The reaction self-force of a classical point particle actually does not diverge

Kenneth Sandale

Abstract The problem with the standard approach to the treatment of the self-force of a point charge is that it incorrectly tacitly assumes that the self-force is generated by acceleration induced by the external field, when in reality the self-force is generated by acceleration induced by the sum of the external field and the self-force field itself. When treated correctly the undesirable infinity does not occur.

Keywords self force · radiation reaction · electromagnetic mass · classical electromagnetism · singularity · paradox

1 Introduction

A charged particle undergoing non-uniform motion will produce a self-force that can be written as an expansion of terms [1]. The term proportional to the acceleration is of the form $\left(\frac{\alpha q^2}{r}\right) a$, where q is the charge, a is the acceleration, r is the "size" of the particle, and α is determined by the geometric distribution of charge in the particle. This term has long been troubling because when r goes to zero it diverges.

Dirac [2] attempted to remove this divergence by mixing advanced waves with retarded waves. Wheeler and Feynman [3] expanded on Dirac's advanced wave scheme by developing their "absorber theory". Bopp [4] modified the rules of electromagnetism. Born and Infeld [5] also modified the rules of electromagnetism. All of these efforts failed to produce an adequate resolution, and were abandoned [6].

We will show here that if we simply keep careful track of the forces acting on the particle, the divergence problem does not really occur – no modifications of standard electromagnetism are at all needed.

Kenneth Sandale
Allegro Wireless Canada Inc., 1 Eva Road, Toronto, ON, M9C 4Z5, Canada
E-mail: sandale@alum.mit.edu

2 Method of Resolution

From Maxwell's Equations the self-field actually is indeed $\left(\frac{\alpha q}{r}\right) a$, leading to a self-force of $\left(\frac{\alpha q^2}{r}\right) a$. What is standardly done is to use $\frac{F_{external}}{m}$ for a. This gives

$$F_{reaction} = -\left(\frac{\alpha q^2}{r}\right)\left(\frac{F_{external}}{m}\right) \tag{1}$$

This indeed gives a diverging $F_{reaction}$ for a finite external field acting on the point particle. *The problem though is that the field acting on the particle is not only the external field, but rather the sum of the external field plus the reaction field itself.* Being that for the case of a point particle the reaction field appears large (even divergent), neglecting it can be expected to produce seriously quantitatively wrong results. We will now include it, and then proceed using elementary algebra.

$$F_{reaction} = -\left(\frac{\alpha q^2}{r}\right)\left(\frac{F_{external} + F_{reaction}}{m}\right) \tag{2}$$

Using trivial algebra this becomes

$$F_{reaction} + \left(\frac{\alpha q^2}{mr}\right) F_{reaction} = -\left(\frac{\alpha q^2}{mr}\right) F_{external} \tag{3}$$

$$F_{reaction} = -\frac{\left(\frac{\alpha q^2}{mr}\right)}{1 + \left(\frac{\alpha q^2}{mr}\right)} F_{external} \tag{4}$$

If r is large then the denominator of the right-hand side approaches 1, and we get the result standardly assumed that $F_{reaction}$ is $\left(\frac{\alpha q^2}{mr}\right) F_{external}$. But if r approaches zero then Equation 4 takes the form

$$F_{reaction} = F_{external} \lim_{\chi \to \infty} \frac{-\chi}{1 + \chi} \tag{5}$$

where χ is $\frac{\alpha q^2}{mr}$. Of course $\lim_{\chi \to \infty} \frac{-\chi}{1+\chi} = -1$ so in the limit of a point charge we get that the reaction force is -1 times the external force, giving a total force of $F_{external} + F_{reaction} = 0$. The divergence simply does not result.

In reality, no literal point charges exist. From the Uncertainty Principle, a completely localized fundamental particle will immediately become unlocalized. But that does not mean our result is without physical meaning. Classical Physics has validity to an excellent approximation under certain conditions. The result that the self-force will go as $\frac{\left(\frac{\alpha q^2}{mr}\right)}{1+\left(\frac{\alpha q^2}{mr}\right)} F_{external}$ rather than $\left(\frac{\alpha q^2}{mr}\right) F_{external}$ is something that will be a valid approximation within a certain range, and can provide a definitive experimental test of our prediction. Thus we see that the pathological behavior of classical point charges that was thought to occur does not occur.

3 Conclusion

The classical self-force of a particle in a uniform electric field goes as $\frac{\left(\frac{\alpha q^2}{mr}\right)}{1+\left(\frac{\alpha q^2}{mr}\right)} F_{external}$ rather than $\left(\frac{\alpha q^2}{mr}\right) F_{external}$. This quantity remains finite as r goes to zero.

Acknowledgements Financial support from Allegro Research is gratefully acknowledged.

References

1. Feynman, R.P., Leighton, R. B., Sands, M.: The Feynman Lectures on Physics, vol. 2, Chapter 28, Section 28-4, Equation 28.9, AddisonWesley, Reading, Massachusetts (1964)
2. Dirac, P. A. M.,:Proc. Roy. Soc. London A 167, 148-169 (1938)
3. Wheeler, J. A., Feynman, R. P.: Rev. Mod. Phys. 17, 157181 (1945)
4. Bopp, F.: Z. Physik 169, 45-52 (1962)
5. Born, M., Infeld, L.:Proc. Roy. Soc. London A 144, 425-451 (1934)
6. Feynman, R.P., Leighton, R. B., Sands, M.: The Feynman Lectures on Physics, vol. 2, Chapter 28, Section 28-5, Equation 28.9, AddisonWesley, Reading, Massachusetts (1964)