Do Heavy Gauge Bosons Exist?

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Abstract

Recently, transverse and longitudinal solutions were found for the $W^{\pm}$ and Z^0 bosons. They satisfy the nonlinear cubic wave equation. Here, new solutions are given which correspond to particles with mass $m'_W = \sqrt{2}\, m_W = 114$ GeV and $m'_Z = \sqrt{2}\, m_Z = 129$ GeV. This work may shed light upon the discoveries at CERN and Fermilab.

Previously, it was shown that the gauge bosons $W^{\pm}_{\mu} = (W^1_{\mu} \mp W^2_{\mu})/\sqrt{2}$ and the scalar field Φ are coupled, according to the equations [1]

$$\partial_\mu \partial^\mu W^\nu + \frac{1}{4}\, g^2 W^\nu \phi^2 \;=\; 0 \tag{1}$$

$$\partial_\mu \partial^\mu \phi - \frac{1}{4}\, g^2 W_\mu W^\mu \phi \;=\; 0 \tag{2}$$

where W_ν is either W^1_ν or W^2_ν. These equations derive from the standard electroweak Lagrangian. The unitary gauge is chosen

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 \\ \phi/\sqrt{2} \end{pmatrix} \tag{3}$$

where $\phi(x)$ is real. Traveling solutions exist which are polarized, $W^\mu(u) = \epsilon^\mu \phi(u)$, where $u = -k_\mu x^\mu$ and ϵ^μ is a real polarization vector. Such vectors are space-like, $\epsilon_\mu \epsilon^\mu = -1$, so that both (1) and (2) will be satisfied if

$$\partial_\mu \partial^\mu \phi(u) + \frac{1}{4} g^2 \phi^3(u) \;=\; k_W^2 \frac{d^2\phi}{du^2} + \frac{1}{4} g^2 \phi^3(u) \;=\; 0 \tag{4}$$

This cubic wave equation is solved by the elliptic function $\mathrm{cn}(u, \frac{1}{2})$

$$\phi(u) = \frac{2k_W}{g}\, \mathrm{cn}(-k_\mu x^\mu) \tag{5}$$

The amplitude of this traveling wave is not arbitrary, but is fixed by the value of $2k_W/g$. The coupling between Z^0 and Φ is described by similar equations

$$\partial_\mu \partial^\mu Z^\nu + \frac{1}{4}(g^2 + g'^2)\, Z^\nu \phi^2 \;=\; 0 \tag{6}$$

$$\partial_\mu \partial^\mu \phi - \frac{1}{4}(g^2 + g'^2)\, Z_\mu Z^\mu \phi \;=\; 0 \tag{7}$$

Once again, the solutions are polarized, $Z^\mu(u) = \epsilon^\mu\, \phi(u)$, where

$$\phi(u) = \frac{2k_Z}{\sqrt{g^2 + g'^2}}\, \mathrm{cn}(-k_\mu x^\mu) \tag{8}$$

There is another solution to the cubic wave equation. It takes the form of the elliptic function $\mathrm{dn}(u, m)$, with derivatives [2]

$$\frac{d\,\mathrm{dn}(u,m)}{du} = -m\,\mathrm{sn}(u,m)\mathrm{cn}(u,m) \quad (9)$$

$$\frac{d^2\mathrm{dn}(u,m)}{du^2} = -m\,\mathrm{dn}(u,m)\{1-2\,\mathrm{sn}^2(u,m)\} \quad (10)$$

Substitute $\phi(u) = a\,\mathrm{dn}(u,m)$ into (4) to find that the equation is satisfied, if the parameter $m = 2$ and $a = 2\sqrt{2}\,k_W/g$

$$\phi(u) = \frac{2\sqrt{2}\,k_W}{g}\,\mathrm{dn}(u,2) \quad (11)$$

The period of an elliptic function is well-defined, if $0 < m < 1$. The function $\mathrm{dn}(u,2)$ may be converted by means of the reciprocal formula [2]

$$\mathrm{dn}(u,2) = \mathrm{cn}(v,\frac{1}{2}) \quad (12)$$

where $v = \sqrt{2}\,u$. It follows that

$$\phi(v) = \frac{2\sqrt{2}\,k_W}{g}\,\mathrm{cn}(-\sqrt{2}\,k_\mu x^\mu) \quad (13)$$

with $W^\mu(v) = \epsilon^\mu\phi(v)$. The period of this solution is identical to that of (5), i.e., since $m = \frac{1}{2}$, the quarter-period is $K \doteq 1.85$. However, the argument and amplitude are both greater than those in (5) by the factor $\sqrt{2}$. Since k_W is directly proportional to the mass m_W, the new solution represents a charged vector boson with mass $m'_W = \sqrt{2}\,m_W$. Similarly, the Z^0 equations (6) and (7) admit the new solution

$$\phi(v) = \frac{2\sqrt{2}\,k_Z}{\sqrt{g^2+g'^2}}\,\mathrm{cn}(-\sqrt{2}\,k_\mu x^\mu) \quad (14)$$

with $Z^\mu(v) = \epsilon^\mu\,\phi(v)$. This represents a neutral vector boson of mass $m'_Z = \sqrt{2}\,m_Z$. Substitute the known values $m_W = 80.4$ GeV and $m_Z = 91.2$ GeV to find $m'_W = 114$ GeV and $m'_Z = 129$ GeV. A neutral boson has been found at CERN near 129 GeV, while Fermilab is reporting activity in the range 115-135 GeV. It remains to be seen whether charged bosons will be discovered at 114 GeV.

References

1. K. Dalton, "On the $W^{\pm}$ and Z^0 Masses", http://vixra.org/abs/1208.0041

2. M. Abramowitz and I. Stegun, *Handbook of Mathematical Functions*, (Dover, New York, 1965) p. 569. Also, http://www.dlmf.nist.gov/22