Heavy gravitons on–shell decay of the Higgs boson at high scales of energy

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Abstract: We propose a gauge theory of gravity with heavy gravitons. In this framework, we examine whether renormalization effects can cause Newton's constant to change dramatically with energy, perhaps even reducing the scale of quantum gravity to the TeV region. For the Standard Model (SM) Higgs boson, with mass 125GeV, we derive the invisible width of Higgs boson decay into heavy gravitons and calculate the ratios of partial widths and couplings for the Higgs decays into heavy gravitons and W vector bosons. This proposition may reveal Higgs and graviton properties for new physics Beyond Standard Model (BSM).

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1 Introduction

Various workers have attempted to derive General Relativity from a gauge-like principle, involving invariance of physics under transformations of the locally (i.e., in the tangent space at each point) acting Lorentz or Poincare group. ([1], [2], [3], [4]).

Gauge theory of gravity is based on the principle of local gauge invariance. Since the model requires strict local gravitational gauge symmetry ([4]), gauge theory is a pertubatively renormalizable quantum model. In the original model, all gauge gravitons are massless ([5]).

The massive gravitons were described effectively by Fierz and Pauli ([6]). According to their theory, the existence of massive gravitons would violate the local gauge symmetry of the Lagrangian ([7]).

Building on this earlier work, N. Wu proposed a mechanism which introduces massive gravitons without violating the local gauge symmetry of the Lagrangian ([4], [8], [9], and [10]).

According to Wu's theory, the third-order gravitational gauge field $C^a_{3\mu}$ is heavy if mass is very large. Such a field has no contribution to the long-range gravitational force. Long-range gravitational force results exclusively from the contribution of the fourth-order gravitational gauge field $C^a_{4\mu}$ and obeys inverse square law. However, it is possible that heavy graviton $C^a_{3\mu}$ field has a contribution to the Standard Model (SM) at high scale of energy [11-14].

If the mass term of gravitational gauge field is extremely small, however, the third-order gravitational gauge field $C^a_{3\mu}$ will contribute to the middle range gravitational force with approximate range $L \approx \frac{hc}{m}$ (where h is the Plank constant and c is the speed of light). For graviton mass $2\times10^{-7}eV$ the gravitational force range will be about one meter.

Recent results from cosmological observation, especially from Cosmic Microwave Background (CMB) temperature anisotropy, suggest that our Universe is essentially flat and that it consists mainly of dark matter and dark energy [15]. A theory about the origin of dark matter and dark energy is to regard them as consisting of massive gravitons. There are indications that those massive gravitons with mass $2\times10^{-7}eV$ can produce today's acceleration of the Universe ([16], [17]).

A major achievement has been announced by the European Laboratory for Particle Physics (CERN) scientists [52]: it was revealed that data from the Large Hadron Collider (LHC) [53] has confirmed the existence of a particle consistent with the Higgs boson, with approximately the 5-sigma certainty required to confirm this as a discovery [52]. This is a tentative confirmation of the existence of the Higgs boson.

Theories of gravity include vector particles. In particular, such gravi-vectors [18-21] appear in flexible brane world models, in which a four dimensional space-time is embedded in a higher dimensional space-time, thus breaking the extra dimensional spatial translation symmetries [22-26]. A variety of flexible brane world models contain massive, stable gravi-vector fields [18, 27-29]. The decay modes of the Higgs boson which contain a pair of such gravi-vectors have been studied [29].

In the traditional gauge treatment of gravity the Lorentz group is localized. The gravitational field is, thus, not represented by gauge potential, but by the metric field $g_{\mu\nu}$.

Here, we propose an alternative understanding of gravity, resulting from the extension of N.Wu's gauge theory of gravity with heavy gravitons at high scale of energy. In this framework, we examine whether renormalization effects can cause Newton's constant to change dramatically with energy, perhaps even reducing the scale of quantum gravity to the TeV region.

In this paper we derive the complete set of formulas for the invisible decay widths of the Higgs boson in heavy graviton. The formulas are valid both for the (SM) and for any arbitrary extension [30].

Furthermore, the bare Newton coupling constant of $(10^{-3} TeV^{-2})$ which is well within the currently allowed range of the proposed heavy graviton mass , 100 GeV [31], [32], is appropriate for a Higgs boson of 125 GeV [33], [34]. The ratios of partial widths and couplings for the on–shell Higgs boson decays to heavy gravitons and vector gauge bosons are calculated.

This proposition may reveal Higgs and graviton properties for new physics Beyond Standard Model (BSM) [30], [51].

2 Gauge theory of gravity with heavy graviton

Taking Wu's gauge model as our starting point ([4], [8], [9], [10], [35-41]), we introduce two gravitational gauge fields $(C_\mu^a, C_{2\mu}^a)$ simultaneously. Since $(C_\mu^a, C_{2\mu}^a)$ are vectors in Lie algebra ([8], [9], [35-41]), they can be expanded as

$$C_\mu = C_\mu^a \hat{P}_a \quad , \; C_{2\mu} = C_{2\mu}^a \hat{P}_a \,. \tag{1}$$

These correspond with two gauge covariant derivatives

$$D_\mu = \partial_\mu - igC_\mu(x) \quad , \; D_{2\mu} = \partial_\mu + iagC_{2\mu}(x) \tag{2}$$

and two different field strengths, given by

$$\begin{aligned} F_{\mu\nu} &= \frac{1}{-ig}[D_\mu, D_\nu] \\ F_{2\mu\nu} &= \frac{1}{iag}[D_{2\mu}, D_{2\nu}] \end{aligned} \tag{3}$$

The Lagrangian of the system is given by

$$\begin{aligned} \Im_0 = &-\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F_{\mu\nu}^b F_{\rho\sigma}^c - \frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F_{2\mu\nu}^b F_{2\rho\sigma}^c \\ &- \frac{m^2}{2(1+a^2)}\eta^{\mu\nu} g_{bc}(C_\mu^b + aC_{2\mu}^b)(C_\nu^c + aC_{2\nu}^c) \end{aligned} \tag{4}$$

where m is the constant mass parameter.

The action of the system is given by

$$S = \int d^4 x J(C) \Im_0 \tag{5}$$

where

$$J(C) = \sqrt{-\det(g_{\alpha\beta})}, \tag{6}$$

and

$$g_{\alpha\beta} = \eta_{\mu\nu} (G^{-1})^{\mu}_{\alpha} (G^{-1})^{\nu}_{\beta}. \tag{7}$$

For the definition of G-matrix and its inverse matrix G^{-1}–matrix see [41]. In this model, space-time is always flat and space-time metric is always Minkowski metric, so $g_{\alpha\beta}$ is no longer space-time metric [41].

Equation (4) gives the mass term in gravitational gauge fields. To obtain the eigenstates of mass matrix the following rotation is needed

$$\begin{aligned} C_{3\mu} &= \cos\theta C_{\mu} + \sin\theta C_{2\mu} \\ C_{4\mu} &= -\sin\theta C_{\mu} + \cos\theta C_{2\mu} \end{aligned} \tag{8}$$

where the angle θ is given by

$$\cos\theta = \frac{1}{\sqrt{1+a^2}}, \quad \sin\theta = \frac{a}{\sqrt{1+a^2}} \tag{9}$$

After transformation (8), the Lagrangian of the system is given by

$$\begin{aligned} \Im_0 = &-\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F^{b}_{30\mu\nu} F^{c}_{30\rho\sigma} - \frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma} g_{bc} F^{b}_{40\mu\nu} F^{c}_{40\rho\sigma} \\ &- \frac{m_c^2}{2}\eta^{\mu\nu} g_{bc} C^{b}_{3\mu} C^{c}_{3\nu} + \Im_I \end{aligned} \tag{10}$$

where $F^{a}{}_{30\mu\nu}$ $F^{a}{}_{40\mu\nu}$ are given by:

$$\begin{aligned} F^{a}_{30\mu\nu} &= \partial_{\mu} C^{a}_{3\nu} - \partial_{\nu} C^{a}_{3\mu} \\ F^{a}_{40\mu\nu} &= \partial_{\mu} C^{a}_{4\nu} - \partial_{\nu} C^{a}_{4\mu} \end{aligned} \tag{11}$$

From the above follows that the gauge field $C^{a}_{3\mu}$ have mass m_c, whereas the gravitational gauge field $C^{a}_{4\mu}$, is massless.

Here, gravitational gauge field $C^{a}_{3\mu}$ is heavy. Such a field contributes to the short-range gravitational force. Long-range gravitational force results exclusively from the contribution of the massless gravitational gauge field $C^{a}_{4\mu}$ and obeys inverse square law. It is possible that the heavy graviton $C^{a}_{3\mu}$ field has a contribution to the Standard Model (SM) at high scale of energy [23-25].

3 The propagators of the model

Using eq. (10), we can deduce propagator of massless gravitational gauge fields and heavy gravitational fields ([41]). First, after a partial integration, we change the form of eq. (10), as follows

$$\int d^4x\Im_0 = \int d^4x\{\frac{1}{2}(C^{\alpha}_{4\mu}O^{\mu\nu(1)}_{\alpha\beta}(x)C^{\beta}_{4\nu} + C^{\alpha}_{3\mu}O^{\mu\nu(2)}_{\alpha\beta}(x)C^{\beta}_{3\nu})\}, \tag{12}$$

where the operators $O^{\mu\nu(1)}_{\alpha\beta}(x)$, $O^{\mu\nu(2)}_{\alpha\beta}(x)$ are defined by

$$O^{\mu\nu(1)}_{\alpha\beta}(x) = \frac{1}{2}\eta^{\mu\nu}\eta_{\alpha\beta}\partial^{\rho}\partial_{\rho} - \frac{1}{2}\eta_{\alpha\beta}\left(1-\frac{4}{a}\right)\partial^{\mu}\partial^{\mu} - \frac{1}{2}\eta^{\mu\nu}\partial_{\alpha}\partial_{\beta}.\ ([41]) \tag{13}$$

$$O^{\mu\nu(2)}_{\alpha\beta}(x) = \frac{1}{2}\eta^{\mu\nu}\eta_{\alpha\beta}(\partial^{\rho}\partial_{\rho} + m_c^2) - \frac{1}{2}\eta_{\alpha\beta}\left(1-\frac{4}{a}\right)\frac{\partial^{\mu}\partial^{\mu}}{m_c^2} - \frac{1}{2}\eta^{\mu\nu}\partial_{\alpha}\partial_{\beta}. \tag{14}$$

We denote the propagator of massless gravitational gauge field as

$$-iD^{\alpha\beta}_{F\mu\nu}(x) \tag{15}$$

and the propagator of heavy gravitational field as

$$-i\Delta^{\alpha\beta}_{F\mu\nu}(x) \tag{16}$$

The propagators (15) and (16) satisfy the following equation:

$$-O^{\mu\nu}_{\alpha\beta}(x)D^{\beta\gamma}_{F\nu\rho}(x-y) = \tilde{\eta}^{\mu\gamma(1)}_{\alpha\rho}(x)\delta(x-y), \tag{17}$$

$$-O^{\mu\nu}_{\alpha\beta}(x)\Delta^{\beta\gamma}_{F\nu\rho}(x-y) = \tilde{\eta}^{\mu\gamma(2)}_{\alpha\rho}(x)\delta(x-y), \tag{18}$$

where $\tilde{\eta}^{\mu\gamma(1)}_{\alpha\rho}(x)$, $\tilde{\eta}^{\mu\gamma(2)}_{\alpha\rho}(x)$ are defined by

$$\tilde{\eta}^{\mu\gamma(1)}_{\alpha\rho}(x) = \frac{1}{2}\left(\tilde{\eta}^{\mu\gamma(1)}(x)\tilde{\eta}^{(1)}_{\alpha\rho}(x)\right),\ ([41]) \tag{19}$$

$$\tilde{\eta}^{\mu\gamma(2)}_{\alpha\rho}(x) = \frac{1}{2}\left(\tilde{\eta}^{\mu\gamma(2)}(x)\tilde{\eta}^{(2)}_{\alpha\rho}(x)\right). \tag{20}$$

In the above definition, $\tilde{\eta}^{\mu\gamma(1)}(x),\tilde{\eta}^{(1)}_{\alpha\rho}(x)$ ([41]) and $\tilde{\eta}^{\mu\gamma(2)}(x),\tilde{\eta}^{(2)}_{\alpha\rho}(x)$ are defined by

$$\tilde{\eta}^{\mu\gamma(1)}(x)=\eta^{\mu\gamma}-\frac{\partial^{\mu}\partial^{\gamma}}{\square+i\varepsilon}, \tag{21}$$

$$\tilde{\eta}^{(1)}_{\alpha\rho}(x)=\eta_{\alpha\rho}-\frac{\partial_{\alpha}\partial_{\rho}}{\square+i\varepsilon}, \tag{22}$$

$$\tilde{\eta}^{\mu\gamma(2)}(x)=\eta^{\mu\gamma}-\frac{\partial^{\mu}\partial^{\gamma}}{\square+m_c^2+i\varepsilon}, \tag{23}$$

$$\tilde{\eta}^{(2)}_{\alpha\rho}(x)=\eta_{\alpha\rho}-\frac{\partial_{\alpha}\partial_{\rho}}{\square+m_c^2+i\varepsilon}, \tag{24}$$

where

$$\square=\partial^2=\partial^{\mu}\partial_{\mu}=\eta^{\mu\nu}\partial_{\mu}\partial_{\nu}, \tag{25}$$

and m_c the mass of heavy gravitational field.

Fourier transformations to momentum space are as follows:

$$-iD^{\alpha\beta}_{F\mu\nu}(x)=\int\frac{d^4k}{(2\pi)^4}(-i)\tilde{D}^{\alpha\beta}_{F\mu\nu}(k)e^{ikx}, \tag{26}$$

$$-i\Delta^{\alpha\beta}_{F\mu\nu}(x)=\int\frac{d^4k}{(2\pi)^4}(-i)\tilde{\Delta}^{\alpha\beta}_{F\mu\nu}(k)e^{ikx}, \tag{27}$$

where $-i\tilde{D}^{\alpha\beta}_{F\mu\nu}(k)$ and $-i\tilde{\Delta}^{\alpha\beta}_{F\mu\nu}(k)$ are the corresponding propagators in momentum space.

These Fourier transformations satisfy the following equations,

$$-O^{\mu\nu(1)}_{\alpha\beta}(k)\tilde{D}^{\beta\gamma}_{F\nu\rho}(k)=\tilde{\eta}^{\mu\gamma(1)}_{\alpha\rho}(k), \tag{28}$$

$$-O^{\mu\nu(2)}_{\alpha\beta}(k)\tilde{\Delta}^{\beta\gamma}_{F\nu\rho}(k)=\tilde{\eta}^{\mu\gamma(2)}_{\alpha\rho}(k), \tag{29}$$

where the operators $O^{\mu\nu(1)}_{\alpha\beta}(k)$ ([41]) and $O^{\mu\nu(2)}_{\alpha\beta}(k)$ are defined by

$$O^{\mu\nu(1)}_{\alpha\beta}(k)=-\frac{1}{2}\eta^{\mu\nu}\eta_{\alpha\beta}k^2+\frac{1}{2}\eta_{\alpha\beta}\left(1-\frac{2}{a}\right)k^{\mu}k^{\nu}+\frac{1}{2}\eta^{\mu\nu}k_{\alpha}k_{\beta}, \tag{30}$$

$$O^{\mu\nu(2)}_{\alpha\beta}(k)=-\frac{1}{2}\eta^{\mu\nu}\eta_{\alpha\beta}(k^2+m_c^2)+\frac{1}{2}\eta_{\alpha\beta}\left(1-\frac{2}{a}\right)\frac{k^{\mu}k^{\nu}}{m_c^2}+\frac{1}{2}\eta^{\mu\nu}k_{\alpha}k_{\beta} \tag{31}$$

and $\tilde{\eta}^{\mu\gamma(1)}_{\alpha\rho}(k)$, $\tilde{\eta}^{\mu\gamma(2)}_{\alpha\rho}(k)$ are defined by

$$\tilde{\eta}^{\mu\gamma(1)}_{\alpha\rho}(k)=\frac{1}{2}(\tilde{\eta}^{\mu\gamma(1)}(k)\tilde{\eta}^{(1)}_{\alpha\rho}(k)), \tag{32}$$

$$\tilde{\eta}^{\mu\gamma(1)}_{\alpha\rho}(k)=\frac{1}{2}(\tilde{\eta}^{\mu\gamma(2)}(k)\tilde{\eta}^{(2)}_{\alpha\rho}(k)). \tag{33}$$

The operators $O^{\mu\nu(1)}_{\alpha\beta}(k)$ and $O^{\nu\mu(2)}_{\beta\alpha}(k)$ have the following symmetric property:

$$O^{\mu\nu(1)}_{\alpha\beta}(k)=O^{\nu\mu(1)}_{\beta\alpha}(k), \tag{34}$$

$$O^{\mu\nu(2)}_{\alpha\beta}(k)=O^{\nu\mu(2)}_{\beta\alpha}(k). \tag{35}$$

In the above relation, $\tilde{\eta}^{\mu\gamma(1)}(k),\tilde{\eta}^{(1)}_{\alpha\rho}(k)$ ([41]) and $\tilde{\eta}^{\mu\gamma(2)}(k),\tilde{\eta}^{(2)}_{\alpha\rho}(k)$ are defined by

$$\tilde{\eta}^{\mu\gamma(1)}(k)=\eta^{\mu\gamma}-\frac{k^{\mu}k^{\gamma}}{k^2-i\varepsilon}, \tag{36}$$

$$\tilde{\eta}^{(1)}_{\alpha\rho}(k)=\eta_{\alpha\rho}-\frac{k_{\alpha}k_{\rho}}{k^2-i\varepsilon}, \tag{37}$$

$$\tilde{\eta}^{\mu\gamma(2)}(k)=\eta^{\mu\gamma}-\frac{k^{\mu}k^{\gamma}}{k^2+m_c^2-i\varepsilon}, \tag{38}$$

$$\tilde{\eta}^{(2)}_{\alpha\rho}(k)=\eta_{\alpha\rho}-\frac{k_{\alpha}k_{\rho}}{k^2+m_c^2-i\varepsilon}, \tag{39}$$

where m_c is the mass of heavy gravitational field.

The solutions to the two propagator equations (28) and (29) give out the propagators in momentum space,

$$-i\tilde{D}^{\alpha\beta}_{F\mu\nu}(k)=\frac{-i\tilde{\eta}_{\mu\nu}(k)}{k^2-i\varepsilon}\tilde{\eta}^{\alpha\beta}(k), \tag{40}$$

$$-i\tilde{\Delta}^{\alpha\beta}_{F\mu\nu}(k)=\frac{-i\tilde{\eta}^{\alpha\beta}(k)}{k^2+m_c^2-i\varepsilon}\left(\tilde{\eta}_{\mu\nu}(k)+\frac{k_{\mu}k_{\nu}}{m_c^2}\right). \tag{41}$$

Here, space-time is always flat and space-time metric is always Minkowski metric [41], so the propagators (40) and (41) is no longer reflects the space-time curvature.

Unlike Einstein's general theory of relativity, the cornerstone of this model is the gauge principle, not the principle of equivalence [41]. For non-relativistic problems, the proposed model can return to Newton's classical theory of gravity [41].

In order to quantize the suggested gauge field theory of gravity in the path integral formulation, we first have to select gauge conditions [35], [39]. To fix the degree of freedom of the gauge transformation, we must select two gauge conditions simultaneously: one for the heavy gauge field $C_{3\mu}$ and another for the massless gauge field $C_{4\mu}$. For instance, if we select temporal gauge condition for massless gauge field $C_{4\mu}$,

$$C_4 = 0, \tag{42}$$

there still exists a remainder gauge transformation degree of freedom, because the temporal gauge condition is unchanged under the following local gauge transformation:

$$C_4 \to U C_4 U^{-1} + \left(1 / ig\sin\theta\right) U \partial_\mu U^{-1}, \tag{43}$$

where

$$\partial_t U = 0, \qquad U = U(\vec{x}). \tag{44}$$

In order to make this remainder gauge transformation degree of freedom completely fixed, we have to select another gauge condition for gauge field $C_{3\mu}$. For instance, we can select the following gauge condition for gauge field $C_{3\mu}$,

$$\partial^\mu C_{3\mu} = 0. \tag{45}$$

If we select two gauge conditions simultaneously, when we quantize the theory in path integral formulation there will be two gauge fixing terms in the effective Lagrangian. The effective Lagrangian can then be written as:

$$\Im_{eff} = \Im - \frac{1}{2a_1} f_1^a f_1^a - \frac{1}{2a_2} f_2^a f_2^a + \bar{\eta}_1 M_{f_1} \eta_1 + \bar{\eta}_2 M_{f_2} \eta_2, \tag{46}$$

where

$$f_1^a = f_1^a(C_{4\mu}), \qquad f_2^a = f_2^a(C_{3\mu}). \tag{47}$$

If we select

$$f_2^a = \partial^\mu C_{3\mu}^\alpha, \tag{48}$$

then the propagator for heavy gauge field $C_{3\mu}$ is:

$$-i\tilde{\Delta}_{F\mu\nu}^{\alpha\beta}(k) = -i\tilde{\eta}^{\alpha\beta}(k) / (k^2 + m_c^2 - i\varepsilon) \times \left[\tilde{\eta}_{\mu\nu}(k) - (1 - a_1) k_\mu k_\nu / (k^2 + a_2 m_c^2)\right]. \tag{49}$$

If we let k approach infinity, then

$$\tilde{\Delta}^{\alpha\beta}_{F\mu\nu}(k) = \frac{1}{k^2}. \tag{50}$$

In this case, and according to the power-counting law, the gauge field theory of gravity with heavy gravitons suggested in this paper is a kind of renormalizable theory [35], [39].

4 The redefinition of Newton's coupling constant

It has become a convention to interpret the Planck scale M_P as a fundamental scale of Nature, indeed as the scale at which quantum gravitational effects become important. However, Newton's constant $G = M_{pl}^{-2}$ in natural units $\hbar = c = 1$ is measured in very low-energy experiments, and its connection to physics at short distances – in particular, quantum gravity - is tenuous. If the strength of gravitational interactions is scale-dependent, the true scale μ_* at which quantum gravity effects are large is one at which

$$G(\mu_*) \approx \mu_*^{-2} \tag{51}$$

This condition implies that gravity at length scales μ_*^{-1} will be unsuppressed. Below we will show that condition (51) can be satisfied in models with μ_* as small as a TeV ([42]).

We consider one scalar field coupled to gravity and adopt the following notation:

$$\begin{aligned} S = \int d^4x J(C)\{(&\frac{1}{128\pi G}\eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{30\mu\nu}F^c_{30\rho\sigma} - \frac{1}{128\pi G}\eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{40\mu\nu}F^c_{40\rho\sigma} \\ &-\frac{m_c^2}{64\pi G}\eta^{\mu\nu}g_{bc}C^b_{3\mu}C^c_{3\nu}) + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi)\} \end{aligned} \tag{52}$$

Consider the gravitational potential between two heavy, non-relativistic sources, which arises through graviton exchange ([42]). The leading term in the gravitational Lagrangian (52) is

$$\begin{aligned} &G^{-1}(\eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{30\mu\nu}F^c_{30\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{40\mu\nu}F^c_{40\rho\sigma} \\ &-\eta^{\mu\nu}g_{\mu\nu}C^b_{3\mu}C^c_{3\nu}) = G^{-1}q^2 \end{aligned}, \tag{53}$$

where

$$\begin{aligned} &(\eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{30\mu\nu}F^c_{30\rho\sigma} - \eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{40\mu\nu}F^c_{40\rho\sigma} \\ &-\eta^{\mu\nu}g_{\mu\nu}C^b_{3\mu}C^c_{3\nu}) = q^2. \end{aligned} \tag{54}$$

Here, the terms

$$\begin{aligned} &(\eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{40\mu\nu}F^c_{40\rho\sigma}), \\ &(\eta^{\mu\rho}\eta^{\nu\sigma}g_{bc}F^b_{30\mu\nu}F^c_{30\rho\sigma} - \eta^{\mu\nu}g_{\mu\nu}C^b_{3\mu}C^c_{3\nu}) \end{aligned} \tag{55}$$

corresponding to massless and heavy gravitons, are of orders k^2 and k^2+m^2 respectively.

We can interpret quantum corrections to either massless or heavy gravitons propagators from the loop (similar to those given in reference [42]) as a renormalization of G. Neglecting the index structure, the massless graviton propagator given by equation (40) with one-loop correction is

$$D(k)_c \approx \frac{iG}{k^2} + \frac{iG}{k^2}\Sigma\frac{iG}{k^2} + ...,\tag{56}$$

where q is the momentum carried by the graviton in this model. The term in the Feynman diagram Σ proportional to k^2 can be interpreted as a renormalization of G, and is easily estimated from the Feynman diagram:

$$\Sigma \approx -ik^2\int^{\Lambda} d^4pD(p)^2 p^2 + ...,\tag{57}$$

where D(p) is the propagator of the particle in the loop. In the case of a scalar field, the loop integral is quadratically divergent. By incorporating this into a redefinition of G one obtains an equation of the form

$$\frac{1}{G_{ren}} = \frac{1}{G_{bar}} + c\Lambda^2,\tag{58}$$

where Λ is the ultraviolet cutoff of the loop and $c = 1/16\pi^2$. G_{ren} is the renormalized Newton constant measured in low-energy experiments.

Fermions contribute with the same sign to the running of Newton's constant, whereas gauge bosons contribute with the opposite sign than scalars. Taking $\Lambda = \mu_*$ (so that the loop cutoff coincides with the onset of quantum gravity) gives $G_{bare} = G(\Lambda) = \mu_*^{-2}$. Then, demanding that $G_{ren} = M_{Pl}^{-2}$ implies that μ_* cannot be very different from the Planck scale M_{Pl}, unless c is very large. For example, to have $\mu_* = 14TeV$, for example, requires that $c = 10^{30}$: it takes 10^{30} ordinary scalars or fermions with masses below 1 TeV (which can run in the loop). This observation has already been made by Dvali et al. [43, 44, 45], although in [43] the argument is expressed in terms of a consistency condition from black hole evaporation rather than as renormalization group behavior [42].

The functional derivation of equation (58) that follows shows that the sign of the contribution of the scalar fields to the running of Newton's constant is not an artifact of the crude (non-covariant) regularization procedure we used earlier. Consider the contribution of a scalar field minimally coupled to the suggested gravity. We follow the presentation of Larsen and Wilczek [46] (see also [47, 48, 42]). The one-loop effective action W is defined through

$$e^{-W} = \int D\phi e^{-\frac{1}{8\pi}\int\phi(-\Delta+m^2)\phi} = [\det(-\Delta+m^2)]^{-1/2}\tag{59}$$

We define the heat kernel

$$H(\tau) = Tre^{-\tau\Lambda} = \sum_i e^{-\tau\lambda_i}\tag{60}$$

where λ_i are the eigenvalues of $\Lambda = -\Delta + m^2$. Then the effective action reads

$$W = \frac{1}{2}\ln\det\Lambda = \frac{1}{2}\sum_i \ln\lambda_i = -\frac{1}{2}\int_{\varepsilon^2}^{\infty} d\tau \frac{H(\tau)}{\tau} \tag{61}$$

The integral over τ is divergent and has to be regulated by an ultraviolet cutoff ε^2 . The heat kernel method can be used to regularize the leading divergence of this integral. This technique does not violate general coordinate invariance. One can write

$$H(\tau) = \int dx G(x, x', \tau), \tag{62}$$

where the Green's function $G(x, x', \tau)$ satisfies the differential equation

$$\left(\frac{\partial}{\partial\tau} - \Delta_x\right) G(x, x', \tau) = 0; \tag{63}$$

$$G(x, x', 0) = \delta(x - x'). \tag{64}$$

In flat space one has

$$G_0(x, x', \tau) = \left(\frac{1}{4\pi\tau}\right)^2 \exp\left(-\frac{1}{4\tau}(x - x')\right), \tag{65}$$

but, in general, one must express the covariant Laplacian in local coordinates and expand for small field strengths. The result is similar to that given in [49]

$$\begin{aligned} H(\tau) = \frac{1}{(4\pi\tau)^2}\{ &\int d^4xJ(C) + \frac{\tau}{6}\int d^4xJ(C)(\eta^{\mu\rho}\eta^{\nu\sigma}g_{\alpha\beta}F^{\alpha}_{30\mu\nu}F^{\beta}_{30\rho\sigma} \\ &-\eta^{\mu\rho}\eta^{\nu\sigma}g_{\alpha\beta}F^{\alpha}_{40\mu\nu}F^{\beta}_{40\rho\sigma} - \frac{m_c^2}{2}\eta^{\mu\nu}g_{\alpha\beta}C^{\alpha}_{3\mu}C^{\beta}_{3\nu}) + O(\tau^{3/2})\}. \end{aligned} \tag{66}$$

Plugging this back into (61) and comparing with (52), one obtains the renormalized Newton constant

$$\frac{1}{G_{ren}} = \frac{1}{G_{bare}} + \frac{1}{12\pi\varepsilon^2}, \tag{67}$$

In this, G_{ren}, relevant for long-distance measurements, is much smaller than the bare value if the scalar field is integrated out $(\varepsilon \to 0)$.

Up to this point our results have been in terms of old fashioned renormalization: we give a relation between the physical observable G_{ren} and the bare coupling G_{bare}. A modern Wilsonian effective theory would describe modes with momenta $|k| < \mu$. Modes with $|k| > \mu$ have been integrated out and their virtual effects already absorbed in effective couplings $g(\mu)$.
In this language, $G_{ren} = G(\mu = 0)$ is appropriate for astrophysical and other long-distance measurements of the strength of gravity.

A Wilsonian Newton constant $G(\mu)$ can be calculated via a modified version of the previous method, this time with an infrared cutoff μ. For example, (9) is modified to

$$W = -\frac{1}{2}\int_{\varepsilon^2}^{\mu^{-2}} d\tau \frac{H(\tau)}{\tau}. \tag{68}$$

The resulting Wilsonian running of Newton's constant for N scalars or Weyl fermions, as shown by a similar functional calculation, is

$$\frac{1}{G(\mu)} = \frac{1}{G(0)} - \frac{\mu^2}{12\pi}, \tag{69}$$

$$\frac{1}{G(\mu)} = \frac{1}{G(0)} - N\frac{\mu^2}{12\pi} \tag{70}$$

This is comparable with Larsen and Wilczek [46], who also derive the opposite sign in the gauge boson case. [42].

As we approach the scale of strong quantum gravity, we lose control of the model. However, it seems implausible that the sign of the beta function for Newton's constant will reverse, so the qualitative prediction of weaker gravity at low energies should still hold.

5 Heavy graviton on–shell decay of the Higgs boson

In the recent announcement of a major achievement by CERN scientists, it was revealed that the data from the LHC [53] has confirmed the existence of a particle consistent with the Higgs boson, within the 5-sigma certainty required to confirm this as a discovery [52]. This is adequate to tentatively confirm the existence of the Higgs boson.

We propose the on–shell decay of Higgs boson to two heavy gravitons (denoted as $H \to CC$). The invisible differential width is given by [50]

$$d\Gamma_{cc} = \frac{1}{32\pi^2}\sum_{pol}|M|^2 \frac{|k_1|}{M_H^2} d\Omega_{k_1}, \tag{71}$$

where

$$M = i\sqrt{16\pi G_N}\, m_c \varepsilon^{\mu}(k_1)\varepsilon^{\nu}(k_2)\tilde{\eta}_{\mu\nu}(k) \tag{72}$$

is the amplitude for the Higgs decay to two heavy gravitons, $G_N = G_{bare} = \mu_*^{-2}$ $(\mu_* = 14TeV)$ the bare Newton coupling constant, defined by equations (67), (70).

The tensor $\tilde{\eta}_{\mu\nu}(k)$ in the amplitude (72), must be constructed out of $\eta_{\mu\nu}$ and the independent wave numbers k_1 and k_2 $(k = k_1 + k_2)$. We write

$$\tilde{\eta}_{\mu\nu}(k) = \eta_{\mu\nu} + T^{c}_{\mu\nu}(k), \tag{73}$$

where

$$T_{\mu\nu}^{c}(k) = -\frac{k_\mu k_\nu}{k^2 + m_c^2}. \tag{74}$$

are the extra contributions from new physics BSM due to heavy gravitons.

We write the amplitude (72) in terms of the extra contributions $T_{\mu\nu}^{c}(k)$ as follows

$$M = i\sqrt{16\pi G_N}\, m_c \varepsilon^{\mu}(k_1)\varepsilon^{\nu}(k_2)\tilde{\eta}_{\mu\nu}(k) = i\sqrt{16\pi G_N}\, m_c \varepsilon^{\mu}(k_1)\varepsilon^{\nu}(k_2)(\eta_{\mu\nu} + T_{\mu\nu}^{c}(k)) \tag{75}$$

We get, therefore,

$$\sum_{pol} |M|^2 = (\sqrt{16\pi G_N}\, m_c)^2 \left(-\eta^{\mu\alpha} + \frac{k_1^{\mu} k_2^{\alpha}}{m_c^2}\right)\left(-\eta^{\nu\beta} + \frac{k_1^{\nu} k_2^{\beta}}{m_c^2}\right)(\eta_{\mu\nu} + T_{\mu\nu}^{c}(k))(\eta_{\alpha\beta} + T_{\alpha\beta}^{c}(k))$$

$$= [2 + \frac{(k_1 \cdot k_2)^2}{m_c^4} + 2T_{\alpha}^{c\alpha}(k) - 2\frac{k_1^{\alpha} k_1^{\beta}}{m_c^4} T_{\alpha\beta}^{c}(k) - 2\frac{k_2^{\alpha} k_2^{\beta}}{m_c^4} T_{\alpha\beta}^{c}(k) + 2\frac{k_1 \cdot k_2 k_1^{\alpha} k_2^{\beta}}{m_c^4} T_{\alpha\beta}^{c}(k)$$

$$+ T_{\mu\nu}^{c}(k) T^{c\mu\nu}(k) - \frac{k_{1\mu} k_1^{\alpha}}{m_c^2} T^{\mu\nu}(k) T_{\alpha\nu}^{c}(k) - \frac{k_{2\nu} k_2^{\beta}}{m_c^2} T^{\alpha\nu}(k) T_{\alpha\beta}^{c}(k)$$

$$+ \frac{k_1^{\mu} k_2^{\nu}}{m_c^2} T_{\mu\nu}^{c}(k) \frac{k_1^{\alpha} k_2^{\beta}}{m_c^2} T_{\alpha\beta}^{c}(k)] \tag{76}$$

Now, using

$$k_1 \cdot k_2 = \frac{1}{2}\left(M_H^2 - 2m_c^2\right) = \frac{1}{2}\sqrt{M_H^4 \lambda(m_c^2, m_c^2; M_H^2) + 4M_c^2} \tag{77}$$

where

$$\lambda(x, y; z) = \left(1 - \frac{x}{z} - \frac{y}{z}\right)^2 - 4\frac{xy}{z^2} \tag{78}$$

and defining

$$X(p_1, p_2, M_H, T^c(p)) = 4[2\frac{p_1^2 p_2^2}{M_H^4} T_{\alpha}^{c\alpha}(p) - 2\frac{p_2^2}{M_H^2}\frac{p_1^{\alpha} p_1^{\beta}}{M_H^2} T_{\alpha\beta}^{c}(p) - 2\frac{p_1^2}{M_H^2}\frac{p_2^{\alpha} p_2^{\beta}}{M_H^2} T_{\alpha\beta}^{c}(p)$$

$$+ 2\frac{p_1 \cdot p_2 p_1^{\alpha} p_2^{\beta}}{M_H^4} T_{\alpha\beta}^{c}(p) + \frac{p_1^2 p_2^2}{M_H^4} T_{\mu\nu}^{c}(p) T^{c\mu\nu}(p) - \frac{p_2^2}{M_H^2}\frac{p_{1\mu} p_1^{\alpha}}{M_H^2} T^{c\mu\nu}(p) T_{\alpha\nu}^{c}(p)$$

$$- \frac{p_1^2}{M_H^2}\frac{p_{2\nu} p_2^{\beta}}{M_H^2} T^{c\alpha\nu}(p) T_{\alpha\beta}^{c}(p) + \frac{p_1^{\mu} p_2^{\nu}}{M_H^2} T_{\mu\nu}^{c}(p) \frac{p_1^{\alpha} p_2^{\beta}}{M_H^2} T_{\alpha\beta}^{c}(p)] \quad , \tag{79}$$

we can write ,

$$\sum_{pol} | M |^2 = 4(\sqrt{16\pi G_N} m_c)^2 \frac{M_H^4}{4m_c^4} \left[\lambda(m_c^2, m_c^2; M_H^2) + 12 \frac{m_c^4}{M_H^4} + X(k_1, k_2, M_H, T^c(k)) \right], \quad (80)$$

It is easy to see that the 4–momenta k_1 and k_2 will only appear in the square bracket of equation (80) as scalar products, such as $k_1 \cdot k_2, P \cdot k_1$ and $P \cdot k_2$. These can all be written in terms of particle masses; therefore, there is no angular dependence on $d\Gamma$.

Noticing also that

$$| k_1 | = \frac{1}{2} M_H \sqrt{\lambda(m_c^2, m_c^2; M_H^2)} \quad , \quad (81)$$

we can finally write:

$$\Gamma_{cc} = G_N M_H^3 \sqrt{\lambda(m_c^2, m_c^2; M_H^2)} \left[\lambda(m_c^2, m_c^2; M_H^2) + 12 \frac{m_c^4}{M_H^4} + X(k_1, k_2, M_H, T^c(k)) \right] \quad (82)$$

The term proportional to X represents the extra contributions from physics BSM and is in agreement with the results of K. Hagiwara, R. Szalapski and D. Zeppenfeld [51].

6 A comparison of the on–shell decays $H \to WW$ and $H \to CC$

We consider the most general coupling of the Higgs H with W vector gauge boson (denoted as HW), where H the Higgs gauge boson and C the heavy graviton. This is

$$igM_W (g_{\mu\nu} + T_{\mu\nu}^W) \quad (83)$$

where g is the weak coupling constant and $T_{\mu\nu}^W$ is the extra contributions from new physics BSM.

The differential width of the on-shell $H \to WW$ decay is given by [50]

$$\Gamma_{ww} = \frac{g^2 M_H^3}{64\pi} \sqrt{\lambda(M_W^2, M_W^2; M_H^2)} \left[\lambda(M_W^2, M_W^2; M_H^2) + \frac{M_W^4}{M_H^4} + X(k_1, k_2, M_H, T^W) \right] \quad (84)$$

where

$$\lambda(x, y; z) = \left(1 - \frac{x}{z} - \frac{y}{z} \right)^2 - 4 \frac{xy}{z^2} \quad ([50]) \quad (85)$$

and

$$X(p_1, p_2, M_H, T^W) = 4[2 \frac{p_1^2 p_2^2}{M_H^4} T_\alpha^{W\alpha} - 2 \frac{p_2^2}{M_H^2} \frac{p_1^\alpha p_1^\beta}{M_H^2} T_{\alpha\beta}^W - 2 \frac{p_1^2}{M_H^2} \frac{p_2^\alpha p_2^\beta}{M_H^2} T_{\alpha\beta}^W$$

$$+2\frac{p_1 \cdot p_2 p_1^{\alpha} p_2^{\beta}}{M_H^4} T^W_{\alpha\beta} + \frac{p_1^2 p_2^2}{M_H^4} T^W_{\mu\nu} T^{W\mu\nu} - \frac{p_2^2}{M_H^2} \frac{p_{1\mu} p_1^{\alpha}}{M_H^2} T^{W\mu\nu} T^W_{\alpha\nu} \quad ([50])$$

$$-\frac{p_1^2}{M_H^2} \frac{p_{2\nu} p_2^{\beta}}{M_H^2} T^{W\alpha\nu} T^W_{\alpha\beta} + \frac{p_1^{\mu} p_2^{\nu}}{M_H^2} T^W_{\mu\nu} \frac{p_1^{\alpha} p_2^{\beta}}{M_H^2} T^W_{\alpha\beta}] \tag{86}$$

Using equations (82), (84), we derive the ratios of partial widths and couplings for the on–shell decays $H \to WW$ and $H \to CC$.

$$\frac{\Delta(\Gamma_c / \Gamma_W)}{\Gamma_c / \Gamma_W}, \tag{87}$$

$$\frac{\Delta(g^2(H,C) / g^2(H,W))}{g^2(H,C) / g^2(H,W)}. \tag{88}$$

Here, the bare Newton coupling constant of $(10^{-3} TeV^{-2})$ is well within the currently allowed range of the proposed heavy graviton mass 100 GeV [31], [32] and is appropriate for a Higgs boson of 125 GeV.

Using equations (87) and (88), we have calculated the ratios of partial widths and couplings for the on–shell decays $H \to WW$ and $H \to CC$, for integrating luminosity in the range of (30-300) fb^{-1} (Tables 1, 2).

Tables.1. Ratios of partial widths for Higgs decay to WW and CC

Integrating luminosity (fb^{-1})	Ratios of partial widths (Γ_c/Γ_w)
30	0.17
300	0.105

Tables.2 Ratios of couplings for Higgs decay to WW and CC

Integrating luminosity (fb^{-1})	Ratios of couplings $(g_c/g_w)^2$
30	0.21
300	0.07

7 Conclusions

We propose an alternative understanding of gravity resulting from the extension of N. Wu's gauge theory of gravity with heavy gravitons at high scale of energy. In this framework, we show that renormalization effects can cause Newton's constant to change dramatically with energy, perhaps even reducing the scale of quantum gravity to the TeV region.

We derive the complete set of formulas for the invisible decay widths of the Higgs boson in heavy graviton. The formulas are valid both for the SM and for any arbitrary extension.

The bare Newton coupling constant of $(10^{-3} TeV^{-2})$ is well within the currently allowed range of the proposed heavy graviton mass 100 GeV and is appropriate for a Higgs boson of 125 GeV. Using equations (87) and (88), we calculate the ratios of partial widths and couplings for the on–shell decays $H \to WW$ and $H \to CC$, for integrating luminosity in the range of (30-300) fb^{-1}.

This proposition may reveal Higgs and graviton properties for new physic BSM.

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