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Analysis of Piezoelectric Actuator for Vibration Control of Thin Cylindrical Shells

V. K. Srivastava

Department of Mechanical Engineering, Institute of Technology, Banaras Hindu University, Varanasi –221005, INDIA, vijayks210@gmail.com

Abstract. An analytical method is proposed for selecting the best suited actuator which can preferably be used in a variety of structural control applications. The selection is based on matching performance characteristics of the actuator, such as force and displacement, to the requirements of the given task. Relations between the mid surface strains and the strain induced in piezoelectric actuator due to application of electric field are derived to optimize the thickness of piezoelectric layer.

Keywords: Piezoelectric actuator, PZT, Vibration Suppression, Cylindrical Shells, Mid-surface strains, PVDF.

1 Introduction

Several authors [1-3] have been observed that the piezoelectric materials are useful for smart adaptive structures since more than a decade. This technology have been proposed and conceived experimentally for the engineering applications, such as for active vibration suppression. Adaptive structures using piezoelectric materials usually employ lead zirconium titanate (PZT) ceramic sensors and actuators to detect and mechanically deform a structure. Piezopolymer films (e.g. polyvinylidene fluoride (PVDF)) are not usually preferred because they lack the stiffness requirements to achieve high actuation authority. The addition of the PZT material allows the structure to sense and react to its environment. Crawley and Anderson [4] suggested that the conventional adaptive structures require a network of these actuators and sensors to be bonded to the surfaces or embedded within the structure.

But since the piezoelectric actuators show very low strain in elastic limit (0.1%) it is desirable to study the relationship between the strain produced due to unbalanced force and moments in cylindrical shell and that produced due to application of electrical voltage. Also recently Shen et al. [5] proposed RAINBOW actuators made of PZT capable of showing large deflections in elastic limit. The manufacturing of these actuators need very simple process. The aim here is to find out the relationship between the strain produced due to unbalanced forces and moments in cylindrical shell and that produced due to application of electrical voltage so that the voltage required for control of vibrations can be eliminated. The contributions of this work include the development of relations between the mid surface strains and the strain induced in piezoelectric actuator due to application of electric field. The stress strain relations in a shell structure had been studied for decades. The simplest membrane shell model was put forth by Love [6] in which the transverse forces, bending and twisting moments are negligible. Such model is suitable for thin shell structures in which only normal and shear forces acting in the mid-surface of the shell are considered. Although it is a low-order shell model, it is easy to present the essential features of the shell, and it provides basic model for higher-order shell model in which shear and twisting effects are considered. Mersky and Herrmann [7] included shear effects in both the axial and circumferential direction and rotary-inertia effects in the study of axially symmetric waves in a cylindrical shell. Naghdi and Cooper [8] presented a theory including shear effects and rotary-inertia for non-axially symmetric motion of shell structures. Several researchers including Timoshenko et al [9], Heyliger and Brooks [10], Heyliger and Saravancos [11] Chen and Shen [12] and Vel and Batra [13] have presented exact solutions for the static deformation, free and forced vibration of piezoelectric plates and shells.

But very limited literature is available about the physical limitation of piezoelectric materials as actuators in various settings. In this work, a relation is derived between the mid surface strains and the strain induced in piezoelectric actuator due to application of electric field, so that it is possible to optimize the thickness of piezoelectric layer as per the requirement. Here optimization of thickness of piezoelectric actuator layer is necessary because there is a limitation on strain a piezoelectric material can produce within elastic limit. The relations can also be used for calculation of voltage that should be applied for vibration control as well as for selection of suitable piezoelectric material and actuation scheme for particular application. For this, therefore, thin cylindrical shell theory has been utilized.

2 Problem Formulation

A long piezoelectric laminated cylindrical shell with two layers is shown in Fig. 1, where the inner layer is of host material and outer layer is that of the piezoelectric material. The curvilinear coordinate system (x, θ ,z) is selected, where x axis is length direction of piezoelectric laminated shells, the θ axis is the circumferential direction of piezoelectric shell, and the z axis is the radial direction of piezoelectric laminated shell. The surface defined by z=0 is set on the geometrical middle plane of piezoelectric laminated shell and R is the radius of the middle plane.

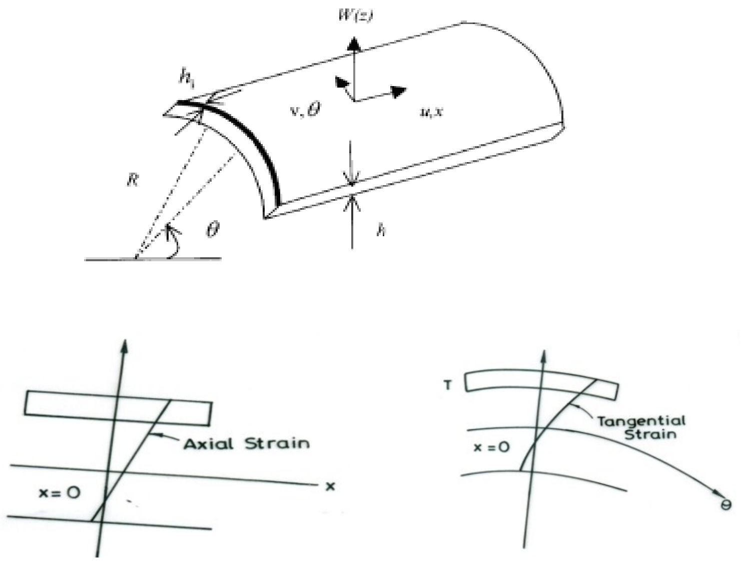

Figure1. Model of the shell with piezoelectric layer

2.1. Cylindrical Shell Equations

2.1.1. Stress strain Relations

The stress strain relations in more general form for elastic and homogeneous materials formulated by Vel and Bailargeon [14], that can be given as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{\theta z} \\ \sigma_{zx} \\ \sigma_{x\theta} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_z \\ \varepsilon_{\theta z} \\ \varepsilon_{zx} \\ \varepsilon_{x\theta} \end{Bmatrix} - \begin{bmatrix} e_{11} & 0 & 0 \\ e_{12} & 0 & 0 \\ e_{13} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & e_{35} \\ 0 & e_{26} & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_\theta \\ E_z \end{Bmatrix}$$

(1)

Where, σ_i, ε_i (i=x,θ,z,θz,zx,xθ) represent the stress and strain respectively; c_{ij}, e_{ij} (i,j= 1,2……………,6) denote the elastic constants and piezoelectric constants respectively. The above relation is general equation for stress and strain.

Based on plane stress problem of thin shell theory, $\sigma_z=0$, $\sigma_{zx}=0$, $\sigma_{z\theta}=0$ in equation-1. Also it is assumed for the reason of simplification that material is elastic and isotropic. It is also assumed that normal strain component $€_z$ is small in comparison with other strains derived by Love [6]. Then, Banks et al. [14] expressed stress-strain relations as given below,

$$\sigma_x = \frac{E}{1-\mu^2}\left(\varepsilon_x + \mu\varepsilon_\theta\right)$$

$$\sigma_\theta = \frac{E}{1-\mu^2}\left(\varepsilon_\theta + \mu\varepsilon_x\right)$$

(2)

$$\sigma_{x\theta} = \frac{E}{2(1+\mu)}\varepsilon_{x\theta}$$

Where the constants E and μ are young's modulus and Poisson ratio for the host material of the shell.

2.1.2. Strain-Displacement Relations

For thin shell, the displacements in plane are much smaller than transverse deflection. Thus, the effect of the in-plane displacements on nonlinear strains are usually neglected, only the nonlinear strains due to the transverse deflection 'w' are considered. Therefore, Banks et al. [15] expresses the nonlinear geometrical relation as given below,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \\ \varepsilon_{x\theta} \end{Bmatrix} = \begin{Bmatrix} e_x \\ e_\theta \\ e_{x\theta} \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_\theta \\ k_{x\theta} \end{Bmatrix} \tag{3}$$

Where k_x, $k_\theta, and.k_{x\theta}$ are the change in curvature of middle surface in x, θ and $x\theta$ directions respectively. In terms of axial, tangential and radial displacements u,v,w, respectively the Donnell-Mushtari expressions for the mid surface strains and changes in curvature for the cylindrical shell are

$$e_x = \frac{\partial u}{\partial x}, \; e_\theta = \frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{w}{R}, \; e_\theta = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}$$

$$k_x = -\frac{\partial^2 w}{\partial x^2}, \; k_\theta = -\frac{1}{R}\frac{\partial^2 w}{\partial \theta^2}, k_{x\theta} = -\frac{2}{R}\frac{\partial^2 w}{\partial x \partial \theta} \tag{4}$$

2.1.3. Force and Moment Resultants

By integrating the stresses over the face of a fundamental element, Banks et al [15] derived the force resultants can be expressed as,

$$(N_x) = \int_{-h/2}^{h/2} (\sigma_x)\left(1 + \frac{z}{R}\right) dz$$

$$\begin{Bmatrix} N_\theta \\ N_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix} dz \tag{5}$$

Similarly the moment resultants are

$$(M_x) = \int_{-h/2}^{h/2_1} (\sigma_x)\left(1 + \frac{z}{R}\right) z dz$$

$$\begin{Bmatrix} M_\theta \\ M_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} z. \begin{Bmatrix} \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix} dz \tag{6}$$

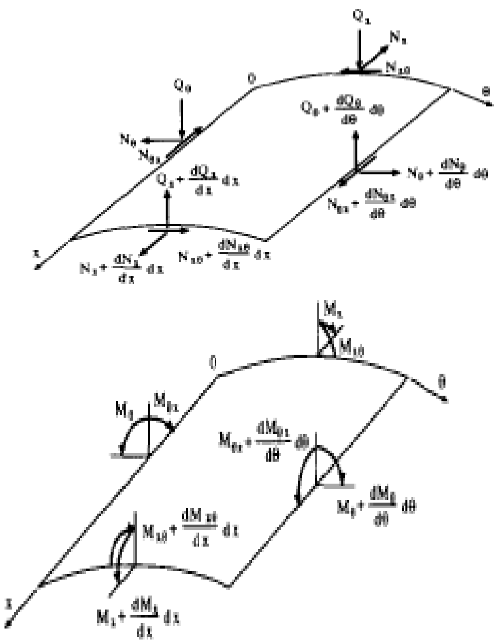

Figure 2. Force and moment resultants for the shell.

The equations of dynamic equilibrium of the element are obtained by balancing the internal forces and moment resultants as shown in Fig. 2 with any externally applied forces and moments. A thin shell surface bonded by a piezoelectric layer is shown in Fig. 1. This shell with bending resistance is under Love's shear-rigidity assumption. This assumption indicates that a certain plane perpendicular to the mid-plane will still remain perpendicular to the mid-plane after deformation. The coordinate is set to indicate the coordinates x for the direction along the shell, θ for the direction of polar angel, and r for the radial direction. The stress analysis on an infinitesimal element of the shell structure is shown in Fig. 2. The governing equations of motion in the longitudinal, tangential, and radial directions are respectively [9]

$$R\frac{\partial N_x}{\partial x}+\frac{\partial N_{\theta x}}{\partial \theta}+R\hat{q}_x=0$$

$$\frac{\partial N_\theta}{\partial \theta}+R\frac{\partial N_{x\theta}}{\partial x}+Q_\theta+R\hat{q}_\theta=0$$

$$R\frac{\partial Qx}{\partial x}+\frac{\partial Q_\theta}{\partial \theta}-N_\theta+R\hat{q}_n=0$$

$$R\frac{\partial Mx}{\partial x}+\frac{\partial M_{\theta x}}{\partial \theta}-RQ_x+R\hat{m}_\theta=0$$

$$\frac{\partial M\theta}{\partial \theta}+\frac{\partial M_{x\theta}}{\partial \theta}-RQ_\theta+R\hat{m}_x=0$$

$$N_{x\theta}-N_{\theta x}-\frac{M_{\theta x}}{R}=0 \tag{7}$$

The surface loadings, $\hat{q}_x, \hat{q}_\theta, \hat{q}_n$, and $\hat{m}_x, \hat{m}_\theta$ have units of force and moment per unit area of middle surface, respectively, and are generated in our problem by the activation of the piezoelectric actuation.

2.2. Piezoelectric Layer Equation

If the piezoelectric layer and cylindrical shell are thin in comparison with the radius of curvature of the shell, then it is reasonable to assume that the relationship in equation-3 for normal strain is maintained throughout the combined thickness h + h_1. Hence

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_\theta \end{Bmatrix}_{pe} = \begin{Bmatrix} e_x \\ e_\theta \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_\theta \end{Bmatrix} \tag{8}$$

Where $(\varepsilon_x)_{pe}$ and $(\varepsilon_\theta)_{pe}$ are the normal strains at an arbitrary point on the piezoelectric layer (the shear strain does not affect the layer moment and forces of interest and thus ignored). It should be noted that this assumption implies that the strains at interface are continuous and that the centers for the radii of curvature for shell and layer remain the same.

With the strains thus defined, we now want to find the corresponding stress distribution in the actuator and note that it will contain contributions from both the free piezoelectric actuator strain and the strain distribution in equation-5. At this point we assume that when voltage is applied and the actuator is activated, in accordance with basic shell theory, equal strains are induced in the x and θ directions and the radius of curvature is not changed in either direction. An actuator satisfying this assumption could be made, for example, by taking a portion of a thin-walled tubular piezoelectric element. The magnitude of the induced free strains is then taken to be

$$\varepsilon_{pe} = (e_x)_{pe} = (e_\theta)_{pe} = \frac{d_{31}}{h_1} V \tag{9}$$

Where d_{31} is a piezoelectric strain constant and V is the applied voltage. The stress distribution in the actuator is given by

$$(\sigma_x)_{pe} = \frac{E_{pe}}{1-\mu_{pe}^2}\left(\varepsilon_x + \mu_{pe}\varepsilon_\theta - (1+\mu_{pe})\varepsilon_{pe}\right) \tag{10}$$

$$(\sigma_\theta)_{pe} = \frac{E_{pe}}{1-\mu_{pe}^2}\left(\varepsilon_\theta + \mu_{pe}\varepsilon_x - (1+\mu_{pe})\varepsilon_{pe}\right) \tag{11}$$

with the negative signs resulting from conservation of forces.

2.3. Forces and Moments

By integrating the stresses over the face of a fundamental element, it follows that the moment and forces for the layer can be expressed as

$$(M_x)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_x)_{pe} \left(1+\frac{z}{R}\right) z\,dz$$

$$(M_\theta)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_\theta)_{pe}\, z\,dz \qquad (12)$$

$$(N_x)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_x)_{pe} \left(1+\frac{z}{R}\right) dz$$

$$(N_\theta)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_x)_{pe}\, dz$$

(12)

Since here we are considering thin cylindrical shell, therefore, the term $\left(1+\frac{z}{R}\right)$ can be approximated to 1 as $\frac{z}{R} \leq \frac{1}{25}$. Therefore equation- 12 can be written as

$$(M_\theta)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_\theta)_{pe} \, zdz$$

$$(M_x)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_x)_{pe} \, zdz$$

(13)

$$(N_x)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_x)_{pe} \, dz$$

$$(N_\theta)_{pe} = \int_{h/2}^{h/2+h_1} (\sigma_x)_{pe} \, dz$$

Now substituting the values of stress from equations- 2, 10 and 11 into equation-13 we get

$$(M_x)_{pe} = \frac{E_{pe}}{(1-\mu_{pe}^2)}\left[\left(\frac{e_x}{2}+\frac{\mu_{pe}e_\theta}{2}-\frac{(1+\mu_{pe})e_{pe}}{2}\right)\left\{\left(\frac{h}{2}+h_1\right)^2-\left(\frac{h}{2}\right)^2\right\}+\left(\frac{k_x+\mu_{pe}k_\theta}{3}\right)\left\{\left(\frac{h}{2}+h_1\right)^3-\left(\frac{h}{2}\right)^3\right\}\right]$$

$$(M_\theta)_{pe} = \frac{E_{pe}}{(1-\mu_{pe}^2)}\left[\left(\frac{e_\theta}{2}+\frac{\mu_{pe}e_x}{2}-\frac{(1+\mu_{pe})e_{pe}}{2}\right)\left\{\left(\frac{h}{2}+h_1\right)^2-\left(\frac{h}{2}\right)^2\right\}+\left(\frac{k_\theta+\mu_{pe}k_x}{3}\right)\left\{\left(\frac{h}{2}+h_1\right)^3-\left(\frac{h}{2}\right)^3\right\}\right]$$

$$(N_x)_{pe} = \frac{E_{pe}}{(1-\mu_{pe}^2)}\left[\left(e_x+\mu_{pe}e_\theta-(1+\mu_{pe})e_{pe}\right)h_1+\left(\frac{k_x+\mu_{pe}k_\theta}{2}\right)\left\{\left(\frac{h}{2}+h_1\right)^2-\left(\frac{h}{2}\right)^2\right\}\right]$$

$$(N_\theta)_{pe} = \frac{E_{pe}}{(1-\mu_{pe}^2)}\left[\left(e_\theta+\mu_{pe}e_x-(1+\mu_{pe})e_{pe}\right)h_1+\left(\frac{k_\theta+\mu_{pe}k_x}{2}\right)\left\{\left(\frac{h}{2}+h_1\right)^2-\left(\frac{h}{2}\right)^2\right\}\right]$$

(14)

Because these resultants are functions of material properties as well as mid surface characteristics, they can be easily constructed once mid surface characteristics have been determined.

3. Force and Moment Balancing

The application of moment equilibrium about the middle surface of the shell yields the two conditions

$$\int_{-h/2}^{h/2} (\sigma_x)\left(1+\frac{z}{R}\right)zdz + \int_{-h/2}^{h/2+h_1} (\sigma_x)_{pe}\left(1+\frac{z}{R}\right)zdz = 0$$

$$\int_{-h/2}^{h/2} (\sigma_\theta)zdz + \int_{-h/2}^{h/2+h_1} (\sigma_\theta)_{pe} \, zdz = 0$$

(15)

Similarly, force equilibrium in x and θ directions yields

$$\int_{-h/2}^{h/2} (\sigma_x)\left(1+\frac{z}{R}\right)dz + \int_{-h/2}^{h/2+h_1} (\sigma_x)_{pe}\left(1+\frac{z}{R}\right)dz = 0$$

$$\int_{-h/2}^{h/2}(\sigma_\theta)dz+\int_{-h/2}^{h/2+h_1}(\sigma_\theta)_{pe}\,dz=0$$

(16)

Now, from first equation of equation-15 we get

$$\frac{E}{(1-\mu^2)}\left[(e_x+\mu e_\theta)\frac{h^3}{12R}+(k_x+\mu k_\theta)\frac{h^3}{12}\right]+\frac{E_{pe}}{(1-\mu_{pe}^2)}$$

$$\left[\left(\frac{z^2}{2}+\frac{z^3}{3R}\right)_{\frac{h}{2}}^{\left(\frac{h}{2}+h_1\right)}\left(e_x+\mu_{pe}e_\theta-(1+\mu_{pe})e_{pe}\right)+(k_x+\mu_{pe}k_\theta)\left(\frac{z^3}{3}+\frac{z^4}{4R}\right)_{\frac{h}{2}}^{\left(\frac{h}{2}+h_1\right)}\right]=0$$

(17)

This above equation can be written as

$$(a_{11}+a_{12})e_x+(\mu a_{11}+\mu_{pe}a_{12})e_\theta+(a_{13}+a_{14})k_x+(\mu a_{13}+\mu_{pe}a_{14})k_\theta=a_{12}(1+\mu_{pe})e_{pe}$$

(18)

Where

$$a_{11}=\frac{E}{(1-\mu^2)}\cdot\frac{h^3}{12R}$$

$$a_{12}=\frac{E}{(1-\mu_{pe}^2)}\{H_1+H_2\}$$

$$a_{13}=R\cdot a_{11}$$

$$a_{14}=\frac{E_{pe}}{(1-\mu_{pe}^2)}(H_2\cdot R+H_3)$$

and

$$H_1=\frac{1}{2}\left\{\left(\frac{h}{2}+h_1\right)^2-\left(\frac{h}{2}\right)^2\right\}$$

$$H_2=\frac{1}{3R}\left\{\left(\frac{h}{2}+h_1\right)^3-\left(\frac{h}{2}\right)^3\right\}$$

$$H_3=\frac{1}{4R}\left\{\left(\frac{h}{2}+h_1\right)^4-\left(\frac{h}{2}\right)^4\right\}$$

Now from second equation of equation- 15 we have

$$(\mu b_{11}+\mu_{pe}b_{12})e_x+(b_{11}+b_{12})e_\theta+(b_{13}+b_{14})k_x+(\mu b_{13}+\mu_{pe}b_{14})k_\theta=b_{12}(1+\mu_{pe})e_{pe}$$

(19)

Where

$$b_{11} = 0$$

$$b_{12} = \frac{E_{pe}}{\left(1-\mu_{pe}^2\right)}\left\{\left(\frac{h}{2}+h_1\right)^2 - \left(\frac{h}{2}\right)^2\right\}\cdot\frac{1}{2}$$

$$b_{13} = \frac{E}{\left(1-\mu^2\right)}\cdot\frac{h^3}{12} = a_{11}\cdot R$$

$$b_{14} = \frac{E_{pe}}{\left(1-\mu_{pe}^2\right)}\left\{\left(\frac{h}{2}+h_1\right)^3 - \left(\frac{h}{2}\right)^3\right\}\cdot\frac{1}{3}$$

Now from first equation of equation-16 we have

$$(c_{11}+c_{12})e_x + (\mu c_{11}+\mu_{pe}c_{12})e_\theta + (c_{13}+c_{14})k_x + (\mu c_{13}+\mu_{pe}c_{14})k_\theta = c_{12}(1+\mu_{pe})e_{pe} \quad (20)$$

Where

$$c_{11} = \frac{E}{\left(1-\mu^2\right)}\cdot h$$

$$c_{12} = \frac{E_{pe}}{\left(1-\mu_{pe}^2\right)}\left(h_1 + \frac{H_1}{R}\right)$$

$$c_{13} = \frac{E}{\left(1-\mu^2\right)}\cdot\frac{h^3}{12R}$$

$$c_{14} = \frac{E_{pe}}{\left(1-\mu_{pe}^2\right)}\left(H_1 + H_2\right)$$

Now from second equation of equation- 16 we have

$$(\mu d_{11}+\mu_{pe}d_{12})e_x + (d_{11}+d_{12})e_\theta + (d_{13}+d_{14})k_x + (\mu d_{13}+\mu_{pe}d_{14})k_\theta = d_{12}(1+\mu_{pe})e_{pe} \quad (21)$$

Where

$$d_{11} = \frac{E}{\left(1-\mu^2\right)} = c_{11}$$

$$d_{12} = \frac{E_{pe}}{\left(1-\mu_{pe}^2\right)}\cdot h_1$$

$$d_{13} = 0 = b_{11}$$

$$d_{14} = \frac{E_{pe}}{\left(1-\mu_{pe}^2\right)}\left\{\left(\frac{h}{2}+h_1\right)^2 - \left(\frac{h}{2}\right)^2\right\}\cdot\frac{1}{2} = b_{12}$$

Now we write

$$\begin{aligned}
&(a_{11}+a_{12})e_x + (\mu a_{11}+\mu_{pe}a_{12})e_\theta + (a_{13}+a_{14})k_x + (\mu a_{13}+\mu_{pe}a_{14})k_\theta = a_{12}(1+\mu_{pe})e_{pe}\\
&(\mu b_{11}+\mu_{pe}b_{12})e_x + (b_{11}+b_{12})e_\theta + (b_{13}+b_{14})k_x + (\mu b_{13}+\mu_{pe}b_{14})k_\theta = b_{12}(1+\mu_{pe})e_{pe}\\
&(c_{11}+c_{12})e_x + (\mu c_{11}+\mu_{pe}c_{12})e_\theta + (c_{13}+c_{14})k_x + (\mu c_{13}+\mu_{pe}c_{14})k_\theta = c_{12}(1+\mu_{pe})e_{pe}\\
&(\mu d_{11}+\mu_{pe}d_{12})e_x + (d_{11}+d_{12})e_\theta + (d_{13}+d_{14})k_x + (\mu d_{13}+\mu_{pe}d_{14})k_\theta = d_{12}(1+\mu_{pe})e_{pe}
\end{aligned} \quad (22)$$

Or in reduced notation

$$A_{11}e_x + A_{12}e_\theta + A_{13}k_x + A_{14}k_\theta = f_{11}e_{pe}$$
$$A_{21}e_x + A_{22}e_\theta + A_{23}k_x + A_{24}k_\theta = f_{22}e_{pe}$$
$$A_{31}e_x + A_{32}e_\theta + A_{33}k_x + A_{34}k_\theta = f_{33}e_{pe}$$
$$A_{41}e_x + A_{42}e_\theta + A_{43}k_x + A_{44}k_\theta = f_{44}e_{pe}$$
(23)

Now above equation can be written in matrix notation as

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{42} \end{bmatrix} \begin{Bmatrix} e_x \\ e_\theta \\ k_x \\ k_\theta \end{Bmatrix} = \begin{Bmatrix} f_{11} \\ f_{22} \\ f_{33} \\ f_{44} \end{Bmatrix} \cdot e_{pe}$$
(24)

Now we can write above equation as

$$\begin{Bmatrix} e_x \\ e_\theta \\ k_x \\ k_\theta \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{42} \end{bmatrix}^{-1} \cdot \begin{Bmatrix} f_{11} \\ f_{22} \\ f_{33} \\ f_{44} \end{Bmatrix} \cdot e_{pe}$$
(25)

From equation-25, we can calculate the value of strain produced in piezoelectric actuator from where it is possible to calculate the magnitude and phase of voltage using equation- 9 that should be applied for minimization of vibrations. Obviously the frequency of the applied voltage should be in accordance with the first natural frequency of structure for optimum [14,15].

4. Discussion

For the purpose of analyzing the relationship between mid surface strains and strain produced due to piezoelectric actuation, here cylindrical glass shell is taken with a layer of piezofilm from self PVDF (polyvinylidene fluoride), mono oriented, and aluminum metal electrodes deposited on both sides [16].

Thickness $60\mu m$
Piezoelectric constant d_{31}=23pc/N
Radius of cylindrical shell R=0.055m
Thickness of shell h= $2\times10^{-3}m$
Thickness of piezoelectric layer h_1=0.15h= $3\times10^{-4}m$
Young's modulus of glass shell $E = 7.2\times10^{10}Pa$
Poisson ratio of glass shell $\mu = 0$

Young's modulus of PVDF layer $E_{pe} = 2\times10^{9}Pa$
Poisson ratio of PVDF $\mu_{pe} = 0.35$

Then after substituting the values of material constants in equation- 25 following expression is obtained

$$\begin{Bmatrix} e_x \\ e_\theta \\ k_x \\ k_\theta \end{Bmatrix} = \begin{bmatrix} 0.01080 \\ 0.01080 \\ 0.03548 \\ 0.03548 \end{bmatrix} \cdot e_{pe} \quad (26)$$

From equation-26 it is clear that for control of vibrations the applied voltage will cause a strain in piezoelectric actuator of the order of 10^2 to that of the mid surface strain. Care must be taken so that while minimizing the vibration the actuator do not approach to plastic deformations, because almost all piezoelectric materials commonly used show a elastic behavior within a limit of 0.1% strain value only. Also as the value of mid surface stains and curvature may be different in actual and complex cases so an optimum value of voltage must be decided as per the requirement. This model can be used for the purpose of selection of best suited material as per the value of strains produced in application as well as for the purpose of economic analysis of manufacturing of special high strain piezoelectric materials.

5. Conclusions

Based on the analytical model following conclusions can be drawn;
*The model can be used for determining the forces and moments generated by the activation of piezoelectric layer which has been bonded to a thin cylindrical shell.
*After knowing amount of line moment and forces it is possible to use the loads in higher order shell models.
*Equation-25 gives a relation, which can be used for the purpose of prediction of strain produced in piezoelectric actuator.
*The model discussed in this work can also be used a cost-effective analysis of special high strain materials.

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