IS A NON ZERO "LOWER" ORDER GRAVITON MASS DERIVABLE IN BRANE THEORY SETTINGS AND KALUZA KLEIN GRAVITON MODELS?

Andrew Beckwith
Chongqing University Department of Physics;
e-mail: abeckwith@uh.edu
Chongqing, PRC, 400044

Abstract

The lowest order mass for a KK graviton, as a non zero product of two branes interacting via a situation similar to Steinhardt's ekpyrotic universe is obtained, as to an alternative to the present dogma specifying that gravitons must be massless. The relative positions as to the branes gives a dynamical picture as to how lowest order KK gravitons could be affected by contraction and then subsequent expansion. And the solution picked is for with a non zero lowest order mass for a KK graviton permits modeling of gravitons via a dynamical Casmir effect which we generalize using Durrer's 2007 work. Anti-de-Sitter braneworld.construction is what is used to model the Casmir effect. This analysis is taken in AdS_5 geometry while assuming that blue spectrum treatment of gravitons no longer holds.
Keywords: Gravitons, KK theory, Casmir effect; AdS_5

1 Introduction

We make use of work done by Ruser and Durrer [1] [2] which is essentially a re do of the Steinhardt model of the *ekpyrotic* universe [2] [3]. With two branes. One of which is viewed to be stationary and the other is moving toward and away from the stationary brane.

The construction used, largely based upon the Ruser and Durrer [1] article makes used of a set of differential equations based on the Sturm Liouville method which in the case of the order mass being zero have in usual parlance a zero value to lowest order KK graviton mass [1].We will turn this idea on its head by having a non zero graviton mass, zeroth order in the KK construction as to show how graviton mass, lowest order is affected by a Casmir plate treatment of graviton dynamics. As written, this is for an AdS_5 system.

2 Setting up a Casmir effect for lowest order 'massive' KK gravitons.

What we will do is to examine via figure 1 from [1] below the dynamics of the two branes with one stationary and the other moving, which influence a analytical form solution of the lowest order graviton mass problem.

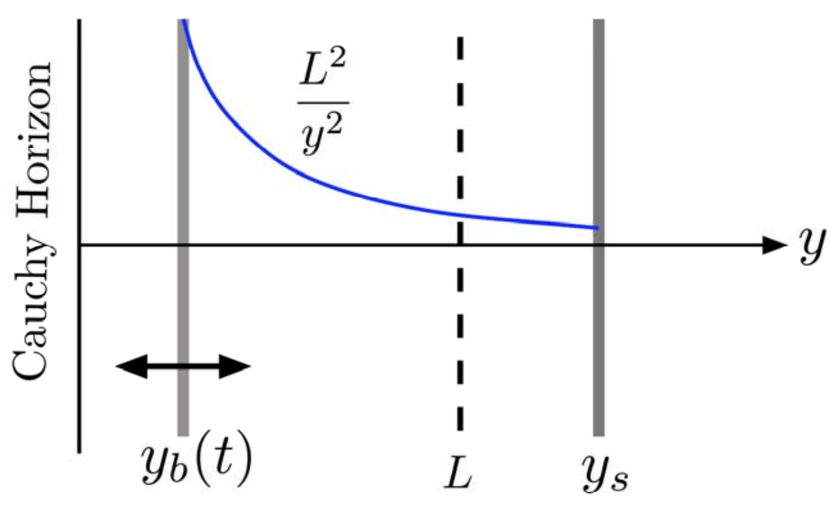

Figure 1, from [1]

Using [1] what we find is that there are two branes on the AdS_5 space-time so that with one moving and one stationary, we can look at figure 1 which is part of the geometry used in the spatial decomposition of the differential operator acting upon the $h_\bullet$ Fourier modes of the h_{ij} operator [1] . As given by [1], we have that

$$\left[\partial_t^2 + k^2 - \partial_y^2 + \frac{3}{y}\cdot\partial_y\right]h_\bullet = 0 \qquad (1)$$

Using [2] (and also [1]) the solution to (1) above takes the form of having

$$h_\bullet = H_{ij} = e_{ij}\cdot\exp[i\cdot\omega\cdot t]\cdot(m\cdot y)^2\cdot A\cdot J_2\left(m\cdot y\right) \qquad (2)$$

e_{ij} is a polarization tensor, and the function $J_2\left(my\right)$ is a 2nd order Bessel function [3] . A generalization offered by Durrer et al. [1], [2] leads to

$$h = \left\{\exp[i\cdot\omega\cdot t]\cdot(m\cdot y)^2\cdot A\cdot J_2\left(m\cdot y\right)\right\}\cdot\left(1+\frac{\pi}{4}\cdot(m\cdot\ell)^2\right) \qquad (3)$$

With the factor of $\left(1+\frac{\pi}{4}\cdot(m\cdot\ell)^2\right)$ coming in due to a boundary condition upon the wall of a brane put in, i.e. looking at [1]. With the right hand side of (4) due to a domain wall tension of a brane.

$$-2\cdot\partial_y H_{ij} = \kappa_5\cdot\pi_{ij}^{(T)} \to 0 \qquad (4)$$

This will be in our example set as not equal to zero, in the right hand side, but equal to an extremely small parameter, namely

$$\partial_y H_{ij}\Big|_{y=yb} = \kappa_5\cdot\pi_{ij}^{(T)} \sim \xi^+ \qquad (5)$$

With this turned into

$$\partial_y h\Big|_{y=yb} \sim \delta^+ \qquad (6)$$

The right hand side of (6) represents very small brane tension, which is understandable. Then using [1],[2],[3] , i.e.

$$\partial_y h\Big|_{y=yb} = \partial_y\left\{\exp[i\omega t]\cdot(my)^2\cdot A\cdot J_2\left(my\right)\right\}\cdot\left(1+\frac{\pi}{4}\cdot(m\cdot\ell)^2\right)\Bigg|_{y=yb} \sim \delta^+ \qquad (7)$$

And

$$J_2\left(my\right) = \frac{(my)^2}{2^2 \cdot 2!} \cdot \left(1 - \frac{(my)^2}{2^2 \cdot 3} + \frac{(my)^4}{2^4 \cdot 2! \cdot 3 \cdot 4} - \frac{(my)^6}{2^6 \cdot 4! \cdot 3 \cdot 4 \cdot 5} + ... \right) \tag{8}$$

The upshot is, that afterwards,

$$\frac{(my)^4}{2^2 \cdot 2!} \cdot \frac{1}{y} \left[\begin{array}{l} \left(1 - \frac{(my)^2}{2^2 \cdot 3} + \frac{(my)^4}{2^4 \cdot 2! \cdot 3 \cdot 4} - \frac{(my)^6}{2^6 \cdot 4! \cdot 3 \cdot 4 \cdot 5} + ... \right) \\ - \left(\frac{2 \cdot (my)^2}{2^2 \cdot 3} + \frac{4 \cdot (my)^4}{2^4 \cdot 2! \cdot 3 \cdot 4} - \frac{6 \cdot (my)^6}{2^6 \cdot 4! \cdot 3 \cdot 4 \cdot 5} + ... \right) \end{array} \right] \tag{9}$$
$$= \frac{\delta^+ \cdot \exp\left[\mp i\omega t\right]}{A} \cdot \left[1 - \frac{\pi}{4} \cdot \left(m \cdot \ell \right)^2 \right]$$

Should the term

$$\frac{\delta^+ \cdot \exp\left[\mp i\omega t\right]}{A} \cdot \left[1 - \frac{\pi}{4} \cdot \left(m \cdot \ell \right)^2 \right] \xrightarrow[\delta^+ \to 0]{} 0 \tag{10}$$

Then, (9) is acting much as in [1], and [2], whereas, one is recovering a simple numerical exercise as to obtain a suitable solution as given by (4) due to [1] where the domain tension of the brane vanishes. The novelty as to this approach given in (9) is to obtain a time dependent behavior of the mass of the graviton, so affected by a numerical root finder, of which the author is aware of several programs, which could perform the job. The point is that in a root finder solution to (9) that one is directly getting a time variation of the mass, m, as in (9) due to (5), and (6) which would have useful implications.

Note that in (9) according to the AdS_5 construction as in Figure 1 above, the y co – ordinate will have time varying behavior. This is the genesis of the remark made that in lieu of (10) collapsing (9) to the condition given in [1] and [2], that having a non zero my expression, of which this is for a function f(t) as a function of time implied by (9)

$$(my) = f(t) \Leftrightarrow m \equiv \frac{f(t)}{y} \tag{11}$$

Needless to say, (9) can only be solved for, numerically, i.e. fourth order polynomial solutions for quartic equations still give over simplified dynamics, especially if (11) holds, and makes things more complicated.

3 **In particular the blue spectrum for (massless gravitons), no longer holds, if gravitons have a slight mass with consequences for observational astrophysics. If (9) holds, the spectrum for light mass gravitons has a different character.**

We refer to (5) and (9) as giving a non zero value of the zeroth order mass of a graviton in KK theory, and then try to re focus upon the more traditional 4 space definition of GW expansion in order to come up with normal modes. To do this, look at the mode equation in 4 space and its analogy to higher dimensions [1]. In 4 space, the mode equation reads as

$$\ddot{\chi}_k + \left[k^2 - \frac{\ddot{a}}{a}\right]\chi_k \sim \ddot{\chi}_k + \left[k^2 - m_0^2\right]\chi_k = 0 \qquad (12)$$

Usually $m_0 = 0$, but if it is not equal to zero, then the (12) equation has a more subtle meaning. Consider from Ruser and Durrer [1] what (12) is turned into, in a more general setting. It gets exotic, namely

$$\ddot{q}_{\alpha,k,\bullet} + \left[k^2 - m_\alpha^2\right] q_{\alpha,k,\bullet} + \sum_\beta \left[M_{\beta,\alpha} - M_{\alpha,\beta}\right]\cdot \dot{q}_{\beta,k,\bullet} + \sum_\beta \left[\dot{M}_{\alpha,\beta} - N_{\alpha,\beta}\right]\cdot q_{\beta,k,\bullet} = 0 \qquad (13)$$

The obvious connection between the two (12) and (13) is that one will have if $\alpha = 0$, then one observes

$$\sum_\beta \left[M_{\beta,\alpha=0} - M_{\alpha=0,\beta}\right]\cdot \dot{q}_{\beta,k,\bullet} + \sum_\beta \left[\dot{M}_{\alpha=0,\beta} - N_{\alpha=0,\beta}\right]\cdot q_{\beta,k,\bullet} = 0 \qquad (14)$$

So, does one have, then, that we can ask if the coefficients in (14) are going to be zero? i.e. can we say that

$$\left[M_{\beta,\alpha=0} - M_{\alpha=0,\beta}\right] = \left[\dot{M}_{\alpha=0,\beta} - N_{\alpha=0,\beta}\right] = 0 \qquad (15)$$

For them to become zero, then we should note by Ruser and Durrer [1] , that by [6]

$M_{\beta,\alpha=0}, M_{\beta,\alpha\neq 0}, M_{\alpha=0,\beta}, M_{\alpha\neq 0,\beta}, N_{\alpha=0,\beta}, N_{\alpha\neq 0,\beta}$ have been already derived in detail in [6] . Furthermore matrix M is defined by brane motion and

$$N = M^T M \tag{16}$$

The claim we have is that **if (9) holds, then (15), and (16) does not hold.** We claim that if (15) does not hold, one is observing conditions for which the blue spectrum for massless gravitons cannot be true, if the initially massless zeroth order KK gravitons becomes massive. In order to understand this though, we should look at what an expert had to say about massive gravitons, h_{ij} and the formation of $h_{\bullet}$. The main point we are emphasizing is that the determination of mass so indicated by (9) above as a result of looking at (5) and (6) which has a very semi classical flavor to it, is no more startling than the work shown in [7] indicating how semi classical methods can indeed, retrieve what is usually asumed to be a quantum result.

5 Conclusion, a necessary Review of Physics of linkage between $h_{\bullet}, h_{ij}$ and massive Gravitons

First of all, review the details of a massive graviton imprint upon h_{ij}, and then we will review the linkage between that and certain limits upon $h_{\bullet}$

As read from Hinterbichler [8],if $r = \sqrt{x_i x_i}$, and we look at a mass induced h_{ij} suppression factor put in of $\exp(-m \cdot r)$, then if

$$h_{00}(x) = \frac{2M}{3M_{Planck}} \cdot \frac{\exp(-m \cdot r)}{4\pi \cdot r} \tag{17}$$

$$h_{0i}(x) = 0 \tag{18}$$

$$h_{ij}(x) = \left[\frac{M}{3M_{Planck}} \cdot \frac{\exp(-m \cdot r)}{4\pi \cdot r} \right] \cdot \left(\frac{1 + m \cdot r + m^2 \cdot r^2}{m^2 \cdot r^2} \cdot \delta_{ij} - \left[\frac{3 + 3m \cdot r + m^2 \cdot r^2}{m^2 \cdot r^4} \right] \cdot x_i \cdot x_j \right) \tag{19}$$

Here, we have that these h_{ij} values are solutions to the following equation, as given by [8], [9], with D a dimensions value put in.

$$\left(\partial^2 - m^2\right)h_{\mu\nu} = -\kappa \cdot \left[T_{uv} - \frac{1}{D-1}\cdot\left(\eta_{uv} - \frac{\partial_\mu \partial_v}{m^2}\right)\cdot T\right] \tag{20}$$

To understand the import of the above equations, set

$$\begin{aligned} M &= 10^{50} \cdot 10^{-27} g \equiv 10^{23} g \propto 10^{61} - 10^{62} eV \\ M_{Plank} &= 1.22 \times 10^{28} eV \end{aligned} \tag{21}$$

We should use the $m_{massive-graviton} \sim 10^{-26} eV$ value in (21) above.If the h_{ij} massive graviton values are understood, then we hope we can make sense out of the general uncertainty relationship given by [10]

$$\left\langle \left(\delta g_{uv}\right)^2 \left(\hat{T}^{uv}\right)^2 \right\rangle \geq \frac{\hbar^2}{V_{vol}{}^2} \tag{22}$$

In reviewing what was said about (22) we should keep in mind the overall Fourier decomposition linkage between $h_{\bullet}, h_{ij}$ which is written up as

$$h_{ij}\left(t, x; k\right) = \frac{1}{\left(2\pi\right)^{3/2}} \int d^3k \sum_{\bullet = +, \otimes} e^{ik\cdot x} e_{ij}^{\bullet} h_{\bullet}\left(t, y; k\right) \tag{23}$$

The bottom line is that the simple de composition with a basis in two polarization states, of $+, \otimes$ will have to be amended and adjusted , if one is looking at massive graviton states, and if we are going to have a coupling as given by (6), 4 dimensional zeroth order mass massive graviton values, and the input of information given in (17) to (21) as given by [1] Having a a simple set of polarization states as given by $+, \otimes$ will have to be replaced, mathematically by a different de composition structure, with the limit of massive gravitons approaching zero reducing to the simpler $+, \otimes$ basis states.

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