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Four Poission-Laplace Theory of Gravitation (I)

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ABSTRACT

The Poisson-Laplace equation ($\nabla^2\Phi = 4\pi G\rho$) is a working and acceptable equation of gravitation which is mostly used or applied in its differential form in Magneto-Hydro-Dynamic (MHD) modelling. From a general relativistic standpoint, it describes gravitational fields in the region of low spacetime curvature as it emerges in the weak field limit. For none-static gravitational fields *i.e.* $\Phi = \Phi(\boldsymbol{r}, t)$, this equation is not generally covariant. On the requirements of general covariance, this equation can be extended to include a time dependent component, in which case, one is led to the Four Poisson-Laplace equation ($\Box\Phi = 4\pi G\rho$). We solve the Four Poisson-Laplace equation for radial solutions *i.e.* $\Phi = \Phi(r, t)$, and apart from the Newtonian gravitational pole, we obtain four new solutions leading to four new gravitational poles capable (*in-principle*) of explaining *e.g.* the rotation curves of galaxies, the Pioneer anomaly, the Titius-Bode Law and the formation of planetary rings. In this letter, we focus only on writing down these solutions. The task to show that these new solutions might explain the aforesaid gravitational anomalies, has been left for separate future readings.

Key words: astrometry – celestial mechanics – ephemerides – planets and satellites: formation.

1 INTRODUCTION

The Azimuthally Symmetric Theory *of* Gravitation (herafter ASTG-model) set out in Nyambuya (2010*a*) and preliminarily explored in Nyambuya (2010*b*, 2010*c*), is based on the Poisson-Laplace equation:

$$\nabla^2\Phi = 4\pi G\rho. \tag{1}$$

When we embarked on the ASTG-model, it was to seek for a possible solution to the radiation problem thought to bedevil massive stars during their process of formation (*see*, Nyambuya 2010*b*, 2010*c*). In the present letter, and more that are (expected) to come, we continue the quest to seek further ground for the ASTG-model, *i.e.*, demonstrate its latent power and hidden potency.

From the vantage point of the Poisson-Laplace equation (1), Newtonian gravitation is born out of the Poisson-Laplace (1) on the assumption that $\Phi = \Phi(r)$ and $\dot{G} \equiv 0$. It has been argued in Nyambuya (2010*a*) that for a spinning gravitating body, we must have $\Phi = \Phi(r, \theta)$. Solving the Poisson-Laplace equation in Nyambuya (2010*a*) for the setting $\Phi = \Phi(r, \theta)$, it was shown that in the case of empty space ($\rho = 0$), we have:

$$\Phi(r, \theta) = -\sum_{\ell=0}^{\infty} \lambda_\ell c^2 \left(\frac{G\mathcal{M}}{rc^2}\right)^{\ell+1} P_\ell(\cos\theta). \tag{2}$$

Under normal Poisson-Laplace theory, the λ's are pure constants. The ingenuity and novelty of the ASTG-model is to posit these λ's as dynamic parameters, they now depend on the gravitating object's mass $\mathcal{M}$, physical radius $\mathcal{R}$ and spin angular frequency ω, *i.e.*:

$$\lambda_\ell = \left(\frac{(-1)^{\ell+1}}{(\ell^\ell)!(\ell^\ell)}\right)\lambda_1 \quad \text{where,} \quad \lambda_1 = \left(\frac{\mathcal{S}}{\mathcal{S}_*}\right)^\zeta \frac{\mathcal{R}}{\mathcal{R}_s}. \tag{3}$$

In the above, $\zeta = 1.71$, $\mathcal{S} = \mathcal{R}^2\omega$ is the specific spin of the gravitating body *i.e.* spin per unit mass and $\mathcal{S}_* = \sqrt{G\mathcal{M}\mathcal{R}}$ is the critical spin of an object of mass $\mathcal{M}$ and radius $\mathcal{R}$; it has been argued in Nyambuya (2010*c*), that once the spin of the object exceeds this critical spin, the repulsive polar gravitational field will switch on and the gravitating body in question will begin an out-pour of matter in the polar regions. This out-pour of matter reeds the star of the excess spin angular momentum which would otherwise tear the fast spinning star apart *via* the strong centrifugal forces. This is one of the most outstanding features of the ASTG-model that make the it a unique theory of gravitation.

The above λ_1-parameter is consistent with the Solar spin properties. This formula ought to be universal, thus a test of the theory would be to apply the theory to as many gravitating system as can be and; if they give results corresponding with experience for these gravitating systems, it would be a further indicator to the possible correctness of the theory.

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From a general covariance stand point, one major set-back and fallout of the Poisson-Laplace equation (1) is that for none static gravitational fields – *i.e.* $\Phi = \Phi(\boldsymbol{r}, t)$; this equation is not covariant. If this equation hopes to stand-up as a true Law *of* Nature, then, it must successfully meet this seemingly sacrosanct requirement. This letter's endeavour is to address this matter.

2 THEORY

From a purely and strictly general relativistic standpoint, the Poisson-Laplace equation cannot be accepted as a true Law *of* Nature as it is neither Lorentz nor coordinate invariant for none-static gravitational fields *i.e.* $\Phi = \Phi(\boldsymbol{r}, t)$. In this sense – *i.e.*, of its none-general covariance, it is more like the highly successful Schrödinger equation, which is a down-graded version of a more generally covariant equation – the Klein-Gordon equation. Yes, the Schrödinger equation is successful, and true as-well is that, it does not meet the strict requirements of general covariance, so it is only but a good approximation to the real Law *of* Nature that gives rise to it. It is just but a very good approximation.

Lorentz and coordinate invariance are sacrosanct minimum requirements for any law that seek the status of a Law of *Nature*. In order for the Poisson-Laplace equation (1) to fulfil the Principle *of* Relativity, it is necessary to supplement it with a time dependent term, *i.e.*:

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 4\pi G\rho, \tag{4}$$

where (here and after) $G = 6.667\times10^{-11}\mathrm{kg}^{-1}\mathrm{m}^3\mathrm{s}^{-2}$ is Newton's universal constant of gravitation, $c = 2.99792458 \times 10^8\,\mathrm{ms}^{-1}$ is the universal speed of light in vacuum. In view of (4), the Poisson-Laplace equation (1) can be viewed as the case where $\ddot{\Phi} = 0$. This law (*i.e.* 4), satisfies the Principle *of* Relativity as it can be derived from Einstein's equation of gravitation as a first order approximation in the weak field limit (Einstein 1915). This law published in 1915 by Albert Einstein (1879 – 1955) is given by:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{5}$$

where $R_{\mu\nu}$, is the Ricci tensor; R, is the Ricci scaler; $g_{\mu\nu}$, is the metric of spacetime; $T_{\mu\nu}$, is the matter stress-energy-momentum tensor; $\kappa = 8\pi G/c^4$ and; Λ, is Einstein's controversial cosmological constant which he introduced to "stop" the Universe from expanding. Here, we shall assume that this constant (Λ) vanishes identically (*i.e.* $\Lambda \equiv 0$).

Now, in the weak field approximation, the metric is given by $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where $\eta_{\mu\nu}$ is the flat spacetime Minkowiski metric and $h_{\mu\nu}$ are the first order terms of the metric; inserting this into (5), one is led to: $\Box h_{\mu\nu} = \kappa T_{\mu\nu}$. In this weak field approximation, the dominant term is the 00-component of this equation, and these components, to first order approximation, they are given by $h_{00} = 2\Phi/c^2$ and $T_{00} = \rho c^2$. Inserting these into $\Box h_{\mu\nu} = \kappa T_{\mu\nu}$, one is led to the four Poisson-Laplace equation (4). Thus, if one accepts the Einstein field equation (5), then, automatically, they also accept (4), thus this equation is an acceptable equation of gravitation.

From a purely general relativistic standpoint, this equation (*i.e.* 4) is but an approximation only applicable in the weak field limit. The ASTG-model, together with the Four Poisson-Laplace theory here being advanced, are not a subset of Einstein's General Theory *of* Relativity (GTR); *see* Nyambuya (2011*b*). Further, we would like to stress at this point that, on a much broader scheme beyond the scope of the present letter, we are not tackling this equation from a general relativistic stand point but from a more generalized relativistic standpoint as set out in Nyambuya (2011*a*). In this work *i.e.* Nyambuya (2011*a*), equation (4) emerges as an equation describing gravitation. In this theory set out in Nyambuya (2011*a*), gravitation is described by a scaler field (Φ) while the metric tensor field ($g_{\mu\nu}$) describes the nuclear forces – *i.e.*, the Electromagnetic, the Weak and the Strong force. This rather ambitious work (Nyambuya 2011*a*), is an on-going attempt at an all-encompassing *Unified Field Theory* of all the four fundamental forces of *Nature*, uniting not just the fundamental forces, but both the classical and quantum phenomenon into one grand unified picture.

However, inorder to accept (4), one does not have to be in agreement with the work presented in Nyambuya (2011*a*), but merely accept (4) on the basis of requiring that the Poisson-Laplace equation obeys the Principle *of* Relativity and, that, it must submit to the general covariance principle. We have mentioned the work presented in Nyambuya (2011*a*), only for the interested reader who has to make their own assessment and judgement of this work. Otherwise, they just have to take matters from (4) and simple forget about the work presented in Nyambuya (2011*a*).

3 DERIVATION OF THE FIELD POTENTIALS

As will be demonstrated shortly, an interesting feature of this law (*i.e.* eqn 4) is that the time dependent potential $\Phi(t)$ can be associated or attributed to a time variable gravitational constant $G(t) = G\Phi(t)$. To see this, we have to solve the equation first. We will do so by separation of variables in §(3.1) where three solutions will be obtained. In §(3.2), we shall solve this same equation for two none-separable solutions. In total, five solutions are obtained.

In solving this equation (*i.e.* eqn 4), we shall do is to solve the empty space equation, *i.e.* $\rho = 0 :\Longrightarrow \Box\Phi = 0$. This empty space equation applies for the case where the mass $\mathcal{M}$ of the central gravitating body is constant. To obtain a solution for the case where a star or the gravitating mass is immersed in a pool of gas like, *e.g.*, a star inside a core where the mass is dependent on the radial distance *i.e.* $\mathcal{M}(r)$, what one needs to do is to replace $\mathcal{M}$ in the empty space solution with $\mathcal{M}(r) = \int_{r_0}^{r}\int_0^{2\pi}\int_0^{2\pi} r^2\rho drd\theta d\varphi$ *i.e.* $\mathcal{M} \longmapsto \mathcal{M}(r)$. The arguments justifying this have been laid down in Nyambuya (2010*c*).

3.1 Separable Solutions

As afore-stated, our focus in this letter is on the radial solutions, thus we are going to solve the equation $\Box\Phi = 0$, only for the radial solutions by the method of separation of variables where $\Phi(r, t) = \Phi(r)\Phi(t)$. From this, it follows that:

$$\frac{1}{r^2\Phi(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi(r)}{\partial r}\right) = \frac{1}{c^2\Phi(t)}\frac{\partial^2\Phi(t)}{\partial t^2} = \mu^2, \tag{6}$$

where μ is a dimensional constant with units of m^{-1}. Obviously, this differential equation will have to be solved for three cases, *i.e.* $(\mu^2 = 0, \mu^2 > 0, \mu^2 < 0)$. The constant μ is a universal constant because it depends neither on any of the space coordinates (r, θ, φ), nor time (t). Clearly, because of this non-dependence on space and time coordinates, this constant must be an important universal and fundamental constant of *Nature*, having (perhaps) the same status as, *e.g.*, the speed of light c. In the subsequent section, we will consider separately the cases $(\mu^2 = 0)$, $(\mu^2 > 0)$ and $(\mu^2 < 0)$ where the first three solutions are laid down and in §(3.2) the last two solutions are presented. For easy referencing, we are going to label them with a subscript which runs from 1 to 5 *i.e.* $\Phi = \Phi_j : j = 1, 2, \dots, 5$.

3.1.1 *First Gravitational Pole:* $(\mu^2 = 0)$

Newtonian Component: In the light of the time dependence just introduced in the Poisson-Laplace equation, we are forced to formally go through the "derivation" of the Newtonian gravitational potential, *albeit*, with the important difference that the gravitational constant forthwith seizes to be a constant; it is now time dependent. Assuming separability *i.e.* $\Phi(r, t) = \Phi(r)\Phi(t)$, the radial solution $\Phi(r)$ of (6) is the well known Newtonian gravitational potential, *i.e.*: $\Phi(r) = -G\mathcal{M}/r$, where (here and after) $\mathcal{M}$ is the mass of the gravitating body in question and r is the radial distance from this body. The solution to the time dependent part is $\Phi(t) = A + Bt$, where (A, B) are constants. Since $\Phi(r, t) = \Phi(r)\Phi(t)$, it follows that:

$$\Phi(r, t) = -\frac{G\Phi(t)\mathcal{M}}{r}. \qquad (7)$$

From the above, clearly, the time dependent component of the gravitational field $\Phi(t)$ can be absorbed into the gravitational constant G as follows: $G(t) = G\Phi(t) = G(A + Bct)$. This means, we can now write (7) with the linear time dependent term having been absorbed into the the gravitational constant as follows:

$$\Phi_1(r, t) = -\frac{G_1(t)\mathcal{M}}{r}, \qquad (8)$$

where, as part of the labelling scheme stated earlier, we now have inserted the subscript "1" onto $\Phi(r, t)$ and $G(t)$. Now, if $G_1(0)$ is the gravitational constant when the cosmic time was equal to zero, and $\dot{G}_1$ is the time rate of change of the gravitational constant, it follows that:

$$G_1(t) = G_1(0) + \dot{G}_1 t. \qquad (9)$$

For the first gravitational pole, throughout this letter, $G_1(0)$ shall represent the gravitational constant when the cosmic time was equal to zero, and $\dot{G}_1$ the time rate of change of the gravitational constant. Because naturally we expect that the strength of the gravitational force should diminish with time; for this to be so, we must have $[\dot{G}_1(0) < 0, G_1(0) > 0]$.

3.1.2 *Second Gravitational Pole:* $(\mu^2 > 0)$

Pioneer Component (I) (Yukawa Potential): Rather in an *ad hoc* manner, the Yukawa type gravitational potential has long been considered as a possible cause of the Pioneer anomaly (see *e.g.* Brownstein & Moffat 2006; Minguzzi 2006; Moffat 2004; Anderson *et al.* 2002) and a contender to explaining the seemingly anomalous rotation curve of galaxies (see *e.g.* Sanders 2006, 1972). In all these considerations, no fundamental justification for the Yukawa type potential has been put forward; thus far, the only justification is compliance empirical evidence. What we shall do here is basically give justification for them. We believe that this will give credence to the efforts (by *e.g.* Brownstein & Moffat 2006; Minguzzi 2006; Sanders 2006; Moffat 2004; Anderson *et al.* 2002; Sanders 1972) where the Yukawa potential has been introduced in a rather *ad hoc* manner to explain the Pioneer anomaly and the seemingly anomalous rotation curves of galaxies. In the afore-cited studies, the gravitational potential is given by:

$$\Phi(r) = \underbrace{-\frac{G\mathcal{M}}{r}}_{\textbf{Newtonian Term}} - \overbrace{\frac{\alpha_* G\mathcal{M}e^{-\mu_* r}}{r}}^{\textbf{Yukawa Term}}. \qquad (10)$$

The Pioneer anomaly and rotation curves of galaxies is then explained by the extra Yukawa term. In the Yukawa term α_* and μ_* are constants that are determined by fitting the theory to the data. The question is: "Other than the fact that this term can fit the data, is there any fundamental justification for this *ad hoc* term? Can it be derived rather than inserted *via* the lathe of hand? As stated, our core-mission in this section is to justify its inclusion in matters of gravitation by deriving it from some credible soils of gravitation.

Now, without wasting much time and space, assuming separability *i.e.* $\Phi(r, t) = \Phi(r)\Phi(t)$, the solutions to (6), are: $\Phi(r) = \frac{Ae^{-\mu r} + Be^{\mu r}}{r}$ and $\Phi(t) = ae^{-\mu ct} + be^{\mu ct}$, where (A, B, a, b) are constants. If $(B, b) \neq 0$, we will obtain a gravitational potential that is not in tandem with physical and natural reality as we know it because the terms with the coefficients (B, b) will lead to a gravitational field that only gets stronger with increasing distance and the progression of time. To avoid this, we must have $(B, b) \equiv 0$. Now, we shall make the setting $A = -G_2(0)\mathcal{M}$ where $G_2(0)$ is the associated gravitational constant at time $t = 0$. Further setting $G_2(t) = G_2(0)\Phi(t) = G_2(0)e^{-\mu_2 ct}$ and putting everything together, the gravitational Yukawa potential should then be given by:

$$\Phi_2(r, t) = -\frac{G_2(t)\mathcal{M}e^{-\mu_2(t)r}}{r}. \qquad (11)$$

In a future reading that only awaits the publication of the the present letter, it will be shown that the potential $\Phi_2(r, t)$, together with $\Phi_4(r, t)$; can *in-principle* explain the Pioneer anomaly. In anticipation, we have coined these two potentials "Pioneer Component (I) and (II)" respectively.

3.1.3 *Third Gravitational Pole:* $(\mu^2 < 0)$

Planetary Ring Component: We are now going to generate our third gravitational pole. This pole gives rise to a ring structure around a central massive gravitating body. These rings are such that they are equally spaced. Given this, and as-well that planets not do exhibit such an even spacing, this ring structure "can not explain" the origins of planets as we know them. If one can conceive of the possibility that in those places where a ring is expected and there is none, then, hypothetical, this ring is considered missing, then, the theory has not failed but predicted a missing ring. That said, let us go onto the derivation of this gravitational pole.

As one can verify for themselves, the general solution to (6) under the constraint ($\mu^2 < 0$), is: $\Phi(r) = \frac{Ae^{i|\mu|r}+Be^{-i|\mu|r}}{r}$, and $\Phi(t) = ae^{-|\mu|ct} + be^{|\mu|ct}$, where $|\mu|$ is the magnitude of μ: remember μ is a complex number, hence $|\mu|$. Now, because $\Phi(r)$ and $\Phi(t)$ must be real, we must have $A = B$ and $a = b$. With this setting, one will have the complex parts of $\Phi(r)$ being identically equal to zero. Let us set $A = B = -G_3\mathcal{M}$, so that the final solution is:

$$\Phi_3(r,t) = -\frac{G_3(t)\mathcal{M}\cos(|\mu_3|r)}{r}, \tag{12}$$

where $G_3(t) = G_3(0)\cos(|\mu_3|ct)$. Making a brief snap-shot of this solution, we note that, in space, this potential has its minimum-points when $\cos(|\mu_3|r) = -1$ and, this occurs when $|\mu_3|r = \pi + \pi n$. From Lagrangian mechanics, we know that a system will tend to settle in regions where its Lagrangian is minimum. One can easily show that the regions defined by the rings: $r_n = (n+1)\left(\frac{\pi}{|\mu_3|}\right) = (n+1)\mathcal{R}_*$, where $\mathcal{R}_* = \frac{\pi}{|\mu_3|}$; will be regions of minimum Lagrangian, therefore, matter will tend to settle in these rings thus giving rise to a ring structure, hence, the potential (12) should most certainly explain the existence of rings around planetary bodies such as the planet Saturn.

3.2 None Separable Solutions

If one was only seeking separable solutions, then, they would have to end with the solution $\Phi_3(r,t)$. With the two "new" solutions $\Phi_2(r,t)$ and $\Phi_3(r,t)$, they will – at least; be in a position to justify the inclusion of the Yukawa potential in gravitational physics and as-well to explain the existence of planetary rings. They will however find themselves not in a position to consistently explain the Pioneer anomaly and the rotation curves of galaxies; and as-well, the Titius-Bode Law that posits the logarithmic placing of planets in planetary systems.

As will be shown shortly, there exists two none-separable solutions. For these none-separable solutions we assume $\Phi(r,t) = \phi(r,t)\Phi(r)$ where $\phi(r,t)$ is the part with the none separable space and time coordinates. The none-separable part $\Phi(r)$ is such that:

$$\frac{1}{\Phi(r)}\frac{\partial}{\partial r}\left[r^2\frac{\partial\Phi(r)}{\partial r}\right] = \nu^2, \tag{13}$$

where $\nu^2 \neq 0$ is a dimensionless constant and the none-separable part is given by:

$$\begin{aligned}\frac{1}{\phi(r,t)}\frac{\partial}{\partial r}\left[r^2\frac{\partial\phi(r,t)}{\partial r}\right] + \frac{2}{\phi(r,t)\Phi(r)}\frac{\partial\Phi(r)}{\partial r}\frac{\partial\phi(r,t)}{\partial r} \\ -\frac{r^2}{c^2\phi(r,t)}\frac{\partial^2\phi(r,t)}{\partial t^2} = -\nu^2\end{aligned}. \tag{14}$$

This none-separable part has no exact solution. One will have to solve this (perhaps) by numerical method. Given the limited space of this letter, we can not embark on this task to find the numerical solution. So, what one will have to do is to solve for $\Phi(r)$ and insert this into (14) and then the proceed to solve the resulting equation by numerical means to obtain $\phi(r,t)$. The solution $\phi(r,t)$ belongs to the gravitational constant. Therefore, G_4 and G_5 will both have a spacial and temporal variation. Now, in the subsequent sections, we shall consider the two cases ($\nu^2 > 0$ and $\nu^2 < 0$).

3.2.1 Fourth Gravitational Pole: ($\nu^2 > 0$)

Pioneer Component (II): Let us set $\nu^2 = \alpha(\alpha+1)$, where α is some dimensionless quantity. Now, from this relationship $\nu^2 = \alpha(\alpha+1)$, it follows that: $\alpha = (-1 \pm \sqrt{1+4\nu^2})/2 = (\alpha_1, \alpha_2)$. The two solutions of α are represented by (α_1, α_2). Note that for a real ν, the discriminant of (3.2.1) [*i.e.* the term $(1+4\nu^2)$, under the square root sign] is positive definite.

Our step in solving (13) and (14) is the following: (1) We solve for $\Phi_4(r)$ in (13). (2) We know that the general solution to this equation in the case $\nu^2 = \alpha(\alpha+1)$, is $\Phi_4(r) = A\left[\left(\frac{r}{\mathcal{R}_4}\right)^{\alpha_1} + \left(\frac{r}{\mathcal{R}_4}\right)^{\alpha_2}\right] + B\left[\left(\frac{r}{\mathcal{R}_4}\right)^{-(\alpha_1+1)} + \left(\frac{r}{\mathcal{R}_4}\right)^{-(\alpha_2+1)}\right]$, where $\mathcal{R}_4$ is a constant with the dimensions of length and (A, B) are some dimensional constants. (3) The next step now is to introduce a time dependence into this expression *i.e.* $\Phi(r)$, by numerically solving for $\phi(r,t)$ in (14). Off cause, as stated, we are not going to do conduct this exercise by assume that this solution exists. (4) The is step is to set $A = B = -G_4(0)\mathcal{M}$. Now, since $\Phi_4(r,t) = \phi_4(r,t)\Phi_4(r)$, it follows that this solution can be written as:

$$\begin{aligned}\Phi_4(r,t) = -G_4(r,t)\mathcal{M}\Bigg[\left(\frac{r}{\mathcal{R}_4}\right)^{\alpha_1} + \left(\frac{r}{\mathcal{R}_4}\right)^{\alpha_2} + \\ \left(\frac{r}{\mathcal{R}_4}\right)^{-(\alpha_1+1)} + \left(\frac{r}{\mathcal{R}_4}\right)^{-(\alpha_2+1)}\Bigg]\end{aligned}, \tag{15}$$

where $G_4(r,t) = G_4(0)\phi(r,t)$.

As will be seen a future reading that tackles the Pioneer anomaly, the potential $\Phi_4(r,t)$ – together with $\Phi_2(r,t)$, can *in-principle*, explain the Pioneer anomaly, hence we have termed it [*i.e.* $\Phi_4(r,t)$] the Pioneer Component (II). Now, we proceed to the fifth and final solution. It is for this reason that we strong believe this gravitational potential must exist. Off cause, for it to exist, it must flow from a legitimate equation [such as (4)] from which gravitational potentials can be derived.

3.2.2 Fifth Gravitational Pole: ($\nu^2 < 0$)

Titius-Bode Component: As before, let us set $\nu^2 = \alpha(\alpha+1)$ where we strictly assume that ($|\nu| > 1/2$). Now, from the foregoing, it follows that: $\alpha = \left(-1 \pm i\sqrt{4|\nu|^2-1}\right)/2 = -\frac{1}{2} \pm \frac{i}{2}a = (\alpha_1, \alpha_2)$, where $a = \sqrt{4|\nu|^2-1}$ and (α_1, α_2) are the two solutions. Now, to achieve our desired objective, we proceed by substituting (α_1, α_2) into $\Phi_5(r) = A\left[\left(\frac{r}{\mathcal{R}_5}\right)^{\alpha_1} + \left(\frac{r}{\mathcal{R}_5}\right)^{\alpha_2}\right] + B\left[\left(\frac{r}{\mathcal{R}_5}\right)^{-(\alpha_1+1)} + \left(\frac{r}{\mathcal{R}_5}\right)^{-(\alpha_2+1)}\right]$. In this solution $\Phi_5(r)$, we shall drop the term whose coefficient is B. The reason for doing this is that this term will only add a phase factor ϕ_0 in the final solution – so, to save space, we have to drop it. Now, let us set $A = -G_5(0)\mathcal{M}/\mathcal{R}_5$. The resultant potential is:

$$\Phi(r,t) = -\frac{G\mathcal{M}}{r}\left[\kappa_1 \underbrace{+ \kappa_2 e^{-\mu_2 r} + \kappa_4\left[\left(\frac{\mathcal{R}_4}{r}\right)^{\alpha_1} + \left(\frac{\mathcal{R}_4}{r}\right)^{\alpha_2} + \left(\frac{\mathcal{R}_4}{r}\right)^{\alpha_1+1} + \left(\frac{\mathcal{R}_4}{r}\right)^{\alpha_2+1}\right]}_{\textbf{Pioneer Anomaly \& (perhaps) Darkmatter}} + \underbrace{\overbrace{\kappa_3\cos(\mu_3 r)}^{\textbf{Linear Spacing}} \overbrace{+\kappa_5\left(\frac{\mathcal{R}_5}{r}\right)^{-\frac{1}{2}}\cos\ln\left(\frac{\mathcal{R}_5}{r}\right)^{a}}^{\textbf{Logarithmic Spacing}}}_{\textbf{Ring and Planet Formation}}\right] ($$

$$\Phi_5(r) = A\left(\frac{\mathcal{R}_5}{r}\right)^{\frac{1}{2}}\left[\left(\frac{\mathcal{R}_5}{r}\right)^{ia} + \left(\frac{\mathcal{R}_5}{r}\right)^{-ia}\right]. \tag{17}$$

Using the relation $a^x = e^{x\ln(a)}$, equation (17) can be written as: $\Phi_4(r) = A\left(\mathcal{R}_5/r\right)^{\frac{1}{2}}\left[e^{ia\ln(\mathcal{R}_5/r)} + e^{-ia\ln(\mathcal{R}_5/r)}\right]$. Since $\Phi_5(r,t) = \phi_5(r,t)\Phi_5(r)$ and from the Euler relation $e^{ix}\cos(x) + i\sin(x)$, if follows that $\Phi_5(r,t)$ becomes:

$$\Phi_5(r,t) = -\left(\frac{G_5(r,t)\mathcal{M}}{\mathcal{R}_5}\right)\left(\frac{\mathcal{R}_5}{r}\right)^{\frac{1}{2}}\cos\left[\ln\left(\frac{\mathcal{R}_5}{r}\right)^{a} + \phi_0\right], \tag{18}$$

where $G_5(r,t) = G_5(0)\phi_5(r,t)$. As was done with $\phi_4(r,t)$, the function $\phi_5(r,t)$ must be subject to the constraint (14). The phase factor ϕ_0 has been added to take into account the term in $\Phi_5(r)$ whose coefficient is B. It will be shown in a future reading that the solution (18), can in-principle, explain the Titius-Bode Law which hypothesis that planets follow a logarithm spacing from their central star (for an exposition of this rather strange "law", see *e.g.* Chang 2010; Lovis *et al.* 2010; Poveda & Lara 2008).

4 MULTI-COMPONENT GRAVITATIONAL FIELD

The five solutions *i.e.* equations (8, 11, 15, 12 & 18) all emerge from the equation (4), they are thus legitimate gravitational potentials. The questions is, "How does *Nature* select one solution over the other?" We hypothesize that *Nature* employs all the five solutions concurrently on every gravitating body at al-times. If this is the case, the total gravitational potential is then given by a superposition of all the five potentials, *i.e.*:

$$\Phi(r,t) = \sum_{k=1}^{5}\Phi_k(r,t). \tag{19}$$

Written in full, the total radial gravitational potential is as given in (16). In this expression (16), $\kappa_j : j = 1, 2, \ldots, 5$, is defined as: $\kappa_j = \kappa_j(t) = G_j(t)/G_1(t)$. In this formula (16), $G = G_1(t)$. Of the five gravitational poles *i.e.*, $\Phi_j : j = 1, 2, \ldots, 5$; obviously, the Newtonian gravitational pole (Φ_1) must be the dominant pole in (16). If this total potential (16) is to tally with reality, then, the other poles should act in such a manner as to give structure formation on the Solar and perhaps galactic scale.

5 DISCUSSION AND CONCLUSION

This letter has shown that the Four Poisson-Laplace equation (4) has five radial solutions $\Phi_j(r,t) : j = 1, 2, \ldots, 5$. According to the order of our presentation, the first of the five solutions is the usual Newtonian law of gravitation. The other four are very interesting as they hold the promise to answer questions on a number of gravitational anomalies including the formation of structure in the Universe. In the present letter, our thrust has mainly been to present these solutions so as to lay down the ground for future exploratory work on these solutions.

An interesting outcome of the Four Poisson-Laplace equation (4) is that the gravitational constant G emerges as a time dependent constant. Since it was first proposed that the gravitational constant G might vary with time (Milne 1935; Dirac 1937, 1938), there has never been a solid theoretical foundation to furnish this hypothesis. If what we have presented is anything to go by – as we strongly believe it to be; then, the foundations of a time variable G have been found. Not only does the Four Poisson-Laplace equation (4) predict a time variable-G, it also predicts about three forms of G (linear, exponential and sinusoidal). Further, for the gravitational constants (G_4, G_5) associated with the none-separable solutions, it is seen that these constants have a spatial dependence.

In-closing, we would like to say that, more could have been presented and said, but because of the limited space [5-pages], this has not been possible. It is hoped that full research articles, that are currently at an advanced stage of preparation; will furnish this part.

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