The theory of electrodynamic space-time relativity (Revision 1)

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Abstract: The theory of electrodynamic space-time relativity is the study of the transformation of time and space between two electrodynamic inertial reference frames, which have both inertial velocity difference and electric potential difference. It is a fundamental theory of theoretic physics based on the Einstein's special theory of relativity and the high-precision experimental facts of the inversion proportional square law of Coulomb's force. It founded new physical space-time concepts, for example, that our space-time is five-dimensional which is composed of quaternion space, and time. It also proposed some new basic concepts of physics, such as electric potential limit, quaternion electric potential and etc., revealing the inherent relationships between electric potential-velocity and time-space. This paper discusses in detail the process of establishing the theory of complex variable electrodynamic space-time relativity and theory of quaternion electrodynamic space-time relativity as well as their various conversions and transformations. This is one of the basic theories among author's other series of related papers.

Keyword: theory of electrodynamic space-time relativity; theory of complex electrodynamic space-time relativity; theory of quaternion electrodynamic space-time relativity; theory of electric potential relativity; special theory of relativity; electric potential limit; complex velocity; complex electric potential; quaternion velocity; quaternion electric potential; quaternion; five-dimensional space-time

The two pillars of modern physics are quantum mechanics and the theory of relativity which were both proven through numerous experiments. However, on the fundamental understandings, such as the "actuality" of physics, there are profound contradictions between quantum mechanics and general relativity. So much so that Einstein was convinced that quantum physics is an incomplete theory, and pursued a unified field theory. Dirac also believed that in the future there will be an improved quantum mechanics, which would make it return to the determinism and prove that Einstein's view was correct. However, this can only be achieved by giving up some basic ideas [1]. For the past century, many attempts to unite quantum mechanics and general relativity have not been successful.

It is well-known that the Dirac equation of quantum mechanics was built upon the relationship between energy and momentum of the special theory of relativity. The general theory of relativity was also developed through advancement of the basic postulates of the special theory of relativity. If there are unsolvable conflicts between quantum mechanics and general relativity, the root cause may be related to special theory of relativity. However, special theory of relativity has been well proven through abundant experiments, and its correctness is sufficiently confirmed. Hence one cannot help but to doubt the completeness of special theory of relativity. In another word, our current understanding of time, space, and momentum may not be complete. To discover what the "incompleteness" of special theory of relativity is and to establish more complete space-time relativity are of great significance for deepening our understanding on the basic physical concepts such as space-time, substance and motion. This will resolving a series of basic problems in current physics and accelerating development of physics.

Part 1. The theory of complex electrodynamic space-time relativity

Einstein's special theory of relativity is based on two basic assumptions [2] about inertial motion, that is

1. The principle of relativity: physical laws should be the same in every inertial frame of reference
2. The principle of invariant speed of light: in any inertial system, the speed of light in vacuum is a constant.

According to these two assumptions, the famous Lorentz transform equation [2] can be derived.

$$X' = \gamma(X - V_X t) \tag{1}$$

$$Y' = Y \tag{2}$$

$$Z' = Z \tag{3}$$

$$t' = \gamma\left(t - \frac{V_X}{{C_0}^2}X\right) \tag{4}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{V_X}^2}{{C_0}^2}}} \tag{5}$$

The Lorentz transformation equations here are scalar equations, where V_X is the speed along x-axis of inertial frame $F'(X', Y', Z', t')$ relative to the inertial frame $F(X, Y, Z, t)$. C_0 is the speed of light. X, Y, Z, t and X', Y', Z', t' are the length and time of the observing system and the observed system, respectively.

Upon analyzing the basic assumptions of the special theory of relativity, a question can be asked: is our understanding of motion complete? Is there another physical reference frame besides the inertial reference frame, where the same the physical laws still hold in each frame of reference when it is under different states? In this physical reference frame, the state of such reference frame cannot be determined through any experiment. Does the related physical quantity have a limit? And would such limit lead to the relativistic effect of time and space?

After further study, it was found that equipotential bodies are of such physical reference systems. Based on Coulomb's inverse proportional square law and its high-precision experiments [3], we know that within a confined conductor of any shape, the electric potential at any point is the same regardless of how much electric charge its surface carries. In addition, the interior electric field strength E is zero. Thus, a thought experiment can be carried out: there are two identical metal confined carriages A and B, and they are motionless relative to each other and are insulated from each other with only carriage B being grounded. Suppose there is an electrode of an ultra-high voltage static electric generator connected to B, and the other electrode of the ultra-high voltage static electric generator is connected to the carriage A. Let the potential of the ground be zero. Once the generator starts running and continues to charge the carriage A with electrical charges (either positive or negative), the electric potential on the surface and in the interior of the carriage increases with it. When the surface electric charges reach Q, the interior and surface potential is ϕ and is the same everywhere. At the same time, the electric field intensity is zero. Therefore, for the observer in the carriage A, it is impossible to know the magnitude of electric potential or whether it is positive or negative relative to carriage B through any experiment. Same can be said for the observer in the carriage B where the electric potential is zero, even though there is a very high electric potential difference between them. Please note that same experimental results can be obtained in the inertial frames of reference. Therefore, we can put forward two assumptions of equipotential reference system:

3. Relative principle of electric potential: physical law has the same form in any electric equipotential frame of reference;

4. Postulate of electric potential limit: in any electric potential frame of reference, any point in vacuum has an electric potential limit constant Φ_0.

The value for such electric potential difference limit Φ_0 needs to be determined by experiment. Its possible value could be exactly the Planck voltage which is 1.04295×10^{27} volts, or of similar magnitude.

By comparing the relative principle of electric potential and the postulate of electric potential limit with the two postulates of the special theory of relativity, it can be seen that their forms are very similar. When the two postulates are established, it leads to the possibility of a new theory of relativity such is the electric potential relativity. It has symmetric relationship with the special theory of relativity. Further reasoning will reveal that both special theory of relativity and the theory of electric potential relativity are special cases of a higher theory of relativity. This higher relativity would be the study of space-time relationship between two frames of reference with both velocity difference and electric potential difference. In order to establish such “higher theory of relativity”, both the electric charge state (electric potential) and the motion state (speed) of the same frame of reference must be unified. In modern physics, however, the concepts of electric potential and speed have no direct correlation. Through in-depth study, it was discovered that in order to solve this contradiction, new physical concepts such as imaginary motion, complex motion and etc. must be introduced.

To investigate this problem, suppose at random point P in the space of equipotential ϕ carriage A:

$$\beta = \frac{\phi}{\Phi_0}$$

Where Φ_0 is the electric potential limit, ϕ is the electric potential difference between two frames of reference, and β is the ratio between the said potential difference and potential limit.

Multiply both the numerator and denominator of equation 6 with imaginary speed of light iC_0, where $i = \sqrt{-1}$,

$$\beta = \frac{\frac{\phi i C_0}{\Phi_0}}{iC_0} \tag{6}$$

The numerator $\frac{\phi i C_0}{\Phi_0}$ on the right side of equation (6) is an imaginary number with the dimension of velocity, represented by $V_\phi i$ whose sign (positive or negative) is arbitrarily defined. In view of complex number, $V_\phi i$ is called imaginary motion or imaginary speed. V_ϕ is not velocity, but speed. However, it is customarily referred to as imaginary velocity. That is:

$$\frac{\phi}{\Phi_0} = \frac{V_\phi}{C_0}$$

$$V_\phi i = \frac{\phi C_0}{\Phi_0} i = K\phi \tag{7}$$

Here, $K = \frac{C_0}{\Phi_0} i$, is an imaginary constant, and is the electrodynamic conversion factor.

From this it can be seen, every point in the equipotential carriage A is in the same imaginary dynamic state (imaginary state) $V_\phi i$. At the same time, every point is also in the same state of real motion (real state) V_X. When the equipotential carriage A is stationary, where $V_X = 0$, however, they are two completely different states of motion. This is the exact purpose for introducing imaginary factor i. To further investigate their space-time relationship, we must extend our concept of motion and space-time from the real number domain to the complex number domain. The more general motion can be abstractly understood as the motion state in the complex plane. If the motion in the coordinate reference system possesses both real and imaginary motion, then we call its motion state the complex motion state. The frame of reference situated in the complex motion state is called complex inertial electrodynamic frame of reference; it has both equipotential and inertial motion. The theory to describe this space-time relation of such motion is called the theory of complex electrodynamic space-time relativity.

Although in mathematics, planar space can be describe using complex number and vectors, but one will see that the physics fundamentals can only be accurately expressed using complex number. Motion can be abstractly understood as different states of the complex space. Because the moduli of the complex number or vector are equal when describing the same subject, therefore depending on the actual case, the equation of complex number can be converted into the equation of vector motion. In the below derivation steps, vector is only used here as supplementary physical quantity to help our understanding and derivation. For example, the dot product of vector is scalar and can be used in complex equation. Here, bold letters and symbols are used to represent vectors. In addition, in order to accommodate both the understanding of precise physical concept expressions and the ease of narration, it is defined here that any complex noun or vector noun combinations, such as complex velocity and complex displacement, or imaginary velocity and imaginary displacement, their physical meanings represents their state of motion and position of the state, respectively. They are complex number, not vector. Their more accurate expressions are complex speed, complex distance, imaginary speed, imaginary distance and etc. Also, please note that the Lorentz's transformation is also a scalar expression. The purpose of presenting all physical quantities in their scalar forms is purely to abstract physical problems into mathematical problems on the complex plane.

As shown in Figure 1, let there be two complex coordinate systems of reference FOX and $F'O'X'$ in the same complex plane. Their imaginary axis F and F', real axis X and X' are all parallel each other. Let FOX be the stationary observing reference frame that is, the imaginary velocity is zero (electrical potential is zero) and real velocity is also zero. While $F'O'X'$ is the observed reference frame and in complex motion state relative to FOX, its complex velocity is V_w, its imaginary velocity is $V_\phi i$, and real velocity is V_X, θ is the complex angle.

$$V_w = V_X + V_\phi i = |V_w|\, e^{\theta i} \tag{8}$$

The moduli of complex velocity is: $|V_w| = \sqrt{{V_X}^2 + {V_\phi}^2}$ (9)

Where, $V_\phi = \frac{\phi C_0}{\Phi_0}$ (10)

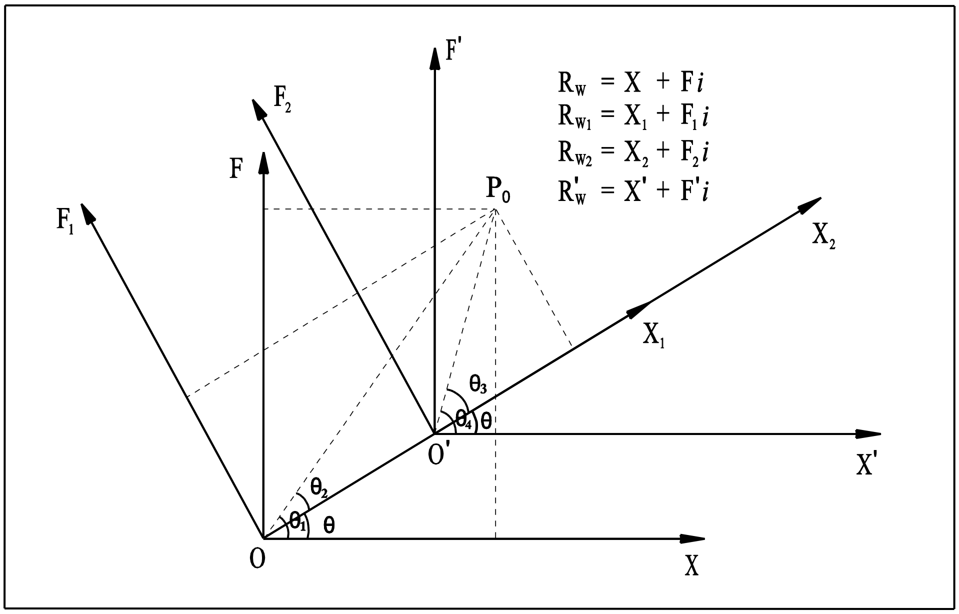

Figure 1

Because of the corresponding relationship between complex plane and two-dimensional vector, so for the ease of derivation and expression, here the moduli of complex number vector is introduced. Let's consider the complex coordinate system FOX and $F'O'X'$ as Cartesian coordination systems represented by $\boldsymbol{FOX}$ and $\boldsymbol{F'O'X'}$. Therefore $\boldsymbol{F'O'X'}$ is moving relative to the system $\boldsymbol{FOX}$**,** with velocity $\mathbf{V_\theta}$, where $\mathbf{V_\theta}$ is equal to the moduli of complex velocity.

The direction of $\mathbf{V_\theta}$ is the direction of positive direction of axis X_1, and is called the moduli velocity of the complex vector. Its components in the coordinate system $\boldsymbol{FOX}$ are $\mathbf{V_X}$ and $\mathbf{V_\phi}$**.** Hence,

$$\mathbf{V_\theta} = \mathbf{V_X} + \mathbf{V_\phi} \tag{11}$$

$$V_\theta = |\mathbf{V_\theta}| = |V_w| \tag{12}$$

Suppose at a random point P_0 on the plane in the complex coordinate system FOX, the magnitude of the corresponding moduli of complex displacement $|R_w|$ is equals to the magnitude of displacement vector $\mathbf{R_p}$**.** Therefore, there is $|\mathbf{R_p}| = |R_w|$, where the direction of $\mathbf{R_p}$ is the original direction point of the coordinate $\boldsymbol{FOX}$. $\mathbf{R_p}$ is called the displacement of complex number moduli.

Its components in the coordinate system $\boldsymbol{FOX}$ are **X** and **F**, where,

$$\mathbf{R_p} = \mathbf{X} + \mathbf{F} \tag{13}$$

Hence, their dot products are

$$\mathbf{R_p} \cdot \mathbf{V}_\theta = \mathrm{XV_X} + \mathrm{FV_\phi} \tag{14}$$

Since real electric potential can be converted into imaginary velocity, then by symmetry principle, real velocity can be converted into imaginary electric potential $\phi_X i$. Therefore, $\phi_X i = V_X \frac{1}{K} = -\frac{V_X \Phi_0}{C_0} i$, where real electric potential ϕ and imaginary electric potential $\phi_X i$ together forms complex electric potential ϕ_w:

$$\phi_w = \phi + \phi_X i \tag{15}$$

The moduli of complex electric potential is:

$$|\phi_w| = \sqrt{\phi^2 + {\phi_X}^2} \tag{16}$$

Where, $\phi_X = -\frac{V_X \Phi_0}{C_0}$ (17)

Multiplying both sides of the equation (15) with K, and comparing with equation (8), we get:

$$\phi_w K = V_w \tag{18}$$

This shows that complex velocity and the complex electric potential are inter-convertible. The complex reference frame of electric potential is a type of complex electrodynamic inertial reference frame. To take one step further, the above four basic assumptions will be combined into two basic assumptions of complex electrodynamic space time relativity theory. Since complex numbers cannot be compared in magnitude, but their modulus can, hence we have:

5. The principle of the complex electrodynamic space-time relativity: physical law has the same form in any complex electrodynamic inertial frame of reference;

6. The postulate of complex electrodynamic time-space limit: in any electrodynamic inertial reference system, the limit of complex velocity's modulus of any point in vacuum is a constant, C_0; or the limit of complex electric potential's moduli of any point is a constant , Φ_0.

Where, C_0 is the speed of light in the vacuum with zero electric potential, and is equal to 299,792,458 m/sec. The limit of complex electric potential's moduli Φ_0, needs to be determined by experiment. Its possible value could be exactly the Planck voltage which is 1.04295×10^{27} volts, or of similar magnitude.

One of the inference can be derived from postulate 6 is that in the frame of reference where electric potential is not zero, the speed of light in the real space is less than C_0. However, the complex velocity's moduli of the speed of light in the complex space is C_0. (This is proven in subsequent paper regarding expansion of the Maxwell equations).

When $V_X = 0,\ V_w = V_\phi i$, the two above postulates become the fundamental postulates of the theory of electric potential relativity. When $V_\phi = 0,\ V_w = V_X$, the two above postulates become the postulates of the special theory of relativity. Hence, special theory of relativity and the theory of electric potential relativity are two special cases of the theory of complex electrodynamic relativity.

The special theory of relativity is commonly referred to the motion of the observed system relative to the observing system along an axis and is called one dimension special theory of relativity. In fact, such motion can have two-dimension or three-dimension form, and the corresponding special theory of relativity become more intricate but also more universal. There are already detailed discussions in literatures on this matter, showing that in the real space two-dimensional [4] and three-dimensional special theory of relativity in arbitrary direction can be derived through rotation and translation of the coordinate system of one-dimensional special theory of relativity [5]; or three-dimensional special theory of relativity can be derived through vector transformation of one-dimensional special theory of relativity [6], [7]. Although both are different in their derivation methods, there is a common point, that is, through the use of one-dimension special theory of relativity and appropriate mathematical approach, the higher real form of special theory of relativity can be derived. Therefore, based on the above mentioned postulates, there may be a transformation method that can derive the complex electrodynamic relativity of space-time by utilizing the special theory of relativity in combination with some transformation methods of complex coordinate system.

As shown in Figure 1, let there be a point P_0 be in the complex plane. In different coordinate systems, it can be represented by different coordinate parameters. In the complex coordinate system FOX, the coordinate of P_0 is represented using complex number $R_w = X + Fi$. The complex angle is θ_1, in the coordinate system $F'O'X'$, the coordinate of point P_0 is represented as complex number $R_w{}' = X' + F'i$, complex angle θ_4. Referring to two dimensional coordinate transformation of real numbers [4], and expanding it into the complex planar space, reference system $F'O'X'$ can be obtained through three coordinate transformation from reference system FOX.

(1). In the complex coordinate system FOX, its time is t, rotating the coordinate system by θ degree counterclockwise, we get complex coordinate system F_1OX_1, whose time is t_1;

(2). Complex coordinate system F_1OX_1 is translated along axis X_1 of real number in the quantity equal to the moduli of the complex velocity $|V_w|$, we get complex coordinate system $F_2O'X_2$, whose time is t_2;

(3). Complex coordinate system $F_2O'X_2$ is rotated by θ degree clockwise, we get complex coordinate system $F'O'X'$, whose time is t'.

The detailed derivation steps, numbered corresponding to the above, are as follow:

(1). The coordinate system FOX is related by θ degree counterclockwise, making the real axis X_1 of the coordinate system F_1OX_1 pass through the origin O' of the reference system $F'O'X'$. Point P_0 in FOX complex coordinate system is complex number R_w, whose complex angle is θ_1. Point P_0 in the coordinate system F_1OX_1 is complex number R_{w_1}, whose complex angle is θ_2. That is,

$$R_w = X + Fi = |R_w|e^{\theta_1 i} \tag{19}$$

$$R_{w_1} = X_1 + F_1 i = |R_{w_1}|e^{\theta_2 i} \tag{20}$$

Also, $|R_w| = \left|R_{w_1}\right|$, therefore,

$$R_{w_1} = R_w\, e^{(\theta_2 - \theta_1)i}$$

$$R_{w_1} = R_w\, e^{-\theta i} \tag{21}$$

$$R_{w_1} = (X + Fi)(\cos\theta - i\sin\theta) \tag{22}$$

From equations (22) and (20),

$$X_1 = X\cos\theta + F\sin\theta \tag{23}$$
$$F_1 = F\cos\theta - X\sin\theta \tag{24}$$

(2). Let F_1OX_1 be the stationary frame of reference, its time is t_1, $F_2O'X_2$ is the moving reference system whose time is t_2, and real axes $X_1$and X_2 are overlapping. The reference system $F_2O'X_2$ moves along the positive direction relative to F_1OX_1 with the magnitude of $|V_w|$, hence $F_2 = F_1$. Since X_1 is the real number axis of F_1OX_1, it can be imagined as the real motion. This is the same state that the special theory of relativity studies. Hence results that have the same form as that of special theory of relativity can be directly obtained.

Because the time is scalar quantity, and isotropic in the complex space, that is, time is not related to the rotary mathematic transformation of the coordinates, therefore $t = t_1$, $t_2 = t'$. From it,

$$X_2 = \gamma(X_1 - |V_w|t) \tag{25}$$
$$F_2 = F_1 \tag{26}$$
$$t' = \gamma\left(t - \frac{|V_w|}{C_0{}^2}X_1\right) \tag{27}$$
$$\text{Where, } \gamma = \frac{1}{\sqrt{1-\frac{|V_w|^2}{C_0{}^2}}} \tag{28}$$

(3). In complex coordinate reference system $F_2O'X_2$, P_0 is represented by complex number R_{w_2}, complex angle θ_3, rotating clockwise θ degree, we get coordinate system $F'O'X'$. In $F'O'X'$, P_0 is represented by complex number $R_w{}'$, complex angle θ_4:

$$R_{w_2} = X_2 + F_2 i = |R_{w_2}|e^{\theta_3 i}$$
$$R_w{}' = X' + F' i = |R_w{}'|e^{\theta_4 i}$$

And also, $|R_w{}'| = \left|R_{w_2}\right|$, hence,

$$R_w{}' = R_{w_2}\, e^{(\theta_4-\theta_3)i} = R_{w_2}\, e^{\theta i} \tag{29}$$

Because $R_{w_2} - R_{w_1} = X_2 - X_1$, therefore,

$$R_w{}' = \left(R_{w_1} - (X_1 - X_2)\right) e^{\theta i} \tag{30}$$

Substituting equation (21) into equation (30),

$$R_w{}' = R_w - (X_1 - X_2)\, e^{\theta i}$$

Because $V_w = |V_w|e^{\theta i}$, therefore,

$$R_w{}' = R_w - V_w \frac{(X_1 - X_2)}{|V_w|} \tag{31}$$
$$\text{Let } t_w = \frac{(X_1 - X_2)}{|V_w|} \tag{32}$$

t_w is a real number, and can be called the system time of complex electrodynamic inertial reference frames. It is not a special time of any particular reference frame, but related to the time of both the observing system and the observed system. It is the time needed to solve fundamental physic problems.

To sum up the above discussion, we obtain the basic relationship between time and space of the complex electrodynamic space-time relativity as the following:

$$R_w' = R_w - V_w t_w \tag{33}$$

$$t' = \gamma\left(t - \frac{|V_w|}{C_0{}^2} X_1\right) \tag{34}$$

Where,， $\gamma = \frac{1}{\sqrt{1-\frac{|V_w|^2}{C_0{}^2}}}$ (35)

$$t_w = \frac{(X_1 - X_2)}{|V_w|} \tag{36}$$

$$X_2 = \gamma(X_1 - |V_w| t) \tag{37}$$

$$X_1 = X\cos\theta + F\sin\theta \tag{38}$$

Depending on different applications, the above relationship can be further expanded into different forms.

Substituting equation (32) into (33):

$$R_w' = R_w - \frac{V_w}{|V_w|}(X_1 - X_2) \tag{39}$$

Since $\cos\theta = \frac{V_X}{V_\theta}$， substituting $\sin\theta = \frac{V_\phi}{V_\theta}$ into (38), and from equation (14), we can get:

$$X_1 = \frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{V_\theta} \tag{40}$$

By substituting equation (40) into equation (37), we get:

$$X_2 = \gamma\left(\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} - |V_w| t\right) \tag{41}$$

And from equation (40) and equation (41), we have:

$$X_1 - X_2 = (1-\gamma)\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t \tag{42}$$

Therefore, by substituting equation (42) into equation (39), then substituting equations (40) and (12) into (34), we can obtain equations describing the complex velocity of the complex of electrodynamic space-time:

$$R_w' = R_w - \frac{V_w}{|V_w|}\left((1-\gamma)\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t\right) \tag{43}$$

$$t' = \gamma\left(t - \frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{C_0{}^2}\right) \tag{44}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|V_w|^2}{C_0{}^2}}}$ (45)

$$\frac{V_w}{|V_w|} = e^{\theta i} = (\cos\theta + i\sin\theta) \tag{46}$$

Speed and electric potential has a corresponding relationship. Therefore, the theory of complex electric potential space-time relativity should also have an equation set for expressing complex electric potential. This will be discussed later in the quaternion electrodynamic space-time relativity.

By decomposing (43) into equations of real number and imaginary number, we obtain:

$$X' = X - \frac{V_X}{|V_w|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t\right) \tag{47}$$

$$F'i = Fi - i\frac{V_\phi}{|V_w|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t\right) \tag{48}$$

Therefore when $\theta = 0$, $V_\phi = 0$, $|V_w| = V_X$, $\mathbf{R_p}\cdot\mathbf{V_\theta} = XV_X$, the form of the special theory of relativity can be obtained:

$$X' = \gamma(X - V_X t)$$

$$t' = \gamma\left(t - \frac{V_X}{{C_0}^2}X\right)$$

$$F' = F$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{V_X}^2}{{C_0}^2}}}$$

When $\theta = \frac{\pi}{2}$, $V_X = 0$, $|V_w| = V_\phi$, $\mathbf{R_p}\cdot\mathbf{V_\theta} = FV_\phi$, $|V_w| = \frac{C_0}{\Phi_0}\phi$, a new kind theory of relativity can be obtained,

$$F'i = \gamma\left(Fi - \frac{C_0}{\Phi_0}\phi i t\right) \tag{49}$$

$$X' = X \tag{50}$$

$$t' = \gamma\left(t - \frac{\phi}{\Phi_0 C_0}F\right) \tag{51}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{\phi^2}{{\Phi_0}^2}}} \tag{52}$$

This is the aforementioned relativity of the electric potential, whose form can be seen here as symmetric to that of the special theory of relativity.

By dividing the both side of equation (48) by i, an equation of real number can be obtained:

$$F' = F - \frac{V_\phi}{|V_w|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t\right) \tag{53}$$

If V_ϕ is understood as V_Y, which is the speed in the direction of Y-axis that is perpendicular to the X-axis, and F', F, $|V_w|$ are to be respectively understood as Y', Y, V_θ, then equations (47), (53), (44) and (45) become the general form of the scalar two-dimensional special theory of relativity. They can be further transformed into the vector equation forms of the two-dimensional special theory of relativity.

Part 2. The theory of quaternion relativity of electrodynamic space-time

The theory of complex electrodynamic space-time relativity describes relationship between the space-time and the state of complex velocity of the reference frames in the complex plane, but the real axis in the complex plane is one-dimensional. In reality, our real number space is three-dimensional. Therefore two-dimensional space of the complex electrodynamic space-time relativity must be expanded into four dimensions. In mathematics, the higher form of complex number is quaternion. Thus the complex electrodynamic space-time relativity can be developed into quaternion electrodynamic space-time relativity. Although the elements of quaternion can have many variations, they can all be converted uniformly into quaternion velocity or quaternion electric potential. Their corresponding frame of reference can be called equi-quaternion velocity or equi-quaternion electric potential frames of reference, or in general be called the quaternion electric inertial frame of reference

Hence, the two basic postulates 5 and 6 of the complex electrodynamic space-time relativity can be further expanded into the basic postulates of quaternion of electrodynamic space-time relativity:

7. Relative principle of quaternion electrodynamic space-time: in any quaternion electrodynamic inertial reference system, physical laws have the same form;

8. Postulate of quaternion electrodynamic space-time limit: in any quaternion electrodynamic inertial reference system, the limit of quaternion velocity's modulus of any point in vacuum is a constant, C_0; or the limit of quaternion electric potential's moduli of any point is a constant , Φ_0.

Where C_0 is the speed of light in the vacuum with zero electric potential, and is equal to $299{,}792{,}458\,\mathrm{m/ses}$. The limit of quaternion electric potential's moduli Φ_0, needs to be determined by experiment. Its possible value could be exactly the Planck voltage which is 1.04295×10^{27} volts, or of similar magnitude.

According to the basic equation (33) of the complex electrodynamic space-time relativity theory, it can be expanded and separated into equation of real number equation and equation of imaginary number equation:

$$\mathrm{F}'i = \mathrm{F}i - \mathrm{V}_{\phi} i \mathrm{t}_{\mathrm{w}} \tag{54}$$

$$\mathrm{X}' = \mathrm{X} - \mathrm{V}_{\mathrm{X}} \mathrm{t}_{\mathrm{w}} \tag{55}$$

Notice that equation (55) is a real scalar expression. However, the fact is that the observed reference frame $\mathrm{F'O'X'}$ moves along the real axis X of the observing reference frame FOX with vector velocity $\mathbf{V_x}$. Let its unit vector be $\mathbf{e}_1$ because axes X′ and X are the same direction as $\mathbf{e}_1$. Multiply both sides of equation (55) by $\mathbf{e}_1$, equation (56) becomes a vector equation:

$$\mathbf{X}' = \mathbf{X} - \mathbf{V_x} \mathrm{t}_{\mathrm{w}} \tag{56}$$

Equations (54) and (56) describe the physical nature more objectively than the complex equation (33). Also, equation (56) can be expanded into three-dimensional. If the vector $\mathbf{X}'$ and $\mathbf{X}$ in equation (56) are defined as vector $\mathbf{r}'$ and $\mathbf{r}$ in the three-dimensional space $\mathrm{X}', \mathrm{Y}', \mathrm{Z}'$ and $\mathrm{X, Y, Z}$, corresponding velocity $\mathbf{V_x}$ is defined as $\mathbf{V_r}$ and have the same direction as $\mathbf{r}'$ and $\mathbf{r}$. Let $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ be the unit vectors in the coordinate of three-dimensional space along the axis X, Y, Z, respectively, we have:

$$\mathbf{r} = \mathrm{X}\mathbf{e_1} + \mathrm{Y}\mathbf{e_2} + \mathrm{Z}\mathbf{e_3} \tag{57}$$

$$\mathbf{r}' = \mathrm{X}'\mathbf{e_1} + \mathrm{Y}'\mathbf{e_2} + \mathrm{Z}'\mathbf{e_3} \tag{58}$$

$$\mathbf{V_r} = \mathrm{V_X}\mathbf{e_1} + \mathrm{V_Y}\mathbf{e_2} + \mathrm{V_Z}\mathbf{e_3} \tag{59}$$

$\mathbf{X}', \mathbf{X}, \mathbf{V_x}$ becomes $\mathbf{r}', \mathbf{r}, \mathbf{V_r}$. At the same time, let $\mathrm{t_w}$ become $\mathrm{t_q}$ accordingly. Therefore, equation (54) and (56) can be transformed into:

$$\mathrm{F}'i = \mathrm{F}i - \mathrm{V}_{\phi} i \mathrm{t}_{\mathrm{q}} \tag{60}$$

$$\mathbf{r}' = \mathbf{r} - \mathbf{V_r} \mathrm{t}_{\mathrm{q}} \tag{61}$$

If the vector quantities in equation (61) become scalar quantities, the series of $F-X$ complex coordinate plane in Figure 1 becomes a series of $F-r$ complex coordinate plane. Then, V_X, X', X_1, X_2 and X becomes V_r, r', r_1, r_2 and r accordingly.

Equation (60) and (61) forms a quaternion space system composed of three real number vectors and one imaginary number, and is called type Y_i quaternion space. Obviously it is not Hamiltonian quaternion. This expands the state of motion of the reference system in the complex plane into motion of the type Y_i quaternion space, that is, the reference frame $F'_q(F', \mathbf{X'}, \mathbf{Y'}, \mathbf{Z'})$ moves with velocity $V_q(V_\phi, \mathbf{V_X}, \mathbf{V_Y}, \mathbf{V_Z})$ relative to reference frame $F_q(F, \mathbf{X}, \mathbf{Y}, \mathbf{Z})$. Suppose R'_q and R_q are quaternion displacement coordinates at any point P_0 in the inertial reference system F'_q and F_q in the Y_i type quaternion space, respectively, then:

$$R_q = Fi + \mathbf{r} = Fi + X\mathbf{e_1} + Y\mathbf{e_2} + Z\mathbf{e_3} \tag{62}$$

The moduli of R_q is $|R_q|$, that is $|R_q| = \sqrt{F^2 + X^2 + Y^2 + Z^2}$ (63)

$$R'_q = F'i + \mathbf{r}' = F'i + X'\mathbf{e_1} + Y'\mathbf{e_2} + Z'\mathbf{e_3} \tag{64}$$

The moduli of R'_q is $|R'_q|$, i.e., $|R'_q| = \sqrt{F'^2 + X'^2 + Y'^2 + Z'^2}$ (65)

Type Y_i quaternion velocity V_q and its moduli $|V_q|$ are:

$$V_q = V_\phi i + \mathbf{V_r} = V_\phi i + V_X\mathbf{e_1} + V_Y\mathbf{e_2} + V_Z\mathbf{e_3} \tag{66}$$

$$|V_q| = \sqrt{{V_\phi}^2 + {V_r}^2} = \sqrt{{V_\phi}^2 + {V_X}^2 + {V_Y}^2 + {V_Z}^2} \tag{67}$$

Where $i = \sqrt{-1}$

Adding equation (60) with (61), and substituting (62), (64) and (66) into it to obtain equation (68). Also, replacing X', X_1, X_2, X and $|V_w|$ of equations (34), (35), (36), (37) and (38) with r', r_1, r_2, r and $|V_q|$, the basic space-time equations of Y_i type quaternion relativity theory can be obtained.

$$R_q' = R_q - V_q t_q \tag{68}$$

$$t' = \gamma\left(t - \frac{|V_q|}{{C_0}^2} r_1\right) \tag{69}$$

where, $\gamma = \frac{1}{\sqrt{1-\frac{|V_q|^2}{{C_0}^2}}}$ (70)

$$r_1 = r\cos\theta + F\sin\theta \tag{71}$$

$$r_2 = \gamma(r_1 - |V_q|t) \tag{72}$$

$$t_q = \frac{(r_1 - r_2)}{|V_q|} \tag{73}$$

Depending on the need, the above basic equation can be transformed into different variations.

Substituting equation (73) into (68), we have:

$$R_q' = R_q - \frac{V_q}{|V_q|}(r_1 - r_2) \tag{74}$$

Suppose there are displacement vector $\mathbf{R_p} = \mathbf{F} + \mathbf{X} + \mathbf{Y} + \mathbf{Z}$ and velocity vector $\mathbf{V_\theta} = \mathbf{V_\phi} + \mathbf{V_X} + \mathbf{V_Y} + \mathbf{V_Z}$

$$V_\theta = \sqrt{{V_\phi}^2 + {V_X}^2 + {V_Y}^2 + {V_Z}^2} = |V_q|$$

Then their dot product is $\mathbf{R_p} \cdot \mathbf{V}_\theta = \mathrm{XV_X} + \mathrm{YV_Y} + \mathrm{ZV_Z} + \mathrm{FV_\phi}$

Because $\cos\theta = \frac{V_r}{|V_q|}$, $\sin\theta = \frac{V_\phi}{|V_q|}$, $r = X\frac{V_X}{V_r} + Y\frac{V_Y}{V_r} + Z\frac{V_Z}{V_r}$, therefore:

$r_1 = r\cos\theta + F\sin\theta = \frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|}$

$r_2 = \gamma(\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|} - |V_q|t)$

$r_1 - r_2 = (1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|} + V\gamma t$

Hence the velocity equations of the type Y_i quaternion electrodynamic space-time relativity is:

$$R_q{}' = R_q - \frac{V_q}{|V_q|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|} + |V_q|\gamma t\right) \quad (75)$$

$$t' = \gamma\left(t - \frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{C_0{}^2}\right) \quad (76)$$

$$\text{Where, } \gamma = \frac{1}{\sqrt{1-\frac{|V_q|^2}{C_0{}^2}}} \quad (77)$$

When $\theta = 0$, $\mathbf{V}_\theta = \mathbf{V_r}$, $\mathbf{R_p} = \mathbf{r}$, $V_\phi = 0$, $R_q{}' = \mathbf{r}'$, $R_q = \mathbf{r}$, $\frac{V_q}{|V_q|} = \frac{\mathbf{V_r}}{|\mathbf{V_r}|}$, then the special theory of relativity in any direction $\mathbf{r}(\mathbf{x},\mathbf{y},\mathbf{z})$ in three-dimensional space can be obtained [5],[6]:

$$\mathbf{r}' = \mathbf{r} - \frac{\mathbf{V_r}}{|\mathbf{V_r}|}\left((1-\gamma)\frac{\mathbf{r}\cdot\mathbf{V_r}}{V_r} + V_r\gamma t\right)$$

$$t' = \gamma\left(t - \frac{\mathbf{r}\cdot\mathbf{V_r}}{C_0{}^2}\right)$$

$$\text{Where, } \gamma = \frac{1}{\sqrt{1-\frac{V_r{}^2}{C_0{}^2}}}$$

Because imaginary velocity V_ϕ and real electric potential ϕ are inter-convertible, and real velocity can be converted into imaginary electric potential vector, hence the equation of the velocity of the type Y_i quaternion electrodynamic relativity can be converted into another type of electric potential equation of quaternion electrodynamic relativity. Let this new quaternion be H-type quaternion.

Multiplying both sides of equation (75) by $-i$, we get:

$$R_q{}'(-i) = R_q(-i) - \frac{V_q}{|V_q|}(-i)\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|} + |V_q|\gamma t\right) \quad (78)$$

Let H-type quaternion electric potential be ϕ_h, then:

$\phi_h = \frac{V_q}{K} = (V_\phi i + V_X\mathbf{e_1} + V_Y\mathbf{e_2} + V_Z\mathbf{e_3})\frac{\Phi_0}{C_0}(-i)$

Let $\phi = \frac{V_\phi\Phi_0}{C_0}$, $\phi_X = \frac{V_X\Phi_0}{C_0}$, $\phi_Y = \frac{V_Y\Phi_0}{C_0}$, $\phi_Z = \frac{V_Z\Phi_0}{C_0}$, then:

$$\phi_h = (\phi i + \phi_X\mathbf{e_1} + \phi_Y\mathbf{e_2} + \phi_Z\mathbf{e_3})(-i) \quad (79)$$

Let $\phi_q = \phi i + \phi_X\mathbf{e_1} + \phi_Y\mathbf{e_2} + \phi_Z\mathbf{e_3}$, and ϕ_q is the quaternion electric potential of type Y_i, hence:

$$\phi_h = (-i)\phi_q \quad (80)$$

$|\phi_h| = |\phi_q|$

$$\text{Let } \mathbf{e_1}(-i) = \mathbf{i},\ \mathbf{e_2}(-i) = \mathbf{j},\ \mathbf{e_3}(-i) = \mathbf{k} \quad (81)$$

Also let $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ be the imaginary unit vector, and let corresponding real unit vector be $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$, which are perpendicular to each other**,** therefore $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are three imaginary unit vector perpendicular to each other. Substituting them into equation (79), we get:

$$\phi_h = \phi + \phi_X\mathbf{i} + \phi_Y\mathbf{j} + \phi_Z\mathbf{k} \tag{82}$$

This shows that H-type quaternion potential ϕ_h is composed of one scalar electric potential and three imaginary vectors of electric potential. For the same reason, the displacement of H-type quaternion is $R_h{}'$ and R_h:

$$R_h{}' = R_q{}'(-i) = F' + X'\mathbf{j} + Y'\mathbf{j} + Z'\mathbf{k} \tag{83}$$

$$R_h = R_q\,(-i) = F + X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k} \tag{84}$$

Multiplying $\frac{1}{K}$ with both numerator and denominator of $\frac{V_q}{|V_q|}$, and also from the definition of ϕ_h, we get:

$$\frac{V_q}{|V_q|}(-i) = \frac{\phi_h}{|\phi_h|} \tag{85}$$

Because $|V_q| = \frac{C_0}{\Phi_0}|\phi_h|$ (86)

Let, $\boldsymbol{\phi_\theta} = \frac{C_0}{\Phi_0}\mathbf{V_\theta}$ (87)

Then, $\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|} = \frac{\mathbf{R_p}\cdot\boldsymbol{\phi_\theta}}{|\phi_h|}$ (88)

Substituting equation(83),(84) ,(85),(86) and (88) into(78), (76), (77) obtaining the electric potential equation for the H-type quaternion electrodynamic space-time relativity theory:

$$R_h{}' = R_h - \frac{\boldsymbol{\phi_h}}{|\phi_h|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\boldsymbol{\phi_\theta}}{|\phi_h|} + \frac{C_0}{\Phi_0}|\phi_h|\gamma t\right) \tag{89}$$

$$t' = \gamma\left(t - \frac{\mathbf{R_p}\cdot\boldsymbol{\phi_\theta}}{\Phi_0 C_0}\right) \tag{90}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\phi_h|^2}{\Phi_0{}^2}}}$, (91)

By analyzing the H-type quaternion, it was discovered that not only is it composed of one scalar and three imaginary vectors, but also the sum of square of the modulus of the four components is equal to the square of the modulus of the H-type quaternion. And there is:

$$\mathbf{i^2} = \mathbf{j^2} = \mathbf{k^2} = -1 \tag{92}$$

$$\mathbf{ij} = -\mathbf{ji} = \mathbf{k},\ \ \mathbf{jk} = -\mathbf{kj} = \mathbf{i},\ \ \mathbf{ki} = -\mathbf{ik} = \mathbf{j} \tag{93}$$

Hence, H-type Quaternion *is* the Hamilton quaternion. According to the nature of the Hamiltonian quaternion **[8]**, we have:

$$\frac{\boldsymbol{\phi_h}}{|\phi_h|} = e^{\alpha\mathbf{I}} \tag{94}$$

Where, $\mathbf{I} = \frac{\phi_X \mathbf{i} + \phi_Y \mathbf{j} + \phi_Z \mathbf{k}}{\sqrt{{\phi_X}^2 + {\phi_Y}^2 + {\phi_Z}^2}}$ (95)

$\alpha = \arctan\left(\frac{\sqrt{{\phi_X}^2 + {\phi_Y}^2 + {\phi_Z}^2}}{\phi}\right)$ (96)

Because the dot product Hamilton quaternion is equal to a dot product of four-dimensional vector,

$\mathbf{R_p} \cdot \boldsymbol{\phi_\theta} = \mathrm{R_h} \cdot \phi_\mathrm{h}$ (97)

Therefore, the potential space-time of the Hamiltonian quaternion electrodynamic time- space relativity can be expressed using Hamilton quaternion's variable equation. The electric potential relationship of the H-type quaternion electrodynamic space-time relativity can be simplified into electric potential equations of the complex electrodynamic relativity. At the same time, H-type and Y_i type quaternion theory of relativity can have various transformation variations according to the need the research.

This shows that our space-time is five dimensional which is composed of quaternion space and time. It has two basic forms: quaternion velocity form and quaternion electric potential form. They are closely linked and are inter-convertible. The relativity of electrodynamic time-space reveals their intrinsic nature and their mutual relations. Also the theory unites inertial motion, electromagnetic motion, time and space. The theory of electrodynamic space-time relativity is the generalized name for the family of relativity theories. Special theory of relativity is a special case of this family. The family of relativity theories includes the theory of electric potential relativity, the theory of complex relativity, the theory of quaternion relativity and other combinations and transformations of relativities. These theories of relativity have close relationships, each describing a corresponding space-time. Space-time is the foundation and core of physics. Electrodynamic space-time relativity establishes a new space-time concept, which will greatly expand our current understanding and revolutionize modern physics. Subsequent papers will further explore the changes and applications this new concept will bring about.

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