Why Baryons May Be Yang-Mills Magnetic Monopoles

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***Abstract**: We demonstrate that Yang-Mills Magnetic Monopoles naturally confine their gauge fields, naturally contain three colored fermions in a color singlet, and that mesons also in color singlets are the only particles they are allowed to emit or absorb. This makes them worthy of serious consideration as baryons.*

Introduction and Summary

The thesis of this paper is simple: magnetic monopole densities which come into existence in non-Abelian Yang-Mills gauge theory are synonymous with baryon densities. Baryons are Yang-Mills magnetic monopoles! We examine three pillars of support for this: a) Yang-Mills magnetic monopoles naturally confine their gauge fields (section 1); b) they naturally contain exactly three fermions which we identify with colored quarks in a color-neutral singlet (sections 2 and 3); and c) the only particles crossing their surface or observed as decay products are mesons also in color-neutral singlets (section 4). Section 5 makes brief concluding remarks about chiral properties of these proposed monopole baryons, for further development which may provide touchstones for experimental validation.

1. Gauge Field Confinement

First, we demonstrate how Yang-Mills magnetic monopoles naturally confine their gauge fields. We use the language of differential forms, and assume the reader has sufficient familiarity so no tutorial explanations are required.

In an Abelian (commuting field) gauge theory such as QED, the field strength tensor *F* is specified in relation to the vector potential gauge field (e.g., photon) *A* according to $F = dA$. The magnetic monopole source density *P* is then specified classically (for high-action $S(\varphi) = \int d^4x \mathfrak{L}(\varphi) >> \hbar$ where the Euler Lagrange equation may be applied) by the field equation $P = dF = ddA = 0$. This makes use of the geometric law that the exterior derivative of an exterior derivative is zero, $dd = 0$. In integral form, this becomes $\iiint P = \iiint dF = \iiint ddG = \oiint F = \oiint dA = 0$. All of the foregoing "zeros" are what tell us that there are no magnetic monopoles in an Abelian gauge theory such as QED. This absence of magnetic monopole charges at all attainable experimental energies is well borne out in the 140 or so years since James Clerk Maxwell published his 1873 *A Treatise on Electricity and Magnetism.*

In a non-Abelian (non-commuting field) Yang-Mills gauge theory such as QCD, the fundamental difference is that the field strength tensor *F* is now specified in relation to the vector potential gauge field G (e.g., gluon in QCD) according to $F = dG - iG^2$. In this relationship, $G^2 = [G^\mu, G^\nu] dx_\mu dx_\nu$ expresses the non-commuting nature of the gauge fields and the non-linearity of Yang-Mills gauge theory. Therefore, although $ddG = 0$ as always because of the exterior geometry, the classical (high-action) magnetic monopole density becomes $P = dF = d\left(dG - iG^2\right) = -idG^2$, which is non-zero. In integral form, using Gauss'/ Stokes' law, this becomes:

$$\iiint P = \iiint dF = \iiint d\left(dG - iG^2\right) = -i\iiint dG^2 = \oiint F = \oiint dG - i\oiint G^2 = -i\oiint G^2 \ , \tag{1.1}$$

and from the last two terms in the above, we may also derive the companion equation:

$$\oiint dG = 0 \,. \tag{1.2}$$

Of course, (1.2), albeit with the different field name, is just the relationship $\oiint dA = 0$ which tells us that there are no magnetic monopoles in Abelian gauge theory. But in light of (1.1), which provides us with a non-zero magnetic monopole $\iiint P = -i\oiint G^2 \neq 0$, what can we learn from (1.2), which is the Yang-Mills analogue to the Abelian "no magnetic monopole" relationship $\oiint dA = 0$?

If we perform a local transformation $F \rightarrow F' = F - dG$ on the field strength F, which in expanded form is written as $F^{\mu\nu} \rightarrow F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$, then we find from (1.1) as a direct and immediate result of the Abelian "no monopole" relationship $\oiint dG = 0$ in (1.2), that:

$$\iiint P = \oiint F \rightarrow \oiint F' = \oiint (F - dG) = \oiint F \,. \tag{1.3}$$

This means that the flow of the field strength $\oiint F = -i\oiint G^2$ across a two dimensional surface is invariant under the local gauge-like transformation $F^{\mu\nu} \rightarrow F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$. We know in QED that invariance under the similar transformation $A^{\mu} \rightarrow A^{\mu}{}' = A^{\mu} + \partial^{\mu}\Lambda$ means the gauge parameter Λ is not a physical observable. We know in gravitational theory that invariance under $g^{\mu\nu} \rightarrow g^{\mu\nu}{}' = g^{\mu\nu} + \partial^{\{\mu}\Lambda^{\nu\}}$ likewise means the gauge vector Λ^{ν} is not a physical observable. In this case, the invariance of $\oiint F$ under the transformation $F^{\mu\nu} \rightarrow F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$ tells us the gauge field G^{μ} is not an observable over the surface through which the field $\oiint F = -i\oiint G^2$ is flowing. But G^{μ} is simply the gauge field, which in QED, is the gluon field. So, simply put: <u>the Yang-Mills gauge fields G^{μ}, including gluons in $SU(3)_C$, are not observables across any closed surface surrounding a magnetic monopole density *P*</u>. Whatever goes on inside the volume represented by $\iiint P$, the gauge fields remain confined.

Taking this a step further, we see that the origins of this gauge field confinement rest in the 140-year old mystery as to why there are no magnetic monopoles in Abelian gauge theory. In differential forms, the statement of this is $ddG = 0$. In integral form, this becomes $\oiint dG = 0$, equation (1.2). Yet it is precisely this same "zero" which renders $\oiint F \rightarrow \oiint F' = \oiint F$ invariant under $F^{\mu\nu} \rightarrow F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$ in (1.3). So the physical observation that there are no magnetic monopoles in Abelian gauge theory translates into a symmetry condition in non-Abelian gauge theory that gauge boson flow is not an observable over the surface of a magnetic charge. Again: In Abelian gauge theory there are no magnetic monopoles. In non-Abelian theory, this absence of Abelian magnetic monopoles translates into there being no flow of gauge bosons (e.g., gluons) across any closed surface surrounding a Yang-Mills magnetic monopole. Consequently, *the absence of gluon flux, hence color, across surfaces surrounding non-Abelian chromo-magnetic monopoles is fundamentally equivalent to the absence of magnetic monopoles in Abelian gauge theory.* And, because this is turn originates in $dd=0$, we see that *this confinement is geometrically mandated, imposed by spacetime*. The very same "zero" which in Abelian gauge theory says that there are no magnetic monopoles, in non-Abelian gauge theory says that there is no observable flux of Yang-Mills gauge fields across a closed surface surrounding a Yang-Mills magnetic monopole. We do not find a free gluon in Yang-Mills gauge theory any more than we find an Abelian magnetic monopole in electrodynamics, for identical geometric reasons.

2. Natural Three-Fermion System: Part I

While color confinement is necessary prerequisite for Yang-Mills magnetic monopoles to be considered as baryon "candidates," it is not sufficient. At minimum, we must also show that these monopoles are capable of naturally containing three fermions in suitable color eigenstates, because we know that baryons contain three colored quarks.

For this purpose, we employ the classical field equations ($D^\mu \equiv \partial^\mu - iG^\mu$):

$$J^\nu = \partial_\mu F^{\mu\nu} = \partial_\mu D^{[\mu}G^{\nu]} = \partial_\mu D^\mu G^\nu - \partial_\mu D^\nu G^\mu = \left(g^{\mu\nu}\partial_\sigma D^\sigma - \partial^\mu D^\nu\right)G_\mu \tag{2.1}$$

$$P^{\sigma\mu\nu} = \partial^\sigma F^{\mu\nu} + \partial^\mu F^{\nu\sigma} + \partial^\nu F^{\sigma\mu} \tag{2.2}$$

together with the Yang-Mills field strength tensor:

$$F^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu - i\left[G^\mu, G^\nu\right] = D^\mu G^\nu - D^\nu G^\mu = D^{[\mu}G^{\nu]} \tag{2.3}$$

where the group generators T^i are related by the group structure $f^{ijk}T_i = -i\left[T^j, T^k\right]$, and where $F^{\mu\nu} \equiv T^i F_i^{\mu\nu}$ and $G^\mu \equiv T^i G_i^{\mu}$ are NxN matrices for any given SU(N). Above, (2.2) and (2.3) respectively are just expanded restatements of the classical field relationships $P = dF$ and $F = dG - iG^2$ which we used in (1.1).

As soon as one substitutes the non-Abelian (2.3) into Maxwell's equation (2.2), while the terms based on $\partial^\mu G^\nu - \partial^\nu G^\mu$ continue to zero out by identity in the usual way (via $dd=0$ which as shown in section 1 confines the gauge fields), one nonetheless arrives at a residual non-zero magnetic charge:

$$\begin{aligned} P^{\sigma\mu\nu} &= -i\left(\partial^\sigma\left[G^\mu, G^\nu\right] + \partial^\mu\left[G^\nu, G^\sigma\right] + \partial^\nu\left[G^\sigma, G^\mu\right]\right) \\ &= -i\left(\left[\partial^\sigma G^\mu, G^\nu\right] + \left[G^\mu, \partial^\sigma G^\nu\right] + \left[\partial^\mu G^\nu, G^\sigma\right] + \left[G^\nu, \partial^\mu G^\sigma\right] + \left[\partial^\nu G^\sigma, G^\mu\right] + \left[G^\sigma, \partial^\nu G^\mu\right]\right) \end{aligned} \tag{2.4}$$

This is a longhand version of $P = -idG^2 = -2idG$ used in (1.1). Let's now study this $P^{\sigma\mu\nu}$ closely.

To begin, we make use of the commutator relationship $\partial^\sigma G^\mu = i\left[k^\sigma, G^\mu\right]$ to replace the various $\partial^\sigma G^\mu$ in (2.4). Expanding, $G^\mu k^\sigma G^\nu - G^\mu k^\sigma G^\nu$ appears throughout, so these terms drop out. Re-consolidating yields:

$$P^{\sigma\mu\nu} = -\left(\left[\left[G^{\mu}, G^{\nu}\right], k^{\sigma}\right] + \left[\left[G^{\nu}, G^{\sigma}\right], k^{\mu}\right] + \left[\left[G^{\sigma}, G^{\mu}\right], k^{\nu}\right]\right). \tag{2.5}$$

Now, we seek an inverse relation $G_{\nu} \equiv I_{\sigma\nu} J^{\sigma}$ to replace each G^{μ} above with a J^{μ}, which can then be used to introduce fermion wavefunctions via $J^{\mu} = \overline{\psi}\gamma^{\mu}\psi$. Again using $\partial^{\sigma} G^{\mu} = i\left[k^{\sigma}, G^{\mu}\right]$, inverse $I_{\sigma\nu}$ is specified in terms of a $\mu \leftrightarrow \sigma$ symmetrized configuration space operator $g^{\mu\nu}\partial_{\sigma} D^{\sigma} - \partial^{\mu} D^{\nu}$ in (2.1), with a hand-added Proca mass, by:

$$I_{\sigma\nu}\left(-g^{\mu\sigma}\left(k^{\alpha}k_{\alpha} + i\left[k^{\alpha}, G_{\alpha}\right] - m^{2}\right) + k^{\mu}k^{\sigma} + \tfrac{1}{2} i\left[k^{\{\mu}, G^{\sigma\}}\right]\right) = \delta^{\mu}{}_{\nu}. \tag{2.6}$$

We also use a $\sigma \leftrightarrow \nu$ symmetrized $I_{\sigma\nu} \equiv A g_{\sigma\nu} + B k_{\sigma} k_{\nu} + \tfrac{1}{2} C i\left[k_{\{\sigma}, G_{\nu\}}\right]$ to calculate $I_{\sigma\nu}$. In doing so, we keep in mind that the G^{σ} is an NxN matrix for the Yang-Mills gauge group SU(N), so anytime G^{σ} appears in a denominator we must actually form a *Yang-Mills matrix inverse*. So that expressions we develop have a similar "look" to familiar expressions from QED, we will use a "quoted denominator" notation $1/"M" \equiv M^{-1}$ to designate a Yang-Mills matrix inverse. Thus, $G^{\sigma^{-1}} = 1/"G^{\sigma}"$, etc. This inverse is calculated to be:

$$I_{\sigma\nu} = \frac{-g_{\sigma\nu} + \dfrac{k_{\sigma}k_{\nu} + \frac{1}{2} i\left[k_{\{\sigma}, G_{\nu\}}\right]}{"m^{2} - k^{\alpha}k_{\alpha} - i\left[k^{\alpha}, G_{\alpha}\right]"}}{"k^{\alpha}k_{\alpha} - m^{2} + i\left[k^{\alpha}, G_{\alpha}\right]"}, \tag{2.7}$$

and can only be formed if we simultaneously impose the covariant gauge condition, in configuration space:

$$\left(\partial_{\sigma}\partial_{\nu} - \tfrac{1}{2}\partial_{\{\sigma} G_{\nu\}}\right)\left(\partial^{\mu}\partial^{\sigma} - \tfrac{1}{2}\partial^{\{\mu} G^{\sigma\}}\right) = 0. \tag{2.8}$$

Note that the often-employed $i\left[k^{\sigma}, G_{\sigma}\right] = \partial^{\sigma} G_{\sigma} = 0$ is not a gauge condition here; this is replaced by (2.8).

Now, inverse (2.7) has many interesting properties which we shall not take the time to explore here. Special cases of interest include $i\left[k_{\nu}, G_{\sigma}\right] = \partial_{\nu} G_{\sigma} \to 0$; $m = 0$; both $\partial_{\nu} G_{\sigma} \to 0$ and $m = 0$; and on shell $k^{\alpha}k_{\alpha} - m^{2} = 0$ for $m \neq 0$ or $k^{\alpha}k_{\alpha} = 0$ for $m = 0$. We will also note that when working towards a quantum path integral formulation, $i\left[k^{\sigma}, G_{\sigma}\right] = \partial^{\sigma} G_{\sigma}$ in (2.7) is replace by a gauge-invariant perturbation $-V = \left(\partial^{\sigma} G_{\sigma} + G_{\sigma}\partial^{\sigma}\right) + G^{\sigma} G_{\sigma}$. But our interest at the moment is in the low-perturbation limit $i\left[k_{\nu}, G_{\sigma}\right] = \partial_{\nu} G_{\sigma} \to 0$. Thus, using (2.7) in inverse relation $G_{\nu} = I_{\sigma\nu} J^{\sigma}$ with $i\left[k_{\nu}, G_{\sigma}\right] = 0$, all the quoted denominators become ordinary denominators, and we obtain:

$$G_{\nu} = -\frac{g_{\sigma\nu}}{k^{\alpha}k_{\alpha} - m^{2}} J^{\sigma}. \tag{2.9}$$

We have reduced this using the fact that in momentum space, current conservation $\partial_{\mu} J^{\mu}(x) = 0$ becomes $k_{\mu} J^{\mu}(k) = 0$ (see [1] after I.5(4)). The above is just like the expressions we encounter for inverses with a Proca mass in QED. It says, not unexpectedly, that in the low-perturbation limit, QCD looks like QED.

The point of developing this inverse, is to be able to use (2.9) in (2.5) and then deploy fermion wavefunctions via $J^{\mu} = \overline{\psi}\gamma^{\mu}\psi$. Because (2.5) contains six different appearances of G_{ν}, there are six independent substitutions of

(2.9) into (2.5), and what we must presume to be six independent Proca masses *m*. To track this, we will use the first six letters of the Greek alphabet $\alpha,\beta,\gamma,\delta,\varepsilon,\zeta$ to carry out the internal index summations and to label each of these six Proca masses. This substitution yields:

$$P^{\sigma\mu\nu}=-\left[\left[\frac{g^{\alpha\mu}J_\alpha}{k^\alpha k_\alpha-m_{(\alpha)}{}^2},\frac{g^{\nu\beta}J_\beta}{k^\beta k_\beta-m_{(\beta)}{}^2}\right],k^\sigma\right]-\left[\left[\frac{g^{\gamma\nu}J_\gamma}{k^\gamma k_\gamma-m_{(\gamma)}{}^2},\frac{g^{\sigma\delta}J_\delta}{k^\delta k_\delta-m_{(\delta)}{}^2}\right],k^\mu\right]-\left[\left[\frac{g^{\varepsilon\sigma}J_\varepsilon}{k^\varepsilon k_\varepsilon-m_{(\varepsilon)}{}^2},\frac{g^{\mu\zeta}J_\zeta}{k^\zeta k_\zeta-m_{(\zeta)}{}^2}\right],k^\nu\right]. \quad (2.10)$$

Here, we see six massive vector boson propagators each coupled with a current vector J_α. We raise the indexes on all the currents and absorb the $g^{\alpha\mu}$. We use $J^\mu=T^iJ_i{}^\mu$, $i=1,2,3...N^2-1$ to explicitly introduce the SU(N) generators. We factor out the resulting commutators $\left[T^i,T^j\right]$. And finally, we employ $J_i{}^\mu=\overline{\psi}T_i\gamma^\mu\psi$ and the like to introduce fermion wavefunctions. With this, and moving all currents into the same numerator, (2.10) becomes:

$$P^{\sigma\mu\nu}=-\left[T^i,T^j\right]\left(\begin{array}{l}\left[\left(\dfrac{1}{k^\alpha k_\alpha-m_{(\alpha)}{}^2}\dfrac{\overline{\psi}T_i\gamma^\mu\psi\overline{\psi}T_j\gamma^\nu\psi}{k^\beta k_\beta-m_{(\beta)}{}^2}\right),k^\sigma\right]\\ +\left[\left(\dfrac{1}{k^\gamma k_\gamma-m_{(\gamma)}{}^2}\dfrac{\overline{\psi}T_i\gamma^\nu\psi\overline{\psi}T_j\gamma^\sigma\psi}{k^\delta k_\delta-m_{(\delta)}{}^2}\right),k^\mu\right]\\ +\left[\left(\dfrac{1}{k^\varepsilon k_\varepsilon-m_{(\varepsilon)}{}^2}\dfrac{\overline{\psi}T_i\gamma^\sigma\psi\overline{\psi}T_j\gamma^\mu\psi}{k^\zeta k_\zeta-m_{(\zeta)}{}^2}\right),k^\nu\right]\end{array}\right). \quad (2.11)$$

The above now shows fermion wavefunctions, and is the starting point for the next stage of development.

3. Natural Three-Fermion System: Part II

Now, we make the following sequence of substitutions for $\overline{\psi}T_i\gamma^\mu\psi\overline{\psi}T_j\gamma^\nu\psi/\left(k^\beta k_\beta-m_{(\beta)}{}^2\right)$ and the other two like terms in (2.11) above:

$$\begin{aligned}\frac{\overline{\psi}T_i\gamma^\mu\psi\overline{\psi}T_j\gamma^\nu\psi}{k^\beta k_\beta-m_{(\beta)}{}^2}&=\frac{\overline{\psi}T_i\gamma^\mu\sum u\overline{u}T_j\gamma^\nu\psi}{k^\beta k_\beta-m_{(\beta)}{}^2}=\frac{\left|N^2\right|}{E+m}\frac{\overline{\psi}T_i\gamma^\mu\left(\not{p}+m\right)T_j\gamma^\nu\psi}{k^\beta k_\beta-m_{(\beta)}{}^2}=\frac{\left|N_{(\beta)}{}^2\right|}{E_{(\beta)}+m_{(\beta)}}\frac{\overline{\psi}T_i\gamma^\mu\left(\not{p}_{(\beta)}+m_{(\beta)}\right)T_j\gamma^\nu\psi}{p^\beta p_\beta-m_{(\beta)}{}^2}\\ &=\frac{\left|N_{(\beta)}{}^2\right|}{E_{(\beta)}+m_{(\beta)}}\frac{\overline{\psi}T_i\gamma^\mu T_j\gamma^\nu\psi}{"\not{p}_{(\beta)}-m_{(\beta)}"}=\frac{\left|N_{(\beta)}{}^2\right|}{E_{(\beta)}+m_{(\beta)}}\overline{\psi}T_i\gamma^\mu T_j\gamma^\nu\psi\times\left(\not{p}_{(\beta)}-m_{(\beta)}\right)^{-1}\end{aligned}. \quad (3.1)$$

Let us now explain each step in the sequence. In the first step, we use the Dirac spinors $u\overline{u}=\psi\overline{\psi}$ and sum over spin states. Often, the spin sum is written as $\sum_{spins}u\overline{u}=\not{p}+m$ (see [2], section 5.5). But there is an implied covariant normalization $\left|N^2\right|=E+m$ in this expression. To be explicit, this should really be written ([2], problem solution 5.9):

$$\sum_{spins}u\overline{u}=\left|N^2\right|\left(\not{p}+m\right)/\left(E+m\right), \quad (3.2)$$

So in the second step, we apply (3.2) in (3.1).

Next, we take the *affirmative step* (which as we will shortly discuss requires some accounting for degrees of freedom that will render the gauge bosons massless) of identifying the rest mass in the resultant $\not{p}+m$ with the labeled mass $m_{(\beta)}$ in the denominator, so we now set $m=m_{(\beta)}$. This $m_{(\beta)}$, of course, started out in (2.10) as a gauge boson mass in a gauge boson propagator denominator, but by this step we turn it into a fermion rest mass. And we simultaneously promote $k^\beta \to p^\beta$ into the momentum four-vector p^β for this fermion with mass. And, we label $E=E_{(\beta)}$ and $N=N_{(\beta)}$. Finally, we use the well-known relationship:

$$\frac{\not{p}_{(\beta)}+m_{(\beta)}}{p^\beta p_\beta - m_{(\beta)}{}^2} = \frac{1}{"\not{p}_{(\beta)}-m_{(\beta)}"} = \left(\not{p}_{(\beta)}-m_{(\beta)}\right)^{-1}, \tag{3.3}$$

but employ an inverse in recognition of the fact that whenever an SU(N) matrix (including the $\sum u\bar{u} = \not{p}+m$) needs to go into a "denominator," we must form its inverse. Thus, applying (3.1) to (2.11) yields:

$$P^{\sigma\mu\nu} = -\left[T^i,T^j\right]\left(\begin{array}{l} \left[\left(\dfrac{1}{k^\alpha k_\alpha - m_{(\alpha)}{}^2}\dfrac{\left|N_{(\beta)}{}^2\right|}{E_{(\beta)}+m_{(\beta)}}\dfrac{\bar{\psi}T_i\gamma^\mu T_j\gamma^\nu\psi}{"\not{p}_{(\beta)}-m_{(\beta)}"}\right),k^\sigma\right] \\ +\left[\left(\dfrac{1}{k^\gamma k_\gamma - m_{(\gamma)}{}^2}\dfrac{\left|N_{(\delta)}{}^2\right|}{E_{(\delta)}+m_{(\delta)}}\dfrac{\bar{\psi}T_i\gamma^\nu T_j\gamma^\sigma\psi}{"\not{p}_{(\delta)}-m_{(\delta)}"}\right),k^\mu\right] \\ +\left[\left(\dfrac{1}{k^\varepsilon k_\varepsilon - m_{(\varepsilon)}{}^2}\dfrac{\left|N_{(\zeta)}{}^2\right|}{E_{(\zeta)}+m_{(\zeta)}}\dfrac{\bar{\psi}T_i\gamma^\alpha T_j\gamma^\mu\psi}{"\not{p}_{(\zeta)}-m_{(\zeta)}"}\right),k^\nu\right] \end{array}\right). \tag{3.4}$$

But there is one final piece of the puzzle that is required to make this all work properly. We must balance the degrees of freedom used to turn (2.11) into (3.4), and in particular, to turn boson rest masses into fermion rest masses. In (2.10), we started with six vector bosons with presumed Proca masses $m_{(\alpha)}, m_{(\beta)}, m_{(\gamma)}, m_{(\lambda)}, m_{(\varepsilon)}, m_{(\zeta)}$. A massive vector boson has three degrees of freedom, so the six bosons in (2.10) brought 3x6=18 degrees of freedom into $P^{\sigma\mu\nu}$. But then we took three of those boson masses and turned them into fermion masses. Massive fermions, however, have <u>four</u> degrees of freedom, not three. So to promote a massive boson mass into a fermion mass, we must transfer one degree of freedom over from the boson to the fermion. So these bosons must drop down to <u>two</u> degrees of freedom apiece and thus become massless, i.e., that we must now set these to zero, $m_{(\alpha)}, m_{(\gamma)}, m_{(\varepsilon)} = 0$. Now, the 18 degrees of freedom that initially belonged three apiece to six massive vector bosons have been redistributed: 12 of these now belong to the 3 fermions, and only 6 belong to the 3 remaining bosons. This should seem very familiar, as this is the same way in which massless gauge bosons first become massive by swallowing a degree of freedom from a scalar field via the Goldstone mechanism. Here, fermions swallow a degree of freedom from bosons.

Looking closely at (3.4), we now also see a path to choosing normalizations for N which simultaneously are covariant, retain the original mass dimensionality of +3 for $\overline{u}u$, and greatly simplify (3.4). Specifically, we now *choose* the covariant, mass dimension-preserving normalizations:

$$\left|N_{(\beta)}{}^2\right| = \left(E_{(\beta)} + m_{(\beta)}\right)k^\alpha k_\alpha; \quad \left|N_{(\delta)}{}^2\right| = \left(E_{(\delta)} + m_{(\delta)}\right)k^\gamma k_\gamma; \quad \left|N_{(\zeta)}{}^2\right| = \left(E_{(\zeta)} + m_{(\zeta)}\right)k^\varepsilon k_\varepsilon . \tag{3.5}$$

Using these together with $m_{(\alpha)}, m_{(\gamma)}, m_{(\varepsilon)} = 0$ in (3.4) yields the vastly simplified:

$$P^{\sigma\mu\nu} = -\left[T^i, T^j\right]\left(\left[\frac{\overline{\psi}T_i\gamma^\mu T_j\gamma^\nu\psi}{"\not{p}_{(\beta)} - m_{(\beta)}"}, k^\sigma\right] + \left[\frac{\overline{\psi}T_i\gamma^\nu T_j\gamma^\sigma\psi}{"\not{p}_{(\delta)} - m_{(\delta)}"}, k^\mu\right] + \left[\frac{\overline{\psi}T_i\gamma^\alpha T_j\gamma^\mu\psi}{"\not{p}_{(\zeta)} - m_{(\zeta)}"}, k^\nu\right]\right). \tag{3.6}$$

Proceeding apace, the commutator $\left[T^i, T^j\right]$ operates to commute the vertices $\left(T_i\gamma^\mu\right)\left(T_j\gamma^\nu\right)$, and in particular, the operation it performs is $\left[T^i, T^j\right]\overline{\psi}\left(T_i\gamma^\mu\right)\left(T_j\gamma^\nu\right)\psi = \overline{\psi}\left[\gamma^\mu, \gamma^\nu\right]\psi$. This is the same commutation $\left[G^\mu, G^\nu\right]$ of free indexes μ, ν with which everything started back in (2.5), and even further back, in the underlying field density $F^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu - i\left[G^\mu, G^\nu\right]$ of (2.3) which is the heart of Yang-Mills theory. So, (3.6) now becomes:

$$P^{\sigma\mu\nu} = -\left(\left[\frac{\overline{\psi}\left[\gamma^\mu, \gamma^\nu\right]\psi}{"\not{p}_{(\beta)} - m_{(\beta)}"}, k^\sigma\right] + \left[\frac{\overline{\psi}\left[\gamma^\nu, \gamma^\sigma\right]\psi}{"\not{p}_{(\delta)} - m_{(\delta)}"}, k^\mu\right] + \left[\frac{\overline{\psi}\left[\gamma^\sigma, \gamma^\mu\right]\psi}{"\not{p}_{(\zeta)} - m_{(\zeta)}"}, k^\nu\right]\right). \tag{3.7}$$

All that now remains in (3.7) is the final commutator with momentum terms such as k^σ. Going back to $\partial^\sigma G^\mu = i\left[k^\sigma, G^\mu\right]$ which tells us that commuting a spacetime field with k^σ is just a clever way to take its derivatives, we can similarly write $\partial^\sigma M^{\mu\nu} = i\left[k^\sigma, M^{\mu\nu}\right]$ for a second rank tensor field $M^{\mu\nu}(x^\sigma)$. So, if we also make use of the second rank Dirac covariant $-2i\sigma^{\mu\nu} = \left[\gamma^\mu, \gamma^\nu\right]$, and also relabel $\beta \to R, \delta \to G, \zeta \to B$ with similar labeling of the associated wavefunctions, (3.7) now becomes:

$$P^{\sigma\mu\nu} = -2\left(\partial^\sigma \frac{\overline{\psi}_R\sigma^{\mu\nu}\psi_R}{"\not{p}_R - m_R"} + \partial^\mu \frac{\overline{\psi}_G\sigma^{\nu\sigma}\psi_G}{"\not{p}_G - m_G"} + \partial^\nu \frac{\overline{\psi}_B\sigma^{\sigma\mu}\psi_B}{"\not{p}_B - m_B"}\right). \tag{3.8}$$

Now let’s explain what we have done. We deduce leading to (3.8) that a Yang-Mills magnetic monopole density naturally contains three fermion wavefunctions ψ and related propagators. But for any SU(N), these ψ are N-component column vectors. So because there are three ψ, we introduce the $SU(3)_C$ gauge group of QCD with group generators $T^i = \lambda^i; i = 1...8$ normalized to $tr\left(\lambda^i\right)^2 = \frac{1}{2}$, and associate each of the three ψ with a quark in an eigenstate of color. Thus, $\psi_R \equiv \left|R\right\rangle \equiv \begin{pmatrix}1 & 0 & 0\end{pmatrix}^T = \left|\lambda^8 = \frac{1}{\sqrt{3}}; \lambda^3 = 0\right\rangle$, $\psi_G \equiv \left|G\right\rangle \equiv \begin{pmatrix}0 & 1 & 0\end{pmatrix}^T = \left|\lambda^8 = -\frac{1}{2\sqrt{3}}; \lambda^3 = \frac{1}{2}\right\rangle$ and $\psi_B \equiv \left|B\right\rangle \equiv \begin{pmatrix}0 & 0 & 1\end{pmatrix}^T = \left|\lambda^8 = -\frac{1}{2\sqrt{3}}; \lambda^3 = -\frac{1}{2}\right\rangle$. This simultaneously forces exclusion so that no two quarks in this system have the exact same quantum numbers. If one then associates each color eigenstate with the spacetime index in the related ∂^σ operator in (3.8), i.e., $\sigma \sim R$, $\mu \sim G$ and $\nu \sim B$, and keeps in mind that $P^{\sigma\mu\nu}$ is antisymmetric in all indexes, then we may express this antisymmetry with wedge products as $\sigma \wedge \mu \wedge \nu \sim R \wedge G \wedge B$. So the natural antisymmetry of the magnetic monopole $P^{\sigma\mu\nu}$ (compare the top line of (2.4)) leads straight to the required antisymmetric color singlet wavefunction configuration $R[G,B]+G[B,R]+B[R,G]$ for a baryon (see [2] equation [2.70]). And, because of what we did to get to the fermion masses in (3.4), we are required to keep the SU(3)$_C$ gauge bosons massless, just as is also required in QCD. The above (3.8), which also confines its gauge fields as developed in section 1, we therefore interpret as a baryon.

Finally, by contrasting (3.8) with (2.4), we see that the field commutator $\left[G^\mu, G^\nu\right]$ at the heart of Yang-Mills theory in the field strength $F^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu - i\left[G^\mu, G^\nu\right]$ of (2.4), has now turned into three Dirac tensors:

$$\frac{i}{2}\left[G^\mu, G^\nu\right] = \frac{\overline{\psi}_R \sigma^{\mu\nu} \psi_R}{"\not p_R - m_R"}; \quad \frac{i}{2}\left[G^\nu, G^\sigma\right] = \frac{\overline{\psi}_G \sigma^{\nu\sigma} \psi_G}{"\not p_G - m_G"}; \quad \frac{i}{2}\left[G^\sigma, G^\mu\right] = \frac{\overline{\psi}_B \sigma^{\sigma\mu} \psi_B}{"\not p_B - m_B"}. \tag{3.9}$$

4. Mesons

Another hallmark of hadron interaction are mesons, so now let's look for those. A meson is a particle / antiparticle (conjugate) pair, so let's start with the well-known Dirac conjugation relationships $\psi_C = C\overline{\psi}^T$, $\overline{\psi}_C = -\psi^T C^{-1}$, $\gamma^\nu C = C\left(-\gamma^\nu\right)^T$, and $C^{-1}\gamma^\mu C = \left(-\gamma^\mu\right)^T$. One may deduce from these, that:

$$\overline{\psi}_C \gamma^\mu \gamma^\nu \psi_C = -\psi^T C^{-1} \gamma^\mu \gamma^\nu C \overline{\psi}^T = -\psi^T C^{-1} \gamma^\mu C \left(-\gamma^\nu\right)^T \overline{\psi}^T = -\psi^T \left(-\gamma^\mu\right)^T \left(-\gamma^\nu\right)^T \overline{\psi}^T = -\overline{\psi} \gamma^\nu \gamma^\mu \psi . \tag{4.1}$$

Specifically, $\overline{\psi}_C \gamma^\mu \gamma^\nu \psi_C = -\overline{\psi} \gamma^\nu \gamma^\mu \psi$ simultaneously commutes indexes and reverses sign.

So using $-2i\sigma^{\mu\nu} = \left[\gamma^\mu, \gamma^\nu\right]$ plus the well-known identity $\gamma^\mu \gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}$ derived from combining the fundamental Dirac relationships $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$ and $g^{\mu\nu} = \frac{1}{2}\left\{\gamma^\mu, \gamma^\nu\right\}$, we may decompose one of the numerator terms for the antisymmetric Dirac tensors in (3.8) or (3.9) into:

$$\begin{aligned} \overline{\psi}\sigma^{\mu\nu}\psi &= \tfrac{i}{2}\overline{\psi}\gamma^\mu\gamma^\nu\psi - \tfrac{i}{2}\overline{\psi}\gamma^\nu\gamma^\mu\psi = \tfrac{i}{2}\overline{\psi}\gamma^\mu\gamma^\nu\psi + \tfrac{i}{2}\overline{\psi}_C\gamma^\mu\gamma^\nu\psi_C \\ &= \tfrac{1}{2}\left(\overline{\psi}\sigma^{\mu\nu}\psi + \overline{\psi}_C\sigma^{\mu\nu}\psi_C\right) + \tfrac{i}{2} g^{\mu\nu}\left(\overline{\psi}\psi + \overline{\psi}_C\psi_C\right) \end{aligned} . \tag{4.2}$$

Because $\overline{\psi}\sigma^{\mu\nu}\psi$ is a tensor and $\overline{\psi}_C\sigma^{\mu\nu}\psi_C$ is a conjugate tensor, the additive combination $\overline{\psi}\sigma^{\mu\nu}\psi + \overline{\psi}_C\sigma^{\mu\nu}\psi_C$ may be understood as a *tensor meson*, that is, as a meson particle for which, e.g., $n^{2s+1}l_J = n^3P_2$ with $J^{PC} = 2^{++}$ in which the parallel intrinsic spins totaling $s=1$ are parallel to the orbital excitation $l=1$. Alternatively, this may be $n^{2s+1}l_J = n^3F_2$ also with $J^{PC} = 2^{++}$ in which the $s=1$ is antiparallel to the stretched orbital excitation $l=3$. Further, $\overline{\psi}\psi$ is a scalar and $\overline{\psi}_C\psi_C$ is a conjugate scalar, so the additive combination $\overline{\psi}\psi + \overline{\psi}_C\psi_C$ may be understood as a *scalar meson*, for which $n^{2s+1}l_J = n^3P_0$ with $J^{PC} = 0^{++}$. So, by virtue of (4.2) and (3.9), we see that the commutator $\frac{i}{2}\left[G^{\mu},G^{\nu}\right]$ actually contains a tensor meson plus a scalar meson! (See [3], [4] for a full exposition of experimentally-observed mesons and their spin classifications as scalars, vectors, tensors, etc. and axial variants.)

In fact, contrasting with $\iiint P = \oiint F = -i\oiint G^2$ from (1.1) and noting that $P = P^{\sigma\mu\nu} dx_{\sigma} dx_{\mu} dx_{\nu}$, let us multiply both sides of (3.8) by the anticommuting volume element $dx_{\sigma} dx_{\mu} dx_{\nu}$, take the triple integral, then apply Gauss' / Stokes law to the right hand side and rename indexes. What we get is:

$$\iiint P = \iiint P^{\sigma\mu\nu} dx_{\sigma} dx_{\mu} dx_{\nu} = -2\oiint \left(\frac{\overline{\psi}_R \sigma^{\mu\nu} \psi_R}{"\not{p}_R - m_R"} + \frac{\overline{\psi}_G \sigma^{\mu\nu} \psi_G}{"\not{p}_G - m_G"} + \frac{\overline{\psi}_B \sigma^{\mu\nu} \psi_B}{"\not{p}_B - m_B"} \right) dx_{\mu} dx_{\nu} = \oiint F = -i\oiint G^2 . \qquad (4.3)$$

We showed in (1.3) that invariance of $\oiint F$ under a gauge-like transformation $F^{\mu\nu} \to F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu} G^{\mu]}$ means that there are no gauge bosons G^{μ} allowed to flow across a closed surface surrounding a Yang-Mills magnetic monopole, which means for $SU(3)_C$, its gluons are confined. So far, so good. But that tells us what cannot flow. The above (4.3) tells us what *can and does* flow. What is allowed to flow across any boundary, are spin 2 tensors $\overline{\psi}\sigma^{\mu\nu}\psi$, which via (4.2) may be decomposed into tensor mesons and scalar mesons. Moreover, the Gaussian integration has removed the ∂^{σ} operators, and what remains by inspection in (4.3) is the wavefunction color configuration $\overline{R}R + \overline{G}G + \overline{B}B$, which is precisely the symmetric singlet color combination required for a meson!

So, (4.3) would seem to say that only colorless tensor and scalar mesons flow across a closed surface surrounding a magnetic monopole density *P*. However, contrasting (4.3) with (4.2), we find that the scalars drop out, $g^{\mu\nu}\left(\overline{\psi}\psi + \overline{\psi}_C \psi_C\right) dx_{\mu} dx_{\nu} = 0$, because $g^{\mu\nu}$ is a symmetric tensor while $\left\{dx_{\mu}, dx_{\nu}\right\} = 0$ are anticommuting so that $\frac{1}{2!} g^{\mu\nu} dx_{\mu} \wedge dx_{\nu} = 0$, where we show the wedge product to make this point clear. So the geometry itself acts as a filter (just as it does to confine gluons!) and shuts down the flow of scalar mesons such as $n^1 S_0$ across the boundary, and forces their confinement as well. All that may cross are spin 2 tensor mesons such as $n^3 P_2$ or $n^3 F_2$ both with 2^{++} (or any other $J = 2$ mesons that may be constructed entirely out of quarks and conjugate quarks, e.g., $qq\overline{q}\overline{q}$ with parallel spin alignments $n^5 S_2$ with 2^{-+}). Of course, after they have exited the closed surface, these tensor mesons may thereafter decay into other tensor or vector or scalar or axial meson by-products, and so be observed, as they are, when studying baryons and especially nucleons. But what (4.3) says is that to actually cross a closed surface surrounding a Yang-Mills magnetic monopole density *P*, whatever is inside the $\iiint P$ volume must first be excited into a color-neutral spin 2 tensor (or axial tensor as we shall discuss momentarily) in order to cross through the surface via $\oiint F = -i\oiint G^2$, after which the spin 2 meson may decay into other observed mesons of other spins. *"Spin 2 meson" is the "passport"* *in and out of a magnetic monopole baryon; all other passage is forbidden.* There is no coupling to the geometry in (4.3) that allows a spin 1 meson to pass, spin 1 gluons are not permitted to pass for the same reason that there are no magnetic monopoles in Abelian gauge theory, and spin 0 mesons are filtered by $g^{\mu\nu} dx_\mu \wedge dx_\nu = 0$.

5. Conclusion: Hadronic Chiral Asymmetry and Experimental Validation

We conclude with a brief comment about axial mesons, which are also widely observed in hadron physics, most notably, the $I = 1$, π pseudoscalar mesons with $n^1 S_0$ and 0^{-+}. All such axial objects involve $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ operating on a wavefunction to produce $\psi_A = \gamma^5\psi_V$, where a "vector" (V) wavefunction ψ_V is defined as a wavefunction for which the related current density $J^\mu = \overline{\psi}_V \gamma^\mu \psi_V$ transforms as a Lorentz four-vector in spacetime.

Based on combining the relationship $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ with duality based on the work of Reinich [5] later elaborated by Wheeler [6] which uses the Levi-Civita formalism (see [7] at pages 87-89), it turns out that there is a whole system of "chiral duality" that is an integral, albeit (apparently) heretofore undeveloped feature of the Dirac algebra. For example, given the duality relationship $*A^{\mu\nu} \equiv \frac{1}{2!}\varepsilon^{\mu\nu\alpha\sigma} A_{\alpha\sigma}$, one may write $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ in the alternative form $\sigma^{\mu\nu} = i*\sigma^{\mu\nu}\gamma^5$. Then, one may form $\overline{\psi}_V \sigma^{\mu\nu}\psi_V = i*\overline{\psi}_V \sigma^{\mu\nu}\psi_A$ by sandwiching between V wavefunctions. Further, it is also well known because the second rank duality operator $** = -1$, that one can form continuous (global) rotations using $e^{*\theta} = \cos\theta + *\sin\theta$. For example:

$$\begin{aligned} \overline{\psi}_V \sigma^{\mu\nu}\psi_V &\rightarrow \cos\theta \overline{\psi}_V \sigma^{\mu\nu}\psi_V + i\sin\theta \overline{\psi}_V \sigma^{\mu\nu}\psi_A \\ \overline{\psi}_V \sigma^{\mu\nu}\psi_A &\rightarrow i\sin\theta \overline{\psi}_V \sigma^{\mu\nu}\psi_V + \cos\theta \overline{\psi}_V \sigma^{\mu\nu}\psi_A \end{aligned}. \tag{5.1}$$

Similar transformations may be developed for first / third and even zeroth / fourth rank duality, with the result that tensors mix with axial tensors, vectors with axial vectors, and scalars with pseudoscalars. And, when $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is applied to (3.8) as part of a Gordon decomposition (really, recomposition) of a vector current, it turns out that baryon and meson physics is endemically, organically non-chiral, which is consistent with what is experimentally observed. The duality angle θ comes to be associated with the strength of the running strong coupling α_S, which in turn bears well-studied relationships, [8], [9] to the experimental momentum transfer *Q*.

So, by fully developing the chiral duality of Dirac's equation and applying this to (3.8), it may well become possible to experimentally confirm the thesis that Baryons are Yang-Mills magnetic monopoles: simply probe nucleons at varying energies, study the chiral characteristics of the debris that emerges from those probes, and correlate those chiral properties to the probe energies that were applied.

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