THE RADIUS OF THE PROTON IN THE SELF-CONSISTENT MODEL

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Abstract

Based on the notion of strong gravitation, acting at the level of elementary particles, and on the equality of the magnetic moment of the proton and the limiting magnetic moment of the rotating non-uniformly charged ball, the radius of the proton is found, which conforms to the experimental data. At the same time the dependence is derived of distribution of the mass and charge density inside the proton. The ratio of the density in the center of the proton to the average density is found, which equals 1.57.

Keywords: *strong gravitation; de Broglie waves; magnetic moment; proton radius.*

1 Introduction

Since the discovery of the proton in 1917 the question arose how to determine the radius of this elementary particle. There are many theoretical models to estimate the radius of the proton. Most of these models is associated with the concept of the electromagnetic form factors as the amendment by which the scattering amplitude of particles by proton is different from the scattering amplitude by a point particle. The calculation of the form factors is complex and requires taking into account many factors, including the radial density distribution of charge and magnetic moment, the dynamics of quarks, partons and virtual particles. There may be a variety of approaches – scattering theory, chiral perturbation theory, lattice QCD, etc., description of which can be found in [1], [2]. Form factors are determined from scattering experiments, depend on the energy of the interacting particles, and allow us to find the root mean square of the charge distribution and magnetic moment as a measure of particle's size. Information on the radius of the proton can be extracted from the analysis of the Lamb shift in hydrogen and in a coupled system of a proton and a negative muon [3].

2 Other estimates of proton radius

Consider some simple methods for determining the radius of the proton. One of them is based on the fact that in the particles, when they are excited, standing electromagnetic waves emerge. The maximum energy of these standing waves does not exceed the rest energy in order to avoid the decay of particles. From this it can be derived that the de Broglie waves are electromagnetic oscillations, detectable in the laboratory frame in the interaction of moving particles. To describe these oscillations it is necessary to apply the Lorentz transformations to the standing waves inside the particles and to find their form in the laboratory reference frame [4], [5].

In the simplest case the spherical standing waves are modeled by two waves, one of which runs from the center to the surface of the particle and the other at the same time is moving backwards. We can assume that in the direction of a specified axis, for example OX, there are two counter-propagating waves of the following form:

$$U_1 = U_0 \sin(\omega' t' - K' x' + \varphi_1), \qquad U_2 = U_0 \sin(\omega' t' + K' x' + \varphi_2),$$

$$U = U_1 + U_2 = 2U_0 \cos\left(K'x' - \frac{\varphi_1 - \varphi_2}{2} \right) \sin\left(\omega' t' + \frac{\varphi_1 + \varphi_2}{2} \right), \tag{1}$$

here φ_1, φ_2 are the initial phases of the oscillations with $t' = x' = 0$, U_0 is the amplitude of the periodic function, ω' and K' denote the angular frequency and wave number and the primes over the variables mean that they are considered in the rest frame of the particle.

As U any periodic function can be used, which satisfies the wave equation. For example, it can be the strength or the field potential of the wave. The phases of the waves in (1) must be shifted to π for emerging of the standing wave. If $\varphi_1 = \pi$, $\varphi_2 = 0$, then in the center of the particle with $x' = 0$ there will be always a node as the absence of visible oscillations, and (1) becomes as follows:

$$U = 2U_0 \sin(K'x') \cos(\omega' t'). \tag{2}$$

As a result of oscillations (2) velocities of charges of the particle substance and the field potentials can periodically change inside the particle. This leads inevitably to periodic oscillations of the field potentials also outside the particle in the surrounding space.

Now we shall assume that the particle moves together with its standing wave along the axis OX in the laboratory reference frame at the velocity u. How are the field oscillations modified inside and outside the particle with respect to its movement? We should express in (2) the primed coordinates and the time inside the moving particle through the coordinates and the time in the laboratory reference frame using the Lorentz transformations (c refers to the speed of light):

$$t' = \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}, \qquad x' = \frac{x - ut}{\sqrt{1 - u^2/c^2}}, \qquad y' = y, \qquad z' = z,$$

$$U = 2U_0 \sin\left(\frac{K'(x-ut)}{\sqrt{1-u^2/c^2}}\right)\cos\left(\frac{\omega'(t-ux/c^2)}{\sqrt{1-u^2/c^2}}\right). \quad (3)$$

From (3) we see that as a result of displacement of the standing wave with the particle for the external motionless observer in the laboratory frame the wavelength and the frequency will change. More precisely, on the observed wave additional antinodes appear, with a wavelength between them, differing from the wavelength $\lambda' = \frac{2\pi}{K'}$ in the reference frame of the particle. We shall stop the wave (3) for a moment with $t = 0$ and shall find the wavelengths as the spatial separation between the points of the wave in the same phase. When $x = 0$ the sine in (3) will be zero, while when $x = \lambda_1$ the phase of the sine will change from 0 to 2π. Hence we obtain:

$$\sin\left(\frac{K'\lambda_1}{\sqrt{1-u^2/c^2}}\right) = \sin(2\pi), \qquad \lambda_1 = \frac{2\pi\sqrt{1-u^2/c^2}}{K'} = \lambda'\sqrt{1-u^2/c^2}. \quad (4)$$

Similarly for the wavelength of the cosine in (3) we find:

$$\cos\left(-\frac{\omega' u\lambda_2/c^2}{\sqrt{1-u^2/c^2}}\right) = \cos(-2\pi), \qquad \lambda_2 = \frac{2\pi c^2\sqrt{1-u^2/c^2}}{\omega' u}. \quad (5)$$

We shall now estimate the temporal separation between the points of the wave in one phase with $x = 0$, considering this separation as the corresponding period of the wave:

$$\sin\left(-\frac{K'uT_1}{\sqrt{1-u^2/c^2}}\right) = \sin(-2\pi), \qquad T_1 = \frac{2\pi\sqrt{1-u^2/c^2}}{K'u}. \quad (6)$$

$$\cos\left(\frac{\omega' T_2}{\sqrt{1-u^2/c^2}}\right)=\cos(2\pi), \qquad\qquad T_2 = T'\sqrt{1-u^2/c^2}\,. \qquad (7)$$

From (4) − (7) we obtain the following expressions for the velocities:

$$\frac{\lambda_1}{T_1}=u\,, \qquad\qquad \frac{\lambda_2}{T_2}=\frac{c^2}{u}=\upsilon_b\,. \qquad (8)$$

As we see from (8) the oscillations of the wave (3) associated with the cosine, are propagating at the phase velocity of de Broglie υ_b. Besides, the oscillations of the wave (3) associated with sine, move in space at the same velocity u as the particle itself. The wavelength λ_2 in (5) can be transformed so as to bring it to the standard form for the de Broglie wavelength. We shall associate the angular frequency of the oscillations inside the particle, similarly to the electromagnetic wave, with the energy of oscillations: $W' = \hbar\omega'$, where $\hbar = \frac{h}{2\pi}$ is Dirac constant, h is Planck constant. This gives the following:

$$\lambda_2 = \frac{h c^2 \sqrt{1-u^2/c^2}}{W' u}\,. \qquad (9)$$

Similarly from (4) we have:

$$\lambda_1 = \frac{h c \sqrt{1-u^2/c^2}}{W'}\,. \qquad (10)$$

In the limiting case when the oscillation energy is compared with the rest energy of the particle, $W' = mc^2 = \hbar\omega'$, from (9) it follows:

$$\lambda_{2f} = \frac{h\sqrt{1-u^2/c^2}}{mu} = \frac{h}{\wp}, \qquad (11)$$

where m is the mass of particle, $\wp$ is relativistic momentum of the particle.

The formula (11) defines de Broglie wavelength with the help of particle momentum. We shall note that de Broglie wrote (11) on condition that the energy of the particle $W = \frac{mc^2}{\sqrt{1-u^2/c^2}}$ is equal to the energy of the wave accompanying the particle. According to the obtained expression (9), the wavelength λ_2 must be present in the particle also at low excitation energy W'. In this case as the excitation energy decreases, the wavelength should increase.

As a rule in the experiments only $\lambda_b = \lambda_{2f} = \frac{h}{\wp}$ is found from (11), and not the wavelength λ_2 from (9). This can occur because among the number of interacting particles at the same time there are particles with different excitation energies W' and different λ_2, so that the wave phenomena are blurred. The same is true for the waves with wavelength λ_1 in (10). Only for the most actively interacting particles, the excitation energies W' of which are close to the rest energy of the particles, the limiting value of the wavelength is reached equal to the de Broglie wavelength. Thus this wavelength is revealed in the experiment. When $W' = mc^2 = \hbar\omega'$ we can also predict for the particles the wave phenomena with the critical wavelength $\lambda_{1f} = \frac{h\sqrt{1-u^2/c^2}}{mc}$. In particular, $\lambda_c = \frac{h}{mc}$ is the Compton wavelength, discovered in the Compton effect. According to our point of view, emerging of de Broglie wave should be treated as a purely relativistic effect, which arises as a consequence of the Lorentz transformation of the standing wave, moving with the particle.

As a result, we have to assume that the wave-particle duality is realized in full only in those particular particles, the excitation energies of which reach their rest energies. In this case the difference of particles and field quanta, if they are treated from the point of view of their wave properties, becomes minimal. At low excitation energies the particles can not emit their energy greatly, and the amplitudes of the oscillations of the field potentials near the particles are small. Then the particles would interact with each other not in the wave way, but rather in the usual way, and the wave phenomena become invisible.

If we assume that the length of the standing wave is equal to $\lambda' = 2R$, where R is the radius of the proton, then from the equality of the wave energy and the rest energy of the proton we obtain:

$$\nu' = \frac{c}{\lambda'} = \frac{c}{2R}, \qquad M_p c^2 = h\nu' = \frac{hc}{2R}, \qquad R = \frac{h}{2M_p c} = 6.6 \cdot 10^{-16} \text{ m},$$

here ν' is the oscillation frequency, M_p is the mass of the proton.

Another way to estimate the radius of the proton assumes that the difference between the rest energy of the neutron and the proton is due to the electrical energy of the proton charge. In this case, it should be:

$$(M_n - M_p)c^2 = \frac{ke^2}{4\pi\varepsilon_0 R}, \tag{12}$$

where M_n is the mass of the neutron, e is the elementary charge, ε_0 is the vacuum permittivity.

In (12) for the case of the uniform distribution of the charge in the volume of the proton $k = 0.6$, as a result the estimation of the proton radius gives the value of $R = 6.68 \cdot 10^{-16}$ m.

In [6] and [7], the radius of the proton was found from the condition that the limiting angular momentum of the strong gravitation field inside the proton is equal in magnitude to the spin of the proton. This leads to the following formula:

$$R = \frac{5\Gamma M_p}{21c^2} = 6.7 \cdot 10^{-16} \text{ m}. \tag{13}$$

In (13) the strong gravitational constant Γ is used. According to [4], this constant is determined from the equation of electric force and the force from the strong gravitation field, acting in the hydrogen atom on the electron with the mass M_e, which is located in the ground state on the Bohr radius R_B:

$$\frac{e^2}{4\pi\varepsilon_0 R_B^2} = \frac{\Gamma M_p M_e}{R_B^2}, \qquad \Gamma = \frac{e^2}{4\pi\varepsilon_0 M_p M_e} = 1.514\cdot 10^{29}\ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}, \qquad (14)$$

In addition to the attractive forces from gravitation and the charges of the nucleus and the electron, in the hydrogen atom the electron substance in the form of the rotating disc is influenced by the repulsive forces acting away from the nucleus. One of these forces is the electric force of repulsion of the charged substance of the electron cloud from itself. In the rotating non-inertial reference frame in which an arbitrary part of the electron substance is at rest, there is also the force of inertia in the form of the centrifugal force, which depends on the velocity of rotation of this substance around the nucleus. In the first approximation, these forces are equal in magnitude, which leads to (14).

We shall remind that the idea of strong gravitation was introduced into science in the works of Abdus Salam and his colleagues [8], [9] as the alternative explanation of the strong interaction of the particles. Assuming that hadrons can be represented as Kerr-Newman black holes, they estimated the strong gravitational constant as $6.7\cdot 10^{27}$ $\text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$.

With the help of the strong gravitation constant (14) we can express the fine structure constant:

$$\alpha = \frac{\Gamma M_p M_e}{\hbar c} = \frac{1}{137.035999}.$$

Another estimate of the radius of the proton follows from the equality of the rest energy and the absolute value of the total energy, which, taking into account the virial theorem, is approximately equal to the half of the absolute value of the strong gravitation energy associated with the proton [4]:

$$M_p c^2 = \frac{k\Gamma M_p^2}{2R}. \qquad (15)$$

If we take $k = 0.6$ for the case of the uniform mass distribution, then from (15) it follows that $R = 8.4 \cdot 10^{-16}$ m.

All of the above estimates are based on the classical approach to the proton as to the material object of small size in the form of the ball with the radius R. It is assumed that the strong gravitation acts at the level of elementary particles in the same way as ordinary gravitation at the level of planets and stars.

In the Standard model of elementary particles and in quantum chromodynamics it is assumed that the nucleons and other hadrons consist of quarks, and baryons have three quarks, while mesons have two quarks. Instead of the strong gravitation, the action of gluon fields is assumed to hold the quarks in hadrons. Quarks are considered to be charged elementary particles, therefore as the radius of the proton the charge and magnetic root mean square radii are considered. These radii are determined by the electric and magnetic interactions of the proton and can differ from each other.

The estimate of the proton charge radius can be made with the help of the experiments on the scattering of charged particles on the proton target [10]. In such experiments the total cross sections of interaction of the particles σ are found. For the case of the protons scattering on nucleons with energies more than 10 GeV we can assume that $\sigma = \pi R^2$, and $\sigma = 3.8 \cdot 10^{-30}$ m^2. Hence we obtain $R = 7.8 \cdot 10^{-16}$ m.

3 The self-consistent model

Our aim will be to find a more exact value of the radius of the proton by using classical methods. In the calculations we shall use only the tabular data on the mass, charge and magnetic moment of the proton. The proton will be considered from the standpoint of the theory of infinite nesting of matter [11], in which the analogue of the proton at the level of stars is a magnetar or a charged neutron star with a very large magnetic and gravitational field. Similarly to the magnetar, the substance of the proton must be magnetized and held by a strong gravitation field.

To take into account the non-uniformity of the substance density inside the proton we shall use the simple formula in which the substance density changes linearly increasing to the center:

$$\rho = \rho_c (1 - Ar), \tag{16}$$

where ρ_c is the central density, r is the current radius, $0 < A < \frac{1}{R}$ is the coefficient which should be determined.

Formula (16) should be considered as a first approximation to the actual distribution of the density of matter inside the proton. Approximate linear dependence of the density of matter in neutron stars has been shown in [12], and we assume that this is also true for the proton as an analogue of the neutron stars.

To estimate the values A and the radius R we shall consider the integral for the proton mass in the spherical coordinates:

$$M_p = \int \rho_c (1 - Ar) r^2 dr \sin\theta \, d\theta \, d\varphi = \frac{4\pi R^3 \rho_c}{3} \left(1 - \frac{3AR}{4} \right). \tag{17}$$

For accurate calculation of state of neutron stars, and thus protons as their analogues we should consider the curvature of spacetime in a strong gravitational field, as well as the contribution of the energy of the gravitational field to the total mass-energy. We shall assume that in (16), in dependency of matter density on the radius all relativistic effects are taken into account, and the mass of the proton (17) is the gravitational mass from the point of view of a distant observer.

In (17) there are three unknown quantities, to obtain which two more equations are required. We shall assume the virial theorem to be valid and equate the rest energy of the proton to the half of the absolute value of the energy of the static field of strong gravitation:

$$M_p c^2 = -\frac{1}{2} \int_0^\infty \varepsilon \, dV = \frac{1}{16\pi \Gamma} \int_0^\infty G^2 dV, \tag{18}$$

where $\varepsilon = -\frac{G^2}{8\pi\Gamma}$ is the energy density of the strong gravitation field according to [4], G is the gravitational acceleration or strength of gravitational field.

In (18), the integration of the energy density of the field should be done both inside and outside of the proton. The value G inside the proton can be conveniently found by integrating the equation for the strong gravitation field $\nabla \cdot \boldsymbol{G} = -4\pi\Gamma\rho$, which is part of the equations of the Lorentz-invariant theory of gravitation [13]. After integrating over the spherical volume with the radius $r \le R$, and then using the Gauss theorem, that is making transition to integrating over the area of the indicated sphere inside this proton, in view of (17) we obtain:

$$\int \nabla \cdot \boldsymbol{G}_i \, dV = \oint \boldsymbol{G}_i \cdot \boldsymbol{n} \, dS = 4\pi r^2 G_i = -\int 4\pi\Gamma\rho \, dV \,,$$

$$\boldsymbol{G_i} = -\frac{4\pi\Gamma\rho_c \mathbf{r}}{3}\left(1 - \frac{3Ar}{4}\right). \tag{19}$$

Outside the proton the gravitational acceleration is equal to:

$$\boldsymbol{G_o} = -\frac{\Gamma M_p \boldsymbol{r}}{r^3}. \tag{20}$$

Substituting (19) and (20) in (18), we obtain the relation:

$$M_p c^2 = 4\pi^2 \Gamma \rho_c^2 R^5 \left(\frac{1}{45} - \frac{AR}{36} + \frac{A^2 R^2}{112}\right) + \frac{\Gamma M_p^2}{4R}. \tag{21}$$

In (21) we can eliminate the value ρ_c using (17), which give the dependence of A on R in the form of the quadratic equation:

$$A^2R^2\left(\frac{\Gamma M_p}{14}-\frac{Rc^2}{4}\right)+AR\left(-\frac{7\Gamma M_p}{36}+\frac{2Rc^2}{3}\right)+\frac{2\Gamma M_p}{15}-\frac{4Rc^2}{9}=0.$$

The analysis of this equation shows that it has the following solution:

$$AR=\frac{\frac{7\Gamma M_p}{36}-\frac{2Rc^2}{3}+\sqrt{\frac{\Gamma M_p Rc^2}{945}-\frac{13\Gamma^2 M_p^2}{45360}}}{\frac{\Gamma M_p}{7}-\frac{Rc^2}{2}}, \qquad (22)$$

on condition that when $0.3<\frac{Rc^2}{\Gamma M_p}<\frac{13}{35}\approx 0.371$, then accordingly $0<AR<1$.

We shall now turn to the magnetic moment of the proton. As in [4], we assume that the magnetic moment of the proton is equal to the magnetic moment, which is formed due to the maximum rapid rotation of the charged substance of the proton. In spherical coordinates, the magnetic moment can be approximately calculated as the sum of the elementary magnetic moments of the separate rings with their radius $r\sin\theta$, which have the magnetic moment due to the current di flowing in them from the rotation of the charge:

$$\begin{aligned}P_m&=\int dP_m=\int \pi r^2\sin^2\theta\, di=\int \pi r^2\sin^2\theta\frac{dq}{dt}=\\&=\int \pi r^2\sin^2\theta\,\rho_{qc}(1-Ar)\,r^2dr\sin\theta\, d\theta\frac{d\varphi}{dt}=\frac{4\pi R^5\omega_L\,\rho_{qc}}{15}\left(1-\frac{5AR}{6}\right).\end{aligned} \qquad (23)$$

The angular velocity $\omega_L=\frac{d\varphi}{dt}$ of the maximum rotation of the proton can be found from the condition of limiting rotation, with the equality of the centripetal force and the gravitation force at the equator: $\frac{\Gamma M_p}{R^2}=\omega_L^2R$. Further we believe that for the charge density and the substance density the equation $\frac{\rho_{qc}}{\rho_c}=\frac{e}{M_p}$ holds, and we use (17). This gives the following:

$$P_m=\frac{4e\sqrt{\Gamma M_p R}\,(6-5AR)}{30(4-3AR)}. \qquad (24)$$

4 Conclusions

The relation (24) together with (22) allow us to find the radius of the proton $R=8.73\cdot 10^{-16}$ m, as well as the value $A=\frac{0.48}{R}$. From (17) we obtain then the central substance density $\rho_c=9.4\cdot 10^{17}$ kg/m^3, which exceeds the average density of the proton 1.57 times. The maximum angular velocity of rotation of the proton in view of (23) is equal to $\omega_L=6.17\cdot 10^{23}$ rad/s. At the same time, if the spin of the proton in the approximation of the uniform density of substance would be equal to the standard value for the spin of the fermion $L=0.4M_p R^2\omega=\frac{\hbar}{2}$, then the angular velocity of rotation $\omega=1.03\cdot 10^{23}$ rad/s would correspond to this spin.

For comparison with the experimental data we shall point to the results of calculations of electron scattering from [14], where the charge radius $R_E=8.7\cdot 10^{-16}$ m is obtained taking into account only the scattering on protons, $R_E=8.71\cdot 10^{-16}$ m – taking into account the data on the pion scattering, and $R_E=8.8\cdot 10^{-16}$ m – taking into account the data on the neutron scattering. In [3] the charge radius $R_E=8.4184\cdot 10^{-16}$ m was found in the study of the coupled system of the proton and the negative muon. The study of the scattering cross section of polarized photons by protons [15] gives the charge radius $R_E=8.75\cdot 10^{-16}$ m and the magnetic radius $R_M=8.67\cdot 10^{-16}$ m. The charge radius $R_E=8.77\cdot 10^{-16}$ m and the magnetic radius $R_M=7.77\cdot 10^{-16}$ m of the proton are listed on the site of *Particle data group* [16]. In the database CODATA [17] the proton charge radius is equal to $R_E=8.775\cdot 10^{-16}$ m.

The value $R = 8.73 \cdot 10^{-16}$ m obtained in the framework of the self-consistent model is close to the experimental values of the radius of the proton, which confirms the possibility of applying the idea of strong gravitation to describe the strong interaction of elementary particles.

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