Consensus seeking on moving neighborhood model of random sector graphs

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Abstract

In this paper, we address consensus seeking problem of dynamical agents on random sector graphs. Random sector graphs are directed geometric graphs and have been investigated extensively. Each agent randomly walks on these graphs and communicates with each other if and only if they coincide on a node at the same time. Extensive simulations are performed to show that global consensus can be reached.

Keywords: consensus problem, discrete-time system, random sector graph, random geometric graph.

1 Introduction

The consensus seeking problem is an important topic in the study of multi-agent systems. It has been extensively used to represent systems in many practical applications such as social systems, biological systems, formation control, and large-scale robotic systems. A set of agents aim to make an agreement on some quantities of interest via distributive decision making. The information interactions are based upon local neighboring structure. A consensus is said to be achieved if all agents in the system tend to agree on the quantities of interest as time goes on. For more general backgrounds on consensus problems, we refer the reader to [8, 12, 13, 21] and references therein.

An interesting and simple flocking model was proposed by Vicsek et al. [22] in 1995, where all agents will reach consensus as time goes on, provided the communication graph switching deterministically over time is periodically jointly connected. Later, Ali Jadbabaie et al. [6] provided theoretical proof. Consensus in random multi-agent systems have also been addressed [5, 10]. Recently, a new model called moving neighborhood network is introduced in [19]. In this model, each agent carries an oscillator and diffuses in the environment. The computer simulation shows that synchronization is possible even when the communication network is spatially disconnected in general at any given time instant. Subsequently, several researchers have derived analytical results on the moving neighborhood networks, see e.g. [1, 3, 7, 11, 18, 20].

An interesting random graph model is the random sector graph [4]. This model can be viewed as a directed variant of random geometric graphs [9]. Some graph theoretical properties of random sector graphs have been reported [15, 16, 17]. The aim of this paper is to implement consensus on moving neighborhood network modeled by random sector graph. In this paper, we show that it is possible to reach consensus on them by using moving neighborhood model.

2 Preliminaries

Let $G = (V, E, W)$ be a weighted graph with vertex set V. E is a set of pairs of elements of V called edges. $W = (w_{ij})$ is the weight matrix, in which $w_{ij} > 0$ if $(i, j) \in E$, and $w_{ij} = 0$ otherwise. Consider n identical agents $\{v_1, v_2, \cdots, v_n\}$ as random walkers on G, moving randomly to a neighbor of their current location in G at any given time. For each agent, the random neighbor that is chosen is not affected by the agent's previous trajectory. The n random walk processes are independent to each other. If v_i and v_j meet at the same node simultaneously, then they can interact with each other by sending information.

Let $X_i(t) \in \mathbb{R}$ be the state of agent v_i at time t. We use the following consensus protocol

$$X_i(t+1) = X_i(t) + \varepsilon \sum_{j \in N_i(t)} b_{ij}(t)(X_j(t) - X_i(t)) \tag{1}$$

where $\varepsilon > 0$ and $N_i(t)$ is the index set of neighbors of agent v_i at time t. The factor $b_{ij} > 0$ for $i \neq j$, and $b_{ii} = 0$ for $1 \leq i \leq n$. Let $A(t) = (a_{ij}(t))$ be the adjacency matrix of the moving neighborhood network, whose entries are given by,

$$a_{ij}(t) = \begin{cases} b_{ij}(t), & (v_i, v_j) \in E(t) \\ 0, & otherwise \end{cases}$$

for $1 \leq i, j \leq n$. Suppose that $\triangle := \max_{1 \leq i \leq n}(\sum_{j=1}^{n} b_{ij}(t))$, and we further assume $\varepsilon \in (0, 1/\triangle)$ for all t. We will show that the states of all agents walking on a random sector graph reach consensus as time goes on.

3 Simulation examples

For a randomly generated sector graph, we take the weight matrix as the adjacency matrix. In addition, we take the $b_{ij}(t)$ randomly from a set of basic functions such as e^t, $\sin(t)$, $\cos(t)$ and so on. In Fig. 1,2,3,4 we show that the consensus can be achieved asymptotically.

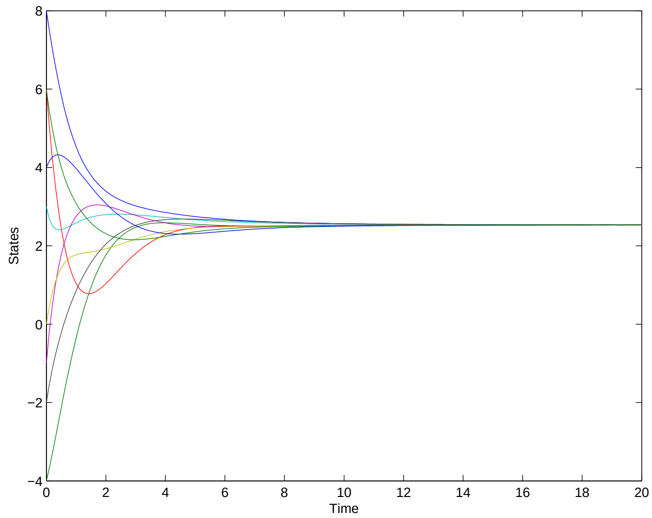

Figure 1: (Color Online) The consensus over moving neighborhood network modeled by a random sector graph.

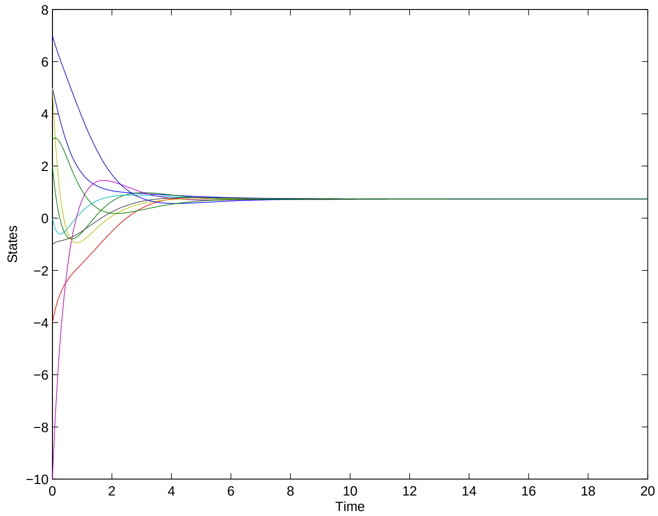

Figure 2: (Color Online) The consensus over moving neighborhood network modeled by a random sector graph.

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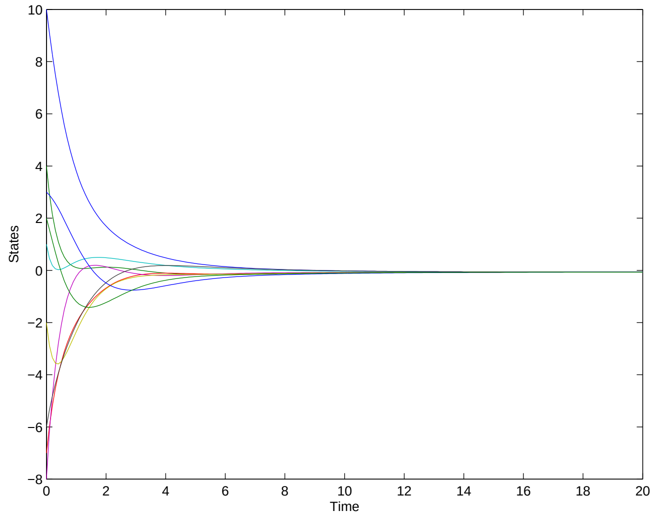

Figure 3: (Color Online) The consensus over moving neighborhood network modeled by a random sector graph.

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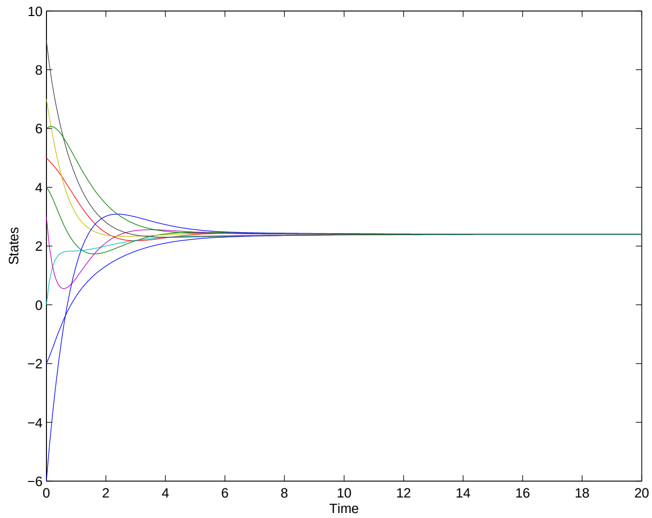

Figure 4: (Color Online) The consensus over moving neighborhood network modeled by a random sector graph.