# F4 and E8:

## Wrong Assumption: E8 cannot unify Fermions and Bosons.
## Useful Truth: F4 and E8 Lie Algebras have both Commutator and AntiCommutator structure.

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Abstract:

Realistic Physics models must describe both commutator Bosons and anticommutator Fermions so that spin and statistics are consistent.
The usual commutator structure of Lie Algebras can only describe Bosons, so a common objection to Physics models that describe both Bosons and Fermions in terms of a single unifiying Lie Algebra (for example, Garrett Lisi's E8 TOE) is that they violate consistency of spin and statistics by using Lie Algebra commutators to describe Fermions.
However,
Pierre Ramond has shown in hep-th/0112261 as shown that the exceptional Lie Algebra F4 can be described using anticommutators as well as commutators.
This essay uses the periodicity property of Real Clifford Algebras to show that E8 can also be described using anticommutators as well as commutators so that it may be possible to construct a realistic Physics model that uses the exceptional Lie Algebra E8 to describe both Bosons and Fermions.

E8 also inherits from F4 Triality-based symmetries between Bosons and Fermions that can give the useful results of SuperSymmetry without requiring conventional SuperPartner particles that are unobserved by LHC.

Realistic Physics models must describe both
integer-spin Bosons whose statistics are described by commutators
(examples are Photons, W and Z bosons, Gluons, Gravitons, Higgs bosons)
and
half-integer-spin Fermions whose statistics are described by anticommutators.
(examples are 3 generations of Electrons, Neutrinos, Quarks and their antiparticles)

Lie Algebra elements are usually described by commutators of their elements
so
if a Physics model attempts to describe both Bosons and Fermions as elements
of a single unifiying Lie Algebra (for example, Garrett Lisi's E8 TOE)
a common objection is:

since the Lie Algebra is described by commutators,
it can only describe Bosons and cannot describe Fermions
therefore
models (such as Garrett Lisi's) using E8 as a single unifying Lie Algebra
violate the consistency of spin and statistics
and are wrong.

However,
Pierre Ramond has shown in hep-th/0112261 as shown that the exceptional Lie Algebra F4 can be described using anticommutators as well as commutators.

The periodicity property of Real Clifford Algebras shows that E8 inherits from F4
a description using anticommutators as well as commutators so that
it may be possible to construct a realistic Physics model that uses
the exceptional Lie Algebra E8 to describe both Bosons and Fermions.

Here are relevant quotes from hep-th/0112261 by Pierre Ramond:
"... exceptional algebras relate tensor and spinor representations
of their orthogonal subgroups,
while Spin-Statistics requires them to be treated differently ...
all representations of the exceptional group F4 are generated by three sets
of oscillators transforming as 26. We label each copy of 26 oscillators as

$$A_{k0}, \quad A_{ki}, i = 1, \ldots, 9, \quad B_{ka}, a = 1, \ldots, 16,$$

and their hermitian conjugates, and where k = 1, 2, 3.
...

One can ... use a coordinate representation of the oscillators by introducing real coordinates
...[ for A_i ]... which transform as transverse space vectors,
...[ for A_0 ]... which transform ... as scalars,
and ...[ for B_a ]... which transform ... as space spinors which satisfy Bose commutation rules
...
Under SO(9), the Ak_i transform as 9, Bk_a transform as 16, and Ak_0 is a scalar. They satisfy the commutation relations of ordinary harmonic oscillators ...
Note that the SO(9) spinor operators satisfy Bose-like commutation relations ... both A_0 and B_a ... obey Bose commutation relations
...
Curiously,
if both ... A_0 and B_a ... are anticommuting, the F4 algebra is still satisfied ...".

To see how the anticommuting property of the 16 B_a elements of F4
can be inherited by some of the elements of E8,
consider that 52-dimensional F4 is made up of:

28-dimensional D4 Lie Algebra Spin(8) (in commutator part of F4)
8-dimensional D4 Vector Representation V8 (in commutator part of F4)
8-dimensional D4 +half-Spinor Representation S+8 (in anticommutator part of F4)
8-dimensional D4 -half-Spinor Representation S-8 (in anticommutator part of F4)

Since 28-dimensional D4 Spin(8) is the BiVector part BV28
of the Real Clifford Algebra Cl(8) with graded structure
Cl(8) = 1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1
and with Spinor structure
Cl(8) = (S+8 + S-8) x (8 + 8)

F4 can be embedded in Cl(8) (blue commutator part, red anticommutator part):

F4 = V8 + BV28 + S+8 + S-8

Note that V8 and S+8 and S-8 are related by the Triality Automorphism.

Also consider the 8-periodicity of Real Clifford Algebras,
according to which for all N

$$Cl(8N) = Cl(8) \text{ x } ...(\text{N times tensor product})... \text{ } Cl(8)$$

so that in particular Cl(16) = Cl(8) x Cl(8)
where Cl(16) graded structure is 1 + 16 + BV120 + 560 + ... + 16 + 1
and Cl(16) Spinor structure is ( (S+64 + S-64) + (64 + 64) ) x (128 + 128)
and Cl(16) contains 248-dimensional E8 as

$$E8 = BV120 + S{+}64 + S{-}64$$

where BV120 = 120-dimensional D8 Lie Algebra Spin(16)
and S+64 + S-64 = 128-dimensional D8 half-Spinor Representation

Consider two copies of F4 embedded into two copies of Cl(8).

**For commutator structure:**

The tensor product of the two copies of Cl(8) can be seen as

$$\begin{gathered} 1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1 \\ \text{x} \\ 1 + V8 + BV28 + 56 + 70 + 56 + 28 + 8 + 1 \end{gathered}$$

which produces the Real Clifford Algebra Cl(16) with graded structure

$$1 + 16 + BV120 + 560 + 1820 + ... + 16 + 1$$

where the Cl(16) BiVector BV120 is made up of 3 parts

$$BV120 = BV28\text{x}1 + 1\text{x}BV28 + V8\text{x}V8$$

that come from the V8 and BV28 commutator parts of the two copies of F4.

This gives the commutator part of E8 as BV120 inheriting commutator structure from the two copies of F4 embedded in two copies of Cl(8) whose tensor product produces Cl(16) containing E8.

**For anticommutator structure:**

The tensor product of the two copies of 256-dim Cl(8) can also be seen as

$$( (S+8 + S-8) \times (8 + 8) )$$
$$\times$$
$$( (S+8 + S-8) \times (8 + 8) )$$

which produces the 2^16 = 65,536 = 256x256-dim Real Clifford Algebra Cl(16)

$$( (S+8 + S-8) \times (S+8 + S-8))$$
$$\times$$
$$( (8 + 8) \times (8 + 8) )$$

with 256-dimensional Spinor structure

$$( (S+8 + S-8) \times (S+8 + S-8)) =$$

$$= ( (S+8 \times S+8) + (S-8 \times S-8) ) + ( (S+8 \times S-8) + (S-8 \times S+8) )$$

that comes from the S+8 and S-8 anticommutator parts of the two copies of F4.

Since the (S+8 x S-8) and (S-8 x S+8) terms inherit mixed helicities from F4

only the (S+8 x S+8) and (S-8 x S-8) terms inherit consistent helicity from F4.

Therefore, define S+64 = (S+8 x S+8) and S-64 = (S-8 x S-8)
so that

( S+64 + S-64 ) = 128-dimensional D8 half-Spinor Representation

This gives the anticommutator part of E8 as S+64 + S-64 inheriting anticommutator structure from the two copies of F4 embedded in two copies of Cl(8) whose tensor product produces Cl(16) containing E8.

The result is that 248-dimensional E8 is made up of:

BV120 = 120-dimensional D8 Lie Algebra Spin(16) (commutator part of E8)

128-dimensional ( S+64 + S-64 ) D8 half-Spinor (anticommutator part of E8)

Note that since the V8 and S+8 and S-8 components of F4 are related by Triality,
and since
the E8 component BV120 contains 64-dimensional V8xV8
and
the 64-dimensional E8 component S+64 = S+8 x S+8
and
the 64-dimensional E8 component S-64 = S-8 x S-8

E8 inherits from the two copies of F4 a Triality relation

V8xV8 = S+64 = S-64

The commutator - anticommutator structure of E8 allows construction of realistic
Physics models that not only unify both Bosons and Fermions within E8
but
also contain Triality-based symmetries between Bosons and Fermions
that can give the useful results of SuperSymmetry
without requiring conventional SuperPartner particles that are unobserved by LHC.

**CONCLUSION:**

Unified E8 Physics models can be constructed without violating spin-statistics,
so that evaluation of such models as Garrett Lisi's E8 TOE
and my E8 Physics model at http://vixra.org/abs/1108.0027 should be based
on other criteria such as consistency with experimental observations.