

Modelling and Simulation of Plate Heat Exchanger

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Abstract. This paper presents simulation investigation of a plate heat exchanger. Basically, it includes the development of a mathematical model to describe its operation and analysis. The model, after testing against the existing experimental data, has been solved to obtain the effect of various parameters like mass flow rate, number of flow channels, plate configuration and flow patterns. Model of a plate heat exchanger has been described by a set of continuity, momentum and energy equations with a number of simplifying assumptions. Heat transfer rate equation has also been included in the energy balance equation to take care of phenomena occurring therein. Mathematical model has been solved by the use of finite difference technique with interval of $\Delta t = 0.005s$ and $\Delta z = 0.005m$ to obtain the transient and steady state behavior.

Keywords : Plate heat exchanger, Finite difference method.

1. INTRODUCTION

During the last few decades, plate heat exchanger has become essential equipment in process, food, dairy and other industries. Extremely high rates of heat transfer, compactness, flexibility in design and operation, ease of cleaning and low liquid hold up are some of the important advantages of plate heat exchanger

NOMENCLATURE [see[1],[2],[3],[8],[11]]

A	Cross sectional area of channel, m^2
C_i	Specific heat capacity of fluid in channel i, $J/kg^\circ C$
E	Constant temperature $^\circ C$
i	Channel number
K_1	Thermal conductivity of fluid 1, $W/m^\circ C$
L	Length of plate, m
m	Mass flow rate of fluid kg/s
m_i	Mass flow rate of fluid in channel i, kg/s
m_c, m_h	Mass flow rate of cold and hot fluid respectively kg/s
T_{1in}, T_{2in}	Inlet temperature of fluid 1 and fluid 2 respectively, $^\circ C$
T_{ci}, T_{hi}	Inlet temperature of cold and hot fluid respectively, $^\circ C$
T_i	Temperature of fluid in channel i. $^\circ C$
T_{ik}^j	Temperature of fluid at node (j,k) in channel i, $^\circ C$
U	Overall heat transfer coefficient in channel, $W/m^2^\circ C$
w	Plate width, m
x	Correlation constant
α	Film heat transfer coefficient of fluid, $W/m^2^\circ C$
∂t	Temperature change in fluid, $^\circ C$
ϵ	Effectiveness of plate heat exchanger
θ	Number of transfer unit

ρ	Fluid density, kg/m^3
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2. MODEL DEVELOPMENT

A plate heat exchanger is considered with n , number of channels. Channels 1,3,5... $n-1$ have cold fluids, whereas 2,4,6... n have hot fluid. Any flow pattern may exist in the plate heat exchanger – loop flow, series flow and complex flow.

The following assumptions are made in the modelling of plate heat exchanger (see [4], [5],[8],[11]).

- (i) All the physical properties of fluids remain constant.
- (ii) Overall heat transfer coefficient is same for all the flow channels in the plate heat exchanger.
- (iii) Turbulent flow condition exists in the heat exchanger.
- (iv) Resistance offered by metal for heat transfer is negligible.
- (v) Mass transfer does not occur in any flow channel.
- (vi) Chemical reaction does not occur in any flow channel.
- (vii) The plates in the plate heat exchanger are clean and there is no fouling/scaling.
- (viii) Convection heat transfer dominates in the flow direction.
- (ix) No heat losses occur from the plate heat exchanger to the surrounding.
- (x) There is no flow in the lateral direction.
- (xi) There is no change of phase in any channel.
- (xii) The fluids are Newtonian.

3. MODELLING TECHNIQUE

The model has been developed by using the shell balance approach. It involves, considering a shell of a finite thickness and applying the principles of conservation of mass, momentum and energy around the shell. The thickness of shell is made to approach zero and a differential equation is set [See [11]].

3.1 MODEL EQUATIONS

In a channel, a shell of finite thickness (Δz) is considered as shown in fig.1, assumptions and conservation principles are applied over it. Flow is in z direction and is considered to be positive, when fluid flows from top ($z = 0$) to bottom ($z = L$). Model equations have been derived only for three channels, the three channels being first, last and

middle channel. All the other channels between the first and last channel resemble the middle channel. The model equations consist of mass, momentum and energy balance equations, and are discussed below.

3.2 CONTINUITY EQUATION

Continuity equation is mass balance equation. Mass balance has been made only in the z direction, which is the flow direction and as fluid enters or leaves the channel, either from top or from the bottom. Continuity equations are presented below (see [8], [11]).

$$\text{Channel 1: } \frac{\partial v_{z1}}{\partial z} = 0 \quad (1)$$

$$\text{Channel i: } \frac{\partial v_{zi}}{\partial z} = 0 \quad \text{for } i = 2 \text{ to } n-1 \quad (2)$$

$$\text{Channel n: } \frac{\partial v_{zn}}{\partial z} = 0 \quad (3)$$

3.3 MOMENTUM BALANCE EQUATION

Momentum balance equations have been made only in the flow direction, as there is no fluid entry or exit into the channel from the sides. Momentum balance equations are presented below (see [8], [11]).

$$\text{Channel 1: } \rho \frac{\partial v_{z1}}{\partial t} = \mu \frac{\partial^2 v_{z1}}{\partial x^2} - \frac{\partial P}{\partial z} + \rho g \quad (4)$$

$$\text{Channel i: } \rho \frac{\partial v_{zi}}{\partial t} = \mu \frac{\partial^2 v_{zi}}{\partial x^2} - \frac{\partial P}{\partial z} + \rho g \quad \text{for } i = 2 \text{ to } n-1 \quad (5)$$

$$\text{Channel n: } \rho \frac{\partial v_{zn}}{\partial t} = \mu \frac{\partial^2 v_{zn}}{\partial x^2} - \frac{\partial P}{\partial z} + \rho g \quad (6)$$

3.4 ENERGY BALANCE EQUATION

Heat enters the shell not only in the flow direction, but also from the side metal walls. Heat entry through sides is taken care by the heat transfer rate equation. The energy balance equations are given below (see [8],[11]).

$$\text{Channel 1: } \frac{\partial T_1}{\partial t} = \pm v_{z1} \frac{\partial T_1}{\partial z} + \alpha_1 (T_2 - T_1) \quad (7)$$

$$\text{Channel i: } \frac{\partial T_i}{\partial t} = \pm v_{zi} \frac{\partial T_i}{\partial z} + \alpha_1 (T_{i+1} - T_{i-1} - 2T_i) \quad \text{for } i=2 \text{ to } n-1 \quad (8)$$

$$\text{Channel n: } \frac{\partial T_n}{\partial t} = \pm v_{zn} \frac{\partial T_n}{\partial z} + \alpha_n (T_{n-1} - T_n) \quad (9)$$

$$\text{Where } \alpha_i = \frac{U v_{zi} w}{m_i c_i}$$

– Sign denotes the flow in positive flow direction, i.e. from top to bottom, whereas the + sign stands for the flow in negative flow direction, i.e. from bottom to top. Equations (1.1 – 3.3) represent a model of plate heat

exchanger for different flow pattern, operating and geometric parameters. To solve the model, initial and boundary conditions are to be specified. They depend upon the flow patter in the heat exchanger.

4. LOOF FLOW PATTERN

In this hot/cold fluid flow through alternate channels and on their exit from the channel, they mix and leave the heat exchanger. It is shown in fig.2. (see [6], [7], [11])

- Velocity at the inlet of each channel is specified. It is,
 For odd channel (cold fluid): $v_{zi} = \frac{m_c}{A(n/2)\rho_c}$ for $i = 1, 3, 5 \dots n-1$.
 For cold channel (hot fluid): $v_{zi} = \frac{m_h}{A(n/2)\rho_h}$ for $i = 2, 4, 6 \dots n$.
- Temperature at the inlet of each channel is specified. It is equal to T_{hi} and T_{ci} for the hot and cold fluid channels respectively.
- The initial temperature profile is also specified. It is,
 Odd channels (cold fluid): $T_1(0, z) = E$, for all $z > 0$.
 Even channels (hot fluid): $T_1(0, z) = E$, for all $z < L$.

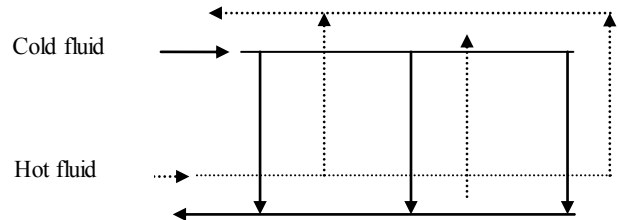
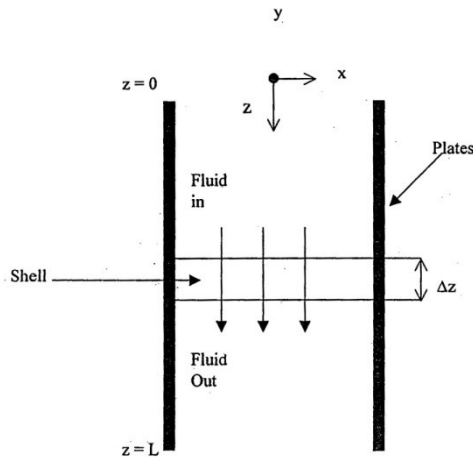


Fig.2. Loop flow pattern

Fig.1. Sketch of a flow channel in a plate heat exchanger

Cold fluid

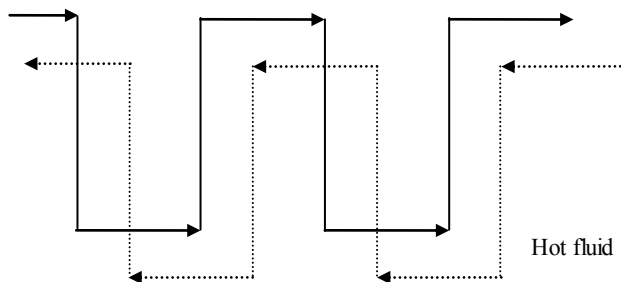


Fig.3. Series flow pattern

5. MODEL SOLUTION

The model equations (1 - 9) are to be solved for the above mentioned conditions. Equation (1 - 3) continuity equations are single variable differential equations, its analytical solution provides velocity v_{zi} to be a constant irrespective of z . This means the in the present analysis, velocity of fluid is considered to be constant in the z direction. With this information, equations (4 - 9) are to be solved simultaneously to obtain the velocity and temperature profiles for transients and steady state conditions. In order to simplify the above situation, it is assumed that the velocity profile in a channel is flat. Meaning by that v_z to be independent of x . This, when applied to moment balance equations (4 - 6) reveals, v_{zi} to be constant. In other words, the velocity of hot/cold fluid in a channel remains unaltered in z and x direction. With this simplification, the model is left only with energy balance equations (6 - 9). These are n , number of simultaneous partial difference equations. The analytical solution of these is quite complex and hence they have to be solved numerically. Finite difference method is used for this purpose. Accordingly the following set of equations is obtained (see [9],[10],[11])

Channel 1:

For flow in positive direction

$$T_{1k}^{j+1} = T_{1k}^j \left(\Delta z - v_{z1} \frac{\Delta t}{\Delta z} - \Delta t a_1 \right) + v_{z1} \frac{\Delta t}{\Delta z} T_{1k-1}^j - \Delta t a_1 T_{2k}^j \quad (10)$$

Channel i:

For flow in positive direction

$$T_{ik}^{j+1} = T_{ik}^j \left(\Delta z - v_{zi} \frac{\Delta t}{\Delta z} - \Delta t a_i \right) + v_{zi} \frac{\Delta t}{\Delta z} T_{ik-1}^j - \Delta t a_i (T_{i+1,k}^j + T_{i-1,k}^j) \quad (11)$$

For $i=2$ to $n-1$

Channel n:

For flow in positive direction

$$T_{nk}^{j+1} = T_{nk}^j \left(\Delta z - v_{zn} \frac{\Delta t}{\Delta z} - \Delta t a_n \right) + v_{zn} \frac{\Delta t}{\Delta z} T_{nk-1}^j - \Delta t a_n T_{n-1k}^j \quad (12)$$

Further, these equations have to be solved, to obtain the temperature profile. To solve these equations, the grid is utilized. At any time $j+1$ in a channel, temperature at all nodes is obtained based on temperature at nodes at time j , as sequence, from channel one to the last channel. This process is repeated till convergence is obtained. Convergence is said to be obtained, when the temperature difference in a node at time j and $j+1$ is not more than 0.001 °C. Computation was made for various values of Δz and Δt . Convergence was obtained only when $\Delta z/\Delta t \geq 2$. The converged values are the steady state temperature profile, all the other non-converged values are considered as unsteady state temperature profiles. The model equations are firstly solved for loop flow than it is also solved for series and complex flow.

Table 1.1: Comparison of model results with the experimental value for hot fluid (see [11])

No of Channels	Hot fluid outlet temperature Calculated (°C)	Experimental value (°C)	% Deviation
4	69.791	69.242	0.793
6	68.825	68.23	0.872
8	68.342	67.724	0.912

10	68.052	67.42	0.937
12	67.859	67.218	0.954
14	67.721	67.073	0.975
16	67.618	66.965	0.975
18	67.537	66.88	0.982
20	67.473	66.813	0.982

Table 1.2: Comparison of model results with the experimental value for cold fluid. (See [11])

No of Channels	Hot fluid outlet temperature Calculated (°C)	Experimental value (°C)	% Deviation
4	30.209	30.793	-2.203
6	31.175	31.77	-1.873
8	31.658	32.276	-1.915
10	31.948	32.58	-1.940
12	32.141	32.782	-1.955
14	32.279	32.927	-1.968
16	32.282	33.035	-2.279
18	32.463	33.12	-1.984
20	32.527	33.187	-1.988

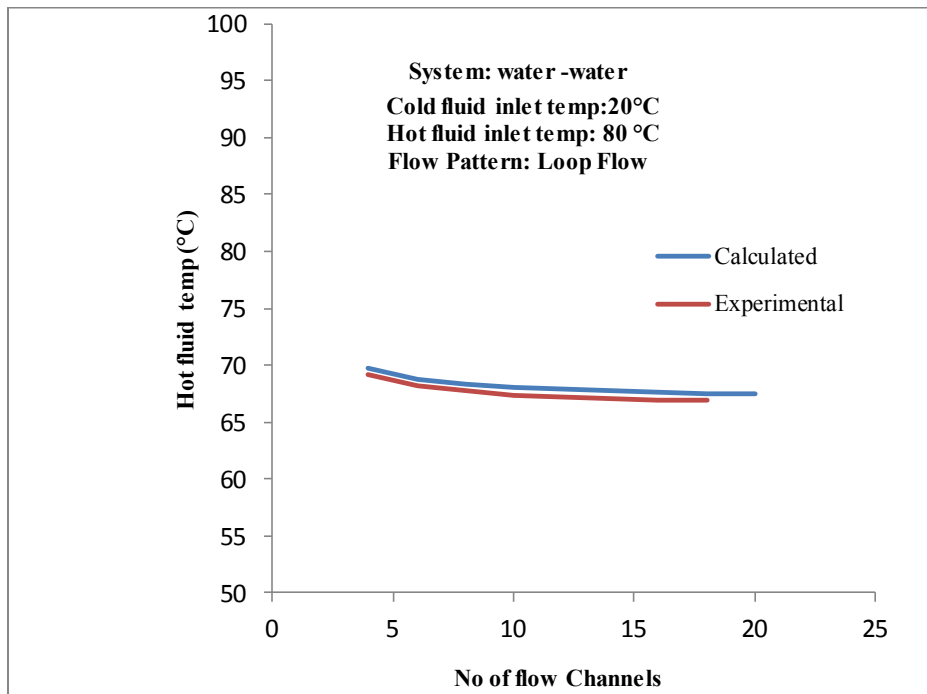


Fig.4. Comparison of model with experimental data for hot fluid (see [11])

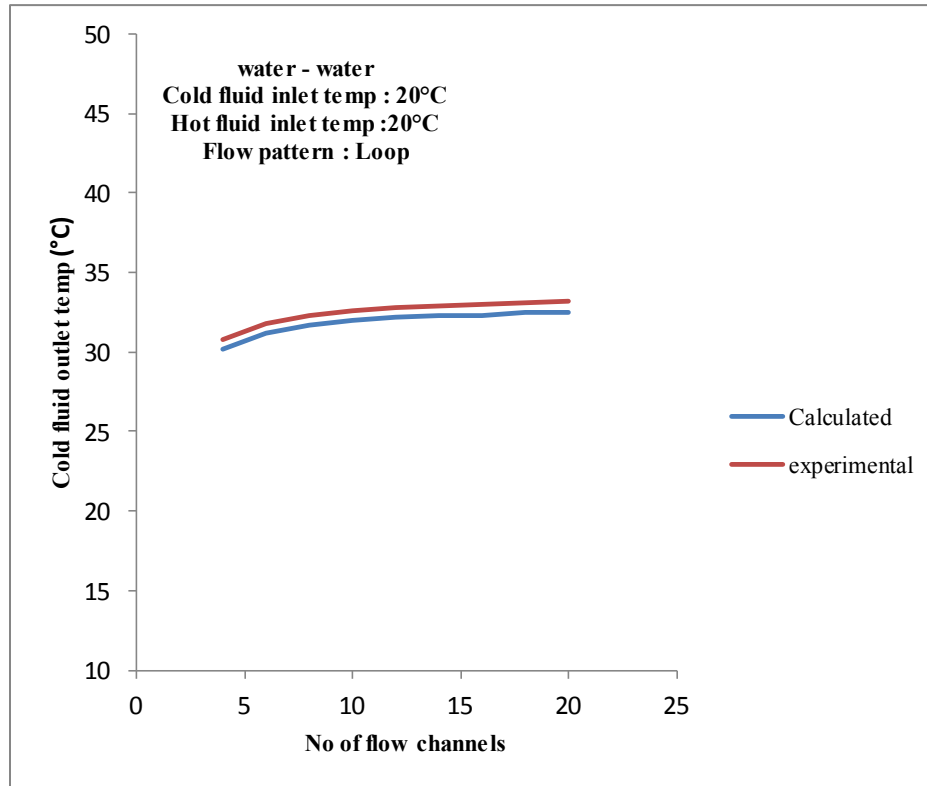


Fig.5. Comparison of model with experimental data for cold fluid (see [11])

6. Results and Discussion

The model equations is solved for various parameters like mass flow rate of cold and hot fluid, hot fluid inlet temperature, flow pattern and plate configuration in a plate heat exchanger, to obtain their effect on its performance. Performance of a plate heat exchanger is given in terms of its heat transfer effectiveness ϵ , defined as [see [11, 12]]:

$$\epsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} \quad \text{or}$$

$$= \frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}} \quad \text{if } m_h C_h < m_c C_c$$

$$\text{Or } \epsilon = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}}$$

EFFECT OF MASS FLOW RATE OF FLUID

In all flow pattern except complex flow, effectiveness decrease with increase in mass flow rate of cold fluid. This is because, as heat capacity of fluid increase temperature change in fluid decreases. Whereas in complex flow, effectiveness increase with cold fluid flow rate, as the cold is in loop flow and takes more heat from the hot fluid when its flow rate is increased. For all the flow patterns, the effectiveness decreases with increase in hot fluid mass flow rate.

EFFECT OF NUMBER OF FLOW CHANNELS

For all flow pattern, effectiveness increases with number of flow channels. This is because, as the number of channel is increased, heat transfer area increases causing greater temperature change to occur in fluids.

EFFECT OF PLTE CONFIGURATION

Plate configuration are classified according to NTU (θ) offered by them. For all flow patterns, effectiveness increases with θ . This is because, when θ increases, the heat transfer capacity of plate also increases and hence more temperature change in fluids. Enhanced turbulence in the flow channel and heat transfer area in the plate is responsible for greater heat transfer.

7. CONCLUSION

1. A mathematical model of a plate heat exchanger has been developed using the equation of continuity, momentum and energy. The model has been solved by the use of finite difference method and results have been compared with experimental data. The maximum deviation is in range of -5.649% to 6.103%.
2. Effectiveness decreases with increase in mass flow rate of cold fluid in loop, and series flow patterns, whereas it increase with increase in mass flow rate of cold fluid in complex flow. However, effectiveness decreases with increase in mass flow rate of hot fluid in loop, series and complex flow patterns.
3. Effectiveness has been found to increase with the increase in number of flow channels in all the flow patterns. However, in the case of loop flow, the maximum value of effectiveness is only 0.218 as against the value of unity in the case of series and complex flow patterns.
4. An increase in plate configuration parameter θ increases the effectiveness, for all the flow patterns considered.

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