International Journal of Mechanical Engineering
ISSN : 2277-7059 Volume 1 Issue 1
http://www.ijmejournal.com/ https://sites.google.com/site/journalijme/

Application of the Hazard Rate Measure in Studying Change of Temperature in a Pulsating Heat Pipe

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Abstract

In this article, we have discussed the importance of applying the hazard rate measure in studying matters related to change of temperature in pulsating heat pipes. It has been found that the hazard rate decreases with increase in diameter of the heat pipe. Finally, it has been validated statistically that the hazard rate increases exponentially as the number of turns in the evaporator section of the heat pipe increases.

Key words: *Heat pipe heat exchanger, waste heat recovery.*

1. Introduction

The Heat Pipe Heat Exchanger has received attention since it was launched into industry at the beginning of the eighties [1]. A heat pipes is a two-phase heat transfer device with high thermal conductivity. As a self-complete energy recovery device that works as a homogeneous flow model, it is very efficient and compact. If some of the waste heat generated from boilers can be recovered, a lot of fuel can be saved.

It is well known that water is excellent as a working fluid for heat pipes for its high latent heat, easy availability, and its high resistance to decomposition and degradation. Water was selected as the working fluid of the heat pipe system developed by Akyurt *et al.* [2]. Because of the problem of incompatibility with iron, copper was selected initially as the container material.

Mathematical models considering the heat transfer effects on operation of a pulsating heat pipe with open end was proposed by Zhang and Faghri [3]. They further studied [4] numerically the oscillatory flow in pulsating heat pipes. Barua *et. al.* ([5], [6]) have recently worked on mathematical modelling of change of temperature in a pulsating heat pipe. It was established that temperature decreases exponentially with respect to time in pulsating heat pipes. In addition, it was found that the rate of change of temperature increases with increase in the number of turns. In this article, we are going to discuss as to how hazard rate, in the sense in which the term is used in reliability theory, increases with respect to number of turns in the evaporator section of a pulsating heat pipe. We are further going to point out that hazard rate decreases with increase in the diameter of the heat pipes.

2. Hazard Rate

While discussing about hazard rate, we shall have to explain a few introductory matters about reliability theory. Reliability is defined as the probability that a device performs properly, for a period of time intended, under acceptable operating conditions ([7], pages 318-319).

Assume that t represents a random variable equal to the time to failure of an item, and assume further that $f(t)$ is the probability density function concerned. Here $t \geq 0$. Let $F(t)$ be the probability distribution function defined as

$$F(t) = \int f(x)\, dx,$$

the integral being taken from $x = 0$ to t. Then reliability $R(t)$ of the item is defined as the complementary distribution function of the random variable. In other words,

$$R(t) = 1 - F(t).$$

If in particular, t follows the exponential probability law, we shall have

$$f(t) = \lambda \exp(-\lambda t),\ t \geq 0,\ \lambda > 0.$$

In this case, we would have

$$F(t) = 1 - \exp(-\lambda t),$$

and therefore,

$$R(t) = \exp(-\lambda t).$$

The hazard rate $h(t)$, known as instantaneous failure rate also, is defined as the probability of an item failing in the next instant of time divided by the reliability to that instant. In other words,

$$h(t) = f(t) / R(t).$$

When $f(t)$ is exponential, the hazard rate becomes equal to λ, a constant. Indeed for a constant hazard rate, the probability law is essentially exponential.

When a quantity decreases exponentially with respect a variable taking positive values only, the function defining the decrease concerned can be seen to be directly proportional to an exponential probability density function. Therefore, in such a case the interpretations can always be expressed in terms of the hazard rate, though not probabilistically. We shall discuss more about this point soon.

We have found that temperature decreases exponentially with respect to time in pulsating heat pipes. We now proceed to explain the matters in terms of hazard rate increasing with number of turns in the evaporator section of pulsating heat pipes. Further, we are going to discuss the application of the hazard rate measure with reference to heat pipes of different diameters.

3. The Experiments

In our experiments, water was selected as the working fluid of the heat pipe system. Copper was selected as the material for the pipes as it has a very high thermal conductivity. Copper pipes of diameter ¼ inch (= 0.6354 cm.) were constructed. The diameter of a pipe would be expressed in inches here because bearing only that unit of measurement, pipes were available. Each pipe was fabricated to take the shape of a square. The length of the condenser section was kept constant at 40 cm. We fabricated 5 different heat pipe setups with number of turns equal to 1, 2, 3, 4 and 5.

Each pipe contained two cut out sections of length 5 cm welded into the main set up. One of them acted as water inlet while the other acts as an exit route for air pockets. The pipes were partially filled with water and then the openings of the inlet and the outlet sections were plugged with a sealing agent and an adhesive tape, the combination of which proved effective enough.

The mathematical models are what we were interested in; we were not really interested on the operability of the entire heat exchanger set up. That is why we have used insulating materials just for minimizing the heat loss without actually evaluating the critical radius of insulation as it would not really affect the mathematical models.

The software called *Daisy Lab* was used for acquisition of data which in our case was temperature. It was connected to a thermo-couple to measure the temperature which gets displayed on a computer screen. The room temperature was fixed at 29^o Centigrade. Every experiment was replicated thrice, and the average temperatures in the condenser section after every minute were noted.

In a second set of experiments, copper pipes of three different diameters viz. ¼ inch (= 0.6354 cm.), ½ inch (= 1.2708 cm.) and 5/8 inch (= 1.5885 cm.) were constructed. Each pipe was made to take the shape of a square. The length of the evaporator section varied for each of the diameter types and was alternately selected as 30 cm, 35 cm and 40 cm respectively. We thus have fabricated nine different heat pipes in this set of experiments.

4. Analysis of the Data

Numerical and statistical analysis of the data collected would now lead us to certain conclusions related to heat transfer. The analysis in detail has been reported in a monograph by the present author [8]. We are going to discuss the statistical analytical matters with reference to mathematical modeling of the data generated. Equations of the type

$$T = C + \alpha \exp(-\lambda \tau),\ \tau \geq 0,\ \alpha > 0,\ \lambda > 0.$$

where T and τ stand for temperature and time respectively, C is a predetermined constant, α and λ being parameters, were hypothesized and fitted.

At this point, we would like to discuss a matter regarding the definition of a random variable. Randomness is one term which is widely misunderstood. Rohatgi and Saleh ([9], page 41) have clearly mentioned that the notion of probability does not enter into the definition of randomness. A random variable is usually said to be one that is associated with some probability law of errors. However, from the measure theoretic standpoint, if we can associate a density function $f(x)$ with the variable X defined in some interval $[a, b]$ such that

$$\int_a^b f(x)\, dx = 1,$$

then X would said to be a random variable with reference to $f(x)$, the concerned density function. In other words, if a variable is probabilistic, it must necessarily be random. However, if a variable is random, it need not be probabilistic in the statistical sense. For example,

$$\int_0^1 2x\, dx = 1,$$

and therefore X here is random variable by definition, but not necessarily probabilistic following some probability law of errors in the statistical sense. This has a very important implication. The broader measure theoretic definition of randomness asserts us that all principles of probability theory would automatically be applicable to a random variable. Now for

$$T = C + \alpha \exp(-\lambda \tau),\ \tau \geq 0,\ \alpha > 0,\ \lambda > 0,$$

it can be seen that the variable

$$\lambda (T - C)/ \alpha = \lambda \exp(-\lambda \tau)$$

is a negative exponential function of τ, which can be seen to be a density function in the measure theoretic sense. Hence, use of the term hazard rate to define λ here would be mathematically valid, though τ here is not probabilistic.

In table- 1 below, we are going to show the estimated values of the hazard rates for different number of turns fitted using the standard method of least squares estimation of linear parameters. The equation is transferable to the form

$$log_e (T - C) = \log_e \alpha - \lambda \tau.$$

The coefficients of determination, which actually are the squares of the correlation coefficients between (T - C) and τ, expressed in percentages, would also be computed. The coefficient of determination expresses the level of acceptability of the linear mathematical model concerned. For example, if the coefficient of determination in a particular case is 95.88, then it can be concluded that 95.88% of the relationship between the two variables concerned, in this case $\log_e (T - C)$ and τ, in the simple linear regression model, can be attributed to mathematical reasons while the rest 4.12% is attributed to randomness. Thus a high coefficient of determination reflects high acceptability of the mathematical model concerned. We are going to tabulate below the equations estimated from the experimental observations and their coefficients of determination concerned.

Table 1: Equations for Different Number of Turns

Number of turns	The equation	Coefficient of determination
1	$T = 29 + 45.98223\ e^{-0.19932\ \tau}$	87.61
2	$T = 29 + 45.79230\ e^{-0.21201\ \tau}$	90.42
3	$T = 29 + 38.63779\ e^{-0.21285\ \tau}$	95.88
4	$T = 29 + 39.26839\ e^{-0.21881\ \tau}$	94.15
5	$T = 29 + 38.32226\ e^{-0.21847\ \tau}$	94.33

It can be seen that the hazard rate increases with increase in the number of turns. As for the fits, it can be seen that the coefficients of determination are very high, thus showing that the log-linear fits are good enough.

Table 2: The Cases of Single Loops with Length 30 cm

Diameter	The equation	Coefficient of determination
¼ inch	$T = 29 + 26.45727e^{-0.33842\ \tau.}$	97.12
1/2 inch	$T = 29 + 33.97826e^{-0.10462\ \tau.}$	98.92
5/8 inch	$T = 29 + 43.91557e^{-0.06439\ \tau.}$	99.44

Table 3: The Cases of Single Loops with Length 35 cm

Diameter	The equation	Coefficient of determination
¼ inch	$T = 29 + 26.21167\ e^{-0.22985\ \tau.}$	99.53
1/2 inch	$T = 29 + 37.28030\ e^{-0.10089\ \tau.}$	99.47
5/8 inch	$T = 29 + 49.14209\ e^{-0.06541\ \tau.}$	99.51

Table 4: The Cases of Single Loops with Length 40 cm

Diameter	The equation	Coefficient of determination
¼ inch	$T = 29 + 29.81405e^{-0.25262\ \tau.}$	99.79

1/2 inch	$T = 29 + 34.32479e^{-0.09192\ \tau.}$	97.85
5/8 inch	$T = 29 + 56.89776e^{-0.05266\ \tau.}$	99.29

In Tables 2, 3 and 4, we have shown the equations concerned for heat pipes with single loops for three different lengths of the evaporator section of the heat pipes. We can clearly see that for any fixed length of the evaporator section of the heat pipe, the hazard rate decreases steeply with increase in the diameter of the pipe. Here too it can be seen that the coefficients of determination are very high indeed, reflecting the fact that the statistical fits are good enough.

We now proceed to find statistically the possible relation between the log linear regression parameter λ and the number n of turns in the evaporator section of the pipes. It can be observed that λ increases as the number of turns increases. As soon as it has been established that decrement of temperature is negative exponential, we are already certain that in any particular number of turns in the evaporator section the hazard rate is constant, independent of time. We hypothesize that the hazard rate λ increases exponentially with respect to number of turns. If this hypothesis is found to be nonrejectable, it would help us in drawing a conclusion that we should not possibly go on increasing the number of turns indefinitely because this would have an adverse effect on cooling. The hypothesized equation in this case is

$$\lambda = \xi\, e^{\psi n}, \xi \geq 0, \psi \geq 0.$$

Once again, as it is transformable to the linear form of the type

$$\log_e \lambda = \log_e \xi + \psi\, n,$$

we can use the method of least squares to fit the equation.

Table 5: Hypothesized Values of the Hazard Rates

Number of Turns	Calculated Hazard Rates	Hypothesized Hazard Rates
1	0.19932	0.20324
2	0.21201	0.20766
3	0.21285	0.21217
4	0.21881	0.21678
5	0.21847	0.22149

We could conclude that the log linear equation fits the data very well. Indeed, it could be seen that the coefficient of determination is 80.84. In other words, 80.84% of the variations are due to this mathematical relationship. Hence, it can be concluded that φ follow the positive exponential law. We therefore conclude that the equation

$$\lambda = 0.19892\ e^{\,0.021504\, n}$$

is statistically valid with coefficient of determination 80.84%. In table- 5 above, we are showing the estimated values of the hazard rates for different number of turns.

We have thus found that hazard rate increases exponentially with respect to number of turns in the evaporator section. When plotted, the curve looks almost like a straight line. However, we have found that an exponential fit does have a very high coefficient of determination, and therefore we can now say that the positive growth is indeed exponential. One point is obvious here. Perfect optimization of the number of turns with respect to hazard rate is not possible because the curve concerned is an increasing one though the rate of increase is very slow.

5. Conclusions

Hazard rate as defined in reliability theory is an important measure in defining certain physical parameters. Particularly when we have an exponential decay curve, the hazard rate will be a constant. In studying the change of temperature in a heat pipe heat exchanger with respect to time, this measure can help us to decide how length of the evaporator section, diameter of the pipe and number of turns in the pipe affects the rate of change of temperature.

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