frac{4AD\alpha^2 c^2}{rt}\sin(\alpha\beta r)\sin(\alpha ct) + \frac{4AD\alpha c}{rt^2}\sin(\alpha ct)\cos(\alpha ct) - \frac{1}{t}\frac{\partial\psi}{\partial t} + \frac{1}{t^2}\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = -\alpha^2 c^2\psi - \frac{1}{t}\left[-\frac{4AD\alpha c}{rt}\sin(\alpha\beta r)\cos(\mathrm{\alpha ct}) - \frac{1}{t}\psi\right] - \frac{1}{t}\frac{\partial\psi}{\partial t}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\alpha^2 c^2\psi - \frac{2}{t}\frac{\partial\psi}{\partial t}$$

Now re-write equation A.1 with the differentials on both sides in expanded form and the inverse terms multiplied through the equation.

Equation A.8

$$\frac{1}{\beta^2}\left[\frac{2}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial r^2}\right] = \frac{1}{c^2}\left[\frac{2}{t}\frac{\partial\psi}{\partial t} + \frac{\partial^2\psi}{\partial t^2}\right]$$

Substitute the simplest forms of the 2'nd derivatives from equations A.6.2 and A.7.2 into A.8.

$$\frac{1}{\beta^2}\left[\frac{2}{r}\frac{\partial\psi}{\partial r} - \alpha^2\beta^2\psi - \frac{2}{r}\frac{\partial\psi}{\partial r}\right] = \frac{1}{c^2}\left[\frac{2}{t}\frac{\partial\psi}{\partial t} - \alpha^2c^2\psi - \frac{2}{t}\frac{\partial\psi}{\partial t}\right]$$

$$-\alpha^2\psi = -\alpha^2\psi$$

This confirms that ψ(r, t) as presented in equation A.5 is a solution to equation A.1.

Appendix B

Begin with the wave equation with both spherical time and space.

Equation B.1

$$\frac{1}{\beta^2}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = \frac{1}{c^2}\frac{1}{t^2}\frac{\partial}{\partial t}\left(t^2\frac{\partial\psi}{\partial t}\right)$$

Assume that the solution is of the following form typical of separation of variables:

Equation B.2

$$\psi(r,t) = R(r)T(t)$$

The author[4] has previously presented a vector solution for the spherical R(r) portion of ψ(r, t). That solution was as follows:

Equation B.3

$$R(r) = \pm\frac{2A}{r}\begin{bmatrix} +\sin(\alpha\beta r) & +\sin(\alpha\beta r) \\ +\sin(\alpha\beta r) & -\sin(\alpha\beta r)\end{bmatrix}\begin{bmatrix}\mathbf{j} \\ \mathbf{k}\end{bmatrix}$$

That same work also presented the following four rotations about the **i** axis:

Equation B.4.1 (counter-clockwise)

$$F_{j,i} = \mathbf{j}e^{-\mathbf{i}\theta_i} = +\mathbf{j}\cos(\theta_i) + \mathbf{k}\sin(\theta_i)$$

Equation B.4.2 (clock-wise)

$$F_{j,i}^{*} = \mathbf{j}e^{+\mathbf{i}\theta_i} = +\mathbf{j}\cos(\theta_i) - \mathbf{k}\sin(\theta_i)$$

Equation B.4.3 (clock-wise)

$$F_{k,i} = \mathbf{k}e^{+\mathbf{i}\theta_i} = +\mathbf{k}\cos(\theta_i) + \mathbf{j}\sin(\theta_i) = \mathbf{j}\sin(\theta_i) + \mathbf{k}\cos(\theta_i)$$

Equation B.4.4 (counter-clockwise)

$$F_{k,i}^{*} = \mathbf{k}e^{-\mathbf{i}\theta_i} = +\mathbf{k}\cos(\theta_i) - \mathbf{j}\sin(\theta_i) = -\mathbf{j}\sin(\theta_i) + \mathbf{k}\cos(\theta_i)$$

The time portion of the solution will be represented as follows:

Equation B.5

$$T(t) = \frac{1}{t}\left(DF_{j,i} - DF_{j,i}^{*} + EF_{k,i} - EF_{k,i}^{*}\right)$$

This formulation was selected to ensure that the cosine terms cancel and only the sine terms remain. As before, this will force the function to have a finite limit at t = 0.

It is likely that symmetry will cause D and E to have equal values although one could be positive and the other could be negative. For D = +1 and E = +1, T(t) = +2**j**sin(θ_i) + 2**k**sin(θ_i). For D = +1 and E = -1, T(t) = -2**j**sin(θ_i) + 2**k**sin(θ_i). For D = -1 and E = +1, T(t) = +2**j**sin(θ_i) – 2**k**sin(θ_i). Lastly, for both D = -1 and E = -1, T(t) = -2**j**sin(θ_i) – 2**k**sin(θ_i).

Next, the rotation angle θ_i is set equal to αct. This can be summarized in the following matrix form:

Equation B.6

$$T(t) = \pm\frac{2D}{t}\begin{bmatrix} +\sin(\alpha ct) & +\sin(\alpha ct) \\ +\sin(\alpha ct) & -\sin(\alpha ct) \end{bmatrix}\begin{bmatrix} \mathbf{j} \\ \mathbf{k} \end{bmatrix}$$

Appendix C

Begin by stating the equation ψ(r, t) in matrix form.

Equation C.1

$$\psi(r,t) = \pm\frac{4AD}{rt}\begin{bmatrix}+\sin(\alpha\beta r) & +\sin(\alpha\beta r)\\ +\sin(\alpha\beta r) & -\sin(\alpha\beta r)\end{bmatrix}\begin{bmatrix}\mathbf{j}\\ \mathbf{k}\end{bmatrix} \times \begin{bmatrix}+\sin(\alpha ct) & +\sin(\alpha ct)\\ +\sin(\alpha ct) & -\sin(\alpha ct)\end{bmatrix}\begin{bmatrix}\mathbf{j}\\ \mathbf{k}\end{bmatrix}$$

Factor out the sin(αβr) and sin(αct).

Equation C.2

$$\psi(r,t) = \pm\frac{4AD\sin(\alpha\beta r)\sin(\alpha ct)}{rt}\begin{bmatrix}+1 & +1\\ +1 & -1\end{bmatrix}\begin{bmatrix}\mathbf{j}\\ \mathbf{k}\end{bmatrix} \times \begin{bmatrix}+1 & +1\\ +1 & -1\end{bmatrix}\begin{bmatrix}\mathbf{j}\\ \mathbf{k}\end{bmatrix}$$

Now there are simply 4 cross product multiplications to perform.

$$(\mathbf{j}+\mathbf{k})(\mathbf{j}+\mathbf{k}) = \mathbf{j}^2 + \mathbf{kj} + \mathbf{jk} + \mathbf{k}^2 = -1 - \mathbf{i} + \mathbf{i} - 1 = -2$$

$$(\mathbf{j}+\mathbf{k})(\mathbf{j}-\mathbf{k}) = \mathbf{j}^2 + \mathbf{kj} - \mathbf{jk} - \mathbf{k}^2 = -1 - \mathbf{i} - \mathbf{i} + 1 = -2\mathbf{i}$$

$$(\mathbf{j}-\mathbf{k})(\mathbf{j}+\mathbf{k}) = \mathbf{j}^2 - \mathbf{kj} + \mathbf{jk} - \mathbf{k}^2 = -1 + \mathbf{i} + \mathbf{i} + 1 = +2\mathbf{i}$$

$$(\mathbf{j}-\mathbf{k})(\mathbf{j}-\mathbf{k}) = \mathbf{j}^2 - \mathbf{kj} - \mathbf{jk} + \mathbf{k}^2 = -1 + \mathbf{i} - \mathbf{i} - 1 = -2$$

Now the question is how to present these 4 or 8 solutions. The constant 2 can simply be lumped in with the pre-multiplication terms. From the matrix multiplication, the result should be a 1x2 matrix. The 1'st two solutions must be added together and the 2'nd two solutions must be added together. This gives (-2 -2i) and (-2 + 2i). This can be represented as follows:

Equation C.3

$$\psi(r,t) = \pm\frac{8AD\sin(\alpha\beta r)\sin(\alpha ct)}{rt}\begin{bmatrix}-1 & -1\\ -1 & +1\end{bmatrix}\begin{bmatrix}1\\ \mathbf{i}\end{bmatrix}$$

Appendix D

Begin by writing the two wave equations.

Equation D.1

$$ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2m}\nabla^2\psi$$

Equation D.2

$$\frac{1}{\beta^2}\nabla^2\psi = \frac{1}{c^2}\frac{1}{t^2}\frac{\partial}{\partial t}\left(t^2\frac{\partial\psi}{\partial t}\right)$$

Solve each equation for the Laplacian and set the Laplacians equal to each other.

Equation D.3

$$\frac{\beta^2}{c^2}\frac{1}{t^2}\frac{\partial}{\partial t}\left(t^2\frac{\partial\psi}{\partial t}\right) = -i\frac{2m}{h}\frac{\partial\psi}{\partial t}$$

$$\frac{\beta^2}{c^2}\left(\frac{2}{t}\frac{\partial\psi}{\partial t} + \frac{\partial^2\psi}{\partial t^2}\right) = -i\frac{2m}{h}\frac{\partial\psi}{\partial t}$$

$$\frac{\partial^2\psi}{\partial t^2} + \left(\frac{2}{t} + i\frac{2m}{h}\frac{c^2}{\beta^2}\right)\frac{\partial\psi}{\partial t} = 0$$

Assume that ψ is of the following form:

Equation D.4

$$\psi(t) = \frac{1}{t}\left[e^{+i\alpha ct} - e^{-i\alpha ct}\right] = i\frac{2}{t}\sin(\alpha ct)$$

This form was chosen to force the function to be finite at t = 0. Now determine the 1'st and 2'nd derivatives.

Equation D.4.1

$$\frac{\partial\psi}{\partial t} = i\frac{2\alpha c}{t}\cos(\alpha ct) - i\frac{2}{t^2}\sin(\alpha ct) = i\frac{2\alpha c}{t}\cos(\alpha ct) - \frac{1}{t}\psi$$

Equation D.4.2

$$\frac{\partial^2\psi}{\partial t^2} = -i\frac{2\alpha^2c^2}{t}\sin(\alpha ct) - i\frac{2\alpha c}{t^2}\cos(\alpha ct) - \frac{1}{t}\frac{\partial\psi}{\partial t} + \frac{1}{t^2}\psi$$

$$\frac{\partial^2\psi}{\partial t^2} = -\alpha^2c^2\psi - \left[\frac{1}{t}\left(\frac{\partial\psi}{\partial t} + \frac{1}{t}\psi\right)\right] - \frac{1}{t}\frac{\partial\psi}{\partial t} + \frac{1}{t^2}\psi = -\alpha^2c^2\psi - \frac{2}{t}\frac{\partial\psi}{\partial t}$$

Substitution of D.4.1 and D.4.2 into equation D.3 gives the following:

$$-\alpha^2c^2\psi - \frac{2}{t}\frac{\partial\psi}{\partial t} + \left(\frac{2}{t} + i\frac{2m}{h}\frac{c^2}{\beta^2}\right)\frac{\partial\psi}{\partial t} = 0$$

$$-\alpha^2c^2\psi + i\frac{2m}{h}\frac{c^2}{\beta^2}\frac{\partial\psi}{\partial t} = 0$$

$$-\alpha^2c^2\psi + i\frac{2m}{h}\frac{c^2}{\beta^2}\left[i\frac{2\alpha c}{t}\cos(\alpha ct) - \frac{1}{t}\psi\right] = 0$$

$$-i\alpha^2c^2\frac{2}{t}\sin(\alpha ct) + i\frac{2m}{h}\frac{c^2}{\beta^2}\left[i\frac{2\alpha c}{t}\cos(\alpha ct) - \frac{1}{t}\psi\right] = 0$$

$$-i\alpha^2c^2 2\sin(\alpha ct) + i\frac{2m}{h}\frac{c^2}{\beta^2}[i2\alpha c\cos(\alpha ct) - \psi] = 0$$

Equation D.5

$$-i\alpha^2c^2 2\sin(\alpha ct) - \frac{2m}{h}\frac{c^2}{\beta^2}2\alpha c\cos(\alpha ct) - i\frac{2m}{h}\frac{c^2}{\beta^2}\psi = 0$$

The next part is a little subtle. If the term $2\alpha^2c^2$ can be made equal to the group of terms that multiply the cosine term in D.5 above, then the isin(x) term can be grouped with the cos(x) term to form a complex exponential.

$$2\alpha^2c^2 = \frac{2m}{h}\frac{c^2}{\beta^2}2\alpha c$$

Equation D.6

$$\alpha c = \frac{2m}{h}\frac{c^2}{\beta^2}$$

Equation D.7

$$\alpha\beta^2 = \frac{2mc}{h}$$

Next, convert equation D.5 into a complex exponential.

$$-2\alpha^2c^2e^{+i\alpha ct} = i\frac{2m}{h}\frac{c^2}{\beta^2}\psi$$

$$2\alpha^2c^2e^{+i\alpha ct} = -i\alpha c\psi$$

$$2\alpha c e^{+i\alpha ct} = -i\psi$$

$$2\alpha c e^{+i\alpha ct} = -i^2 \frac{2}{t} \sin(\alpha ct)$$

Equation D.8

$$\alpha c e^{+i\alpha ct} = +\frac{1}{t} \sin(\alpha ct)$$

Use equation D.6 to replace the αc that multiplies the complex exponential.

$$\frac{2m}{h} \frac{c^2}{\beta^2} e^{+i\alpha ct} = +\frac{1}{t} \sin(\alpha ct)$$

Equation D.9

$$m = \frac{h}{2} \frac{\beta^2}{c^2} \left[e^{-i\alpha ct}\right] \left[\frac{\sin(\alpha ct)}{t}\right]$$

There is at least one possible error or weakness in the derivation above. The term cos(t)/t occurs briefly. This value is undefined at t = 0. The author eliminated the term by multiplying by t as soon as possible.

Appendix E

Begin by defining a hyper-sphere as follows:

Equation E.1

$$c^2t^2 + r^2 + \psi^2 = a^2$$

Rearrange E.1 slightly.

Equation E.2

$$c^2t^2 + r^2 = a^2 - \psi^2$$

The form of equation E.2 presumes that r and t are independent variables and that a and ψ are the dependent variables. Take the 1'st and 2'nd partial derivatives of E.2 with respect to r and t.

Equation E.3.1

$$\frac{\partial}{\partial r} = 2r = 2a\frac{\partial a}{\partial r} - 2\psi\frac{\partial \psi}{\partial r}$$

Equation E.3.2

$$\frac{\partial^2}{\partial r^2} = 2 = 2\left[a\frac{\partial^2 a}{\partial r^2} + \left(\frac{\partial a}{\partial r}\right)^2\right] - 2\left[\psi\frac{\partial^2 \psi}{\partial r^2} + \left(\frac{\partial \psi}{\partial r}\right)^2\right]$$

Equation E.4.1

$$\frac{\partial}{\partial t} = 2c^2t = 2a\frac{\partial a}{\partial t} - 2\psi\frac{\partial \psi}{\partial t}$$

Equation E.4.2

$$\frac{\partial^2}{\partial t^2} = 2c^2 = 2\left[a\frac{\partial^2 a}{\partial t^2} + \left(\frac{\partial a}{\partial t}\right)^2\right] - 2\left[\psi\frac{\partial^2 \psi}{\partial t^2} + \left(\frac{\partial \psi}{\partial t}\right)^2\right]$$

Divide equation E.3.2 by 2 and equation E.4.2 by $2c^2$ and set them equal to each other.

Equation E.5

$$\left[a\frac{\partial^2 a}{\partial r^2} + \left(\frac{\partial a}{\partial r}\right)^2\right] - \left[\psi\frac{\partial^2 \psi}{\partial r^2} + \left(\frac{\partial \psi}{\partial r}\right)^2\right] = 1 = \frac{1}{c^2}\left[a\frac{\partial^2 a}{\partial t^2} + \left(\frac{\partial a}{\partial t}\right)^2\right] - \frac{1}{c^2}\left[\psi\frac{\partial^2 \psi}{\partial t^2} + \left(\frac{\partial \psi}{\partial t}\right)^2\right]$$

The next step is an assumption that the wave media is expanding. Assume the following:

Equation E.6.1

$$\frac{\partial a}{\partial t} = c\,; \;\; \frac{\partial^2 a}{\partial t^2} = 0$$

Equation E.6.2

$$a = ct + \; a_0$$

Taking a_o as zero allows equation E.2 to be simplified to the following:

Equation E.7

$$r^2 = \; -\psi^2$$

Equation E.8

$$\psi = \; \pm ir$$

$$\frac{\psi}{r} = \; \pm i$$

Equation E.8.1

$$\frac{\partial \psi}{\partial r} = \; \pm i$$

At this point, the author may be about to make an error. There is some subtlety concerning partial derivatives and total derivatives. The author has been using partial derivatives thus far in this Appendix. It is necessary to have a time derivative of ψ. But time has been removed from equation E.7. Essentially, r is no longer an independent variable since equation E.8 directly relates r to ψ and ψ is a dependent variable. Therefore equation E.7 will be differentiated with respect to t using the total differential.

Equation E.9.1

$$2r\frac{dr}{dt} = \; -2\psi\frac{d\psi}{dt}$$

$$r\frac{dr}{dt} = \; -\psi\frac{d\psi}{dt}$$

Equation E.9.2

$$\left[r\frac{d^2r}{dt^2} + \left(\frac{dr}{dt}\right)^2\right] = \; -\left[\psi\frac{d^2\psi}{dt^2} + \left(\frac{d\psi}{dt}\right)^2\right]$$

Now it is time to return to equation E.5. The radius of the hyper-sphere is only dependent upon t. Therefore, the 1'st and 2'nd terms are zero. The radius of the hyper-sphere is assumed to be increasing at a constant rate c. Therefore, the 5'th term is zero and the 6'th term is c^2. Making these substitutions produces the following:

Equation E.10

$$-\left[\psi\frac{\partial^2\psi}{\partial r^2}+\left(\frac{\partial\psi}{\partial r}\right)^2\right]=1=\frac{1}{c^2}[\,(c)^2]-\frac{1}{c^2}\left[\psi\frac{\partial^2\psi}{\partial t^2}+\left(\frac{\partial\psi}{\partial t}\right)^2\right]$$

Rearranging gives the following:

$$\frac{1}{c^2}\psi\frac{\partial^2\psi}{\partial t^2}-\psi\frac{\partial^2\psi}{\partial r^2}=1=\ 1+\left(\frac{\partial\psi}{\partial r}\right)^2-\frac{1}{c^2}\left[+\left(\frac{\partial\psi}{\partial t}\right)^2\right]$$

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial r^2}=\frac{1}{\psi}=\frac{1}{\psi}+\frac{1}{\psi}\left\{\left(\frac{\partial\psi}{\partial r}\right)^2-\frac{1}{c^2}\left(\frac{\partial\psi}{\partial t}\right)^2\right\}$$

Lastly, applying equations E.8, E.8.1, and E.9.1 to the terms within the {} above gives the following:

Equation E.11

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial r^2}=\frac{1}{\psi}=\frac{1}{\psi}+\frac{1}{\psi}\left\{-1+\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right\}$$

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial r^2}=\frac{1}{\psi}=\frac{1}{\psi}+\frac{1}{\psi}\left(\frac{v^2}{c^2}-1\right)$$

Equation E.12

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}-\frac{\partial^2\psi}{\partial r^2}=\frac{1}{\psi}\,\frac{v^2}{c^2}$$

Equation E.12 was the primary objective of this Appendix. But there is something more to be gleaned here. Return to equation E.1.

Equation E.1

$$c^2t^2+r^2+\psi^2=a^2$$

Rearrange to become

$$r^2+\psi^2=a^2-c^2t^2$$

Factor out a square on both sides.

$$r^2\left(1+\frac{\psi^2}{r^2}\right)=a^2\left(1-\frac{c^2t^2}{a^2}\right)$$

Now complete the square on the right hand side.

$$r^2\left(1+\frac{\psi^2}{r^2}\right)=a^2\left(1-\frac{c^2t^2}{a^2}\right)+2acti-2acti$$

$$r^2\left(1+\frac{\psi^2}{r^2}\right)+2acti=a^2\left(1-\frac{c^2t^2}{a^2}+2\frac{ct}{a}i\right)$$

$$r^2\left(1+\frac{\psi^2}{r^2}\right)+2acti=a^2\left(1+\frac{ct}{a}i\right)^2$$

Equation E.13

$$r^2\left(1+\frac{\psi^2}{r^2}+2\frac{act}{r^2}i\right)=a^2\left(1+\frac{ct}{a}i\right)^2$$

Determine ψ such that the left hand side of E.13 is a perfect square. The final form must look like one of the following:

Equation E.14.1 (complex)

$$r^2\left(1+\frac{\psi}{r}\right)^2=a^2\left(1+\frac{ct}{a}i\right)^2;\ \frac{\psi}{r}=+\frac{\alpha ct}{r^2}i;\ \psi=+\frac{\alpha ct}{r}i;\ \left(\frac{\psi}{r}\right)^2=-\frac{\alpha^2c^2t^2}{r^4}$$

Equation E.14.2 (real)

$$r^2\left(1+\frac{\psi}{r}i\right)^2=a^2\left(1+\frac{ct}{a}i\right)^2;\ \frac{\psi}{r}=+\frac{\alpha ct}{r^2};\ \psi=+\frac{\alpha ct}{r};\ \left(\frac{\psi}{r}\right)^2=+\frac{\alpha^2c^2t^2}{r^4}$$

If equation E.14.1 is used, the (ψ^2/r^2) term is positive. This is in agreement with equation E.13. If equation E.14.2 is used then the (ψ^2/r^2) term would have to have a negative sign in equation E.13. Therefore, equation E.14.1 completes the square if ψ = +(αct/r)i.

Equation E.15

$$r^2\left(1+\frac{\alpha ct}{r^2}i\right)^2=a^2\left(1+\frac{ct}{a}i\right)^2;\ \psi=+\frac{\alpha ct}{r}i$$

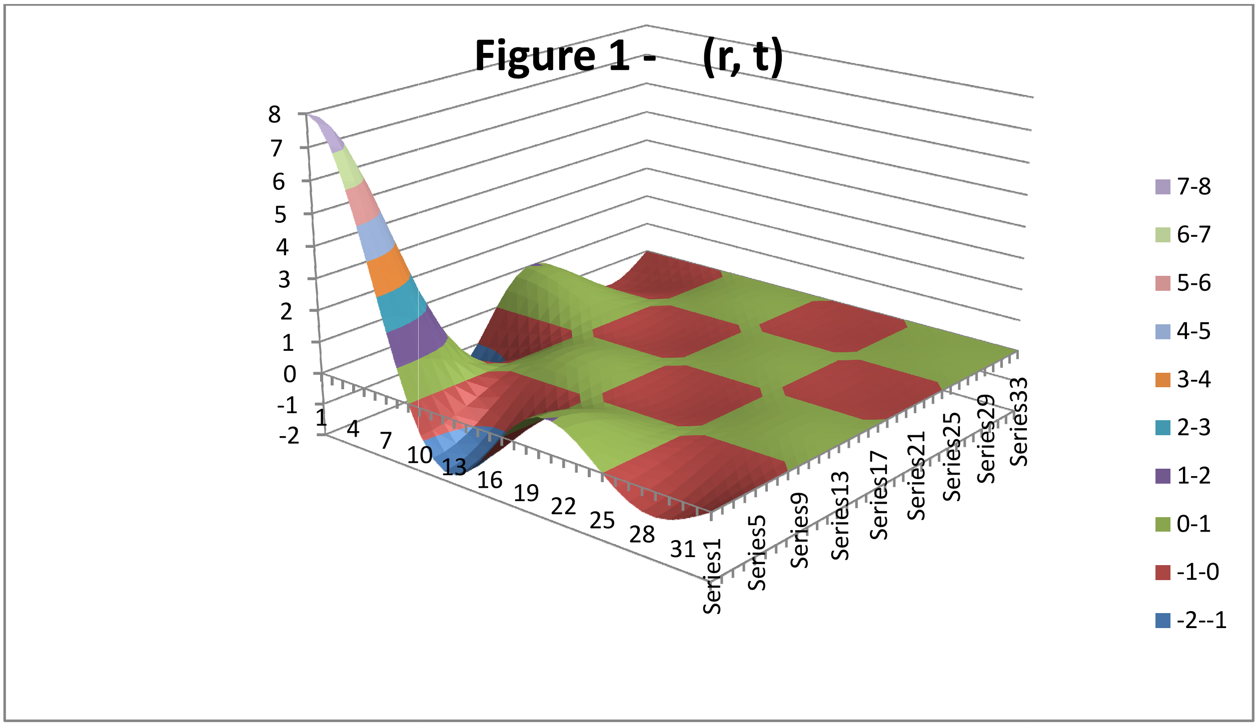

Figure 1 - (r, t)
8
7
6
5
4
3
2
1
0
-1
-2
1
4
7
10
13
16
19
22
25
28
31
Series1
Series5
Series9
Series13
Series17
Series21
Series25
Series29
Series33
7-8
6-7
5-6
4-5
3-4
2-3
1-2
0-1
-1-0
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