The self-accelerated matter in the gravity field

Sangwha Yi

Department of Math , Taejon University 300-716

ABSTRACT

In the general relativity theory, using Einstein's gravity field equation, find new solution of the self-accelerated matter(ex.rocket) in the general relativity theory. The new solution has the acceleration's term. Therefore according to the new solution, treat that the matter self-accelerates continuously in the gravity.

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e-mail address:sangwha1**@nate.com**

Tel:051-624-3953

I.Introduction

This paper is that in the general relativity theory, find new term that treat that the matter(ex,rocket. Ofcourse, an acceleration of a rocket differ each step.) self-accelerates continuously in the gravity.

The general relativity theory's field equation is written completely.

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} R = -\frac{8\pi G}{c^4} T_{\mu\upsilon} \quad (1)$$

Eq (1) multiply $g^{\mu\upsilon}$ and does contraction,

$$g^{\mu\upsilon} R_{\mu\upsilon} - \frac{1}{2} g^{\mu\upsilon} g_{\mu\upsilon} R$$

$$= -R = -\frac{8\pi G}{c^4} T^{\lambda}{}_{\lambda} \quad (2)$$

Therefore, Eq (1) is

$$R_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} \frac{8\pi G}{c^4} T^{\lambda}{}_{\lambda} = -\frac{8\pi G}{c^4} T_{\mu\upsilon}$$

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4} (T_{\mu\upsilon} - \frac{1}{2} g_{\mu\upsilon} T^{\lambda}{}_{\lambda}) \quad (3)$$

In this time, the spherical coordinate system's vacuum solution is by $T_{\mu\upsilon} = 0$

$$R_{\mu\upsilon} = 0 \quad (4)$$

The spherical coordinate system's invariant time is

$$d\tau^2 = A(t,r)dt^2 - \frac{1}{c^2}[B(t,r)dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (5)$$

Using Eq(5)'s metric, save the Riemannian-curvature tensor, and does contraction, save Ricci-tensor.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = 0 \quad (6)$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = 0 \quad (7)$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = 0 \quad (8)$$

$$R_{\phi\phi} = R_{\theta\theta} \sin^2\theta = 0 \quad (9)$$

$$R_{tr} = -\frac{\dot{B}}{Br} = 0 \ (10) \quad R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0 \ (11)$$

In this time, $' = \frac{\partial}{\partial r}$, $\cdot = \frac{1}{c}\frac{\partial}{\partial t}$

II.Additional chapter

By Eq(10),

$$\dot{B} = 0 \quad (12)$$

By Eq(6) and Eq(7),

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}\left(\frac{A'}{A} + \frac{B'}{B}\right) = -\frac{(AB)'}{rAB^2} = 0 \quad (13)$$

Therefore,

$$A = \frac{1}{B} \quad (14)$$

If Eq(8) is inserted by Eq(14),

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + \left(\frac{r}{B}\right)' = 0 \quad (15)$$

If solve Eq(15)

$$\frac{r}{B} = r + C_1 + C_2 \rightarrow \frac{1}{B} = 1 + \frac{C_1}{r} + \frac{C_2}{r} \quad (16)$$

In this time,

$$C_1 = -\frac{2GM}{c^2} \quad (17)$$

$$\frac{1}{B} = 1 - \frac{2GM}{rc^2} + \frac{C_2}{r} \quad (18)$$

In this time, rindler coordinate system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ is

$$d\tau^2 = \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 (d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2] \quad (19)$$

$$\frac{d^2\xi^1}{(d\xi^0)^2} \approx -\frac{1}{2}c^2 \cdot 2\left(1 + \frac{a_0\xi^1}{c^2}\right) \cdot \frac{a_0}{c^2} \approx -a_0 \quad (20)$$

To find new term that treat that the matter self-accelerates in the general relativity theory, in Eq(18),the term C_2 includes the acceleration a_0.

If C_2 is

$$C_2 = \alpha \frac{G^2 M^2}{c^6} a_0 \quad (21)$$

α is non-Dimension number.

G 's Dimension is $m^3 / s^2 \cdot kg$

Therefore, $G^2 M^2 a_0 / c^6$'s Dimension is $[(m^3 / s^2 \cdot kg)^2 \cdot kg^2 \cdot m / s^2] / (m^6 / s^6) = m$

Therefore, Eq(18) is

$$A = \frac{1}{B} = 1 - \frac{2GM}{rc^2} + \alpha \frac{G^2 M^2}{c^6 r} a_0 \qquad (22)$$

To know Eq(22)'s third term, does Newton's approximation

$$\frac{d^2 r}{dt^2} \approx \frac{1}{2} c^2 \frac{\partial(-A)}{\partial r} = -\frac{GM}{r^2} + \alpha\left(\frac{G^2 M^2}{2r^2 c^4}\right) a_0$$

$$\approx -\frac{GM}{r^2} + a_0 \qquad (23)$$

Therefore, non-Dimension number α is ,(consider $a_0 \approx a_0(\frac{R^2}{r^2}), R \approx r$)

$$\alpha \approx \frac{2R^2 c^4}{G^2 M^2} \text{ (24)},$$ R is the star's radius.

In this time, reason that it use the star's radius R in Eq(24) is that R is only the constant element that it's Dimension is m in the gravity theory.

III.Conclusion

Therefore, the spherical coordinate system's invariant time(the vacuum solution) include new term that treat that the matter self-accelerates in the gravity is

$$d\tau^2 \approx \left(1 - \frac{2GM}{rc^2} + \frac{2R^2}{c^2 r} a_0\right) dt^2 - \frac{1}{c^2}\left[\frac{1}{\left(1 - \frac{2GM}{rc^2} + \frac{2R^2}{c^2 r} a_0\right)} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

(25) In this time, $a_0 \approx a_0(\frac{R^2}{r^2}), R \approx r$

According to this theory, the more the r approach the star's radius R ,the more Eq(25) is accurate.
In Eq(25),in case that the space-time does flat Minkowski space, in the condition $M = 0, R = 0$

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2] \text{(27)}$$

The rindler coordinate system $\xi(\xi^0, \xi^1, \xi^2, \xi^3)$ is

$$d\tau^2 = dt^2 - \frac{1}{c^2}[dx^2 + dy^2 + dz^2] = \left(1 + \frac{a_0 \xi^1}{c^2}\right)^2 (d\xi^0)^2 - \frac{1}{c^2}[(d\xi^1)^2 + (d\xi^2)^2 + (d\xi^3)^2]$$

(28)

Therefore, if the gravity disappear, self-accelerated system does the rindler coordinate system in the flat Minkowski space.

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