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Single Machine Scheduling Problem under Fuzzy Processing Time and Fuzzy Due Dates

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Abstract: In this paper, we propose *n*-jobs to be processed on Single Machine Scheduling Problem (SMSP) involving fuzzy processing time and fuzzy due dates. The different due dates for each job be considered which meet the demand of customer with more satisfaction level. The main objective of this paper is the total penalty cost to be minimum in the schedule of the jobs on the single machine. This cost is composed of the total earliness and the total tardiness cost.

Here, an algorithm is developed using Average High Ranking Method (AHRM) which minimizes the total penalty cost due to earliness (lateness) of jobs in fuzzy environment. Finally, numerical example is given to illustrate proposed method.

Key Words: Single machine, Fuzzy Processing time, Fuzzy Due dates, Early/late job, Fuzzy environment, etc.

1. INTRODUCTION

The short- term schedules show an optimal order (sequence) and time in which jobs are processed as well as show time tables for jobs, equipment, people, facilities and all other resources that are needed to support the production plan. The schedules should use resources efficiently to give low costs and high utilizations.Other purpose of scheduling are, minimizing customer wait time for a product, and meeting promised delivery dates, keeping stock levels low, giving preferred working pattern, minimizing waiting time of patients in a hospitals in for different types of tests, and so on.

The general scheduling or sequencing problem may be described as: Let, there are *n* jobs (Tasks) to be performed one at a time on each of *m* machines (Processor) [1].

The study of earliness and tardiness penalties in scheduling models is arelatively recent area of inquiry. For many years scheduling research focused on single performance measures. Most of the literature deals with regular measure such as mean flow, time mean lateness, percentage of jobs tardy, mean tardiness etc. in deterministic time but the environment in modern society is neither fixed nor probabilistic. So, here we are considering fuzzy environment i.e. the processing time of each job is in indeterminist environment. Here fuzzy processing time is considered in three situations *(a, b, c)* where, *a*- in *Favorable (High) condition*, *b- Normal (Medium) condition and c- in Worse (Bad) conditions*. The mean tardiness criterion, in particular, has been a standardway of measuring conformance to due dates, although it ignores the consequences of jobs completing early. [2] Studied sequences with earliness and tardiness penalties In a JIT scheduling environment, jobs that complete early must be held in finished good inventory until their due date, while jobs that complete after their due date may cause a customer to shut down operations. Therefore, an optimal schedule is one in which all jobs finish on their assigned due dates. This can be translated to a scheduling objective in several ways. The most obvious objective is to minimize the deviation of job completion time around these due dates in non-deterministic time.

The concept of penalizing both earliness and tardiness has spawned a new and rapidly developing line of research in the scheduling field. Because the use of both earliness and tardiness penalties in fuzzy environment give rise to a non-regular performance measure, it has led to new methodological issues in the design of solution procedures.

This paper presents a special case of Early/Tardy (E/T) having distinct due dates (DDD) problem, when the earliness and tardiness are penalized at the rates fixed by demand maker for the jobs. The next sections introduce the concept of single machine and the processing time of the jobs in fuzzy environment. The average high ranking and the scheduling of some small systems are determined in the section after. An algorithm based on these arguments is developed here and it is justified by a numerical example.

(a) Concept of Single Machine:

Now days, in competitive and flexible market installing of machines is very expensive, as the technology changes very frequently and the out dated machines can't satisfy the demands of the modern market. Secondly installing of more than one machines of the same type can speed up the work but needs more and more maintenance and supervision. Thirdly, installation of machines demands for more spaceto install, which also increases the idle cost of the project. So to reduce the expenditure, contractor wishes to process the work on single machine using an intelligent scheduling system and for the small systems single machine maximizes the profit of whole the project.

(b) Fuzzy Processing Time:

The processing time of a job can vary in many ways, may be due to environmental factor or due to the different work places. We find that when a contractor takes the work from a department, he/she calculates total expenditure at the time of allotment. But due to many factors like non available of labor, weather not favorable, or sometimes abnormal conditions, cost may vary. Hence due to these reasons work can be completed late and creates due date problem i.e. order can't be delivered on time, on the other hand if the work completes before the due time it arises the inventory problem. So to overcome these factors, the processing time of a job considered here is in three situations- favorable (High), Normal (Medium) and worse (Bad) conditions. In this paper, a new concept of different processing time of each job is considered which helps the contractor to estimate the cost of the work at the time of allotment.

In this paper, different due dates for each of the job be considered which meets the demand maker with more satisfaction level. So using the Average High Ranking Method (AHRM) algorithm developed here, contractor can save the penalty cost and can satisfy the demand maker to great extent.

2. ASSUMPTION AND NOTATION

The machine scheduling problem studied in this paper requires *n* independent jobs J_i *(i= 1,2 ,3.....n)* to be processed on a single machine with the following assumptions:

i) Only one job can be processed on a given machine at a time

ii) All jobs are available for processing at time zero.

iii) The single machine can process at most one job at a time.

iv) No pre-emption is allowed

Let, S Schedule for the n jobs.

a, b, c Processing time of job i on the machine in fuzzy environment.

A_i Average high ranking of the processing time a, b, c
= [3b+ (c-a)] /3.

d_i Due date for the job i.

C_i Completion time of job

T_i Max. $(0, c_i - d_i)$

E_i Max. $(0, d_i - c_i)$

Sl_i Slack time of job i

e_i Penalty per unit time for the earliness of job i.

l_i Penalty per unit time for the tardiness of job i.

An important special case in the family of E/T problems involves minimizing the sum of absolute deviations of job completion time form a DDD having processing time in fuzzy environment. In particular, the objective function can be written as: $f(s) = \Sigma \,|c_i - d_i|\, = \Sigma\,(E_i + T_i)$

When we write the objective function in this form, it is clear that earliness andtardiness are penalized at the rate e_i and l_i for all the jobs. In this paper, processing time of the jobs considered in triangular fuzzy environment.

3. FORMULATION OF SINGLE MACHINE SCHEDULING PROBLEM:

Let us consider the following single machine scheduling problem. There are a set of n jobs (Tasks) $\{J_1, J_2,.....,J_n\}$ to be scheduled non-preemptively for processing on a single machine. The machine is continuously available from $t = 0,$ and can process only one job at a time. The jobs are also continuously available from $t = 0,$ and require positive processing times $\{P_1, P_2,....,P_n\}$ and Due Date(DD) d_i are associated with each job J_i are positive integers. As completion time of job J_i passes between due date d_i and late date $d^*_{i,}$ the customer satisfaction decreases until it vanishes in the delay case.The greater the delay, the lower the satisfaction.

Let C_i denotes completion time of job J_i and membership function $\mu_i\,(C_i)$ denote degree of satisfaction with respect to C_i, which is defined as follows:

$$\mu_i\,(C_i) = \begin{cases} 1, & C_i \le d_i, \\ 1-(C_i - d_i)/(d^*_i - d_i), & d_i < C_i < d^*_i, \\ 0, & d^*_i \le C_i. \end{cases} \qquad (1)$$

If completion time C_i is before due date d_i, there is maximum degree of satisfaction that is one. When completion time after due date d_i level of satisfaction will decrease to zero. Generally, completion time C_i is within the interval [0, 1] [3].

The concepts of fuzzy processing time, fuzzy due date, fuzzy precedence relation etc. are introduced by various researchers. [4], [5] discussed about earliness & lateness of jobs in flow shop scheduling.[6] studied the job sequencing problem when job processing time is represented with fuzzy numbers.[7], [8] and [9],[10] has fuzzified the scheduling problems by using a fixed due date. Thus the fuzzy due date is directly related to the earliness and tardiness penalty in conventional scheduling problems. In this paper, different due dates for each of the jobs is considered. Next jobs are scheduled in increasing order of their slack time.

This paper investigates a different approach to single machine under fuzzy environment with bi-objective criteria. On one side it minimizes the penalty cost of the tardy jobs and on the other side it minimizes the total flow time of all the jobs.

Here consider the processing time in fuzzy environment (a, b, c), which is real time situation and is defuzzified by Average High Ranking Method.

i. e. AHR = [3b+ (c-a)] /3. (2)

4. SOLUTION PROCEDURE

Average High Ranking Methodology (AHRM)

Step1. Find Average High Ranking (AHR) of the fuzzy processing time (a, b, c) of all the jobs.

Step2. Find the slack time of all the jobs $Sl_i = | A_i - d_i |$

Step3. Arrange the jobs in increasing order of their slack time. If two jobs have the same slack time then considers the jobs of lowest processing time at the earlier position.

Step4. Using the sequence obtained in step 3 find the total penalty of all the jobs using earliness (e_i) and lateness (l_i) penalty cost.

5. NUMERICAL EXAMPLE

Consider 7- jobs having fuzzy processing time, single machine and distinct due dates. Penalty cost (e_i) for earliness is also given.

Job	P_i	AHR	d	Sl_i	e_i	l_i
1	3,4,6	5	6	1	2	3
2	5,7,9	25/3	8	1/3	2	3
3	11,13,15	43/3	9	16/3	2	3
4	7,9,11	31/3	12	5/3	2	3
5	6,8,10	28/3	10	2/3	2	3
6	8,9,11	10	17	7	2	3
7	12,14,16	46/3	12	10/3	2	3

7-jobs having fuzzy processing time (a, b, c) are converted in to Average High Ranking (AHR) by using equation (2) i.e. AHR = [3b + (c - a)] /3 and as per mentioned in our algorithm the near optimal schedule is**:**

S = 2 > 5 > 1 > 4 > 7 > 3 > 6.

Job	Processing Time	d	Sl_i	Cost
2	0-25	8	1/3	1/3*2
5	25/3 - 55/3	10	2/3	2/3*3
1	55/3 - 70/3	6	1	1*3
4	70/3 - 101/3	12	5/3	3/3*3
7	101/3 - 49	12	10/3	10/3*3
3	49 - 190/3	9	16/3	16/3*3
6	190/3 - 220/3	17	7	7*3

Total Penalty Cost is = **43.6**

6. CONCLUSION

We considered single machine scheduling problem (SMSP) with fuzzy processing time and fuzzy due dates.To minimizes the total penalty cost for each jobs being late (the penalty is independent of the magnitude of the lateness).We have shown how to determine the optimal schedule according to the proposed fuzzy AHR algorithm (Criteria) approach. This method is very easy to understand and will help the Decision Maker (D.M) in determining a best schedule for a given sets of jobs effectively to control penalty cost and provide a situation of job schedule.

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