Gluon Confinement in Yang-Mills Magnetic Monopoles

This brief two-page paper points out a symmetry property of Yang-Mills magnetic monopoles which makes them plausible baryon candidates. We use the language of differential forms to do so, and will assume the reader has sufficient familiarity with differential forms so that no tutorial explanations are required.

In an Abelian (commuting field) gauge theory such as QED, the field strength tensor *F* is specified in relation to the vector potential gauge field (e.g., photon) *A* according to $F = dA$. The magnetic monopole source density *P* is then specified classically (for high-action circumstances $S(\varphi) = \int d^4 x \mathcal{L}(\varphi) >> \hbar$ where the Euler Lagrange equation may be applied) by $P = dF = ddA = 0$. This makes use of the geometric law that the exterior derivative of an exterior derivative is zero, i.e., $dd = 0$. In integral form, this becomes $\iiint P = \iiint dF = \iiint ddG = \oiint F = \oiint dA = 0$. All of the foregoing is what tells us that there are no magnetic monopoles in an Abelian gauge theory such as QED. This absence of magnetic monopole charges has been well borne out experimentally in the 140 or so years since as James Clerk Maxwell published his 1873 *A Treatise on Electricity and Magnetism*.

In a non-Abelian (non-commuting field) Yang-Mills gauge theory such as (but not limited to) QCD, the fundamental difference is that the field strength tensor *F* is now specified in relation to the vector potential gauge field G (e.g., gluon in QCD) according to $F = dG - iG^2$. In this relationship, $G^2 = [G^\mu, G^\nu] dx_\mu dx_\nu$ expresses the non-commuting nature of the gauge fields. Therefore, although $ddG = 0$ as always because of the exterior geometry, the classical (high-action) magnetic monopole density becomes $P = dF = d(dG - iG^2) = -idG^2$, which is non-zero. In integral form, using Gauss' law, this becomes:

$$\iiint P = \iiint dF = \iiint d(dG - iG^2) = -i\iiint dG^2 = \oiint F = \oiint dG - i\oiint G^2 = -i\oiint G^2 , \tag{1}$$

and from the last two terms in the above, we may also derive the companion equation:

$$\oiint dG = 0 . \tag{2}$$

Of course, (2), albeit with the different field name, is just the relationship $\oiint dA = 0$ which tells us that there are non-magnetic monopoles in Abelian gauge theory. But in light of (1), which provides us with a non-zero magnetic monopole $\iiint P = -i\oiint G^2 \neq 0$, what can we learn from (2), which is the Yang-Mills analogue to the Abelian "no magnetic monopole" relationship $\oiint dA = 0$?

If we perform a local transformation $F \to F' = F - dG$ on the field strength *F*, which in terms of the field density tensor is written as $F^{\mu\nu} \to F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu} G^{\mu]}$, then in integral form we find from (1), as a direct and immediate result of the Abelian "no monopole" relationship $\oiint dG = 0$ in (2), that:

$$\iiint P = \oiint F \to \oiint F' = \oiint (F - dG) = \oiint F . \tag{3}$$

This means that the flow of the field strength $\oiint F = -i\oiint G^2$ across a two dimensional surface is invariant under the local gauge-like transformation $F^{\mu\nu} \to F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$. Now, we know that the invariance of the QED Lagrangian under the similar transformation $A^\mu \to A^\mu{}' = A^\mu + \partial^\mu\Lambda$ means that the gauge parameter Λ is not a physical observable. Similarly, the invariance of the gravitational Lagrangian under $g^{\mu\nu} \to g^{\mu\nu}{}' = g^{\mu\nu} + \partial^{\{\mu}\Lambda^{\nu\}}$ means that the gauge vector Λ^ν is not a physical observable (and we know Λ^ν is in fact connected merely with a coordinate transformation $x^\mu \to x'^\mu = x^\mu - \Lambda^\mu(x^\nu)$). In this case, the invariance of $\oiint F$ under the transformation $F^{\mu\nu} \to F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$ similarly tells us that the gauge field G^μ is not an observable over the surface through which the field $\oiint F = -i\oiint G^2$ is flowing. But G^μ is simply the gauge field, which in QED, is the gluon field. So, simply put: <u>the Yang-Mills gauge fields *G*$^\mu$ (including gluons in SU(3)$_C$) are not observables across any closed surface surrounding a magnetic monopole density *P*</u>. Whatever goes on inside the volume represented by $\iiint P$, the gauge fields remain confined.

Taking this a step further, we see that the origins of this gauge field confinement in fact lie in the 140-year old mystery as to why there are no magnetic monopoles in Abelian gauge theory. In differential forms language, the statement of this is $ddG = 0$. In integral form, this becomes $\oiint dG = 0$, equation (2). And, it is precisely this same "zero" which renders $\oiint F \to \oiint F' = \oiint F$ invariant under $F^{\mu\nu} \to F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$ in (3). So the physical observation that there are no magnetic monopoles in Abelian gauge theory becomes translated into a symmetry condition in non-Abelian gauge theory that gauge boson flow is not an observable over the surface of a magnetic charge. Again: In Abelian gauge theory there are no magnetic monopoles. In non-Abelian theory, this Abelian absence of magnetic monopoles translates into there being no flow of gauge bosons (e.g., gluons) across any closed surface surrounding a Yang-Mills magnetic monopole. Consequently, <u>*the absence of Abelian magnetic monopoles is fundamentally, organically equivalent to the absence of gluon flux, hence color, across surfaces surrounding non-Abelian chromo-magnetic monopoles.*</u> And, because this is turn originates in $dd = 0$, we see that this confinement is geometrically mandated. This makes Yang-Mills magnetic monopoles plausible baryon candidates. The very same "zero" which in Abelian gauge theory says that there are no magnetic monopoles, in non-Abelian gauge theory says that there is no observable flux of Yang-Mills gauge fields across a closed surface surrounding a Yang-Mills magnetic monopole.

We do not find a free gluon (or other gauge field) in Yang-Mills gauge theory any more than we find an Abelian magnetic monopole in electrodynamics, for identical geometric reasons.