Dimensionless Physical Constant Mysteries

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Abstract: Feynman proposed searching for $-\alpha^{1/2} = -0.08542455$ with the $\pm$ sign on the $\alpha^{1/2}$ for the *positive* and *negative* charge, and may be related to π, e, 2 and 5. We found $\alpha^{1/2} \approx \pm\frac{\log e}{\Phi\pi} = \pm 0.0854372$ where $\Phi = \frac{1}{\phi} = 2\cos(\frac{\pi}{5})$. I/FQHE $R_{xy} = \pm Z_0/2\nu_i\alpha$ unveils $\alpha^{1/2} = \log e^{\pm\phi/K\pi}$ where $\Phi - \phi - e - \pi$ in Euler Identity and $K \sim \{3, 37, 61\}$ from $2^{(\mathbf{p}-1)}(\mathbf{p}-1)! \in 2^n n!$ are linked to Quantum theories. The energy-mass formula $E = mc^2$ and special relativistic mass $m = \gamma m_0$ established the particle rest-mass m_0, mass-ratio m_i/m_e, mass-defect $\triangle m$. The rest-mass of a particle can be quantized by the fine structure constant and the proton-electron mass ratio $\beta_{p/e} = (\alpha^{-3/2} - 2\alpha^{1/2} + \alpha^2/\pi\phi^2 - \eta\alpha^3)\ln\pi$. The hydrogen atomic rest-mass is $m_{^1\mathrm{H}} = m_{\mathrm{p}^+} + m_{\mathrm{e}}(1 - \alpha^2 \ln 10)$ in the Quantum Gravity. The high-energy $\mathrm{W}^\pm$ boson $\alpha_{\mathrm{W}}^{1/2} = \pm\frac{\log F}{\Phi\pi}(1 - \alpha \cdot \sin^2\theta_w)$, where Fransén-Robinson constant $F = \int_0^\infty \frac{dt}{\Gamma(t)} = 2.80777\ldots$ replaced $e = \sum\limits_{n=0}^{\infty} \frac{1}{\Gamma(n)} = 2.71828\ldots$ We get the g-factors of particles (Leptons and Baryons) as

$$\begin{pmatrix} \frac{|g_e|}{2} & \frac{|g_p|}{2} \\ \frac{|g_\mu|}{2} & \frac{|g_n|}{2} \end{pmatrix} = \begin{pmatrix} \ln\left\{e^{\mathbf{1}} \cdot \frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2} + \frac{\mathbf{2}}{\mathbf{3}})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2} + \frac{\mathbf{1}}{\mathbf{3}})]} e^{\mathcal{O}_{\mathbf{e}}}\right\} & \ln\left\{e^{\mathbf{3}} \cdot \frac{\exp[\frac{\mathbf{1}}{\mathbf{3}}(\frac{11}{9\pi}\alpha^{1/2})]}{\exp[\frac{\mathbf{1}}{\pi}(8\alpha^{1/2})]} e^{\mathcal{O}_{\mathbf{p}}}\right\} \\ \ln\left\{e^{\mathbf{1}} \cdot \frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2} + \frac{\mathbf{2}}{\mathbf{3}})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2} + \frac{\mathbf{1}}{\mathbf{3}})]} e^{\mathcal{O}_\mu}\right\} & \ln\left\{e^{\mathbf{2}} \cdot \frac{\exp[\frac{\mathbf{2}}{\mathbf{3}}(\frac{11}{9\pi}\alpha^{1/2})]}{\exp[\frac{1}{\mathbf{2}\pi}(8\alpha^{1/2})]} e^{\mathcal{O}_n}\right\} \end{pmatrix}$$

Keywords: Fine structure constant, Quantum Theory, Dirac monopole, I/FQHE, Energy-mass equation, Rest-mass, Mass-ratio, Mass-defect, Quantum Gravity, g-factors, Lepton, Baryons, Spin.

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1 Brief History of the α Math Formulas

1-1 Introduction of the Fine Structure Constant

The elementary charge of electron $\mathbf{e}$ was proposed by Stoney in 1894 and discovered by Thomson in 1896, [1, 2] then Planck introduced the energy quanta $h\nu$ in 1901 and explained as photon $\epsilon = h\nu$ by Einstein in 1905. [3, 4]

Planck first noticed in 1905 that $\mathbf{e}^2/c$ and h have the same dimension. [5] In 1909, Einstein found that there are two fundamental velocities in physics: c and $\mathbf{e}^2/h$ requiring explanation. He said, *"It seems to me that we can conclude from $h = \mathbf{e}^2/c$ that the same modification of theory that contains the elementary quantum $\mathbf{e}$ as a consequence,*

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will also contain as a consequence ***the quantum structure of radiation****.*" [6] Later on, Einstein noticed that "three decimals are missing." He prompted $hc/\mathbf{e}^2 \sim 879$ (i.e., $\alpha^{-1} = 140(5)$), which was subsequently rejected by Lorentz since "900 seems too much." [6] His guesswork was before Millikan's oil-drop experiment to measure the elementary charge $\mathbf{e}$ and Avogadro Constant N_A. [7]

In 1913, Jeans suggested $h/2\pi = (4\pi\mathbf{e})^2/c$, i.e., $\alpha^{-1} = hc/2\pi e^2 = (4\pi)^2 = 157.91367$. [8] In 1914, two American electrochemists, Lewis and Adams proposed that "all universal constants involve only integral numbers and π", and presented [9]

$$h = \frac{\mathbf{e}'^2}{c}\left(\frac{8\pi^5}{15}\right)^{1/3} = \frac{4\,(4\pi\mathbf{e})^2}{c}\left(\frac{\pi^5}{5!}\right)^{1/3} \tag{1.1}$$

where $\mathbf{e}' = 4\pi\mathbf{e}$. From (1.1)

$$\alpha^{-1} = \frac{\hbar c}{\mathbf{e}^2} = 32\pi\left(\frac{\pi^5}{5!}\right)^{1/3} = 137.348 \tag{1.2}$$

which is derived from Stefan-Boltzmann law $\varepsilon_{\mathrm{T}} = a\mathrm{T}^4$. They noticed that the dimension of the radiation constant $a = k^4(\varepsilon/\theta^4)$ is $(energy \times length)^{-3}$ while $\mathbf{e}'^2$ is $(energy \times length)$. $8\pi^5/15$ in (1) can be obtained from the dimensionless integration in 3D blackbody radiation

$$8\pi \cdot J = 8\pi\int_0^\infty \frac{x^3dx}{e^x - 1} = 8\pi\Gamma(4)\zeta(4) = \frac{8\pi^5}{15} = \frac{4^3\pi^5}{5!} \tag{1.3}$$

where *zeta*-function $\zeta(4)$ was solved by Euler using Wallis products. Lewis applied a cube root to (1.3) since it involves a 3D volume. In 1915, Allen rewrote (1.1) as $\alpha = (15/\pi^2)^{1/3}/(4\pi)^2$. [10] Therefore, the study of an α math formula came before Sommerfeld's fine structure constant formula of relativistic discrete H-spectra in 1916, which gives the experimental value $\alpha^2 = 5.30 \times 10^{-5}$ (i.e., $\alpha^{-1} = 137.360563948$). Sommerfeld was the first to pin-down the approximate value of α and also point out that α serves as a bridge in the quantum h to electricity $\mathbf{e}$, relativity c. [11]

1-2 Early Exploration and Wyler's formula

Since (1) involves blackbody radiation, people believed that α may be linked to temperature. In 1931, $\mathrm{T}_0 = -(2/\alpha - 1) = -273$ [K] was erroneously published in *Naturwissenschaften*. [12] The Lewis–Adams conjecture was discussed among physicists. In 1935, Heisenberg wrote to Dirac: "*I do not believe at all any more in your conjecture that the Sommerfeld fine-structure constant may have something to do with the concept of temperature; that is, neither do I any more believe in the Lewis value*". Indeed, Lewis' value is wrong, but his idea lead to another dimensionless constant $\alpha_R = \frac{2}{\pi}\left(\frac{\pi^5}{5!}\right)^{1/3}\alpha = \left[\frac{\zeta(4)}{\zeta(2)}\right]^{1/3}\alpha = \frac{1}{157.555}$, involving the continued spectra of blackbody radiation. [13] Heisenberg wrote to Bohr with a joke formula suggested by Lunn in 1922

$$\alpha^{-1} = 2^4 3^3/\pi = 137.50987 \tag{1.4}$$

Bohr replied with the golden angle $(2\pi/\Phi^2)(180/\pi) = 360\phi^2 = 137.5077643^{\mathrm{o}}$ for studying phyllotaxis in 1909

$$\alpha^{-1} = 360/\Phi^2 = 137.5077643 \tag{1.5}$$

Pauli worked hard in 1935, and once suggested that five-dimensional relativity theory might help to understand the problem. [14]

Following Pauli, Wyler's formula in 1969 exposed a similar pattern with the Lewis formula (2), but in 4th root and in the reciprocal way [15]

$$\begin{aligned}\alpha &= \tfrac{9}{16\pi^3}\left(\tfrac{\pi}{5!}\right)^{1/4} = \tfrac{9}{16\pi^4}\left(\tfrac{\pi^5}{5!}\right)^{1/4} = \left(\tfrac{3}{4\pi^2}\right)^2\left(\tfrac{\pi^5}{5!}\right)^{1/4} \\ &= 1/137.0360824 = 0.00729734813\end{aligned} \tag{1.6}$$

Wyler used Hua's result in "Harmonic analysis of functions of several complex variables in the classical domains", [16] without discussing the physical dimensional analysis, and proposed a new assembling formula according to the 5D Dirac relativistic equation

$$\begin{aligned}&V(D_n) = \tfrac{\pi^n}{2^{n-1}n!}; \quad V(S_{n-1}) = \tfrac{2\pi^{n/2}}{\Gamma(n/2)}; \quad V(C_n) = \tfrac{\pi 2\pi^{n/2}}{\Gamma(n/2)} \leftarrow Hua \\ &\alpha = \tfrac{8\pi\cdot[V(D_n)]^{1/(n-1)}}{V(S_{n-1})V(C_n)} \qquad\qquad \leftarrow Wyler\end{aligned} \tag{1.7}$$

let $n = 5$ in (1.7), then

$$\begin{aligned}&V(D_5) = \tfrac{\pi^5}{2^4 5!}; \quad V(S_{5-1}) = \tfrac{8\pi^2}{3}; \quad V(C_5) = \tfrac{8\pi^3}{3} \leftarrow Hua \\ &\alpha = \tfrac{8\pi(\pi^5/2^4 5!)^{1/4}}{(8\pi^2/3)(8\pi^3/3)} = \tfrac{9}{16\pi^3}\left(\tfrac{\pi}{5!}\right)^{1/4} \leftarrow Wyler\end{aligned} \tag{1.8}$$

The puzzling Wyler formula in (1.6) has many variations, such as

$$\alpha^{-1} = \sqrt[4]{\frac{1^0\cdot 4^0\cdot 5^1\cdot 8^0\cdot 9^0}{2^{19}\cdot 3^7\cdot 6^0\cdot 7^0\cdot 0^0}\pi^{11}} = 2^{-19/4}\cdot 3^{-7/4}\cdot 5^{1/4}\cdot \pi^{11/4} \tag{1.9}$$

He also reported $\beta = m_p/m_e = 3\times 2\pi^5 = 6!(\pi^5/5!) = 1836.1181$. Unfortunately, he failed to provide an origin for his ideas or an explanation of their use. In 1925, Rice suggested $2\pi\alpha^{-1} = hc/e'^2 = \underline{(\frac{8\pi^3}{3})}(R\rho/r^2)$. [17] Born suggested $\alpha^3 = (\frac{2}{\pi})\underline{(\frac{8\pi^2}{3})}(e^2\nu/m_e c^3)$ and tested $\alpha = \dfrac{\mathbf{e}^2}{\hbar c} = \dfrac{2\pi\gamma}{1.236} = \Phi\pi\gamma$ in 1935, and proposed the reciprocity principle in 1939. [18] In 1951, Lenz suggested that $m_p/m_e = 6\pi^5$. [19] Wyler's formula came after these works. The above history shows that this line of study is a combination of the n-space dimensionless volume and blackbody radiation with Pauli's idea. Obviously, 3D (x-y-z), 4D (x-y-z-t) or 5D (x-y-z-t-E) is based on the experimental data *137*. In 1989, Bailey and Ferguson used a supercomputer to check Wyler's formula, and automatically produced several "other relations of comparable complexity with even better accuracy." One example is $\alpha^{-5} = 150\pi(6^5 5^2\pi^3)^8$ (i.e., $\alpha^{-1} = 137.036048362143$). [20] This clearly showed that a Wyler-type formula could not be the *unique* answer for the fine structure constant. Wyler's formula is later discussed in the $E8$ lie groups. In 2006, Castro reviewed the coupling constant with the Complex Domains. [21]

1-3 More Debating Formulas

Wyler's work made people devise simpler ways to obtain the magic number, with no more care given to physical dimensional analysis. Here are some popular examples:

A formula with 4π in 1971 [22]

$$\alpha^{-1} = \pi + \pi^2 + 4\pi^3 = \pi(1 + \pi(1 + 4\pi)) = 137.03630037758 \tag{1.10}$$

A Wyler-type formula in 1971 [22]

$$\alpha^{-1} = 2^{5/3} \cdot 3^{-8/3} \cdot 5^{5/2} \cdot \pi^{7/3} = 137.036006449 \tag{1.11}$$

From Aether Theory in 1972 [23]

$$\alpha^{-1} = 108\pi(8/1843)^{1/6} = 137.03591484 \tag{1.12}$$

From Pythagorean theorem in 1975 [24]

$$\alpha^{-1} = (137^2 + \pi^2)^{1/2} = 137.0360157199 \tag{1.13}$$

From $N = 42$ invariance groups in 1976 [25]

$$\alpha^{-1} = N(N-1)/4\pi = 137.032406 \tag{1.14}$$

In 2004, Stoyan proposed $\alpha = \frac{2}{(n^2+2\pi^2)^{1/2}+n}$, and $\alpha^{-1} = \frac{137+\sqrt{137^2+2\pi^2}}{2} = 137.03601098839$ when $n = 137$ for the regular Tetrahedron geometry. [26] It is one solution from $x^2 - 137x - \pi^2/2 = 0$ but the other one is ignored. They are all in line with the concept that '*all universal constants involve only integral numbers and* π'. The 4π in (1.10) was interesting but went nowhere.[1]

In 2000, Kosinov suggested the more complex but more accurate formula yielding $\alpha^{-1} = 137.03600982$. [27]

$$\alpha^{20} = (\pi\Phi^{14})^{1/13}10^{-43} \tag{1.15}$$

In 2005, Heyrovska proposed [28]

$$\alpha^{-1} = 360\phi^2 - 2\phi^3 = 137.0356280 \tag{1.16}$$

In 2006, Naschia suggested a formula in the E-Infinity Cantorian space-time. [29]

$$\alpha^{-1} = 20\Phi^4 = 137.0820393 \tag{1.17}$$

This type of formula followed Bohr's thought, involving the golden mean, Fibonacci and Lucas numbers.

1-4 Recent Fittings by Gilson and others

In order to fit CODATA-2006 $\alpha^{-1} = 137.035999679(94)$, Gilson developed a well-known formula in the 1990s, using two primes 29 and 137 in the trigonometric functions

$$\alpha = \frac{\cos(\pi/137)}{137}\frac{\tan(\pi/(29\cdot137))}{\pi/(29\cdot137)} = 1/137.0359997867 \tag{1.18}$$

which is based on wave capture circling analysis in 2007. [30] However, CODATA-2010 gives $\alpha = 1/137.035999074(44)$. Kirakosyan gives a similar analysis $\sum I_m/I = \sin\triangle\varphi = \alpha^{1/2}$ and $\alpha^{-1} = \frac{1}{\sin^2\triangle\varphi}$ in 2011. [31][2]

[1] First presented by E. D. Reilly, Jr. in 1971. Note $\beta = \prod_{k=0}^{2}(4\pi - k/\pi) = 4\pi(4\pi - \frac{1}{\pi})(4\pi - \frac{2}{\pi}) = 1836.1517$ and $\beta_{n/e} \approx \beta_{p/e} + \ln(4\pi) = 1838.6827635$ link to $\mu_0 = 4\pi \times 10^{-7}[Wb/Am]$ and $k_e = 1/4\pi\varepsilon_0[Nm^2C^{-2}]$

[2] Other approximate trigonometric functions include: $\tan^{-1}(\alpha^{-1}) \simeq \frac{\pi}{2}$; $\cos^{-1}(\alpha) \simeq \frac{\pi}{2}$; $\cos(\alpha^{-1}) \simeq e^{-1}$; $\sin(\alpha) \simeq \alpha$.

In 2007, Lestone used a blackbody radiation model to get [32]

$$\alpha^{-1} = 16\pi^3/3\zeta(3)g(\sigma_T) = 137.042999$$

In 2009, Markovitch suggested [33]

$$\begin{aligned} \alpha^{-1} &= \tfrac{A^a - D^d}{B^b} + C^c - E^e = 137.036 \\ &= \tfrac{10^3 - 10^{-3}}{3^3} + 10^2 - 10^{-3} \\ &= \tfrac{22^3 - 2^{-1}}{25^2} + 11^2 - 1^{-1} \end{aligned} \tag{1.19}$$

In 2010, Rhodes suggested $\alpha^{-1}_{Rhodes} = 4\alpha_{swe} = 4m_{\nu_\mu}/m_{\nu_e}$ with Precision Cryptographic Calculation using the *prime* sequence to obtain the magic number [34]

$$\begin{aligned} \alpha^{-1} &= 4 \cdot \tfrac{2\cdot 3^2\cdot 5^2\cdot 7\cdot 11\cdot 17\cdot 19\cdot 31\cdot 47\cdot 53\cdot 59\cdot 61\cdot 73\cdot 79\cdot 103\cdot 109\cdot 113\cdot 131\cdot 149}{2\cdot 13\cdot 23\cdot 29\cdot 37\cdot 41\cdot 43\cdot 67\cdot 71\cdot 83\cdot 89\cdot 97\cdot 101\cdot 107\cdot 127\cdot 137\cdot 139\cdot 151} \\ &= 4 \times 4.44 \times 10^{29}/1.52 \times 10^{31} = 137.035999104743744 4154 \end{aligned} \tag{1.20}$$

However, according to the experimental $m_{\nu_\mu}/m_{\nu_e} = 0.8019meV/27.45meV = 34.2312$, then $\alpha^{-1}_{Rhodes} = 136.9248$.

In 2011, Code gives [35]

$$\begin{aligned} \mathcal{L}(\Phi,\psi) &= \Phi^{\log(\psi^{\log(\Phi^{\log(\psi^{\log(\Phi^{\log(\psi^{\log(\Phi^{\log(\psi)})})})})})})} \\ \psi &= \triangle - 137 + 137(\Phi+\phi)Im\left(\frac{i^{i^{\cdot^{\cdot^{\cdot}}}}}{1+\frac{1}{2+\frac{1}{\ddots}}}\right) \end{aligned} \tag{1.21}$$

where $\Phi = 1.618033$, and $\phi = 0.618033$, and $\triangle = A^{-1} - 8A^{-(\triangle^{-1}(\triangle)=33+\frac{4}{137}+\frac{144000}{1209485})}$ and $A = e^{\frac{1}{12}-\zeta'(-1)} = 1.28242712\cdots$, then $\alpha = \mathcal{L}(\phi,\psi) = 1/137.03599908573\cdots$ matches $\alpha^{2008}_{QED} = 1/137.035999084(51)$.

In 2012, Schonfeld proposed a simple but less accurate formula $\alpha^{-1} = \pi^4\sqrt{2}m_{qm}/m_e$ with the bare charge equal to [36]

$$\alpha^{-1} = \pi^4\sqrt{2} = 137.757 \tag{1.22}$$

All the precision study on 137 started after Eddington's *adding-one* formula based on matrix theory in the 1930s. He believed $(10\times 10)+(6\times 6) = 136$ real matrix and $(10\times 6)+(6\times 10) = 120$ imaginary matrix existed in spacetime, and also proposed an additional matrix to match with the experimental value, then obtained a whole number

$$\alpha^{-1} = \tfrac{n^2(n^2+1)}{2} + 1 = \mathbf{136} + \mathbf{1} = \mathbf{137} \tag{1.23}$$

where $n = 4$. He later proposed $\alpha^{-1} = 137\beta^{1/24} = 137.042$ in 1948. [37]

We all learned valuable lessons from the early explorers. After countless efforts, Pauli's simplest question still remains unanswered: "Why 137?" [38–42] In his Nobel Lecture delivered in Stockholm on 13 December 1946, Pauli expressed his goal was to establish a theory "*which will determine the value of the fine-structure constant and will thus explain the* ***atomistic structure of electricity****, which is such an essential quality of all atomic* *sources of electric fields actually occurring in nature.*" [43] As the initialization, "... ***from a physical point of view, that the existence of atomicity, in itself so simple and basic, should also be interpreted in a simple and elementary manner by theory and should not, so to speak, appear as a trick in analysis***." [43] His lifelong search for 137, a millennium puzzle, ended in hospital room 137. [44]

1-5 The Various Experimental Data

The difficulty of finding the correct α formula is partly due to the uncertainty of the experimental values - approximately 137.036. Some experimental data of the fine structure constant is listed in the **Table 1.1**. [45–50]

Table 1.1 The history of the experimental Fine Structure Constant Values

DATE	$1/\alpha$	SOURCE	DATE	$1/\alpha$	SOURCE
1916	137.360563948	A. Sommerfeld	1973	137.03612(15)	CODATA 1973
1929	137.29 ± 0.11	R. Birge	1987	137.0359895(61)	CODATA 1986
1930	136.94 ± 0.15	W. Bond	1998	137.03599883(51)	T. Kinoshita
1932	137.305 ± 0.005	R. Birge	2000	137.03599976(50)	CODATA 1998
1935	137.04 ± 0.02	F. Spedding et al.	2002	137.03599911(46)	CODATA 2002
1941	137.030 ± 0.016	R. Birge	2007	137.035999070(98)	G. Gabrielse et al.
1943	137.033 ± 0.092	U. Stille	2008	137.035999679(94)	CODATA 2006
1949	137.027 ± 0.007	J. DuMond, E. Cohen	2008	137.035999084(51)	D. Hanneke et al.
1949	137.041 ± 0.005	H. Bethe, C. Longmire	2010	137.035999037(91)	R. Bouchendira
1957	137.0371 ± 0.0005	J. Bearden, J. Thomsen	2010	137.035999132(9)(6)(33)	T. Kinoshita et al.
1969	137.03602(21)	CODATA 1969	2011	137.035999074(44)	CODATA 2010

Many people are trying to fit the latest data mathematically, but fail to understand that CODATA is a statistic of experimental data and keeps changing. The QED calculation itself is a supercomputer numeral fitting and also keeps changing. The reality of experimental data is that the fine structure constant measured using different methods will create slight differences. [46, 48–50]

$$\begin{aligned}
\alpha^{-1}(Muonium\,Hyperfine) &= \quad 137.0359997(84) \quad [61ppb]\\
\alpha^{-1}(ac\,Josephson) &= \quad 137.0359875(43) \quad [31ppb]\\
\alpha^{-1}(Quantum\,Hall) &= \quad 137.0360030(25) \quad [18ppb]\\
\alpha^{-1}(Neutron\,Wavelength) &= \quad 137.0360077(28) \quad [21ppb]\\
\alpha^{-1}(Atom\,Interferometry) &= \quad 137.0360001(11) \quad [7.7ppb]\\
\alpha^{-1}(Optical\ Lattice) &= \quad 137.03599883(91) \quad [6.7ppb]\\
\alpha^{-1}(Electron\,g/2\,QED) &= \quad 137.035999084(28) \quad [0.37ppb]
\end{aligned} \tag{1.24}$$

Whether α is variable with time or location in the universe is still in debate. Even stranger, the high-energy $W^{\pm}$ boson $\alpha(m_W)$ is approximately 1/128 compared with the fermion zero-energy value of approximately 1/137. [47]

The α value obtained by the electron g-2 QED in (1.24) is an experiment dependent value, not a theoretical value. Therefore, QED can not offer a true explanation for 137. The *true* α must have a mathematical underpinning with an applicable physical definition, otherwise even the best experiments still can not answer "Why 137?"

1-6 The Clues from Einstein and Feynman

Einstein was a pioneer on searching for an α math formula started in 1909. He said in 1945: "*There are two kinds of constants: apparent and real ones. ... I therefore believe that such number can be only of a basic type, as for instance π or e ...*" [51] In 1954, Turing wrote "Charge= $\frac{e}{\pi}$ arg of character of a 2π rotation" without formulation. [53]

In 1985, Feynman in his lecture "QED: The Strange Theory of Light and Matter" said: [52]

"*If electrons were ideal, ... J would be simply be its "**charge**" (the amplitude for the electron to couple with a photon). ...*

*... J - **theoretical numbers that are not directly observable anyway**; ...*

But no such ideal electrons exist. ... a real electron, which emits and absorbs its own photons from time to time, and therefore depends on the amplitude for coupling, J. ...

... the observed coupling constant e *- **the amplitude for a real electron to emit or absorb a real photon**.*[3] *It is **a simple number** that has been experimentally determined to be close to* -0.08542455. *(My physicist friends ... remember it as the inverse of its square: about* 137.03597 *...).*

*... a **magic number** that comes to us with no understanding by man ...*

A good theory world say the e *is the square root of 3 over 2 pi squared, or something. There have been, from time to time, suggestions as to what* e *is, **but none of them has been useful**. ... Every once in a while, someone notices that a certain combination of **pi**'s and **e**'s (the base of the natural logarithms), and **2**'s and **5**'s produces the mysterious coupling constant, but it is a fact not fully appreciated by people who play with arithmetic that you would be surprised how **many** numbers you can make out of pi's and e's and so on. ...*

*Even though we have to resort to **a dippy process** to calculate J today, it's possible that someday a legitimate mathematical connection between J and* e *will be found. That would mean that J is **the mysterious number, and from it comes*** e" [52]

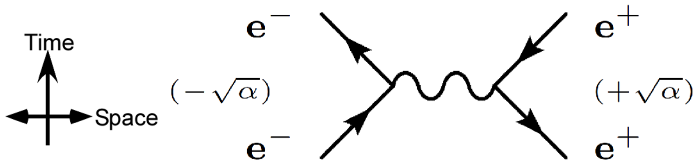

Fig. 1.1: The electron $\mathbf{e}^-$ and positron $\mathbf{e}^+$ as time reversal in the Feynman diagram

Like Pauli and Dirac, Feynman was never satisfied with any math formula for the fine structure constant, but he certainly investigated some different approaches:

(1) Studying $-\alpha^{1/2} = -0.08542455$ or $-\alpha^{-1/2} = -11.70623667\ldots$ instead of $\alpha = 0.007297\ldots$ or $\alpha^{-1} = 137.036\ldots$. (i.e., the ratio $\pm$e$/q_{Plack}$ instead of e$^2/\hbar c =$ (e$/q_{Plack})^2$).

(2) $\alpha^{1/2}$ is a simple number as a Lorentz invariant scalar quantity, but not a vector or a complex number.

(3) $\alpha^{1/2}$ is the physical coupling constant: a real number for a real electron to emit or absorb a real photon.

(4) There is a $\pm$ sign on $\alpha^{1/2}$ for the *positive* and *negative* charge distinguished by Franklin in 1750s, which is clearly presented as the time reversal in the Feynman diagram (**Fig. 1.1**).

[3] where e$= \alpha^{1/2}$ is related to J - "*theoretical numbers that are not directly observable anyway*" - assumed by Feynman [52] p128.

(5) The "magic number" is for the creation of charge and works forever, so that it could be a math limitation from the infinite series/products, or continued fraction, etc.

(6) $\alpha^{1/2}$ may be obtained using a certain combination of π, e, 2 and 5 (e.g., the golden ratio $\Phi = \frac{1}{\phi} = 2\cos(\frac{\pi}{5})$).

(7) There are many π *and* e combinations which are not unique solutions to this important unique physical number.[4]

(8) The α math formula must be useful for solving physical mysteries, mere arithmetic is not appreciated.

(9) There may be an unique mysterious theoretical number J for the unique physically measurable $\pm\alpha^{1/2}$.

(10) No ideal electrons exist. A real electron depends on the coupling, J.

(11) QED calculation is "a dippy process", and we need to probe a legitimate mathematical connection between J and $\pm\alpha^{1/2}$.

Soon after Feynman's lecture, we noticed a simple formula,

$$\alpha^{1/2} \approx \pm\frac{\log e}{\Phi\pi} = \log e^{\pm\phi/\pi} = \frac{\pm 1}{\Phi\pi \ln 10} = 1/\ln 10^{\pm\Phi\pi} = \pm 0.0854372111 \tag{1.25}$$

which yields $\alpha = [\frac{\phi}{\pi}\log e^{\pm 1}]^2 = \frac{1}{136.995532}$, out of range of the experimental values. However, it is embedded with the $\pm$ sign and only involves three basic math constants $\Phi - \phi - e - \pi$ in Euler-type Identity $e^{\pm i\pi} + \Phi = \phi$ ("the most beautiful equation" - Feynman (**Fig. 1.2**)). In this paper, we put this type of formula on debate.

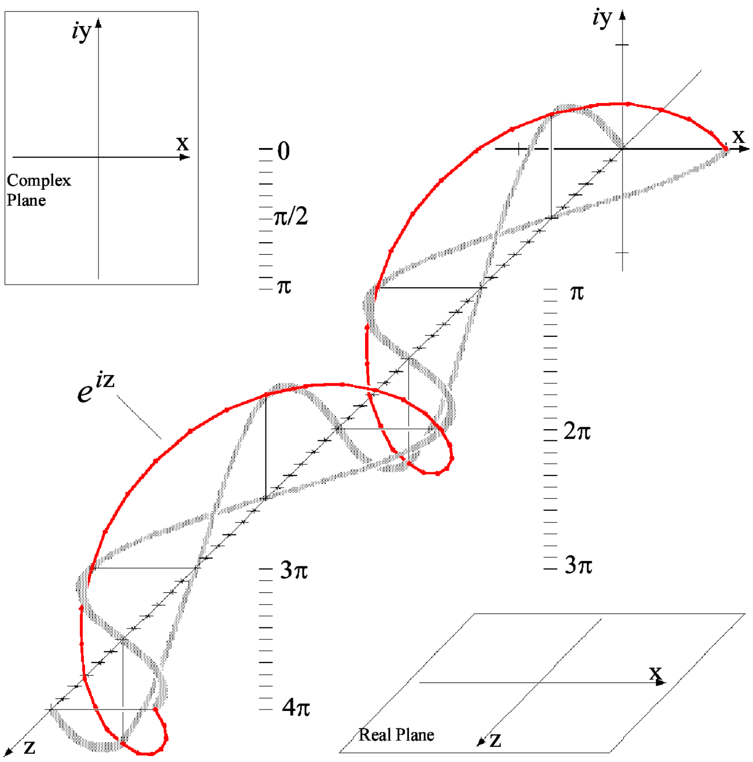

Fig. 1.2: Three-dimensional visualization of Euler's formula - The circular polarization

[4]Beyond $\sqrt{3}/2\pi^2 = 0.0877467$, we can also get $e/\pi^3 = 0.0876687$, $\tan^{-1}(e,\pi)/\pi^2 = 0.08688410$, $e + e^{-1} - 3 = 0.0861612$, $2/e^2\pi = 0.0861571$, $\arg(e + i\pi)/2\Phi\pi = 0.084329$, and $e\pi\% = \frac{e\pi}{100} = \frac{e}{10}\frac{\pi}{10} = \frac{e}{2\times5}\frac{\pi}{2\times5} = \frac{e}{2}\frac{\pi}{2}\frac{1}{(\Phi+\phi)^4} = 0.08539734222$, *etc.* The last one is close to $\alpha^{1/2}$ and $\alpha^{-1}_{e\pi\%} = 137.1233109$ with $\mathbf{e} = \sqrt{\alpha\hbar c} = \frac{e\pi}{100}\sqrt{\hbar c} = \frac{e}{100}\sqrt{\frac{\pi}{2}hc}$ in [esu].

2 Probing of the Fine Structure Constant

2-1 Maxwell Equations and Charge-Pole Quantization

Dimensionless physical constants such as the fine-structure constant $\alpha = \frac{\mathbf{e}^2}{2\varepsilon_0 hc} = \frac{1}{137.036}$ are not simply mathematical coincidences; their values are governed by the deepest numerical theory. [18,53] The *true* α must have a mathematical underpinning with an applicable physical definition, otherwise even the best experiments still can not answer Pauli's question: "Why *137*?" [43,54] Let's start from the free-space characteristics of the electromagnetic wave.

In 1856, Weber and Kohlrausch experimentally discovered that $1/\sqrt{\varepsilon_0\mu_0}$ was very close to the light-speed c measured by Fizeau in 1849 and confirmed by Foucault in 1962. [55] This is finally theoretically approved by the Maxwell equations in 1864. [56]

In the geometric algebra $\boldsymbol{Cl_n}$ with an orthogonal basis $\mathcal{G}(p,q)$ consists of $\sum_{k=0}^{n}\binom{n}{k} = 2^n = 2^{p+q}$ elements, such as $\mathcal{G}(3,0)$ for the 3D Euclidean space with $2^3 = \{1\oplus 3\oplus 3\oplus 1\}$, [57]

$$\{\underbrace{1}_{scalar}, \underbrace{e_1, e_2, e_3}_{vectors}, \underbrace{e_1e_2, e_1e_3, e_2e_3}_{bivectors}, \underbrace{e_1e_2e_3}_{trivector}\} \tag{2.1}$$

the Maxwell four-equations are united into a single equation in the *SI* system

$$\underbrace{(\nabla\cdot\mathbf{E} - \tfrac{\rho}{\varepsilon_0})}_{Gauss(E)} + I\underbrace{c(\nabla\cdot\mathbf{B}\,)}_{Gauss(M)} + I\underbrace{(\nabla\times\mathbf{E} + \tfrac{\partial\mathbf{B}}{\partial t})}_{Faraday} - c\underbrace{(\nabla\times\mathbf{B} - \tfrac{1}{c^2}\tfrac{\partial\mathbf{E}}{\partial t} - \mu_0\mathbf{J})}_{Amp\grave{e}re-Maxwell} = 0 \tag{2.2}$$

i.e., $0+0+0-0=0$ in the light-cone of 4D Minkowski space $\{x,y,z,ict\}$. From (2.2),

$$\underbrace{[\tfrac{1}{c}\partial_t - \nabla]}_{\Box}\underbrace{(\mathbf{E} + I\mu_0 c\mathbf{H}\,)}_{F} - \underbrace{(\tfrac{\rho}{\varepsilon_0} + \mu_0 c\mathbf{J})}_{=J} \;=\; 0 \tag{2.3}$$

where $\nabla\mathbf{X} = \nabla\cdot\mathbf{X} + I\nabla\times\mathbf{X}$ for spatial vectors $\mathbf{X}$, the pseudo-scalar $I^2 = -1$ commutes with all spatial vectors; the electric constant $\varepsilon_0 \approx 8.854187817\ldots\times 10^{-12}\,[\frac{\mathrm{F}}{\mathrm{m}}] = \frac{10^7}{4\pi c^2}\,[\frac{\mathrm{F}}{\mathrm{m}}] \approx \frac{10^{-9}}{36\pi}\,[\frac{\mathrm{s}}{\mathrm{m}\Omega}]$ and the magnetic constant $\mu_0 = 1.2566370614\ldots\times 10^{-6}\,[\frac{\mathrm{H}}{\mathrm{m}}] = 4\pi\times 10^{-7}\,[\frac{\mathrm{s}\Omega}{\mathrm{m}}]$, $\varepsilon_0\mu_0 c^2 = \sigma_0 Z_0 = 1$ with $c = 299792458\,[\frac{\mathrm{m}}{\mathrm{s}}]$. Since Ω and $\frac{1}{\Omega}$ are reciprocals in $[\frac{\mathrm{H}}{\mathrm{m}}] = [\frac{\mathrm{s}\Omega}{\mathrm{m}}]$ and $[\frac{\mathrm{F}}{\mathrm{m}}] = [\frac{\mathrm{s}}{\mathrm{m}\Omega}]$, the characteristic impedance or conductance of vacuum is the ratio of electric and magnetic field strength $(\mathbf{E},\mathbf{H})$ or flux density $(\mathbf{D},\mathbf{B})$

$$\begin{aligned} Z_0 &= \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \frac{|\mathbf{B}|}{|\mathbf{D}|} = 376.730313461\ldots\ [\Omega] \\ \sigma_0 &= \varepsilon_0 c = \sqrt{\frac{\varepsilon_0}{\mu_0}} = \frac{|\mathbf{H}|}{|\mathbf{E}|} = \frac{|\mathbf{D}|}{|\mathbf{B}|} = 2.6544187\times 10^{-3}\ [\mathrm{S}] \end{aligned} \tag{2.4}$$

where $\mathbf{E} = \mathbf{D}/\varepsilon_\mathbf{0}\,[\frac{\mathrm{V}}{\mathrm{m}}]$ and $\mathbf{H} = \mathbf{B}/\mu_\mathbf{0}\,[\frac{\mathrm{A}}{\mathrm{m}}]$ are the amplitude of the electric and the magnetic field; $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic flux densities. The electromagnetic energy density is $u = \frac{1}{2}(\mathbf{E}\cdot\mathbf{D} + \mathbf{B}\cdot\mathbf{H}) = \frac{1}{2}(\varepsilon_0\mathbf{E}^2 + \mu_0\mathbf{H}^2)$ and $\mathbf{E} = c\mathbf{B}$ as the Faraday law. The intensity of radiation $\mathbf{S} = \mathbf{E}\times\mathbf{H}$ is called Poynting vector with the right-hand rule. The asymmetric electromagnetic field $\mathbf{E} > \mathbf{H}$ $(\mathbf{D} < \mathbf{B})$ in **Fig. 2.1** is elliptical but near linear (the ratio of semi-axis is $376.7 : 1 \approx 5!\pi : 1$)[5]

[5]The asymmetric vacuum properties are eliminated in Heaviside-Lorentz units $c = \varepsilon_0 = \mu_0 = \sigma_0 = Z_0 = 1$. If $\hbar = c = 1$ then charge $\boldsymbol{e} = \sqrt{4\pi\alpha} = 0.302822$, which is used in particle physics with a mystery of symmetric broken. The Lorentz-Maxwell equations use the micro-fields $\mathfrak{e}$ and $\mathfrak{h}$ for the single charged particle.

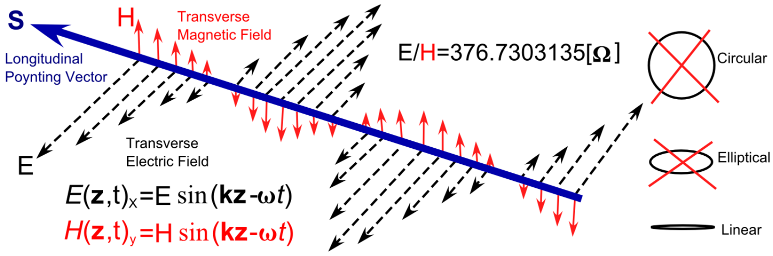

Fig. 2.1: Electromagnetic field with $\mathbf{E} : \mathbf{H} = \mathbf{B} : \mathbf{D} = 376.730 : 1$, it is a near linear polarization.

In the electromagnetic field, an electric elementary charge is accelerated by a Lorentz force $\mathbf{F}_l = \mathbf{e}[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$ without the radiation of photon $h\nu$. The elementary charge $\mathbf{e}$ is defined by the Planck constant h and the fine structure constant α with the electromagnetic properties of free space in (2.4). [58] The α serves as a bridge in the quantum h to electricity $\mathbf{e}$, relativity c, and vacuum $\varepsilon_0, \mu_0, \sigma_0, Z_0$. [59] $\frac{\mathbf{e}^2}{h}$ are particle quanta properties and $\varepsilon_0 c$ are spacetime properties in the fine structure constant. Experimentally,

$$\begin{aligned} \mathbf{e} &= [2\alpha h\varepsilon_0 c]^{1/2} &&= [2\alpha h/\mu_0 c]^{1/2} &&= [2\alpha\sigma_0 h]^{1/2} &&= [2\alpha h/Z_0]^{1/2} \\ &= G_0/K_J &&= 2/K_J R_K &&= G_0\Phi_0 &&= 2\Phi_0/R_K \end{aligned} \tag{2.5}$$

where $K_J = 2\mathbf{e}/h = 1/\Phi_0 = 483597.870(11) \times 10^9\,[\mathrm{HzV}^{-1}]$ is the Josephson constant for the superconductor quantization and $R_K = h/\mathbf{e}^2 = 2/G_0 \overset{\text{def}}{=} 25812.807\,[\Omega]$ is the von Klitzing constant for the quantum Hall quantization, $G_0 = 2\mathbf{e}^2/h = 7.7480917346\,(25) \times 10^{-5}[\mathrm{S}]$ is the conductance quanta and $\Phi_0 = h/2\mathbf{e} = 2.067833758\,(46) \times 10^{-15}\,[\mathrm{Wb}]$ is the magnetic flux quanta. From (2.5), the quantum impedance and conductivity are linked to (2.4) by the fine structure constant

$$\begin{aligned} R_K &= \frac{h}{\mathbf{e}^2} = Z_0/2\alpha = \sigma_K^{-1} = 25812.807\ [\Omega] \\ \sigma_K &= \frac{\mathbf{e}^2}{h} = 2\alpha \cdot \sigma_0 = R_K^{-1} = 3.8740 \times 10^{-5}[\mathrm{S}] \end{aligned} \tag{2.6}$$

Charged particles with high quantum impedance R_K (or low conductivity σ_K) are surrounded by a low impedance of free-space Z_0 (or high conductivity σ_0), and the coupling constant between two zones (Fermion/Boson) is the fine structure constant. In this way, a charged particle becomes a space-time light-trap for the electromagnetic wave. This is the foundation of the photoelectric effect.

The reality of Ohm's law on the atomic scale is a memristor [60]

$$M(q) = \frac{V(t)}{I(t)} = \frac{d\Phi_m}{dQ} = \frac{d\Phi_B}{dq} = \frac{-d\Phi_B/dt}{-dq/dt} \tag{2.7}$$

A memristor is a functional relationship between the time integral of current (related to charge) and the time integral of voltage (related to magnetic flux). During the study of 137, Dirac proposed charge-monopole quantization $\mathbf{e}g = \frac{1}{2}nhc$ in *cgs* unit, which can be expressed as $\mathbf{e}g = nh$ in *SI* unit with $n \in \mathbb{Z}$. **Resistance is the ratio of electric and magnetic properties**, i.e., $R = M(q)$ as constant ratio of charge and monopole.

$$R = M(q) = \frac{d\Phi_B}{dq} = \frac{-\mathbf{B}d\mathbf{S}/dt}{-2\mathbf{e}dN/dt} = \frac{1}{2N}\frac{h}{\mathbf{e}^2} = \frac{Z_0}{4\alpha N} = \frac{R_K}{2N} = \frac{1}{2N}\frac{g}{\mathbf{e}} \tag{2.8}$$

where the magnetic flux $d\Phi_B = \mathbf{B}d\mathbf{S}$ and $R_K = \frac{g}{\mathbf{e}}$. The magnetic coupling constant is defined by the monopole as $\beta_m = \frac{g^2}{\hbar c} = \frac{\hbar c}{4\mathbf{e}^2} = \frac{1}{4\alpha} = \frac{\sigma_0}{G_0} = 34.259$, a magnitude for the unified strong-electroweak coupling constant is defined as $\alpha_{swe} = \frac{m_{\nu_e}}{m_{\nu_\mu}} \approx 34.26^{-1}$. [61]

2-2 Alpha Formula, Charge-sign and Criterion

Feynman talked about $\alpha^{1/2} \sim -0.08542455$ as the number "my physicist friends won't recognize...", [52] because it is better known as $\alpha^{-1} \sim 137.036$. [11] In this paper, we show how I/FQHE exposes the physical information formula

$$|\alpha^{1/2}| \equiv \pm\frac{M\phi}{K\pi} \equiv \frac{\log e^{\pm 1}}{K\Phi\pi} \equiv \log e^{\pm\phi/K\pi} \equiv \frac{1}{\ln 10^{\pm K\Phi\pi}} \tag{2.9}$$

where $M = \log e = 1/\ln 10$ is used in the logarithmic *information entropy* $S = \log e^w$ or $S = \ln 10^w$ in *units* [ban], [hart], [nat], $\Phi - \phi - e - \pi$ each only appear once and can be presented as continued fractions

$$\begin{aligned} \Phi &= 1 + \phi = 1 + \tfrac{1}{1+}\tfrac{1}{1+}\cdots = 1.61803\ldots \\ e &= 2 + \tfrac{1}{1+}\tfrac{1}{2+}\tfrac{2}{3+}\tfrac{3}{4+}\tfrac{4}{5+}\cdots = 2.71828\ldots \\ \pi &= 3 + \tfrac{1^2}{6+}\tfrac{3^2}{6+}\tfrac{5^2}{6+}\tfrac{7^2}{6+}\tfrac{9^2}{6+}\cdots = 3.14159\ldots \end{aligned} \tag{2.10}$$

in the Euler-type Identity

$$e^{\pm i\pi} + \Phi = \phi \tag{2.11}$$

From (2.9), one gets

$$\alpha \equiv \tfrac{\mathbf{e}^2}{2\varepsilon_0 hc} \equiv \tfrac{\mu_0 c\mathbf{e}^2}{2h} \equiv \left(\tfrac{\log e^{\pm 1}}{K\Phi\pi}\right)^2 \equiv \log^2\left(e^{\pm 1/K\Phi\pi}\right) = \tfrac{\log e^{\pm\phi^2}}{\ln 10^{\pm K^2\pi^2}} \tag{2.12}$$

There are only two types of elemental charge, $\mathbf{e}^-$ and $\mathbf{e}^+$, or their combination 0, distinguished by Franklin. [62] Feynman found the time reversal, but the *intrinsic* difference between $\mathbf{e}^-$ and $\mathbf{e}^+$ is still a mystery. The elementary charge is related to $\alpha^{1/2} = \log e^{\pm 1/K\Phi\pi}$ in (2.9). We know Euler's number (the infinite series was first published by Newton in 1669)

$$\begin{aligned} e^{+1} &= \Gamma(1,-1) = \sum_{n=0}^{\infty} \tfrac{(+1)^n}{n!} = 2.71828\cdots \\ e^{-1} &= \Gamma(1,+1) = \sum_{n=0}^{\infty} \tfrac{(-1)^n}{n!} = 0.36787\cdots \end{aligned} \tag{2.13}$$

The $\pm$ sign is auto-formed by $\log e^{\pm 1} = \pm 0.43429$, where the stable negatively charged electron involves an *alternating* series $\frac{1}{e} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = \sum_{2n}^{\infty} \frac{1}{n!} - \sum_{2n+1}^{\infty} \frac{1}{n!} = 1.543080635 - 1.175201194 = 0.367879441$. The photon is an energy quanta with light-speed c and the electron is a slower host (αc) during the photon-electron interaction. It is well known that Euler's number e is linked to the *time* related growth or decay (i.e., time reversal), and to Fibonacci numbers as (alternating $\pm$ at the *odd* factorial term)[6]

$$e^{\pm 1} = \frac{\sum_{k=0}^{\infty} \frac{F(1\pm k)}{k!}}{\sum_{k=0}^{\infty} \frac{F(1\mp k)}{k!}} = \frac{\frac{1}{0!} \pm \frac{1}{1!} + \frac{2}{2!} \pm \frac{3}{3!} + \frac{5}{4!} \pm \frac{8}{5!} + \frac{13}{6!}\cdots}{\frac{1}{0!} \mp \frac{0}{1!} + \frac{1}{2!} \mp \frac{1}{3!} + \frac{2}{4!} \mp \frac{3}{5!} + \frac{5}{6!}\cdots} \tag{2.14}$$

In this way, the elementary charge is frozen in time in the quantum theory and embedded with the $\pm$ sign using logarithms (i.e., $\log \lim_{n\to\infty} (\frac{n}{\sqrt[n]{n!}})^{\pm 1} = \log \lim_{n\to\infty} (1+\frac{1}{n})^{\pm n} = \pm \log e$). More

[6] Note $\phi = \lim_{n\to\infty} \frac{F_n}{F_{n+1}}$, $\Phi = \lim_{n\to\infty} \frac{F_{n+1}}{F_n} = 2\cos(\frac{\pi}{5})$, and $\Phi^n = \Phi^{n-1} + \Phi^{n-2} = \Phi F_n + F_{n-1}$.

importantly, e is statistically an algebraic sum of the arrangement ratio (permutation) between Bosons (spin-1 for photon) and Fermions (spin-$\frac{1}{2}$ for electron), which compel a real electron to emit and absorb a real photon. [52, 54]

$$\begin{aligned} e^{\pm 1} &= \tfrac{(\pm 1)^0}{0!} + \tfrac{(\pm 1)^1}{1!} + \tfrac{(\pm 1)^2}{2!} + \tfrac{(\pm 1)^3}{3!} + \cdots + \tfrac{(\pm 1)^n}{n!} \cdots \\ &= \sum_{n=0}^{\infty} \tfrac{(\pm 1)^n}{n!} = \sum_{n=0}^{\infty} (\pm 1)^n \tfrac{\text{Arrangement of Bosons}}{\text{Arrangement of Fermions}} \end{aligned} \tag{2.15}$$

Bosons are indistinguishable particles, with only ***one*** arrangement no matter how many photons are in the box, while the n distinguishable Fermions (Pauli Principle) must have $n!$ arrangement. [64] In (2.15), the negatively charged electron has an *alternating* series, while the positively charged particle takes a *normal* series.

The arbitrary charge sign $+$ or $-$, given by Franklin in 1750, is interpreted by using the different intrinsic statistics in (2.13)-(2.15). It self-dictates the $\pm$ sign for the charge conservation, e.g., the annihilation of electron and positron ($\mathbf{e}^- + \mathbf{e}^+ = 2\gamma$) or the formation of a Cooper pair ($\mathbf{e}^- + \mathbf{e}^- = 2\mathbf{e}^-$).

$$\begin{aligned} \mathbf{e}^{\pm} + \mathbf{e}^{\mp} &= (2\sigma_0 h)^{1/2} \log e^{\pm\phi/K\pi \mp \phi/K\pi} = 0 \\ \mathbf{e}^{\pm} + \mathbf{e}^{\pm} &= (2\sigma_0 h)^{1/2} \log e^{\pm 2/K\Phi\pi} = 2 \cdot \mathbf{e}^{\pm} \end{aligned} \tag{2.16}$$

(2.16) sets up a simple criterion for the $\alpha^{1/2}$ math formula. The elementary charge is a quanta for an electron particle in the point charge model, and the math formula of $\alpha^{1/2}$ must allow for an algebraic sum. ***Charge sign*** with ***intrinsic characteristics*** and ***conservation*** are three simple criterion for approving the $\alpha^{1/2}$ math formula. This criterion can be used to exclude those mathematical coincidences of the *137* formulas.

2-3 I/FQHE and Variable Charge-Pole Ratio

The elementary charge changes the vacuum property of an electromagnetic field. In fact, $4\alpha = 0.02918941 = 1/34.259$ for the strong interaction is equal to the conductivity ratio

$$4\alpha = \frac{G_0}{\sigma_0} \equiv \frac{2 \cdot \mathbf{e}^2}{\sigma_0 h} \equiv \frac{2Z_0\mathbf{e}^2}{h} \equiv \left(\frac{2}{\pi}\frac{\log e^{\pm 1}}{K\Phi}\right)^2 \tag{2.17}$$

where the conductance quantum $G_0 = 2\mathbf{e}^2/h$ is related to the von Klitzing constant $R_{\mathrm{K}} = h/\mathbf{e}^2$ of IQHE in the *weak* magnetic field. [65] FQHE is still only incompletely described by the theories of 2DEG, Composite Fermion, Topological Order, Anyon, and so on. [66–71] The acid test for (2.9) is whether it can give a good interpretation of the FQHE of GaAs-GaAlAs in the *strong* magnetic field near 0 K (**Fig. 2.2**). This long and hard procedure is absolutely necessary to establish and confirm the basic structure of the fine structure constant math formula in (2.9).

Quantum Hall conductance $\sigma_{xy} = 1/R_{xy}$ is equal to $\mathbf{e}N/B$ and the variable reciprocal von Klitzing constant R_{K}/ν_i, one whole charge $\mathbf{e}$ for conducting and another cycling fractional charge $\mathbf{e}_\nu$ for the variable external magnetic field B.

$$\sigma_{xy}^{h/\mathbf{e}} = \frac{1}{R_{xy}} = \frac{\pm\mathbf{e}N}{B} = \frac{\pm 1}{R_{\mathrm{K}}/\nu_i} = \frac{\pm\mathbf{e}}{h/(\nu_i\mathbf{e})} = \frac{\pm\mathbf{e}}{h/\mathbf{e}_\nu} = \frac{\pm\mathbf{e}}{\Phi^\nu} \tag{2.18}$$

where $N \sim 10^{11}$ [cm^{-2}] is the 2D charge density; $\mathbf{e}_\nu = \nu_i\mathbf{e}$ is the fractional charge; $\Phi^\nu = h/\mathbf{e}_\nu$ is a quantized variable linked to the external variable magnetic field B in [Tesla] and to the magnetic flux quanta Φ_0. Coupling of holes (+) and electrons (−) are found on InAs/GaSb, and graphene at room-temperature. [72, 73] The existence of *quasi*-electron and *quasi*-hole states carrying fractional charge $\mathbf{e}_\nu$ are experimentally confirmed. [74] In (2.18), the conducting multi-charges $\mathbf{e}N$ are treated as a single charge $\mathbf{e}$, and another cycling charge $\mathbf{e}$ is further rearranged as an assembling charge $\mathbf{e}_\nu$, in either Integers or Fractions. In (2.18), only the *divisors* (R_{xy}, B, ν_i, $\mathbf{e}_\nu$, Φ^ν) are variables. $R_{xy} = 1/\sigma_{xy}$ is *proportional* to Φ^ν and *inversely proportional* to ν_i (**Fig. 2.2**).

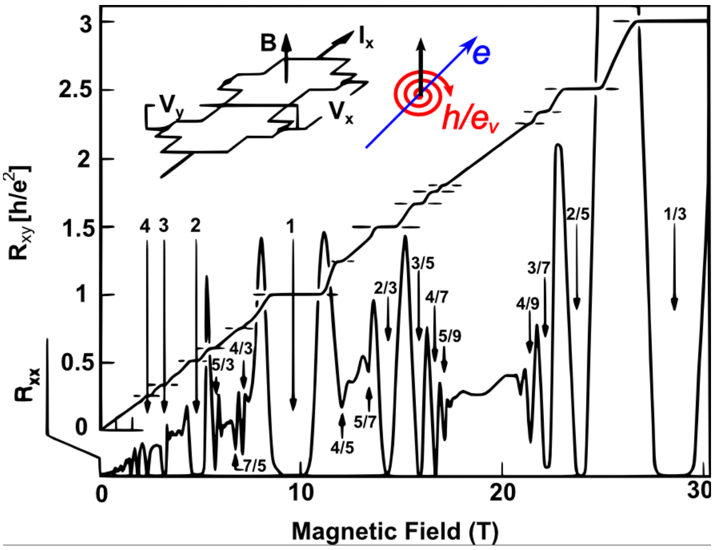

Fig. 2.2: I/FQHE resistance plateaus measured in GaAs-GaAlAs with a perpendicular magnetic field B [T]. $R_{xy} = 1/\sigma_{xy}$ is linked to a conducting charge and the cycling fractional charge is restructured by the external magnetic field B [T].

Physically, due to Lorentz force, it is easy to assume that if more cycling electrons are trapped in higher external magnetic fields B, decreased charge density N makes R_{xy} increase. Therefore, the cycling fractional charge $\mathbf{e}_\nu$ is not for conducting, but is rather allied to the variable external magnetic field B. From Dirac monopole-charge quantization, the elementary conductance $\sigma = \frac{\mathbf{e}}{g} = \frac{\mathbf{e}^2}{h} = \frac{\mathbf{e}}{2\Phi_0}$ (or the elementary resistance (i.e., the von Klitzing constant) $R_\mathrm{K} = \frac{g}{\mathbf{e}} = \frac{h}{\mathbf{e}^2} = \frac{2\Phi_0}{\mathbf{e}}$) is the ratio of the elementary charge and the magnetic monopole. From (2.8) and (2.18)

$$\sigma_{xy}^{h/\mathbf{e}} = \frac{1}{R_{xy}} = \frac{\pm\nu_i \mathbf{e}^2}{h} = \frac{2\nu_i \alpha}{Z_0} = 2\nu_i\alpha\sigma_0 = \pm\nu_i\frac{\mathbf{e}}{g} = \pm\frac{p_i\mathbf{e}}{q_i g} \tag{2.19}$$

In (2.19), if $\nu_i = 1/3$ then the denominator is $3g$. The divisor of the fractional charge is equivalent to the multi-poles, and the dividend of the fractional charge is the multi-charge. The filling factor ν_i is a variable of the charge-pole ratio. Under extremely low temperature, the Ga-As-Al nucleus in I/FQHE must respond to a high magnetic field.

2-4 Self-generation of Filling Factors

The filling factors ν_i obtained from many experiments of the IQHE and FQHE are

$$\begin{aligned} \nu_I &= n, \cdots 8, 7, 6, 5, 4, 3, 2, 1 \\ \nu_F &= \frac{1}{3}, \frac{2}{5} \cdot\cdot \frac{2}{3}, \frac{3}{5} \cdot\cdot \frac{p}{2mp \pm 1} \cdot\cdot \frac{1}{2}, \frac{1}{4} \cdot\cdot \frac{4}{11}, \frac{6}{13} \cdot\cdot \frac{p_i}{q_i} \end{aligned} \tag{2.20}$$

where n, m, and p are all integers, and p_i/q_i is a fraction. Current FQHE theories are all based on the *arbitrary* fractional filling factors but not self-generated.

Next, let's see if the filling factor ν_i can be self-generated by the math formula of $\alpha^{1/2}$ in (2.9). Since $\mathbf{e} = \underline{(2\sigma_0 h\alpha)}^{1/2}$, the *fractional* charge $|\pm \mathbf{e}_\nu| = \nu_i|\pm \mathbf{e}|$ is proportional to a *fractional* fine structure constant $|\alpha_\nu^{1/2}| = \nu_i|\alpha^{1/2}|$. Both ν_i and α are dimensionless numbers. Modifying the Wallis products (1656) for the $\frac{\pi}{2}$ in (2.17) [75]

$$\begin{aligned}
\frac{\pi}{2} &= \frac{2}{1}\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\frac{8}{9}\frac{10}{9}\frac{10}{11}\cdots = \prod_{p=1}^{\infty} 1^2 4\left(\frac{p}{2p-1}\right)\left(\frac{p}{2p+1}\right) \\
&= \left(\frac{2}{1}\frac{2}{3}\odot\frac{6}{5}\frac{6}{7}\odot\frac{10}{9}\frac{10}{11}\odot\cdots\right)\cdot\prod_{p=1}^{\infty} 2^2 4\left(\frac{p}{2\cdot 2p-1}\right)\left(\frac{p}{2\cdot 2p+1}\right) \\
&= \left(\frac{2}{1}\frac{2}{3}\frac{4}{3}\frac{4}{5}\odot\frac{8}{7}\frac{8}{9}\frac{10}{9}\frac{10}{11}\odot\cdots\right)\cdot\prod_{p=1}^{\infty} 3^2 4\left(\frac{p}{2\cdot 3p-1}\right)\left(\frac{p}{2\cdot 3p+1}\right) \\
&= \prod_{n,m,p=1}^{\infty} n^2 4\left(\frac{p}{2mp-1}\right)\left(\frac{p}{2mp+1}\right)
\end{aligned} \tag{2.21}$$

where $\odot$ shows where the twin-fractions are taken out, and n may or may not be equal to m. From (2.9)

$$\begin{aligned}
\alpha^{-1/2} &= \frac{K\Phi\pi}{M} = \frac{2K\Phi}{M}\prod_{p=1}^{\infty} n^2 4\cdot\left(\frac{p}{2mp-1}\right)\left(\frac{p}{2mp+1}\right) \\
&= \left(\frac{p}{2mp\pm 1}\right)\left[\frac{2K\Phi}{M}\prod_{p=1}^{\infty} n^2 4\cdots\right] \\
&= \left(\frac{p}{2mp\pm 1}\right)\left[\alpha_\nu^{-1/2}\right] = \nu_i\cdot\alpha_\nu^{-1/2}
\end{aligned} \tag{2.22}$$

or taking out n in (2.22) as the integer n in (2.20), it yields

$$|\alpha_\nu^{1/2}| = \begin{pmatrix} n & \frac{p}{2mp+1} \\ \frac{p}{2mp-1} & n \end{pmatrix}\cdot\left|\alpha^{1/2}\right| = \nu_i\cdot\left|\alpha^{1/2}\right| \tag{2.23}$$

From (2.17), (2.18) and (2.23), 2D Hall conductivity σ_{xy} is linked to the vacuum conductivity σ_0 by the filling factors ν_i and the fine structure constant α

$$\sigma_{xy} = \pm 2\alpha^{1/2}\alpha_\nu^{1/2}\varepsilon_0 c = \pm 2\nu_i\alpha\sigma_0 \tag{2.24}$$

i.e., $R_{xy} = \pm Z_0/2\alpha_\nu^{1/2}\alpha^{1/2} = \pm Z_0/2\nu_i\alpha$ where $Z_0 = \mu_0 c = 2\alpha h/\mathbf{e}^2 = 376.730313461\,[\Omega]$. Therefore, a good FQHE theory must expose the details of the α information. This gives a clue for derivation of the fine structure constant math formula.

Why does the FQHE only choose $4\,[p/(2mp-1)]\,[p/(2mp+1)]$ or $4nn$ in the Wallis formula? This is because a complex product for the real part of two complexes (z_1, z_2) is equal to $4\mathrm{Re}(z_1)\mathrm{Re}(z_2) = (z_1+\check{z}_1)(z_2+\check{z}_2)$, where $z_1 = (x_1+iy_1)e^{i\omega t}$ and $\check{z}_1 = (x_1-iy_1)e^{-i\omega t}$ are the complex conjugates. It needs $4nn$ or $4\,[p/(2mp-1)]\,[p/(2mp+1)]$ in (2.22). According to Laughlin, the wave function for a filling factor $\nu_i = 1/\mathrm{M}$ is [70]

$$\Psi_{\mathrm{M}}(z_1\cdots z_N) = \prod_{j\langle k\rangle}^{N}(z_j - z_k)^{\mathrm{M}}\exp\left(-\frac{1}{\mathbf{4}}\frac{1}{l_0^2}\sum_j^N |z_j|^2\right) \tag{2.25}$$

where $z_j = x_j + iy_j$ is a complex number representing the position of the j^{th} particle on the plane, $l_0 = \sqrt{\hbar/\mathbf{e}B} = \sqrt{h/2\pi\mathbf{e}B}$ is the magnetic length, and M is an *odd* integer to satisfy the Pauli principle. 4 is also in the exponential term in (2.25).

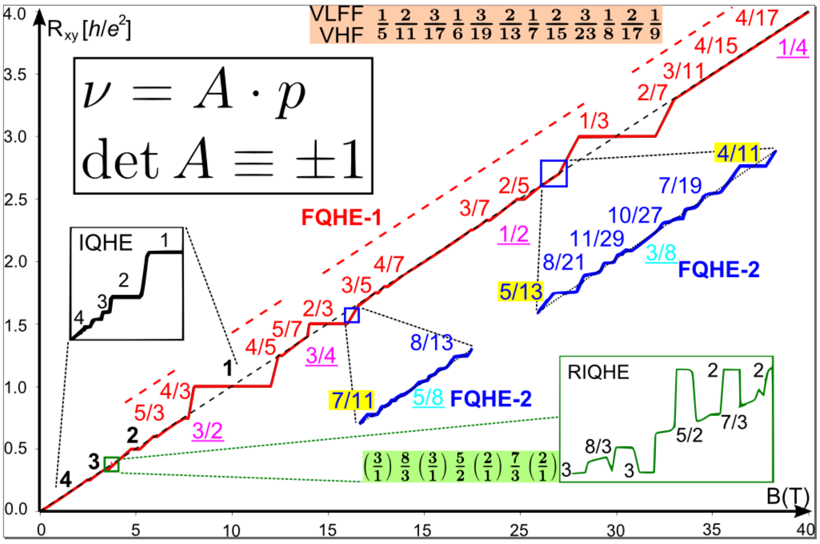

Fig. 2.3: I/FQHE resistance plateaus measured in GaAs-GaAlAs with a perpendicular magnetic field B [T]. The enlargement shows the FQHE-2, IQHE, RIQHE, VLFF-VLF.

In **Fig. 2.3**, we have $R_{xy} = 1/\nu$ in $[h/\mathbf{e}^2]$ unit, where the filling factor ν_i is arranged from largest to smallest, when B is increased from 0 to 40 [T]. These filling factors ν_i are a summation of many experimental reports, which are grouped as IQHE, RIQHE, FQHE-1, FQHE-2, Very Low Filling Factors in Very High Field, the even-denominator fraction, etc.

Charge coupling combines the IQHE and FQHE, i.e., the algebraic sum of ν_I and ν_F in (2.20) and (2.23)

$$\nu_i = \nu_{n,m,p} = \underset{\text{IQHE}}{\pm n} \pm \underset{\text{FQHE}}{\frac{p}{2m \cdot p \pm 1}} = \underset{\text{FQHE}-1}{\frac{(\pm 2nm \pm 1) \cdot p \pm n}{2m \cdot p \pm 1}} \Rightarrow \underset{\text{FQHE}-2}{\frac{\boldsymbol{a} \cdot p \pm \boldsymbol{b}}{\boldsymbol{c} \cdot p \pm \boldsymbol{d}}} = \mathbf{A} \cdot p \tag{2.26}$$

where $n = 0, 1, 2\ldots$, $m = 1, 2, 3\ldots$, and $p = 1, 2, 3\ldots$; $\boldsymbol{a} = \pm 2nm \pm 1$, $\boldsymbol{b} = n$, $\boldsymbol{c} = 2m \cdot p$ and $\boldsymbol{d} = 1$. Next, we give some ν_i examples of (2.26) to compare with **Fig. 2.2** and **Fig. 2.3**.

Let $n = m = +1$ and $p = 1, 2, 3\ldots$ (the limits are $\frac{3}{2}$ and $\frac{1}{2}$)

$$\begin{aligned}
\nu_{+-}^{1,1} &= 2, \tfrac{5}{3}, \tfrac{8}{5} \cdots \left(\tfrac{3p-1}{2p-1}\right) \cdots && [5 \rightarrow 7 \text{ [T]}] \\
\nu_{++}^{1,1} &= \tfrac{4}{3}, \tfrac{7}{5}, \tfrac{10}{7} \cdots \left(\tfrac{3p+1}{2p+1}\right) \cdots && [7 \leftarrow 10 \text{ [T]}] \\
\nu_{-+}^{1,1} &= \tfrac{2}{3}, \tfrac{3}{5}, \tfrac{4}{7} \cdots \left(\tfrac{p+1}{2p+1}\right) \cdots && [15 \rightarrow 20 \text{ [T]}] \\
\nu_{--}^{1,1} &= \underline{\underline{0}}, \tfrac{1}{3}, \tfrac{2}{5} \cdots \left(\tfrac{p-1}{2p-1}\right) \cdots && [20 \leftarrow 30 \text{ [T]}]
\end{aligned} \tag{2.27}$$

Let $n = 2$, $m = 1$, and $p = 1, 2, 3\ldots$ gives the RIQHE region between $\nu = 2 - 3$ (the limits are $\frac{5}{2}$ and $\frac{3}{2}$, at $4 \to 10$ [T])

$$\begin{aligned} \nu_{+-}^{2,1} &= 3, \tfrac{8}{3}, \tfrac{13}{5} \cdots \left(\tfrac{5p-2}{2p-1}\right) \cdots \\ \nu_{++}^{2,1} &= \tfrac{7}{3}, \tfrac{12}{5}, \tfrac{17}{7} \cdots \left(\tfrac{5p+2}{2p+1}\right) \cdots \\ \nu_{-+}^{2,1} &= \tfrac{5}{3}, \tfrac{8}{5}, \tfrac{11}{7} \cdots \left(\tfrac{3p+2}{2p+1}\right) \cdots \\ \nu_{--}^{2,1} &= 1, \tfrac{4}{3}, \tfrac{7}{5} \cdots \left(\tfrac{3p-2}{2p-1}\right) \cdots \end{aligned} \tag{2.28}$$

Let $n = 0$ and $m = 2$, with a limit of $\frac{1}{4}$ ($30 \to 40$ [T]) [76]

$$\begin{aligned} \nu_{-}^{0,2} &= \tfrac{1}{3}, \tfrac{2}{7}, \tfrac{3}{11}, \tfrac{4}{15} \cdots \left(\tfrac{p}{4p-1}\right) \cdots \\ \nu_{+}^{0,2} &= \tfrac{1}{5}, \tfrac{2}{9}, \tfrac{3}{13}, \tfrac{4}{17} \cdots \left(\tfrac{p}{4p+1}\right) \cdots \end{aligned} \tag{2.29}$$

Let $n = 1$ and $m = 2$, with a limit of $\frac{3}{4}$ ($10 \to 15$ [T])

$$\begin{aligned} \nu_{++}^{1,2} &= 1, \tfrac{4}{5}, \tfrac{7}{9} \cdots \left(\tfrac{3p+1}{4p+1}\right) \cdots \\ \nu_{--}^{1,2} &= \tfrac{2}{3}, \tfrac{5}{7}, \tfrac{8}{11} \cdots \left(\tfrac{3p-1}{4p-1}\right) \cdots \end{aligned} \tag{2.30}$$

For both the IQHE and FQHE, Hall resistance usually increases when filling factor ν_i decreases. RIQHE at $n > 1$ shows the alternating switch between the IQHE and FQHE. [77] From (2.27) to (2.30), all *neighboring* fractions obey a transition rule $|p_i q_{i+1} - p_{p+1} q_i| = 1$ (except 0, it is also invalid between the limitations).

The *even*-denominator in the FQHE should not really exist if it is due to limitations (e.g., $5 \times 10^8/10^9 + 1) \approx \frac{1}{2} \neq \frac{1}{2}$). [77] However, the even-denominator fraction occurs in RIQHE and FQHE,[7] and works with *neighboring* ν_i as $|ad - cb| = 1$ (e.g., $\frac{1}{2}$ with $\frac{1}{3}$ or $\frac{2}{3}$ will have $3 - 2 = 4 - 3 = 1$); they are yielded by Euler's infinite product rearranged as same as (2.21) ~ (2.23).

$$\begin{aligned} \frac{\pi}{2} &= \frac{3}{2}\frac{5}{6}\frac{7}{6}\frac{11}{10}\frac{13}{14}\frac{17}{18}\frac{19}{18}\frac{23}{22}\cdots \qquad \left[\frac{\textit{all odd prime}}{\textit{discrete even}}\right] \\ \frac{\pi}{4} &= \frac{3}{4}\frac{5}{4}\frac{7}{8}\frac{11}{12}\frac{13}{12}\frac{17}{16}\frac{19}{20}\frac{23}{24}\cdots = \left(\frac{1}{2}\frac{3}{2}\right)\left(\frac{1}{2}\frac{5}{2}\right)\left(\frac{1}{4}\frac{7}{2}\right)\left(\frac{11}{2}\frac{1}{6}\right)\cdots \end{aligned} \tag{2.31}$$

The *even*-denominators also appear in graphene, and may work for fault-tolerant quantum computations. [78] In fact, each of the *even*-denominator fractions are linked to multi-pairs of approximate *boson*-like states, where $R_{xx} \neq 0$ to form the wide bridge on the H-shaped minima. Due to this effect, $R_{xx} \neq 0$ may occur when $B = 0$ in the IQHE. They can be photon-induced to ZRS in a sample with an ultra-high mobility ($\mu > 10^7$ [cm^2/Vs]) or with a long free path ($\lambda > 300\mu$m), which is also the necessary condition for the FQHE. [79, 80]

It is interesting that the pair $(\frac{1}{3}, \frac{2}{5})$ in (2.27) creates a secondary fraction and generates several higher order filling factors ($\frac{4}{11}$ and $\frac{5}{13}$). [81] It can be $\frac{3p\pm1}{8p\pm3}$ or $\frac{3p\pm2}{8p\pm5}$, both produce a limit of $\frac{3}{8}$. [82] The pair $(\frac{2}{3}, \frac{3}{5})$ in (2.27) has fractions $\frac{7}{11}$ and $\frac{8}{13}$ produced by $\frac{5p\pm2}{8p\pm3}$ *or* $\frac{5p\pm3}{8p\pm5}$ with a limit $\frac{5}{8}$. We can get other secondary fractions in a similar way. Double solutions in Composite Fermion must have different physical construction for FQHE-2. [67] The interpretation of the *fractional* fine structure constant avoids this contradiction.

[7] $\overline{\nu}_{(2n\pm1)/2} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2} \cdots$; $\overline{\nu}_{(2n\pm1)/4} = \frac{1}{4}, \frac{3}{4}, \cdots \frac{17}{4}, \frac{19}{4} \cdots$

2-5 General Criterion of Filling Factors

The geometrical Ford circles on the edge (**Fig. 2.4**(a)) follow the Pythagoras Theorem. [83] The general formula of all *neighboring* fractions a/c and b/d obeys

$$\nu = \frac{\boldsymbol{a} \cdot p \pm \boldsymbol{b}}{\boldsymbol{c} \cdot p \pm \boldsymbol{d}} = \begin{bmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{bmatrix} \cdot p = \mathbf{A} \cdot p \qquad (p = 0, 1, 2 \ldots) \tag{2.32}$$

It yields a limit of a/c with the initial fraction b/d, where c is an even number; d is the odd-denominator of the initial *neighboring* fraction. a, b, c, and d in the 2×2 matrix $\mathbf{A}$ must satisfy $\det \mathbf{A} = ad - cb = \pm 1$. The determinant of 2×2 Pauli metrics have the same criteria $\det(\sigma_i) = -1$. Equation (2.32) belongs to the special linear group $\mathrm{SL}\,(2, \mathrm{R})$ and $|ad - cb| \equiv 1$ is an universal rule wherever the $\pi/2$ fraction occurs. The quantization by using the quantum group $\mathrm{SL}(2, \mathrm{R})$, the moduli space of $SU(2)$ connections ($\mathbf{2} \otimes \mathbf{2} = \mathbf{3} \oplus \mathbf{1}$) on the 2-dimensional *torus*, Möbius transformation, Continued Fraction, Weyl quantization and Lie group $\mathbf{GL}_2(\mathbb{R})$ are all mathematically connected. [84] In the continued model, $y = (ax \pm b)/(cx \pm d)$ is an equilateral hyperbola as shown in **Fig. 2.4**(b), with the rectangular center point at $(-d/c,\, a/c)$.

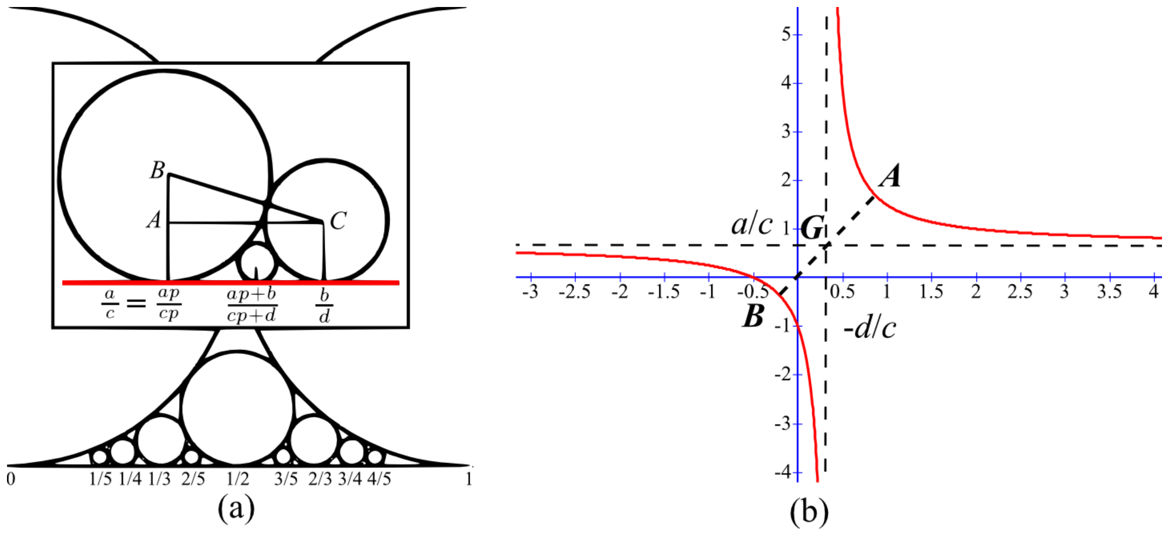

Fig. 2.4: (a) Ford circles for describing the gaskets on edge. Enlargement shows Pythagoras' Theorem. (b) The equilateral hyperbola for $y = (ax \pm b)\,/\,(cx \pm d)$ with a rectangular center G at $(-d/c,\, a/c)$.

VLFF-VHF and RIQHE states in **Fig. 2.3** are a good evidence of this transition rule $\det \mathbf{A} = ad - cb = |p_i q_{i+1} - p_{i+1} q_i| = \pm 1$.[8] [85–87]

$$\nu_{RIQHE} = \left\{ \begin{array}{ccccccc} \frac{4}{1} & \mathbf{\frac{19}{5}} & \frac{4}{1} & \multirow{2}{*}{$\left(\begin{array}{c} \mathbf{\frac{7}{2}} \\ \mathbf{\frac{5}{2}} \end{array}\right)$} & \frac{3}{1} & \mathbf{\frac{16}{5}} & \frac{3}{1} \\ \frac{3}{1} & \mathbf{\frac{8}{3}} & \frac{3}{1} & & \frac{2}{1} & \mathbf{\frac{7}{3}} & \frac{2}{1} \end{array} \right. \tag{2.33}$$

Although a doped semiconductor is not a perfect crystal and may not have all the fractions in one sample, the 256 inequalities of experimental filling factors ν_i in many I/FQHE experiments show that $a/c > b/d$ satisfies

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - cb = \pm 1 \tag{2.34}$$

[8] e.g., $4 \times 5 - 1 \times 19 = 20 - 19 = 1$. It is also valid in the quantum interference of the 2D electron collision.

and there is a symmetric pattern of even-denominators in 256 ν_i, which is graphically illustrated in **Fig. 2.5** on the order from the highest to the lowest filling factor (i.e., $\nu_n > \nu_{n+1}$).[9] [88]

ν_i (10-4) (4-2) (2-1) (1-1/9)

Symmetric Pattern

/2/

4/2/4

8/6/4/10/2/10/4/6/8

8/6/16/26/24/14/4/10/16/14/8/22/12/30/16/2/16/30/12/22/8/14/16/10/4/14/24/26/16/6/8

$\frac{10}{1}$ >	$\frac{19}{2}$ >	$\frac{9}{1}$ >	$\frac{17}{2}$ >	$\frac{8}{1}$ >	$\frac{15}{2}$ >	$\frac{7}{1}$ >	$\frac{13}{2}$ >	$\frac{6}{1}$ >	$\frac{11}{2}$ >	$\frac{5}{1}$ >	$\frac{9}{2}$ >	$\frac{4}{1}$ >	$\frac{19}{5}$ >	$\frac{34}{9}$ >	$\frac{15}{4}$ >
$\frac{11}{3}$ >	$\frac{18}{5}$ >	$\frac{25}{7}$ >	$\frac{7}{2}$ >	$\frac{24}{7}$ >	$\frac{17}{5}$ >	$\frac{10}{3}$ >	$\frac{23}{7}$ >	$\frac{13}{4}$ >	$\frac{22}{9}$ >	$\frac{16}{5}$ >	$\frac{3}{1}$ >	$\frac{14}{5}$ >	$\frac{25}{9}$ >	$\frac{11}{4}$ >	$\frac{19}{7}$ >
$\frac{8}{3}$ >	$\frac{13}{5}$ >	$\frac{18}{7}$ >	$\frac{23}{9}$ >	$\frac{28}{11}$ >	$\frac{33}{13}$ >	$\frac{5}{2}$ >	$\frac{32}{13}$ >	$\frac{27}{11}$ >	$\frac{22}{9}$ >	$\frac{17}{7}$ >	$\frac{12}{5}$ >	$\frac{7}{3}$ >	$\frac{16}{7}$ >	$\frac{9}{4}$ >	$\frac{20}{9}$ >
$\frac{11}{5}$ >	$\frac{2}{1}$ >	$\frac{15}{8}$ >	$\frac{43}{23}$ >	$\frac{28}{15}$ >	$\frac{13}{7}$ >	$\frac{11}{6}$ >	$\frac{9}{5}$ >	$\frac{16}{9}$ >	$\frac{7}{4}$ >	$\frac{19}{11}$ >	$\frac{12}{7}$ >	$\frac{17}{10}$ >	$\frac{5}{3}$ >	$\frac{8}{5}$ >	$\frac{11}{7}$ >
$\frac{14}{9}$ >	$\frac{17}{11}$ >	$\frac{20}{13}$ >	$\frac{3}{2}$ >	$\frac{19}{13}$ >	$\frac{16}{11}$ >	$\frac{13}{9}$ >	$\frac{10}{7}$ >	$\frac{7}{5}$ >	$\frac{4}{3}$ >	$\frac{13}{10}$ >	$\frac{9}{7}$ >	$\frac{14}{11}$ >	$\frac{5}{4}$ >	$\frac{11}{9}$ >	$\frac{6}{5}$ >
$\frac{7}{6}$ >	$\frac{8}{7}$ >	$\frac{17}{25}$ >	$\frac{26}{23}$ >	$\frac{9}{8}$ >	$\frac{1}{1}$ >	$\frac{8}{9}$ >	$\frac{15}{17}$ >	$\frac{7}{8}$ >	$\frac{13}{15}$ >	$\frac{6}{7}$ >	$\frac{11}{13}$ >	$\frac{16}{19}$ >	$\frac{5}{6}$ >	$\frac{19}{23}$ >	$\frac{14}{17}$ >
$\frac{9}{11}$ >	$\frac{22}{27}$ >	$\frac{13}{16}$ >	$\frac{17}{21}$ >	$\frac{21}{26}$ >	$\frac{25}{31}$ >	$\frac{4}{5}$ >	$\frac{23}{29}$ >	$\frac{19}{24}$ >	$\frac{15}{19}$ >	$\frac{11}{14}$ >	$\frac{18}{23}$ >	$\frac{7}{9}$ >	$\frac{10}{13}$ >	$\frac{13}{17}$ >	$\frac{16}{21}$ >
$\frac{19}{25}$ >	$\frac{3}{4}$ >	$\frac{17}{23}$ >	$\frac{14}{19}$ >	$\frac{11}{15}$ >	$\frac{8}{11}$ >	$\frac{5}{7}$ >	$\frac{12}{17}$ >	$\frac{7}{10}$ >	$\frac{16}{23}$ >	$\frac{9}{13}$ >	$\frac{20}{29}$ >	$\frac{11}{16}$ >	$\frac{24}{35}$ >	$\frac{13}{19}$ >	$\frac{2}{3}$ >
$\frac{11}{17}$ >	$\frac{20}{31}$ >	$\frac{9}{14}$ >	$\frac{16}{25}$ >	$\frac{7}{11}$ >	$\frac{12}{19}$ >	$\frac{17}{27}$ >	$\frac{5}{8}$ >	$\frac{18}{29}$ >	$\frac{13}{21}$ >	$\frac{8}{13}$ >	$\frac{3}{5}$ >	$\frac{16}{27}$ >	$\frac{29}{49}$ >	$\frac{13}{22}$ >	$\frac{23}{39}$ >
$\frac{10}{17}$ >	$\frac{17}{29}$ >	$\frac{24}{41}$ >	$\frac{7}{12}$ >	$\frac{25}{43}$ >	$\frac{18}{31}$ >	$\frac{11}{19}$ >	$\frac{4}{7}$ >	$\frac{21}{37}$ >	$\frac{38}{67}$ >	$\frac{17}{30}$ >	$\frac{30}{53}$ >	$\frac{13}{23}$ >	$\frac{22}{39}$ >	$\frac{31}{55}$ >	$\frac{9}{16}$ >
$\frac{32}{57}$ >	$\frac{23}{41}$ >	$\frac{14}{25}$ >	$\frac{5}{9}$ >	$\frac{6}{11}$ >	$\frac{7}{13}$ >	$\frac{8}{15}$ >	$\frac{9}{17}$ >	$\frac{10}{19}$ >	$\frac{11}{21}$ >	$\frac{1}{2}$ >	$\frac{10}{21}$ >	$\frac{9}{19}$ >	$\frac{8}{17}$ >	$\frac{7}{15}$ >	$\frac{6}{13}$ >
$\frac{5}{11}$ >	$\frac{4}{9}$ >	$\frac{11}{25}$ >	$\frac{18}{41}$ >	$\frac{25}{57}$ >	$\frac{7}{16}$ >	$\frac{24}{55}$ >	$\frac{17}{39}$ >	$\frac{10}{23}$ >	$\frac{23}{53}$ >	$\frac{13}{30}$ >	$\frac{29}{67}$ >	$\frac{16}{37}$ >	$\frac{3}{7}$ >	$\frac{8}{19}$ >	$\frac{13}{31}$ >
$\frac{18}{43}$ >	$\frac{5}{12}$ >	$\frac{17}{41}$ >	$\frac{12}{29}$ >	$\frac{7}{17}$ >	$\frac{16}{39}$ >	$\frac{9}{22}$ >	$\frac{20}{49}$ >	$\frac{11}{27}$ >	$\frac{2}{5}$ >	$\frac{5}{13}$ >	$\frac{8}{21}$ >	$\frac{11}{29}$ >	$\frac{3}{8}$ >	$\frac{10}{27}$ >	$\frac{7}{19}$ >
$\frac{4}{11}$ >	$\frac{9}{25}$ >	$\frac{5}{14}$ >	$\frac{11}{31}$ >	$\frac{6}{17}$ >	$\frac{1}{3}$ >	$\frac{6}{19}$ >	$\frac{11}{35}$ >	$\frac{5}{16}$ >	$\frac{9}{29}$ >	$\frac{4}{13}$ >	$\frac{7}{23}$ >	$\frac{3}{10}$ >	$\frac{5}{17}$ >	$\frac{2}{7}$ >	$\frac{3}{11}$ >
$\frac{4}{15}$ >	$\frac{5}{19}$ >	$\frac{6}{23}$ >	$\frac{1}{4}$ >	$\frac{6}{25}$ >	$\frac{5}{21}$ >	$\frac{4}{17}$ >	$\frac{3}{13}$ >	$\frac{2}{9}$ >	$\frac{5}{23}$ >	$\frac{3}{14}$ >	$\frac{4}{19}$ >	$\frac{5}{24}$ >	$\frac{6}{29}$ >	$\frac{1}{5}$ >	$\frac{6}{31}$ >
$\frac{5}{26}$ >	$\frac{4}{21}$ >	$\frac{3}{16}$ >	$\frac{5}{27}$ >	$\frac{2}{11}$ >	$\frac{3}{17}$ >	$\frac{4}{23}$ >	$\frac{1}{6}$ >	$\frac{3}{19}$ >	$\frac{2}{13}$ >	$\frac{1}{7}$ >	$\frac{2}{15}$ >	$\frac{3}{23}$ >	$\frac{1}{8}$ >	$\frac{2}{17}$ >	$\frac{1}{9}$.

Fig. 2.5: The symmetric pattern of even-denominators in 256 ν_i

This is the reality, behind all of the confusing fractions of **I/FQHE**, linked to Euler's convergence improvement transformation of $\pi/2$ [10]

$$
\begin{aligned}
\frac{\pi}{2} &= \sum_{n=0}^{\infty} \frac{n!}{(2n+1)!!} = 1 + \frac{1}{3} + \frac{1}{3}\cdot\frac{2}{5} + \frac{1}{3}\cdot\frac{2}{5}\cdot\frac{3}{7} + \frac{1}{3}\cdot\frac{2}{5}\cdot\frac{3}{7}\cdot\frac{4}{9}\cdots \\
&= \sum_{n=0}^{\infty} \frac{n!\,[2^n n!]}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{n!(2n)!!}{(2n+1)!} = 1 + \frac{1}{3}\Bigl(1 + \frac{2}{5}\Bigl(1 + \frac{3}{7}\Bigl(1 + \frac{4}{9}\Bigl(1 + \cdots\Bigr)\Bigr)\Bigr)\Bigr)
\end{aligned} \tag{2.35}
$$

where all neighboring fractions obey $|ad - cb| \equiv 1$ (e.g., $2 \times 3 - 1 \times 5 = 1$).

Without a doubt, the filling factor ν_i of 2D fractional charge comes from the 2D geometrical constant π, due to the nucleon electric field of spherical symmetry being broken by an external magnetic field. However, the π in $\nu = 2\pi l_0^2 N$ is canceled by the π in $l_0^2 = \hbar/\mathbf{e}B = h/2\pi\mathbf{e}B$ that has no donation to ν_i, and there is no π in $\sigma_{xy} = \nu\mathbf{e}^2/h$ or $\mathbf{e} = [2\alpha\varepsilon_0 hc]^{1/2} = [2\alpha h/\mu_0 c]^{1/2} = [2\alpha\sigma_0 h]^{1/2} = [2\alpha h/Z_0]^{1/2}$. **The I/FQHE experiment clearly shows that the $\alpha^{1/2}$ formula must contain $\log e^{\pm 1}$ for $\pm$ sign and $1/\pi$ for ν_i. If there is $\log e^{\pm 1}$ and $1/\pi$ in the $\alpha^{1/2}$ formula, from $\frac{\log e^{\pm 1}}{\pi\alpha^{1/2}} = 1.618273$, one can easily get ϕ or Φ, then get $K \cong 1$.** This is a strong authorization of (2.9), a simple physical explanation of I/FQHE, beyond the current unsatisfactory theories.

Unlike α being the interaction of two-charges, $\alpha^{1/2}$ is the ratio of elementary charge $\mathbf{e}$ [esu] and Planck charge $\mathbf{q}_{\mathrm{P}} = \sqrt{\hbar c}$ for a single-charge quantization. It has *one* value

[9] Any new fractions can be easily added into the table according to (2.25), e.g., possibly 19/8 between 12/5 and 7/3.

[10] $2^n n! = (2n)!! = 2\cdot4\cdot6\cdots(2n)$ *even product*, $(2n+1)!! = 1\cdot3\cdot5\cdots(2n+1)$ *odd product*

but *many* formats, $\phi - e - \pi$ each appear only once naturally in (2.9). Theoretically, the information entropy requires $\log e$ instead of $\log \Phi$ or $\log \pi$, so nature takes the minimum from $\underline{e^{1/\Phi\pi}} < e^{\Phi/\pi} < e^{\pi/\Phi} < e^{\Phi\pi}$ to build (2.9). The fact that so *many* applications involve $\phi - e - \pi$, however, prompted nature to adopt another *exclusive* number from the infinite natural numbers $(1, 2, \cdots, \infty)$ to pin-down K and the value of $\alpha^{1/2}$ in (2.9). Logically, an *exclusive* $K \cong 1$ should use the same math format as $\Phi - \phi - e - \pi$ in (2.10). From $\alpha^{-1} \approx 137.036\cdots$, $K \approx 1.00014768\ldots$ as continued fraction

$$K(J) = 1 + \tfrac{1}{J+}\tfrac{1}{J+}\tfrac{1}{J+}\cdots = [1, \overline{6771}] \tag{2.36}$$

where $J = 3 \times 37 \times 61 = 6771$ is the "***the mysterious numbers***" for $\alpha^{1/2}$ assumed by Feynman (see 1-6). [52]

2-6 Sphenic Number and Double Factorial

The *true* α must be a theoretical math solution because even the best experimental data can not answer the simplest question: *why did nature choose this number instead of other numbers*? In order to determine the theoretical value of α, one needs to work on the J of $K(J)$ in (2.36). The sphenic number $J = 6771 = 3 \times 37 \times 61$ is experimentally safeguarded by 137.03599 ± 0.000125 ($137.035865 < \alpha^{-1} < 137.036115$). Möbius function $\mu(n) = \sum_{1 \le k \le n}^{\gcd(k,n)=1} e^{i2\pi k/n}$ is the sum of the primitive n^{th} roots of unity, with $\mu(3) = \mu(37) = \mu(61) = -1$ and $\mu(6771) = (-1)^3 = -1$. A set of sphenic numbers $\{1, \mathbf{3}, \mathbf{37}, \mathbf{61}, 111, 183, 2257, \mathbf{6771}\}$ obeys

$$\{1, \boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}, pq, pr, qr, \boldsymbol{pqr}\} \tag{2.37}$$

where a sphenic number $J = \boldsymbol{p} \times \boldsymbol{q} \times \boldsymbol{r}$. (2.37) is similar to the standard basis of geometric algebra in (2.1). Elementary charge is a special relativity invariant and independent of time. However, where do these 3, 37, 61 come from? How can we link them to the quantum theory?

The *infinite* prime double factorial equation

$$\mathbf{P}(\mathbf{p}) \;=\; \frac{2^{(\mathbf{p}-1)}(\mathbf{p}-1)!+1}{\mathbf{p}} = \frac{2(\mathbf{p}-1)!!+1}{\mathbf{p}} \tag{2.38}$$

only has three solutions as prime sets $\mathbf{P}(\mathbf{p}) = \{\cancel{\mathbf{2}}, \mathbf{3}, \mathbf{6.9} \times \mathbf{10^{50}}, \mathbf{1.5} \times \mathbf{10^{98}}\}$[11] when $\mathbf{p} = \{\cancel{\mathbf{1}}, \mathbf{3}, \mathbf{37}, \mathbf{61}\}$[12]. (2.38) covers the entire prime set $(2, 3, 5, 7, 11 \cdots, \infty)$, and is similar to the Wilson quotient $\mathrm{W}(\mathbf{p}) = [(\mathbf{p}-1)!+1]/\mathbf{p}$. The prime double factorial set $\{A\} = 2^{(\mathbf{p}-1)}(\mathbf{p}-1)! = [2(\mathbf{p}-1)]!!$ is a subset of set $\{B\} = 2^n n! = (2n)!!$ $(\{A\} \subseteq \{B\})$. $2^n n! = (2n)!!$ is equal to the total permutation $\Gamma(n+1) = n!$ times the sum of k-combination 2^n (binomial coefficient of Pascal's triangle)[13]

$$\begin{aligned} 2^n n! &= (2n)!! = 2 \cdot 4 \cdot 6 \cdots 2n \\ &= P_n \sum_{k=0}^{n} C_k^n = P_n \sum_{k=0}^{n} \binom{n}{k} \\ &= 2^n \Gamma(n+1) = 2^n n \Gamma(n) \end{aligned} \tag{2.39}$$

[11] $\mathbf{P}(37) = 6.9 \times 10^{50} = 69089693962934762901433148382870696609107857297297 3$

$\mathbf{P}(61) = 1.5 \times 10^{98} = 15726958986616372057199901553556265877348344168210475239941138025072615402440675110441967213114754 1$

[12] M. Bouayoun, http://oeis.org/A091824 (2004). Note 1 is not prime,

[13] $(2n)!! = \frac{(2n)!}{(2n-1)!!} = \frac{(2n+1)!}{(2n+1)!!} = (2n+1)!! \int_0^{\pi/2} \sin^{2n+1}\theta d\theta = 2^n \left(\frac{2}{\pi}\right)^{\sin^2(n\pi)/2} (n-1)!! n!!$

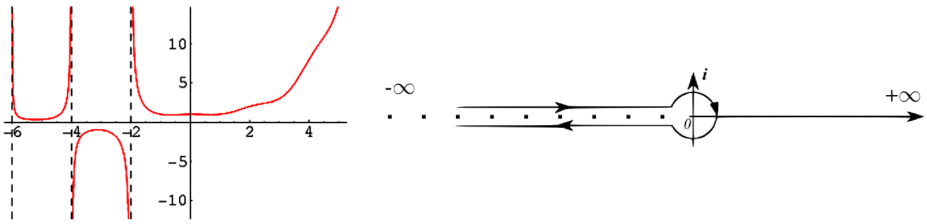

Fig. 2.6: Gamma Function and Hankel contour integral

Since it involves the Gamma function $\Gamma(z) = \frac{1}{z}\prod_{n=1}^{\infty}[(1+\frac{1}{n})^z/(1+\frac{z}{n})]\,(z \neq -n)$ and $\Gamma(n+1) = n!$ in **Fig. 2.6**, the graphic pattern of $(2n)!!$ is similar to the $\Gamma(\frac{n}{2}+1)$ with poles at $-(2n+1) < -2$ $(n = 1, 2, \cdots)$. The *reciprocal Gamma* function is a Hankel contour-integral $\frac{1}{\Gamma(z)} = \frac{1}{2\pi i}\oint_C e^t \cdot t^{-z} dt$ $(|\arg t| < \pi)$ starting from $t = -\infty$ and circling clockwise around the origin *0* before going back to $-\infty$ (similar to an electron absorbing and emitting a photon), and has a simple pole at $z = -n$ for every natural number and zero n; the residues are given by $\textbf{Res}\,(\Gamma, -n) = (-1)^n/n\Gamma(n) = (-1)^n/n!$, and the sum of the residues of these poles equals

$$\sum_{n=0}^{\infty}\textbf{Res}\,(\Gamma,-n) = \sum_{n=0}^{\infty}\frac{(-1)^n}{n!} = \frac{1}{e} = 1 - \frac{1}{1+}\frac{1}{2-}\frac{1}{3+}\frac{1}{2-}\frac{1}{5+}\frac{1}{2-}\frac{1}{7+}\cdots \tag{2.40}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma(n)} = \ln e \times \sum_{n=0}^{\infty}\frac{(-1)^n}{n!} = \ln\left(\frac{e^{\frac{1}{0!}}}{e^{\frac{1}{1!}}}\frac{e^{\frac{1}{2!}}}{e^{\frac{1}{3!}}}\frac{e^{\frac{1}{4!}}}{e^{\frac{1}{5!}}}\cdots\frac{e^{\frac{1}{(2n)!}}}{e^{\frac{1}{(2n+1)!}}}\cdots\right) = e^{-1}$$

which is used as $\Gamma(1,+1) = 1/e = 0.367879\ldots$ for the negative charge electron in (2.13). There are no poles at $z > 0$, so $e = 2.71828\ldots$ does not exist (it also must be written as $e^{+1} = \ln(e^{\frac{1}{0!}}e^{\frac{1}{1!}}e^{\frac{1}{2!}}\cdots e^{\frac{1}{(2n)!}}e^{\frac{1}{(2n+1)!}}\cdots))$.[14] Asymmetry of the *Gamma* function may correspond to the absence of Positrons in nature. For the natural numbers $n > 0$, $(2n)!!$ yields $\{2, \underline{8}, 48, \underline{384}, 3840, \underline{46080}, 645120, 10321920, 1.86\times10^8, \underline{3.72\times10^9}, \ldots\}$, where the subset of $[2(\mathbf{p}-1)]!!$ is shown underlined.

Binomial distributions for $p = q = \frac{1}{2}$ involve $(2k)!!$ and $[2(n-k)]!!$

$$P(k) = \frac{n!}{k!(n-k)!}p^k q^{(n-k)} = \frac{n!}{(2k)!![2(n-k)]!!} \tag{2.41}$$

$(2n)!! = 2^n n!$ appears in the spherical Bessel function for electromagnetic waves, and in the Hermite polynomials for the eigenfunctions of the quantum harmonic oscillator with an eigenvalue of $E_n = (\boldsymbol{n}+\frac{1}{2})\hbar\omega$, $(\boldsymbol{n} = 1, 2, 3\ldots)$

$$\psi_n(x) = \left(2^n n!\sqrt{\pi}\right)^{-1/2} e^{x^2/2}\frac{d^n}{dx^n}e^{-x^2} \tag{2.42}$$

The orthogonal Hermite polynomials have $H_n^{(n)}(x) = 2^n n!$ and

$$\int_{-\infty}^{\infty} H_m(x)\,H_n(x)\,e^{-x^2}dx = \begin{cases} 0 & (m \neq n) \\ 2^n n!\sqrt{\pi} & (m = n) \end{cases} \tag{2.43}$$

[14] The Fransén-Robinson Constant for the contentious Gamma function $F = \int_0^\infty \frac{dx}{\Gamma(x)} = 2.807772420$ does not equal $e = \sum_{n=0}^{\infty}\frac{1}{\Gamma(n)} = 2.718281828$ for the discrete Gamma function, where $\frac{1}{\Gamma(x)} = xe^{\gamma x}\prod_{n=1}^{\infty}\left\{\left(1+\frac{x}{n}\right)e^{-x/n}\right\}$ and $F - e = \int_0^\infty\{e^x[\pi^2+(\ln x)^2]\}^{-1}dx = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2}\exp(\pi\tan\theta - \exp(\pi\tan\theta))d\theta = 0.08948841$. We get $\alpha_{\mathrm{W}'}^{-1/2} = \frac{\Phi\pi}{\log F} = 11.3372868$ and $\alpha_{\mathrm{W}'}^{-1} = (\frac{\Phi\pi}{\log F})^2 = 128.53407$ for the W-boson at high energy scale.

$2^n n!$ also appears in the Rodrigues' formula as $2^l l!$

$$\begin{aligned} P_l(x) &= \frac{1}{2^l l!}\frac{d^l}{dx^l}\left(x^2-1\right)^l \\ 2^l l! &= \left[\frac{d^l}{dx^l}\left(x^2-1\right)^l\right]_{x\to 1} \end{aligned} \tag{2.44}$$

which is linked to the associated Legendre polynomials in spherical harmonic expansion, to solve the Schrödinger equation (yielding $\boldsymbol{l} = 0,\ 1,\ 2\ldots(\boldsymbol{n}-1)$, and $\boldsymbol{m} = 0,\ \pm1,\ldots\pm\boldsymbol{l}$)

$$\begin{aligned} P_l^{|m|}(x) &= \frac{(-1)^m}{2^l l!}\left(1-x^2\right)^{|m|/2}\frac{d^{|m|}}{dx^{|m|}}P_l(x) \\ &= \frac{(-1)^m}{2^l l!}\left(1-x^2\right)^{|m|/2}\frac{d^{l+|m|}}{dx^{l+|m|}}\left(x^2-1\right)^l \end{aligned} \tag{2.45}$$

where $(-1)^m$ is the extrinsic parity of the spherical harmonics. The spherical harmonics are defined by

$$Y_l^m(\theta,\phi) \equiv \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos\theta)e^{im\phi} \tag{2.46}$$

which obeys $Y_l^{-l}(\theta,\phi) = \frac{1}{2^l l!}\cdot\sqrt{\frac{2l+1}{4\pi}}\sin^l\theta e^{-il\phi}$, $Y_l^0(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}}P_l(\sin\theta)$ and $Y_l^{-m}(\theta,\phi) \equiv (-1)^m\overline{Y}_l^m(\theta,\phi)$. It is used for solving the Schrödinger equation for the atomic electron configuration (**Fig. 2.7**). It can be further extended with the Condon-Shortley phase as $Y_{lm} \equiv (-1)^m Y_l^m$, and the Spin-weighted spherical harmonics as ${}_sY_{lm} = \sqrt{\frac{(l\mp s)!}{(l\pm s)!}}\eth Y_{lm}$.

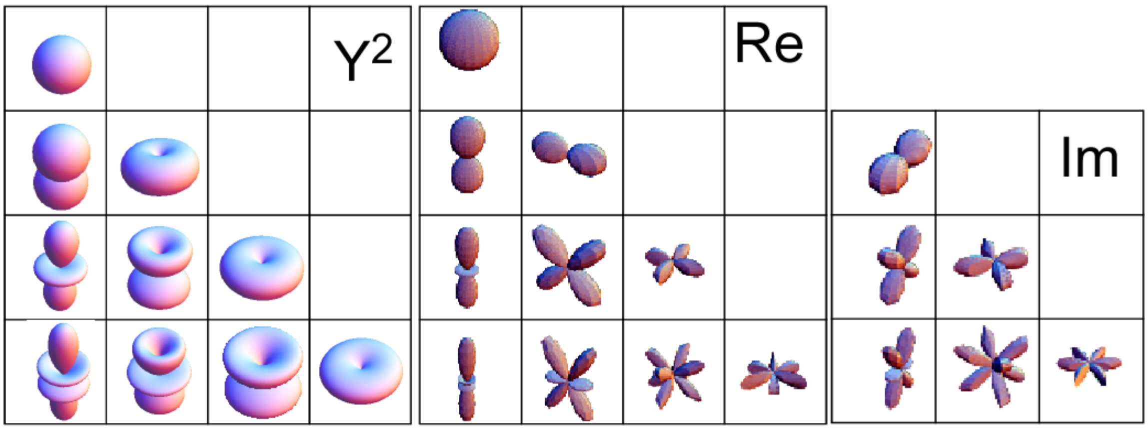

Fig. 2.7: The above illustrations show $|Y_l^m(\theta,\phi)|^2$ (*lift*), $\mathrm{Re}\,[Y_l^m(\theta,\phi)]^2$ (*middle*) and $\mathrm{Im}\,[Y_l^m(\theta,\phi)]^2$ (*right*).

Orthogonal polynomials were initially considered for continued fractions. Their inner products in Hilbert space obey $\langle\psi_n,\psi_m\rangle = \delta_{nm}$ and form the quantum numbers $(\boldsymbol{n},\ \boldsymbol{l},\ \boldsymbol{m},\ \boldsymbol{m_s})$. In the Path-Integral methods of QED, according to Weinberg, $2^n n!$ is also used in the Gaussian Multiple Integrals.

In reality, they all involve spherical symmetry; the Wallis $\pi/2$ formula in 1656 was linked to $2^n n! = (2n)!!$, and to all odd primes $\{3,5,7,11,\ldots\}$ by Euler in 1737, with the countless variations

$$
\begin{aligned}
\frac{\pi}{2} &= \frac{2}{1}\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\frac{8}{9}\frac{10}{9}\frac{10}{11}\cdots && \big(Wallis{-}1656\big) && (2.47)\\
&= \left[\frac{(2n)!!}{(2n+1)!!}\right]^2 = \left[\frac{2^n n!}{(2n+1)!!}\right]^2 && \left\{\frac{even}{odd}\right\}^2 \\
&= \prod_{n=1}^{\infty}\frac{2n}{2n-1}\cdot\frac{2n}{2n+1} = \prod_{n,m,p=1}^{\infty} 4n^2\cdot\frac{\boldsymbol{p}}{\mathbf{2}\boldsymbol{mp}\mathbf{-1}}\cdot\frac{\boldsymbol{p}}{\mathbf{2}\boldsymbol{mp}\mathbf{+1}} \\
\frac{\pi}{2} &= \frac{3}{2}\frac{5}{6}\frac{7}{6}\frac{11}{10}\frac{13}{14}\frac{17}{18}\frac{19}{18}\frac{23}{22}\cdots && \big(Euler{-}1737\big) \\
&= \prod_{n=2}^{\infty}\frac{\mathbf{p}_n}{\mathbf{p}_n+(-1)^{(\mathbf{p}_n-1)/2}} && \left\{\frac{odd\,prime}{even}\right\}
\end{aligned}
$$

where n may or may not equal to m. The infinite product of Wallis has a consecutive sequence whereas Euler's has a discrete sequence in (2.47). In this way, Quantum theory does indirectly involve the discrete prime sequence.

$2^n n!$ also appears in a n-dimensional complex spheres, $V(D_n) = 2\pi^n/2^n n! = 2\pi^n/(2n)!!$, [16] and still exists in the power series of the complete elliptic integral of the first and second kind. The spherical harmonic field gradually stretches into an elliptic and linear field if a particle moves at very high-speed or light-speed c: $\bigcirc \quad \cdots \quad \cdots \quad \longleftrightarrow$. [15] $2^n n!$ disappears in light-speed, therefore, a photon has neither charge nor mass. Spherical harmonics are fundamental in physics theories and applications, such as the electron configuration, gravitation or electromagnetic field theories.

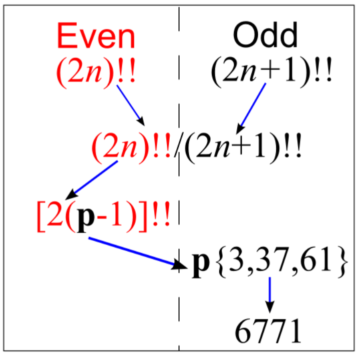

Fig. 2.8: In Nature, *odd* and *even* numbers are equally distributed, which makes π as the coupling constant of *odd* and *even* numbers in (2.47), i.e., $2(2\cdot 4\cdot 6\cdots)^2 = \pi(1\cdot 3\cdot 5\cdots)^2$ or prime~even $(2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13\cdots) = \pi(2\cdot 6\cdot 6\cdot 10\cdot 14\cdots)$. The *even* number double factorial set $\{(2n)!!\}$ contains a subset $\{[2\,(\mathbf{p}-1)]!!\}$. The prime double factorial equation (2.38) yields a *odd prime* set $\{3,\,37,\,61\}$, making the *unique* sphenic number $(2^2-1^2)(4^3-3^3)(5^3-4^3) = 3\times 37\times 61 = 6771$ for the exclusive fine structure constant, where the *even*/*odd* numerical symmetry is broken.

Spherical harmonics involve $2^n n!$, and (2.38) with $2^{(\mathbf{p}-1)}(\mathbf{p}-1)!$ yields an *odd prime* set $\{3, 37, 61\}$ which produces the sphenic number 6771 (**Fig. 2.**8), where *even*/*odd* numerical symmetry is broken. $6771 = 3\times 37\times 61$ to a sphere is just like the stem on an apple. It denotes a unique point on an otherwise perfect sphere. The uniqueness of the fine structure constant is based on this *even*/*odd* numerical symmetry breaking. Perhaps coincidentally, $\frac{37\times 61}{2\times 3} = 376.16\overline{6}$ is close to the value of vacuum impedance.

[15] Special relativity will gradually distort the spherical harmonics. The relativistic Dirac equation can be solved by symmetric distortion, i.e., the wavefunction of *non*-Spherical harmonics which involves $2^l k!$ instead of $2^l l!$. However, charge only disappears at light speed, i.e., photon. The Dirac equation is for fermions running at very high-speed. In polar coordinates, a conic section $r = l/(1\pm e\cos\theta)$ where $l=0$ is a point (*origin* O), if $l>0$ then varying $e\in\{0,\,(0<e<1),\,1,\,(1<e<\infty),\,\infty\}$ gives a circle $(e=0)$, ellipse $(0<e<1)$, parabola $(e=1)$, hyperbola $(e>1)$ or line $(e\to\infty)$.

Since the dimensionless α is exclusive in physics, is there a corresponding natural number in math? Here we show $J = 6771$ is the exclusive number for the exclusive fine-structure constant.

2-7 Variability of the Fine Structure Constant

We are not looking for a simple numeral solution but an explainable math-physical expression. The α variability has been discussed using two approaches: (A) In the zero-energy limitation for fermions, the value is fluctuated around $\alpha^{-1} = 137.036$; (B) In the high-energy running coupling as the electroweak unification in the *beta*-function, it appears in different values $\alpha_{\mathrm{W}}^{-1} \sim 128.952$ for high-energy $\mathrm{W}^{\pm}$ boson. Let's first discuss (A).

(2.9) is a physical equation with the information units [ban], [hart], [nat]. In fact, $K = [1, \overline{6771}]$ in (2.36) can be a limitation of the π-form continued fraction $K(J,H) = 1 + \frac{1^H}{J+}\frac{3^H}{J+}\frac{5^H}{J+}\cdots$ when $H = 0$. The value of $K^2(J,\,H)$ has the same pattern with α^{-1} ($\triangle K^2(J,H) = 6.1 \times 10^{-7}$ in **Fig. 2.9**). We can get the discrete experimental data by variation of H ($\triangle\alpha/\alpha = 6.1 \times 10^{-7}$ in **Fig. 2.9**). For example, let $H = \frac{2}{9}\pi(e^{+1} + e^{-1})^2 = 4\pi\sin^2\theta_w\cosh^2(\pm 1)$ then $\alpha^{-1}_{6.649} = \underline{137.03599907}395$ to match $\alpha^{-1}_{2010} = \underline{137.03599907}4\,(44)$ (*inside the little green circle* in **Fig. 2.9**). In this way, (2.9) gives the origin and the range of domain for α, including the values of CODATA.

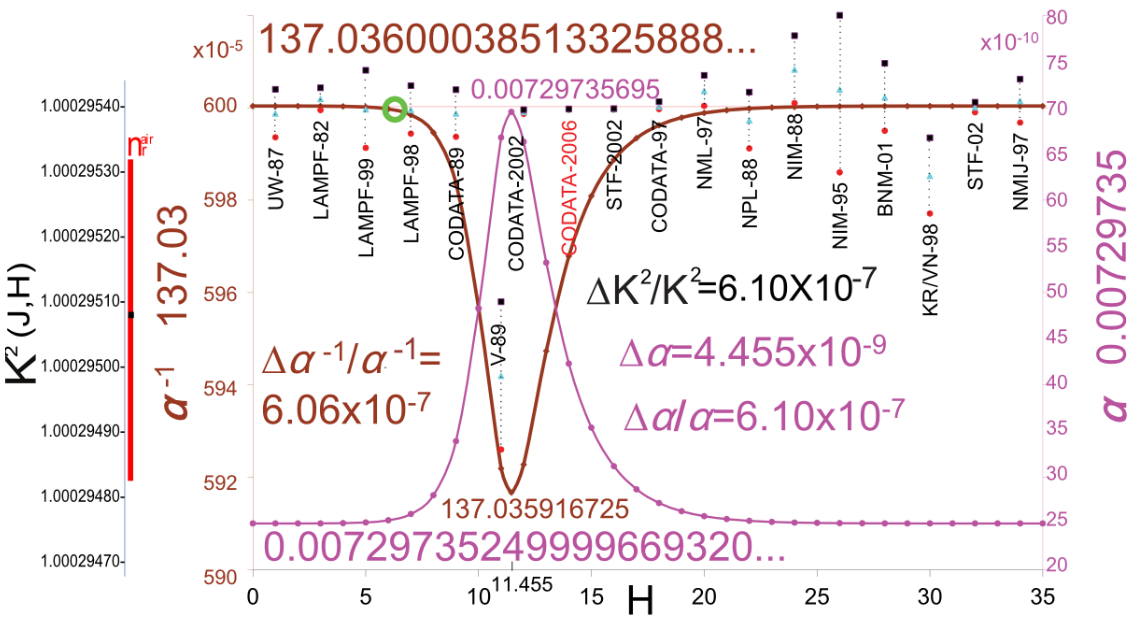

Fig. 2.9: Plot of $K^2(J,H) \sim n_r^{air}$, α^{-1} and α vs variable H with the compiled reports.

The values of CODATA come from selective reports based on the statistic average of discrete data. [48, 89] The CODATA recommended values constantly change and never exclude other experimental data. There are many real experiments yielding slightly different values of $\alpha^{-1} = K^2(J,H)\cdot\underline{\Phi^2\pi^2\ln^2 10} = \frac{c}{v_e}$, where v_e is the velocity of the electron in the first "circular" Bohr orbit. Unfortunately, no report has been given on the reasons for differences arising in controlled experimental conditions.

As we discussed in the section of (2-1), **the fine structure constant is the coupling constant of two zones (electron/space).** In the determination of the elementary charge, the early Millikan oil-drop experiment involved Stokes law with the viscosity of *air*.

[90] In **Fig. 2.9**, the variable region of $K^2(J,H)$ is approximately equal to the refractive index of air $n_r^{air} = \frac{c}{v_p} = (\varepsilon_r^{air} \cdot \mu_r^{air})^{1/2} = 1.00029507 \pm 2.4985 \times 10^{-7}$ (red-bar in **Fig. 2.9**), where $\varepsilon_r^{air} = 1.00058986 \pm 0.00000050$ and $\mu_r^{air} = 1.00000037$ are the relative permittivity and permeability of air (N_2(78.084%) + O_2(20.946%) + $\cdots$ in **e-e** covalent bond). [91] Physically, the effective $K(J,H) \sim n_r^{1/2} = (\mu_r^{e^2} \cdot \varepsilon_r^{e^2})^{1/4}$ is for the *single* electron. There is a single peak at $0 \leq H \lesssim \alpha^{-1}$ in **Fig. 2.9**. Therefore, a modified (2.9) can be linked to the *effective electric charge* that has been widely discussed in the *low-energy* physical theory. According to Feynman, "*But no such ideal electrons exist. ... a real electron, which emits and absorbs its own photons from time to time, and therefore depends on the amplitude for coupling, J.*" [52] The refractive index is equal to the ratio of light-speed in a vacuum and medium. Charge quantization begins during the photon deceleration of light-speed. It is logical to think that the gas model of the *electron* in an atom breaks the vacuum space-time symmetry. We have proved that electrons are the charged oscillators in blackbody radiation. [13] We also shown that the fine structure constant is the corner-store in the quantum theory. [92] However, many factors can make the quantum fluctuation of the *macro*-refractive index, such as density (pressure, temperature, ...), impurity absorption (CO_2, H_2O, ...), and so on. The refractive index of a solid or liquid is more complicated as a tensor, which involves a disturbance of the field and the positions and velocities of the charged particles (electrons) within the material. It is noticed that the monatomic gas of ${}^{133}C_s^{6S1}$ with the *single* 6S1 electron in the photon-recoil measurement of atom interferometry yield $\alpha^{-1}(Atom\,Interferometry) = 137.0360003(10)$, which is close to the theoretical limitation in **Fig. 2.9**. [93] We need more precision *controlled* data for the *solo* photon-electron *micro*-refractive index.

2-8 Information and Physical Reality

Feynman lectured that *"the observed coupling constant* ***e*** *- the amplitude for* ***a real electron to emit or absorb a real photon****. It is* ***a simple number*** *that has been experimentally determined to be close to* -0.08542455*. ... a certain combination of* ***pi****'s and e's (the base of the natural logarithms), and* ***2****'s and* ***5****'s produces the mysterious coupling constant..."* [52]

Here we propose (2.9), alike Feynman's hunch, based on $\Phi - \phi - e - \pi$ in a modified *Euler* identity in (2.11) and $6771 = 3 \times 37 \times 61$ from (2.38), as a simple and physically understandable *unique* α math formula, which yields the mystery of *137* as

$$\begin{aligned}\alpha_0^{-1} &\equiv (\pm K_0 \Phi \pi \ln 10)^2 \equiv [\pm K_0 \cdot 5 \cdot \ln 10 \cdot \prod_{n=1}^{\infty} \tfrac{(10n)^2}{(10n-1)(10n+1)}]^2 \\ &= \{\pm[1,\overline{6771}] \cdot 10 \cdot \tanh^{-1}\left(\tfrac{10-1}{10+1}\right) \cdot \prod_{n=1}^{\infty} \tfrac{(10n)^2}{(10n-1)(10n+1)}\}^2 \\ &= \{\pm[1,\overline{6771}] \cdot 2 \cdot 5 \cdot \tanh^{-1}\left(\tfrac{2\cdot5-1}{2\cdot5+1}\right) \cdot \prod_{n=1}^{\infty} \tfrac{(2\cdot5n)^2}{(2\cdot5n-1)(2\cdot5n+1)}\}^2\end{aligned} \tag{2.48}$$

where $\ln 10 = \ln 2 + \ln 5 = 2\tanh^{-1}\left(\frac{10-1}{10+1}\right)$, and the infinite product of $\Phi\pi$ is based on Euler's infinite product formula $\sin(\pi z) = \pi z \prod_{n=1}^{\infty}(1 - \frac{z^2}{n^2})$, i.e.,[16]

[16] The sinc-function $\mathrm{sinc}(\pi z) = \frac{\sin(\pi z)}{\pi z} = \prod_{n=1}^{\infty}(1 - \frac{z^2}{n^2}) = \frac{1}{z\Gamma(z)\Gamma(1-z)}$ have been used in many physical applications, such as, the double-slit interference. [92]

$$\begin{aligned}\frac{\pi/k}{\sin(\pi/k)} &\equiv \frac{k}{k-1}\frac{k}{k+1}\frac{2k}{2k-1}\frac{2k}{2k+1}\cdots \\ &= \prod_{n=1}^{\infty}\frac{(nk)^2}{(nk-1)(nk+1)}\end{aligned} \tag{2.49}$$

Since $2\sin(\pi/10) = \phi$, let $k = 10$

$$\begin{aligned}\frac{2\cdot(\pi/10)}{2\sin(\pi/10)} &\equiv \frac{\pi\Phi}{5} = \frac{10}{9}\frac{10}{11}\frac{20}{19}\frac{20}{21}\frac{30}{29}\frac{30}{31}\cdots \\ &= \prod_{n=1}^{\infty}\frac{(10n)^2}{(10n-1)(10n+1)}\end{aligned} \tag{2.50}$$

The α is built with continued fractions or infinite products because Nature can't use the decimal $\pi = 3.14159\ldots$ as we do. She can only work with the ratio of integers after the physical quantity is counted as numbers of quanta. However, as an information entropy, the logarithmic format should be used, such as, $\log e^w$, $\ln 10^w$ or $\ln \pi^w$. The information entropy has the basic formula $(a-b+c-d)\ln \mathbf{a} = \ln \mathbf{a}^a - \ln \mathbf{a}^b + \ln \mathbf{a}^c - \ln \mathbf{a}^d = \ln\left(\frac{\mathbf{a}^a\mathbf{a}^c}{\mathbf{a}^b\mathbf{a}^d}\right)$. Now we can understand the importance of an alternating series to the electron, and by extension explain the absence of positions in nature (i.e., (2.13)-(2.15) and (2.41)).

This number *137* serves as a bridge in the quantum (h) to electricity ($\mathbf{e}$), relativity (c), and vacuum (ε_0). [59] It is an *Universal information number* regulating many physical ratios, such as charge $\left(\frac{\mathbf{e}^2}{\hbar c}\right)$, velocity $\left(\frac{v_\mathbf{e}}{c}\right)$, energy $\left(\frac{E_\mathbf{e}}{E_\phi}\right)$, length $\left(\frac{r_\mathbf{e}}{ƛ_\mathrm{c}} = \frac{ƛ_\mathrm{c}}{a_0}\right)$, force $\left(\frac{M_P F_\mathbf{e}(ƛ_\mathrm{c})}{m_e^2 F_P}\right)$, angular momentum $\left(\frac{\mathbf{e}^2/c}{\hbar}\right)$, conductivity $\left(\frac{G_0}{4\sigma_0}\right)$, and impedance $\left(\frac{\mu_o c}{\mathbf{e}^2/h}\right)$, *etc.* As we discussed in the section of (2-1), **the fine structure constant is a coupling ratio between the particle quanta properties and the surrounding spacetime properties, i.e., the coupling between wave and particles**. It frustratingly involves so many physical areas and holds so many physical mysteries. This paper only serves as an initial outlook into the mysteries related to the fine structure constant. This section builds the basic math formula of α by studying I/FQHE, with countless variations of $\phi - e - \pi$ and the core of $J = 6771$ is based on $\{3, 37, 61\}$, by the principles of spherical symmetry and asymmetry. The following sections will investigate this α math formula in other applications. Much more work is needed in the future, including modification, confirmation, correction, or debating other solutions.[17]

All physical equations carry math relationships. At root, there must be a magic group of numbers with basic principles at work in nature, according to which the natural mysteries can be understood. We usually imagine circles when discussing π without realizing that it is a coupling of *even* and *odd* numbers; we think of beauty when talking about ϕ, and don't grasp its statistic meaning; we use e more often in engineering and economics, and don't understand it to be a coupling of boson and fermion; we fixate on *137*, and don't know the uniqueness of $J = 6771$ in the infinite natural numbers. We might be too influenced by our eyes, and poorer at understanding the real logical thinking in the brain. $\alpha^{-1/2}$ in (2.48) can be easily converted to the ellipsoid volume $V = \frac{4}{3}\pi abc$,[18]

[17] such as, $\alpha^{1/2} = \frac{e\pi}{2^2 5^2} = \frac{1}{5^2}\cdot\frac{e}{2}\cdot\frac{\pi}{2} = \frac{1}{25}\left(\frac{2}{1}\right)^{3/2}\left(\frac{2}{3}\cdot\frac{4}{3}\right)^{5/4}\left(\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot\frac{8}{7}\right)^{9/8}\left(\frac{8}{9}\frac{10}{9}\frac{10}{11}\frac{12}{11}\frac{12}{13}\frac{14}{13}\frac{14}{15}\frac{16}{15}\right)^{17/16}\cdots$, i.e., $\alpha = \frac{e^2\pi^2}{2^4 5^4} = \frac{1}{625}\left(\frac{2}{1}\right)^3\left(\frac{2}{3}\cdot\frac{4}{3}\right)^{5/2}\left(\frac{4}{5}\cdot\frac{6}{5}\cdot\frac{6}{7}\cdot\frac{8}{7}\right)^{9/4}\left(\frac{8}{9}\cdot\frac{10}{9}\cdot\frac{10}{11}\cdot\frac{12}{11}\cdot\frac{12}{13}\cdot\frac{14}{13}\cdot\frac{14}{15}\cdot\frac{16}{15}\right)^{17/8}\cdots$

[18] Let $a = \frac{3}{4}\ln 10$; $b = \Phi$; $c = K_0$; then $V_{ellipsoid} = \frac{4}{3}\pi abc = \alpha^{-1/2}$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

and $\Phi\pi$ can be linked to the golden circle. However, it can also be linked to the number $10 = 2 \times 5$.

We probably need to look at both the number theory and topology, and constantly check with the physical experiments. Otherwise, we may run into the trap of metaphysics. There are many α math formulas based on a geometric construction only, the logical analysis of the number theory has been neglected. If we construct a geometric α, the elementary charge and spin of the Fermions simply could not be the Lorentz invariant ($\mathbf{e} = \mathbf{e}_0$ and $\hbar/2 = \hbar_0/2$ v.s. $l = l_0/\gamma$ and $m = \gamma m_0$). We should not refuse to look at the number theory because it has no conventional physical unit.

Einstein was a pioneer on searching for an α math formula. He said: "*It is my conviction that pure mathematical construction enables us to discover the concept and the laws connecting them, which gives us the key to understanding of the phenomena of Nature.*" [94]

3 Probing of Proton to Electron Mass Ratio

3-1 Introduction

Einstein considered: "... inertia originates in a kind of interaction between bodies..." and questioned "Does the Inertia of a Body Depend upon its Energy Content?" [95, 96] According to the energy-mass equation $E = m_0c^2$ and Planck energy quanta $E = h\nu$, if the energy is quantized as the quantum theory, then it is logical to think the inertia mass is also quantized. Elementary particle masses are indeed individual values. However, no formula can calculate the rest-mass of particles, mass-ratio and mass-defect in atomic construction. Born believed that $\alpha \sim 1/137$ and $\beta \sim 1836$ might be related, but no formula was found. [18] This section explores the $\alpha \sim \beta$ relationship, calculation of particle rest-mass, mass ratio, mass defect, and the law of inertia whose origin as entropy.

As common sense, a physical body is a *thing* moving in the surrounding nothingness of geometrical space-time. Kepler found *nothing* is required for planetary motion (R^3/T^2); [97] Galileo showed *nothing* is needed to keep a *thing* at rest or in motion ($y = \frac{1}{2}gt$); [98] Hooke determined an isolated body keeps its speed ($v = constant$) if there is *nothing* to stop it; [99] Newton formulated the law of inertia m when $\sum F \equiv 0$, an external force to accelerate its motion $F = m\boldsymbol{a}$, two opposing actions $F_{2\,on\,1} = -F_{1\,on\,2}$, and Universal Gravity *action-at-a-distance* $F_g = -Gm_1m_2/r^2$ as the inverse-square law (G is determined by Cavendish in 1797), however, "*hypotheses non fingo*" on gravity and inertia. [100] Einstein proved the equivalence principle of gravity and inertia mass $m_G \equiv m_I$, and followed Kepler/Galileo geometric study of gravitation, but used the relativistic spacetime curvature as a tensor equation $R_{ab} - \frac{1}{2}g_{ab}R = \frac{8\pi G}{c^4}T_{ab}$ in General Relativity; He also introduced $E = mc^2$ and $m = \gamma m_0$ in Special Relativity. [101]

In another line of study, Dalton showed all matter consists of atoms; [102] Rutherford showed that an atom is an almost empty space of *nothing*, and most of its mass is held by a tiny nuclei surrounded by cycling electrons; [103] On the fm scale, a lattice of bonding atoms looks like the frame of *nothing*. In the Standard Model, the nucleon is made by quarks (u, d) with QCD strong interaction accounting for 99% of the nucleon mass (see **Table. 3.1**). [104] However, what are quarks and gluons made from? Theoretically, these kind of sub-sub-particle interactions could be an endless game. QCD can not explain the electron mass, the proton/electron mass-ratio, and mass-defect. In this way, mass or inertia will always be "*the will of God*" as Newton said.

Table. 3.1 The development history of mass and inertia

Scientist	**Space-time**	**Mass**	**Formula**	**Mass***	**D**	**Year**
Kepler	absolute	no	$\mu = R^3/T^2$	$\mu \succcurlyeq Gm$	2+1	1619
Galileo	absolute	no	$g = d/t^2$	$w \succcurlyeq mg$	1+1	1638
Newton	absolute	absolute	$g = \mu/r^2$	$F = ma$	3+1	1666
Dalton	absolute	absolute	$\sum m_{atom}$	$N \cdot m_{atom}$	*	1805
Einstein	relativistic	relativistic	$m_g \equiv m_I$	$m = \gamma m_0$	4	1905
Rutherford	relativistic	relativistic	e, p, n	$Am_p + Nm_n + Zm_e$	*	1909
S.M.	relativistic	relativistic	u_{rgb}, d_{rgb}	Quark+Gluon	4	1964

The real space of universe is not *nothing*, it filled with matter/energy such as cosmic rays (90% proton). For example, the Sun emits huge amounts of energy and matter (photon and neutrino) into space, which Earth only receives in very tiny parts (estimated as $\pi r^2_{earth}/4\pi R^2_{Sun-earth} = 4.5\times10^{-10}$, on the power of 4 Kg/s radiation equivalent matter). Even so, the expanding universe is so vast that space still looks like *nothing* is there. Newtonian gravity tells us how masses interact but not how mass is created. Einstein considered that the spacetime curvature is caused by mass, but where does the original mass come from? [105] A gravitational theory without a particle mass formula is like a tree without roots. Mass or inertia is the most fundamental concept of physics. Understanding and calculating the masses of the elementary particles are the prime mystery.

Considering that the particle mass and charge are created simultaneously, the fine structure constant in defining the elementary electric charge should also play a role in particle mass creation. The fine-structure constant α and the proton-to-electron mass ratio β are governed by the deepest Quantum theory. [106] We have proposed

$$|\alpha^{1/2}| \quad \equiv \quad \pm\frac{M\phi}{K\pi} \equiv \frac{\log e^{\pm1}}{K\Phi\pi} \equiv \log e^{\pm\phi/K\pi} \equiv \frac{1}{\ln 10^{\pm K\Phi\pi}} \tag{3.1}$$

where $K = [1, \overline{6771}]$. From (3.1), we get $\alpha = 0.007297352499996 = 1/137.036000385133$, matching $\alpha = 1/137.0360003(10)$ from the photon recoil (atom interferometer). [93] As a new approach, here we discuss a fine structure constant interpretation of the mass-energy equation $E = mc^2$.

A charged particle in the electromagnetic field is accelerated by a Lorentz force $\mathbf{F}_l = \mathbf{e}[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$, which can also be described by Newton's law $\mathbf{F} = m\boldsymbol{a} = m\frac{d\mathbf{v}}{dt}$. Therefore, $m = \frac{dt}{d\mathbf{v}}\mathbf{e}[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]$. In the universe, only two charged individual free particles do not decay: electron $\mathbf{e}^-$ and proton $\mathbf{p}^+$. The confinement neutron $\mathbf{n}^0$ is stable inside the nucleus and a freed neutron will decay $\mathbf{n}^0 \to \mathbf{p}^+ + \mathbf{e}^-_L + \bar{\boldsymbol{\nu}}^R_e$ after 881.5(15) s. Three fermions $\mathbf{e}^-$, $\mathbf{p}^+$, $\mathbf{n}^0$ (spin-1/2) construct atoms, such as H and He, but their rest-mass and mass-ratio are still unaccountable. There is one electron surrounding a proton in ^{1}H, and two electrons surrounding a nucleus of 2p2n in ^{4}He. The mass-defect in the construction of the nucleus of atoms obeys $\triangle E = \triangle mc^2$, let's start on these simple particles and atoms.

3-2 Relationship of α and β

Eddington and Born both believed that the proton-to-electron mass-ratio $\beta \sim 1836$ and the fine structure constant $\alpha \sim 1/137$ are related, [18, 107] but the link has never been found.[19] Physically, since $m_p = h/c\lambda_c^p$ and $m_e = 2R_\infty h/c\alpha^2$, we can get $\alpha^2\beta^{-1} = 2\lambda_c^p R_\infty = 2.9001594 \times 10^{-8}$. Therefore, $\beta = \underline{\alpha^2}/2\lambda_c^p R_\infty = 2\pi\underline{\alpha^1}a_0/\lambda_c^p = 2\pi r_e/\underline{\alpha}\lambda_c^p$. These complicated physical relations could not establish a math relationship between α and β.

From CODATA-2010 $\alpha = 1/137.035999074(44)$ and $\beta = \underline{1836.15267245}(80)$, we find

$$\begin{aligned}\alpha^3\beta^2 \approxeq \ln^2\pi = \left[\int_1^\pi \frac{dt}{t}\right]^2 = -\mathrm{Li}_1^2(1-\pi) = [2\ln\Gamma(\tfrac{1}{2})]^2 \\ = [2\coth^{-1}(\tfrac{\pi+1}{\pi-1})]^2 = [2\tanh^{-1}(\tfrac{\pi+1}{\pi-1})]^2\end{aligned} \tag{3.2}$$

It similar to the Kepler's third law $a^3n^2 = C$, where a is a semi-major axis, $n = 2\pi/T$ is a mean motion. [97] If $\alpha = \frac{r_e}{ƛ_c^e}$ and $T_c^i = \frac{ƛ_c^i}{c} = \frac{\hbar}{m_i c^2}$, $\beta = T_c^e/T_c^p$ and $n_i = 2\pi/T_c^i$, then the ratio of $(\frac{a_1}{a_2})^3(\frac{n_1}{n_2})^2 = \frac{c_1}{c_2}$ is dimensionless. Next, let's try to give a physical picture of (3.2)

(i) Since the mass ratio is linked to the Compton wavelength ratio $\beta = \frac{m_p}{m_e} = \frac{\lambda_c^e}{\lambda_c^p}$ and the charge ratio is linked to the length ratio $\alpha = (\frac{\mathbf{e}}{\mathbf{q_P}})^2 = \frac{r_e}{ƛ_c^e}$, the ratio of mass and charge in (3.2) are linked by the length ratio.

$$\alpha^3 \cdot \beta^2 = \frac{(2\pi r_e)^3}{\lambda_c^e \cdot (\lambda_c^p)^2} = \frac{r_e^3}{ƛ_c^e \cdot (ƛ_c^p)^2} = \frac{r_e^2}{(ƛ_c^p \alpha^{-1/2})^2} \approxeq \ln^2\pi \tag{3.3}$$

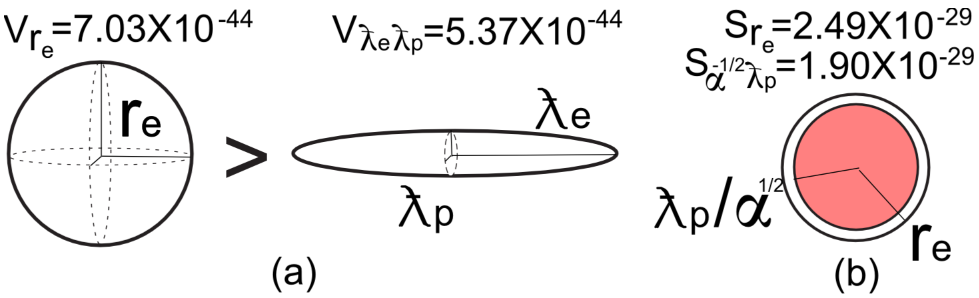

Fig. 3.1: (a) the volume ratio of sphere and ellipsoid, and (b) the surface ratio between proton and electron.

In (3.3), the volume ratio of Sphere/Ellipsoid $V_S/V_E = \frac{4\pi}{3}r_e^3/\frac{4\pi}{3}ƛ_c^e ƛ_c^{p2} \approxeq \ln^2\pi$ (**Fig. 3.1**(a)), or the surface ratio $S_{re}/S_{ƛ_c^p/\alpha} = \pi r_e^2/\pi(ƛ_c^p/\alpha)^2 \approxeq \ln^2\pi$ (**Fig. 3.1**(b)), where $r_e = 2.8179403267\times10^{-15}$ m, $ƛ_c^e = 3.86159268\times10^{-13}$ m, and $ƛ_c^p = 2.1030891047\times10^{-16}$ m.

(ii) From (3.1), (3.3) can be

$$\alpha^2 \cdot \beta^2 = \frac{r_e^2}{(ƛ_c^p)^2} = \frac{r_e^2}{ƛ_c^{e2}}\frac{m_p^2}{m_e^2} = \frac{ƛ_c^{e2}}{a_0^2}(\frac{m_p}{m_e})^2 \approxeq \frac{\ln^2\pi}{\alpha} = (K\Phi\pi\ln 10\ln\pi)^2 \tag{3.4}$$

where $\alpha \cdot \beta = r_e/ƛ_c^p = 13.3990534229$ and $r_e/\ln(\pi)ƛ_c^p = 11.70499136 \approx \Phi\pi\ln(10))$. Physically, it can be a mass balance as **Fig. 3.2**

$$\alpha \cdot \beta = \frac{r_e}{(ƛ_c^p)} = \frac{r_e}{ƛ_c^e}\frac{m_p}{m_e} = \frac{ƛ_c^e}{a_0}\frac{m_p}{m_e} \approxeq \frac{\ln\pi}{\alpha^{1/2}} \tag{3.5}$$

[19] Perles got $\alpha^{-1} = \beta/[2\pi(\pi-1)]$ where $\alpha = 1/137$ and $\beta \sim 1843$ in 1928, Haas got $\beta\alpha = 3/2\pi\sqrt{2}$ in 1938, Lenz got $\beta = 6\pi^5$ in 1951 and Wyler got $\alpha = \left(3/4\pi^2\right)^2\left(\pi^5/5!\right)^{1/4}$ in 1969, and Aspden got $\alpha^{-1} = 108\pi(8/1843)^{1/6}$ in 1972

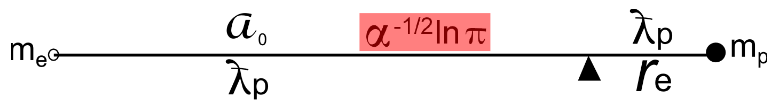

Fig. 3.2: The mass balance of $\alpha \cdot \beta = \frac{r_e}{(\lambda_c^p)} = \frac{r_e}{\lambda_c^e}\frac{m_p}{m_e} = \frac{\lambda_c^e}{a_0}\frac{m_p}{m_e}$

Since $\ln(a)\ln(b) = 4\coth^{-1}(\frac{a+1}{a-1})\coth^{-1}(\frac{b+1}{b-1})$, (3.5) is equal to

$$\alpha \cdot \beta \approxeq K\Phi\pi \ln 10 \cdot \ln \pi = 4K\Phi\pi \coth^{-1}(\tfrac{11}{9})\coth^{-1}(\tfrac{\pi+1}{\pi-1}) \tag{3.6}$$

(iii) Physically, it is impossible for the different types of particles to act at the same distance. (3.3) can also explained as a gravity force ratio between two protons and two electrons at different distances (**Fig. 3.3**(a)).

$$\alpha^3 \cdot \beta^2 = \alpha^1 \cdot \alpha^2 \cdot \beta^2 = \frac{r_e}{\lambda_c^e} \cdot \frac{r_e}{a_0} \cdot (\frac{m_p}{m_e})^2 \cdot \frac{G}{G} = \frac{Gm_p^2/[(a_0\lambda_c^e)^{1/2}]^2}{Gm_e^2/r_e^2} = \frac{F_g^{proton}}{F_g^{electron}} \tag{3.7}$$

where $(a_0\lambda_c^e)^{1/2} = a_0\alpha^{1/2} = a_0 \log e^{\phi/K\pi} = r_e \cdot \alpha^{-3/2} = r_e \cdot (K\Phi\pi \ln 10)^3 = 4.5204721480 \times 10^{-12}$ m is about the extended length of the hydrogen bond X$\cdots$H. If we use two distances $R_\infty^{-1}/4\pi = a_0/\alpha = 7.2516327786 \times 10^{-9}$ m and $r_e = 2.8179403267 \times 10^{-15}$ m, then we get the ratio of work or Torque $rF_g = Gm^2/r$ (**Fig. 3.3**(b))

$$\alpha^3 \cdot \beta^2 = \alpha\frac{r_e}{a_0} \cdot (\frac{m_p}{m_e})^2 \cdot \frac{G}{G} = \frac{Gm_p^2/\alpha^{-1}a_0}{Gm_e^2/r_e} = \frac{(\alpha^{-1}a_0)F_g^{proton}}{r_e F_g^{electron}} = \frac{\tau^{proton}}{\tau^{electron}} \approxeq \ln^2 \pi \tag{3.8}$$

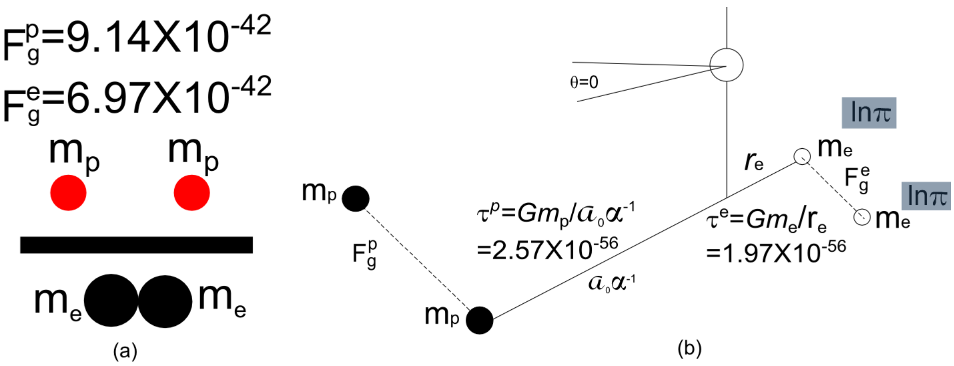

Fig. 3.3: (a) the particle force ratio between proton and electron; and (b) The torsion balance of $\alpha^3 \cdot \beta^2 = \frac{\tau^{proton}}{\tau^{electron}} \approx \ln^2 \pi$ as Cavendish

(iv) The electrostatic force also obeys the inverse-square law, and $\mathbf{p}^+ \cdots \mathbf{e}^-$ should also work with Kepler motion. (3.2) can also expressed as the Coulomb's force ratio between the electron charge and Planck charge

$$\alpha^3 \cdot \beta^2 = \alpha^1 \cdot \alpha^2 \cdot \beta^2 = (\frac{e}{\mathrm{q_P}})^2 \cdot (\frac{r_e}{\lambda_c^e})^2 \cdot (\frac{\lambda_c^e}{\lambda_c^p})^2 \cdot \frac{k_e}{k_e} = \frac{k_e e^2/[\lambda_c^p]^2}{k_e \mathbf{q}_\mathrm{P}^2/r_e^2} = \frac{F_e^{electron}}{F_e^{Planck}} \tag{3.9}$$

where $\beta = \frac{m_p}{m_e} = \frac{\lambda_c^e}{\lambda_c^p} = \frac{\lambda_c^e}{\lambda_c^p}$, $\alpha = (\frac{\mathbf{e}}{\mathbf{q}_\mathrm{P}})^2 = \frac{r_e}{\lambda_c^e}$, $\boldsymbol{e} = 1.602176565 \times 10^{-19}$ C, and $\mathbf{q}_\mathrm{P} = 1.8755459 \times 10^{-19}$ C. Therefore, (3.2) is the unified expression for Newtonian gravity and the Coulomb electrostatic force ratio, and can be converted to other physical ratios.

$$\alpha^3 \cdot \beta^2 = \frac{r_e}{a_0}(\frac{e}{\mathrm{q_P}})^2(\frac{\lambda_c^e}{\lambda_c^p})^2\frac{k_e}{k_e} = \frac{r_e}{a_0}\frac{k_e e^2/[\lambda_c^p]^2}{k_e \mathbf{q}_\mathrm{P}^2/[\lambda_c^e]^2} = \frac{r_e F_e^{electron}}{a_0 F_e^{Planck}} = \frac{\tau^{electron}}{\tau^{Planck}} \tag{3.10}$$

3-3 Proton to Electron Mass Ratio

Following the above physical discussion, we find $\beta = 1836.15267247$ as an approximate function of $\alpha^{-1/2}$ in (3.1) (**Fig. 3.4**)

$$\beta = \frac{m_p}{m_e} \approx \alpha^{-3/2} \ln \pi = (K\Phi\pi \ln 10)^3 \ln \pi = \underline{1836}.348197739 \tag{3.11}$$

(3.11) may not be a numeral coincidence (e.g., $\prod_{k=0}^{2}(4\pi - k/\pi) = 1836.1517$ or $6\pi^5 = 1836.1181$). From $\beta = \frac{m_p}{m_e} \approx (\frac{\mathbf{q}_\mathrm{P}}{\mathbf{e}})^3 \ln \pi$, physically, $\mathbf{e}^3 m_p \approx \mathbf{q}_\mathrm{P}^3 m_e \cdot \ln \pi$. Protons and Neutrons are proportional to the cubical $\alpha^{-1/2}$ that relates to three color charged quarks with eight-gluons. The gluon is a spin-1 vector boson, like the photon. The mass of Baryons are composed primarily of gluons but not quarks, where $\alpha^{-1/2} = K\Phi\pi \ln 10 = 11.706237669941$ is related to the Dirac magnetic monopole $g \equiv \frac{\hbar c}{2\mathbf{e}} = \frac{1}{2}\frac{\mathbf{q}_\mathrm{P}}{\alpha^{1/2}}$. [108, 109] Since $\frac{g}{\mathbf{e}} = \frac{1}{4\alpha^2} \approx 4692$, Dirac noticed that the electron is much easier to generate than the monopole.

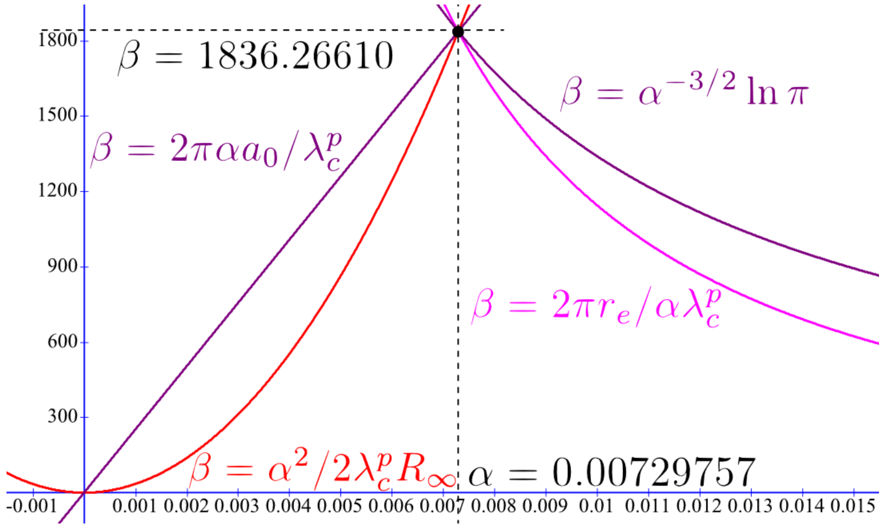

Fig. 3.4: Graphic solution of α and β, the error is due to $\beta = \alpha^{-3/2} \ln \pi$.

We can get β' with $\alpha^{1/2} = \mathbf{e}/\mathbf{q}_\mathrm{P}$ electric modification by the two charges of proton $+\mathbf{e}$ and electron $-\mathbf{e}$

$$\beta' \backsimeq \left(\alpha^{-3/2} - 2\alpha^{1/2}\right) \ln \pi = \underline{1836}.\underline{1526}216852 \tag{3.12}$$

or β'' with $\alpha^2 = r_e/a_0$ geometrical modification for H-atom construction ($\pi\phi^2$ is the area of golden circle). The golden ratio $\phi = 0.618$ plays a significant role in atomic physics where it governs what is known as the Bohr radius [110]

$$\beta'' \approxeq \left(\alpha^{-3/2} - 2\alpha^{1/2} + \frac{\Phi^2}{\pi}\alpha^2\right) \ln \pi = \underline{1836}.\underline{152672}484 \tag{3.13}$$

and $\alpha^4 = (r_e/a_0)^2$ is an additional modification to match with $\beta_{2010} = \underline{1836.15267245}(75)$ as well as $\beta_{2006} = \underline{1836.15267247}(80)$

$$
\begin{aligned}
\beta_{p/e} &= (\alpha^{-2+\frac{1}{2}} - 2\alpha^{0+\frac{1}{2}} + \tfrac{\Phi^2}{\pi}\alpha^2 - \tfrac{9}{2}\alpha^4)\textstyle\int_1^\pi \frac{du}{u} \qquad (3.14)\\
&= (\alpha^{-3/2} - 2\alpha^{1/2} + \tfrac{\Phi^2}{\pi}\alpha^2 - \eta\alpha^3)\ln\pi \\
&\quad \underset{\textbf{Mag}}{\textbf{1836}} \;>\; \underset{\textbf{Ele}}{\textbf{.1526}} \;>\; \underset{\textbf{Geo}}{\textbf{724}} \;>\; \underset{\textbf{EW}}{\textbf{7000}} \\
&= \ln(\pi^{\alpha^{-3/2}} \cdot \pi^{-2\alpha^{1/2}} \cdot \pi^{\alpha^2/\pi\phi^2} \cdot \pi^{-\eta\alpha^3}) \\
&= \underline{1836}.\underline{1526}\,\underline{724}\,\underline{7000}
\end{aligned}
$$

where $\alpha^3 = 4\pi R_\infty r_e$, and $\eta = \frac{9}{2}\alpha = \alpha/\sin^2\theta_w$ is an *electroweak* coupling constant that is discussed in the next section.

Since $(a-b+c-d)\ln\mathbf{a} = \ln\mathbf{a}^a - \ln\mathbf{a}^b + \ln\mathbf{a}^c - \ln\mathbf{a}^d = \ln\left(\frac{\mathbf{a}^a\mathbf{a}^c}{\mathbf{a}^b\mathbf{a}^d}\right)$, (3.14) is written as $\beta_{p/e} = \ln\left(\frac{\pi^a\cdot\pi^c}{\pi^b\cdot\pi^d}\right)$, where $a = \alpha^{-3/2}$, $b = 2\alpha^{1/2}$, $c = \frac{1}{\pi\phi^2}\alpha^2 = \frac{1}{\pi^3K^2\ln^2 10}$ and $d = \frac{9}{2}\alpha^4 = \eta\alpha^3$. The function $\beta_{p/e}(\alpha) = 1836.15267247$ in (3.14) yields an exclusive solution of $\alpha = 0.0072973524999671$, i.e, $\alpha^{-1} = 137.0360003851329$ (**Fig. 3.5**).

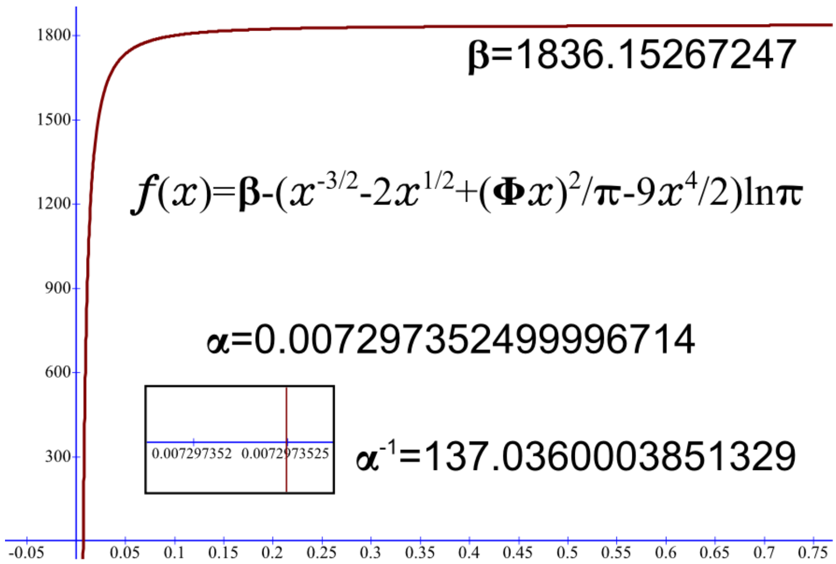

Fig. 3.5: $\beta = \ln[\pi^{\alpha^{-3/2}} \cdot \pi^{-2\alpha^{1/2}} \cdot \pi^{\alpha^2/\pi\phi^2} \cdot \pi^{-\eta\alpha^3}] = 1836.15247247$ and $\alpha = \frac{\log e^{1/K\Phi\pi}}{\ln 10^{K\Phi\pi}} = 0.0072973524999967$ ($\alpha^{-1} = \frac{\ln 10^{K\Phi\pi}}{\log e^{1/K\Phi\pi}} = 137.036000385133$)

In (3.14), the cubical $\alpha^{-1/2} = 2g/\mathbf{q_p} = K\Phi\pi\ln 10$ is related to the strong interaction with **3** color charged quarks and **8** gluons; $2\alpha^{1/2}$ is the electric correlation of the two charges of proton and electron; $\boldsymbol{\alpha^2} = r_e/a_0$ is the geometrical quantization for the H-atom construction (a_0 is the Bohr radius and $\pi\phi^2$ is the area of a golden circle); and $\frac{9}{2}\alpha^4 = (\frac{r_e}{a_0\sin\theta_w})^2$ is an *electroweak* interaction. The physical expression of (3.14) is

$$
\begin{aligned}
\beta_{p/e} &= (\mathbf{8}\tfrac{g^3}{\mathbf{q_p^3}} - \mathbf{2}\tfrac{\mathbf{e}}{\mathbf{q_p}} + \tfrac{1}{\pi\phi^2}\tfrac{r_e}{a_0} - (\tfrac{r_e}{a_0\sin\theta_w})^2)\ln\pi \qquad (3.15)\\
&\quad \textbf{Gluon}>\textbf{Charge}>\textbf{Atom}>\ \textbf{EW} \\
&= \ln(\pi^{\mathbf{8}g^3/\mathbf{q_p^3}} \cdot \pi^{-\mathbf{2e}/\mathbf{q_p}} \cdot \pi^{r_e/a_0\pi\phi^2} \cdot \pi^{-(r_e/a_0\sin\theta_w)^2}) \\
&= \underline{1836}.\underline{15267247000}
\end{aligned}
$$

where the mysterious **magnetic monopole** g **defined by Dirac is inside the nucleon**, which is related to the magnetic-strong coupling and the quantum gravity. In the opinion of Feynman and Wheeler, the proton may hiding 1836/2=918 paired $\mathbf{e^+e^-}$ to justify the missing positron. [111] This energy equivalence of a $\mathbf{e^+e^-}$ pair could be true at least for the gluon. In the standard model, due to the quark confinement, no individual quark or magnetic monopole can be found. [108] In (3.14) and (3.15), the factors of **Mag** > **Ele** > **Geo** > **EW**, i.e., **Gluon** > **Charge** > **Atom** > **EW** are separated by $> 10^3$. Therefore, each term of the alternating series will improve 3~4 digits in the data fitting. This extremely high precision data matching is one of the strongest justifications for the correctness of the theoretical value of α yielded by (3.1). (3.14) and (3.15) not only gives a mathematical solution but also has a clear physical definition. In (3.15), cubical $\frac{g}{\mathbf{q}_\mathrm{p}} = \frac{1}{2\alpha^{1/2}} = 5.853118$ and the spherical radius $r = 7.261970$. Using $E_e = 0.510998928\,\mathrm{MeV}$, $\frac{g}{\mathbf{q}_\mathrm{p}} \ln \pi E_e = 3.4238\,\mathrm{MeV}$ and $r \ln \pi E_e = 4.247931\,\mathrm{MeV}$, matching the experimental quark mass (i.e., $m_u = 1.7 \sim 3.3$ MeV and $m_d = 4.1 \sim 5.8$ MeV). The experimental limit mass of the gluon $< 20\,\mathrm{MeV}$, we get $(\frac{g}{\mathbf{q}_\mathrm{p}})^2 = \frac{1}{4\alpha} = 34.25899$ and $(\frac{g}{\mathbf{q}_\mathrm{p}})^2 \ln \pi E_e = 20.03999\,\mathrm{MeV}$. Note $(\frac{g}{\mathbf{q}_\mathrm{p}})^3 = \frac{1}{8\alpha^{3/2}} = 200.5219$ and $(\frac{g}{\mathbf{q}_\mathrm{p}})^3 \ln \pi E_e = 117.2964449\,\mathrm{MeV}$ matches the mass of the Strange quark.[20] (3.14) and (3.15) are expressed from the viewpoint of stable atoms, and the particle mass creation of the quark model will be discussed in a separate paper.

3-4 Mass of Neutron and Electron

Feynman noticed that the charge and g-factor are different between the proton and neutron. [111] The decaying free neutron has a lifetime $\tau_n = 885.5 \pm 0.1\, s$, the two-steps of β-decay: $\mathbf{n}^0 \to \mathbf{p}^+ + \underline{\mathrm{W}^-} \to \mathbf{p}^+ + \mathbf{e}_L^- + \bar{\boldsymbol{\nu}}_e^R$, with the kinetic energy of a proton and an electron $Q = T_k^e + T_k^p \approx 0.7823329(5)\,\mathrm{MeV}$. The kinetic energy of a proton and electron released from neutron β-decay obeys $\triangle E = \triangle mc^2 = \triangle_{n-p-e} E_e$, as the neutron binding energy

$$\begin{aligned} \triangle_{n-p-e} &= \tfrac{5e \ln 2 \ln 3 \ln \pi}{-3\pi^2} g_n = 1.530899254 \\ &\approx \tfrac{-7}{22} \ln 3 \ln \pi \cdot g_n = -\tfrac{2}{5} g_n = -(\tfrac{\sqrt{2}}{\Phi+\phi})^2 g_n \end{aligned} \tag{3.16}$$

The mass ratio of neutron/electron $\beta_{n/e}^{2010} = \underline{1838.6836605}(11)$ is linked to $\beta_{n/e} = \beta_{p/e} + \triangle_{n-p-e} + 1 = \underline{1838.683660}515$. $|g_n| = \mp 3.826085$ is the neutron/anti-neutron g-factor, which is also a function of the fine structure constant. [111] In (3.14) to (3.16), the Gravitational mass of proton and neutron are constructed from Strong-Electro-Weak interactions. Since each particle has unique S-E-W interactions, no elementary mass is the natural mass unit (**Table 3.2**).

Table 3.2. The natural mass units

Units	Mass	Values	Conversion Factor
Particle Physics	eV/c^2	1.78×10^{-36} kg	
Atom	m_e	$9.10938291(40) \times 10^{-31}$ kg	β
QCD	m_p	$1.672621777(74) \times 10^{-27}$ kg	
Stoney	$\sqrt{k_e e^2/G}$	1.85921×10^{-9} kg	$\sqrt{\alpha}$
Planck	$\sqrt{\hbar c/G}$	2.17645×10^{-8} kg	

[20] $\alpha^2 \ln \pi E_e \sim 3.115\,\mathrm{eV}$ is in the electron neutrino range; $\alpha \ln \pi E_e \sim 4.2\,\mathrm{KeV}$; $\alpha^{1/2} \ln \pi E_e \sim 49.96\,\mathrm{KeV}$; $\alpha^{-1/2} \ln \pi E_e \sim 6.85\,\mathrm{MeV}$ is in the quark range; $\alpha^{-1} \ln \pi E_e \sim 80.16\,\mathrm{MeV}$; $\alpha^{-1.5} \ln \pi E_e \sim 938.37\,\mathrm{MeV}$; $\alpha^{-2} \ln \pi E_e \sim 10.98\,\mathrm{GeV}$; $\alpha^{-2.5} \ln \pi E_e \sim 128.69\,\mathrm{GeV}$ is in the exploring Higgs boson range; $\alpha^{-3} \ln \pi E_e \sim 1.5053\,\mathrm{TeV}$.

In nature, electrons and protons are generally paired as atoms, or as neutrons through electron capture (the inverse β-decay). If $m_e \equiv \log_\pi e \cdot \ln \pi = 1$ as the mass unit, then $m_p \equiv \beta_{p/e}[m_e]$ and *vice-versa*. Physically, we always measure the mass ratio by defining a mass unit. Beyond the compared masses, there is no other way to account for the mass.

Experimentally, the electron and positron can be created by γ-ray photon $\omega_e \geqq 7.763440 \times 10^{20}\,\mathrm{s}^{-1}$. In the Planck mass $m_{Planck} = (\hbar c/G)^{1/2} = 2.17651(13) \times 10^{-8}$ Kg where G with $u_r = 1.2 \times 10^{-4}$, [112] the mass of electron or positron $|m_e| = 9.10938291(40) \times 10^{-31}$ Kg is formulated by (3.1) as

$$
\begin{aligned}
|m_e| &= \ln \pi^\eta \cdot m_{Planck} \cdot \tfrac{\pi^2}{5} \cdot e^{-\phi^2/\alpha} \\
&= \ln \pi^\eta \cdot m_{Planck} \cdot 3\tfrac{\zeta(4)}{\zeta(2)} \cdot e^{-\phi^2/\alpha} \\
&= \ln \pi^\eta \cdot m_{Planck} \cdot 3\prod_{\mathbf{P}} \tfrac{\mathbf{P}^2}{\mathbf{P}^2+1} \cdot e^{-\phi^2/\alpha} \\
&= \ln \pi^\eta \cdot m_{Planck} \cdot 3\tfrac{\alpha_R^3}{\alpha^3} \cdot e^{-\phi^2/\alpha} \\
&= \ln \pi^\eta \cdot m_{Planck} \cdot (\tfrac{\pi}{\Phi+\phi})^2 e^{-(K\pi \ln 10)^2} \\
&= \pm 9.109118 \times 10^{-31}\ \mathrm{Kg}
\end{aligned} \tag{3.17}
$$

where $\eta = \pm 1$ represents the *intrinsic parity*, and $\alpha_R = 1/157.555$ is the Blackbody radiation constant. [13] In this way, nature creates charge by using logarithms and mass by using exponents. Their ratio must involve the very large numbers. Since $\boldsymbol{e} = (\alpha\hbar c/k_e)^{1/2}$, the electron charge-mass ratio is given as

$$
\begin{aligned}
\frac{\boldsymbol{e}}{m_e} &= -\frac{e^{\phi^2/\alpha}}{\ln \pi}\left(\frac{\Phi+\phi}{\pi}\right)^2 \cdot \sqrt{\frac{\alpha G}{k_e}} \\
&= -1.758820 \times 10^{11}\ \tfrac{\mathrm{C}}{\mathrm{kg}}
\end{aligned} \tag{3.18}
$$

In the Planck scale, the gravity and electric force are equal for the Planck mass particle.

$$
F_g^P = \frac{G m_{Planck}^2}{r_{Planck}^2} = F_e^P = \frac{\hbar c}{r_{Planck}^2} = F^P = \frac{c^4}{G} \tag{3.19}
$$

In the real world however, after the electron mass is defined by the fine structure constant in (3.17)

$$
F_g = G\frac{m_e^2}{r^2} \lll F_e = k_e \frac{e^2}{r^2} \tag{3.20}
$$

Therefore, $\frac{F_e}{F_g^e} = \frac{k_e e^2}{G m_e^2} = [\frac{-1}{\ln \pi}(\frac{\Phi+\phi}{\pi})^2 \cdot e^{\phi^2/\alpha}]^2 \cdot \alpha \approx 4.166 \times 10^{42}$ or $\frac{F_e}{F_g^{ep}} = \frac{k_e e^2}{G m_e m_p} = [\frac{-1}{\ln \pi}(\frac{\Phi+\phi}{\pi})^2 \cdot e^{\phi^2/\alpha}]^2 \cdot \frac{\alpha}{\beta} \approx 2.26895 \times 10^{39} = 1/4.40732 \times 10^{-40}$ must be due to the elementary charge and particle mass quantization by the fine structure constant in (3.14) to (3.17). We can get the gravity coupling constant $\alpha_G^e = \frac{G m_e^2}{\hbar c} = (\frac{\pi^2 \cdot \ln \pi}{5})^2 e^{-2\phi^2/\alpha} = 1.75158 \times 10^{-45}$ or $\alpha_G^p = \frac{G m_p^2}{\hbar c} = \beta^2 \alpha_G^e = 5.90539 \times 10^{-39}$. In this way, the Dirac Large Number is accountable. [113, 114]

3-5 Quantum-gravity of Mass-Defect

In the standard model, the gravity of mass is explained by the strong interaction between quarks and gluons. However, where do the quarks and gluons come from? In this deadlock, the gravitational mass can become an endless mystery. (3.15) and (3.16) shows that not only is mass involved in the strong interaction, the electromagnetic and weak interactions are also involved. Any interaction with the energy difference will affect the measurement of a particle mass, which is based on the quantization of $\triangle E = \triangle m \cdot c^2$ by fine structure constant α. Next, we give some examples of the calculation.

Mass depending on the various interactions can be confirmed by the electron-positron annihilation $\mathbf{e}^- + \mathbf{e}^+ = 2\gamma\,(1.02\,\mathrm{MeV})$ in the matter/anti-matter interaction, and the mass of a hydrogen atom in the quantum interaction. The mass of a hydrogen atom is measured as $m_{^1\mathrm{H}} = 1.0078250321(4)\,\mathrm{u}$, i.e., $\beta^{exp}_{^1\mathrm{H}} = 1837.152550166470.$[21] It shows that $m_{^1\mathrm{H}} \neq m_p + m_e$ but involves the fine structure constant in (3.1)

$$\begin{aligned} m_{^1\mathrm{H}} &= m_p + m_e(1 - \alpha^2 \ln 10) \\ &= m_p + m_e\left(1 - \tfrac{\alpha \cdot \log e}{(K\Phi\pi)^2}\right) \end{aligned} \tag{3.21}$$

i.e., $\beta_{^1\mathrm{H}} = \beta_p + 1 - \ln 10 \cdot \alpha^2 = 1837.152549854230$ with $u_r = 1.699 \times 10^{-10}$. It gives

$$\begin{aligned} E_{^1\mathrm{H}} &= E_p + E_e(1 - \alpha^2 \ln 10) \\ &= E_p + E_e - \ln 10 \cdot \mathrm{E_h} \end{aligned} \tag{3.22}$$

where the Hartree energy $\mathrm{E_h} = \frac{m_e \mathbf{e}^4}{\hbar^2} = 2 \cdot \mathrm{Ry} = 2hcR_\infty = m_e(\alpha c)^2 = 27.21138505(60)\,\mathrm{eV}$. The mass-defect in the atomic construction is mainly due to the electromagnetic interaction. That is why Schrödinger's equation with the electric potential $\mathbf{e}^2/r$ can describe the atomic electron configuration. The quantum fluctuation exist on all types of matter that will make $\delta m = \varepsilon_0\mu_0\hbar \cdot \delta\omega$. It may be a mysterious link between the gravity and quantum theory. The gravity and quantum theory have the same mystery of superposition or *action-at-a-distance*. $E = mc^2$ could be the linked reality for both the gravity and quantum theory. In fact, $E = mc^2$ is not a continuous function, it only works for the particle mass and the nuclear fission/fusion. Therefore, $E = mc^2$ must be a quantization, which is not yet completely understood or formulated.

Table 3.3. The Compare of experiment and equation (3.23)

Atom	N	k	β_{exp}	β_{calc}	$U_r \times 10^{-10}$	Abundance
^{1}H	**1**	**1**	1837.1525501665	1837.1525498542	1.69959	**99.985%**
^{2}D	1	2	3671.4827473946	3671.4827199685	74.7003	0.015%
^{3}T	1	1/2	5497.9214757955	5497.9214653921	18.9223	Trace
^{3}He	2	3/2	5497.8850949517	5497.8850914763	6.32120	0.000137%
^{4}He	**2**	**1**	7296.2993684301	7296.2994138842	-62.2975	**99.999863%**

The atomic construction can be formulated as

$$\beta_{atom} = \beta_{nuclear} + \beta_{electron}(N - k\alpha^2 \ln 10) \tag{3.23}$$

where k is about 1~2. The calculation from (3.23) and experimental data on a few atoms are listed in **Table 3.3**. In **Table 3.3**, the stable atom of H and He obeys

$$m_{atom} = m_{nuclear} + m_{electron}(N - \alpha^2 \ln 10) \tag{3.24}$$

the eigenvalue of electrons in hydrogen-like atoms involves $E = -\mathrm{Ry}/n^2 = -\mathrm{E_h}/2n^2 = -13.60569\,\mathrm{eV}/n^2$. This may lead to a progression of quantum theory. Einstein always

[21] http://www.webelements.com/hydrogen/isotopes.html and $\beta_{u/e} = 1822.88839$

believed the quantum theory was not completed, and the current theory still does not represent the reality. Feynman also said that quantum theory can't be taught as simply as Newton's law since it hasn't been dug deep enough. However, the small mass-defect for the atom and molecule are often being ignored, the effective mass correlation is only needed in solid-state physics. If $E = mc^2$ can be quantized by the fine structure constant, both the quantum theory and relativity theory will progress. With $E = mc^2$ linked to the fine structure constant, the atomic mass-defect is quantized. [115] This may be the reality of quantum gravity theory.

The Deuterium/electron ratio $\beta^{2010}_{D/e} = 3670.48296520$, as nuclear $p^+ + n^0 \to D^+ + \gamma$, i.e., $m_D = m_p + m_n + m_{binding}$, we get

$$\begin{aligned} \beta_{D/e} &= \beta_{p/e} + \beta_{n/e} + (g_n + \tfrac{1}{4}\alpha^{1/2} + \tfrac{1}{4}\alpha)\ln\pi \\ &= 3670.482983 \end{aligned} \tag{3.25}$$

The mass-defect in (3.25) is related to

$$\begin{aligned} \triangle_{p^+ + n^0 \to D^+} &\approx (-g_n - \tfrac{1}{4}\alpha^{1/2} - \tfrac{1}{4}\alpha)\ln\pi \\ &= 4.353298979 \end{aligned} \tag{3.26}$$

where the binding energy is $E_b = m_b c^2 = \triangle_{p^+ + n^0 \to D^+} \cdot E_e = 2.22456\,\text{MeV}$, and mainly due to the magnetic g-factor of neutron. This is a direct link between the fusion and magnetic effect. The magnetic confinement of fusion has been confirmed by experiments using Tokamaks and Stellarators. Experimentally, the 2.2 MeV γ-ray line of solar neutron capture has been found (**Fig. 3.6**). [116, 117] This process does not involve the neutrino emission, which could explain the missing solar neutrino.

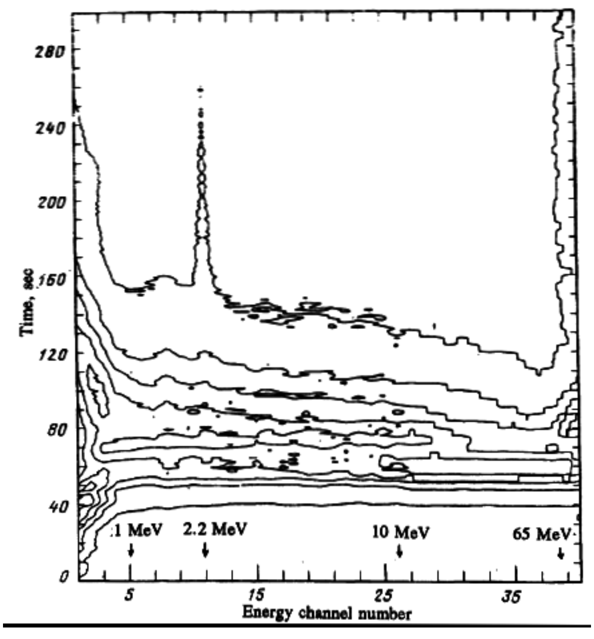

Fig. 3.6: 2.2 MeV γ-ray line on the Solar photon fluxes.

3-6 Quantization of $E = mc^2$

Einstein explained $E = m_0c^2$ and the relativity energy-momentum equation as $(m_0c^2)^2 = E^2 + ||Pc||^2$. However, he worried about how the particle mass is quantized. In 1948, he said that, “It is not good to introduce the concept of the mass $M = m/(1-v^2/c^2)^{1/2}$ of a moving body for which no clear definition can be given. It is better to introduce no other mass concept than the 'rest mass' m. Instead of introducing M it is better to mention the expression for the momentum and energy of a body in motion.” [118] The relativistic mass $M \Rightarrow \infty$ when $v = c$,

$$M = \gamma m_0 = m_0 / \sqrt{1 - (\tfrac{v}{c})^2} = m_0 / \sqrt{1 - v^2 \cdot \varepsilon_0 \mu_0} \tag{3.27}$$

i.e., there is a speed-limitation for a physical body moving in the vacuum $v^2 \cdot \varepsilon_0 \mu_0 < 1$. In (3.27), the relativistic mass M does not work with gravity constant G and changes with the particle's relativistic velocity. The Lorentz transformation γ was originally an modification on the electromagnetic property of space-time, as a hyperbolic rotation of Minkowski space. [119] The more general set of transformations that also includes translations is known as the Poincaré group. Poincaré wrote the $\boldsymbol{M} = S/c^2$ in 1900, where $\boldsymbol{M} = mc$ is “momentum of radiation” and $S = Ec$ is “flux of radiation.” [120] The Poincaré formula $\boldsymbol{M} = S/c^2$ is almost identical with Einstein's $m = L/c^2$ in 1905, however, Einstein directly addressed mass m and energy L. [96] The quantization of $E = m \cdot c^2$ will lead to the real particle rest-mass definition in the vacuum

$$\begin{aligned} m_0^i &= E_i \cdot \varepsilon_0 \mu_0 \\ &= \beta_i \cdot E_e \cdot \varepsilon_0 \mu_0 \\ &= \beta_i \cdot \nu_e \cdot h \varepsilon_0 \mu_0 \end{aligned} \tag{3.28}$$

where the vacuum permeability μ_0 and the vacuum permittivity ε_0 with $\varepsilon_0 \mu_0 = c^{-2}$. In (3.28), $h\varepsilon_0\mu_0 = 7.37249668 \times 10^{-51}\,\mathrm{Kg \cdot s}$, $\nu_e = 1.23559 \times 10^{20}\,\mathrm{Hz} \approx \exp(\frac{9^2}{2^2}\frac{1}{\alpha e^4 \ln 3})\,\mathrm{Hz}$ and $E_e = \mathrm{E_h}/\alpha^2\,\mathrm{eV}$. **The gravitational or inertial mass is created by trapping the electromagnetic energy, and *vice-versa*.**

The Poincaré group is a semi-direct product of the translations and the Lorentz transformations: $R^{1,3} \rtimes O(1,3)$. It has 10 symmetric basic-shifts, which are $1\,time + 3\,space + 3\,rotation + 3\,boost$ in 2-rank tensor. Einstein also used a similar stress-energy tensor in general relativity, which has 10 independent components in a 4-dimensional space. In the Einstein equation $R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab}$, the left-side is the Einstein tensor, a specific divergence-free combination of the Ricci tensor R_{ab}, the metric tensor g_{ab}, the curvature scalar R; and the right-side, T_{ab} is the energy-momentum tensor. The components of the stress-energy tensor is

$$T_{ab} = \begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix} \tag{3.29}$$

where the matrix terms are Energy density T_{00}, Energy flux T_{01}, T_{02}, T_{03}, Momentum density T_{10}, T_{20}, T_{30}, and Momentum flux (three components of pressure T_{11}, T_{22}, T_{33} and six components of shared stress $T_{12}, T_{13}, T_{23}, T_{21}, T_{31}, T_{32}$). However, it is often necessary to work with the covariant form $T_{ab} = g_{a\alpha} g_{b\beta} T^{\alpha\beta}$, where the stress-energy tensor is a diagonal matrix

$$T^{\alpha\beta} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \tag{3.30}$$

The stress-energy tensor is the source of the gravitational field in the Einstein field equations of general relativity. The Einstein equation in the weak gravity field became the Newton-Guass gravity field equation as $\nabla^2\Phi(r) = 4\pi G \cdot \rho(r)$ where the scalar potential $\Phi(r) = -Gm/r$. The object mass m is a sum of the particle mass. Therefore, the general relativity theory is also quantized after the particle mass quantization.

3-7 Discussion

In his third letter to Bentley in 1693, Newton wrote, "*Gravity must be caused by an agent* acting *constant*ly *according to certain laws; but whether this agent be* ***material or immaterial****, I have left to the consideration of my readers.*" [121] His view that light is a *particle* was overturned by Maxwell into an electromagnetic *wave*. Einstein used $\epsilon = h\nu$ as a *Photon* to explain the photoelectric effect, and $E = mc^2$ to address whether the *Inertia* depends on its *Energy Content*. Then, we are running into the paradox of Wave-Particle duality. In this way, the physical picture becomes clearer as scientific history cycles. Newton did not know about atoms, the nucleus, quarks, ..., which have been proved as the building blocks of the material universe and described by the quantum theory. In other words, **the inertia or mass of elementary particles are physically quantized by the fine structure constant**, which is the basic idea of this paper.

Can the structure of physical reality be inferred by a pure mathematician? As Einstein posed: 'Did God have any choice when he created the universe?' Here we show how to use the fine structure constant to calculate the particle mass-ratio and mass-defect, which makes the $E = mc^2$ quantization. Mass appears to be the harmonic motion of charged particles "trapped" within an electromagnetic energy cavity. This is why the most convenient and most often used unit expresses mass in terms of energy: the eV related to the fine structure constant α.

Our calculation is based on the experimental data, "*hypotheses non fingo*," while the formula of the fine structure constant (3.1) is governed by Euler's identity in a continued fraction, Feynman assumed $J = 6771 = 3 \times 37 \times 61$ is a unique solution of the *infinite* prime double factorial equation $\mathbf{P}(\mathbf{p}) = \{[2\,(\mathbf{p}-1)]!! + 1\}\,/\mathbf{p}$. The proton/electron mass ratio $\beta_{p/e} = \ln(\pi^{\alpha^{-3/2}} \cdot \pi^{-2\alpha^{1/2}} \cdot \pi^{\alpha^2/\pi\phi^2} \cdot \pi^{-\eta\alpha^3})$, and other mass-ratio or mass-defect are all in logarithmic format as the information entropy $S = k \ln \Omega$. Imitating the osmotic ray, the artificial generation of particles (both particles and anti-particles) has been achieved in laboratory conditions. In Nature, however, whether the particles are created by Black-holes, Supernovas or the Big Bang remains a question. [122, 123] We may need to search for the energy-mass cycle as well as infinity mechanism to address this question. Again, it should be simple according to Pauli. [43]

4 Probing of the g-factors of Leptons and Baryons

4-1 Spin and the Fine Structure Constant

The particle's magnetic moment is $\mu_i = \frac{g_i}{2}\frac{\mathbf{e}\cdot s}{m_i}$ where $\mathbf{e}$, m_i, s and g_i are the particle's charge, mass, spin and g-factor. We have discussed charge and mass with α. Next, we are going to discuss spin and g-factor of the particles.

The spin and g-factor of particles are also related to α. However, no general formula can calculate the g-factor of particles. We will discuss these issues with the formula of the fine structure constant

$$|\alpha^{1/2}| \equiv \pm\frac{M\phi}{K\pi} \equiv \frac{\log e^{\pm 1}}{K\Phi\pi} \equiv \log e^{\pm\phi/K\pi} \equiv \frac{1}{\ln 10^{\pm K\Phi\pi}} \tag{4.1}$$

where $K(J,H) = 1 + \frac{1^H}{J+}\frac{3^H}{J+}\frac{5^H}{J+}\frac{7^H}{J+}\cdots$, $J = 6771$, $0 \leq H < \alpha^{-1/2}$, and $\Phi - \phi - e - \pi$ in Euler-type Identity $e^{\pm i\pi} + \Phi = \phi$.

Pauli suggested the connection between spin and statistics with *odd-even* numbers. [63] The Bose-Einstein distribution is derived from [124]

$$\frac{\sum\limits_{n=0}^{\infty}(+1)^n\,(n+1)x^{n+1}}{\sum\limits_{n=0}^{\infty}(+1)^n\,x^n} = \frac{\sum\limits_{n=0}^{\infty}(+1)^n\,ne^{nh\nu/kT}}{\sum\limits_{n=0}^{\infty}(+1)^n\,e^{h\nu/kT}} = \frac{1}{e^{h\nu/kT}-1} \tag{4.2}$$

where $x = \exp\left(-E/kT\right)$, i.e., $E = h\nu = -kT\ln x$ for the boson with spin $s = \frac{n}{2}\hbar$ ($n = even$), such as photon or phonon. Fermi-Dirac distribution is derived from [125,126]

$$\frac{\sum\limits_{n=0}^{\infty}(-1)^n\,(n+1)x^{n+1}}{\sum\limits_{n=0}^{\infty}(-1)^n\,x^n} = \frac{\sum\limits_{n=0}^{\infty}(-1)^n\,ne^{\pm nh\nu/kT}}{\sum\limits_{n=0}^{\infty}(-1)^n\,e^{\pm h\nu/kT}} = \frac{1}{e^{h\nu/kT}+1} \tag{4.3}$$

where $x = \exp\left(\pm E/kT\right)$, i.e., $E = mc^2 = \pm kT\ln x$ for the fermion with spin $s = \frac{n}{2}\hbar$ ($n = odd$), such as electron or positron. Unlike the boson in Bose-Einstein distribution which always has a negative chemical-potential, a fermion in Fermi-Dirac distribution can have both $+$ or $-$ chemical-potential. Combining (4.2) and (4.3), we have

$$\frac{\sum\limits_{n=0}^{\infty}(\pm 1)^n\,(n+1)x^{n+1}}{\sum\limits_{n=0}^{\infty}(\pm 1)^n\,x^n} = \frac{x}{1\mp x} = \frac{1}{e^{E/kT}\mp 1} \tag{4.4}$$

where $(\pm 1)^n$ gives the $+$ for Bose-Einstein distribution and $-$ for Fermi-Dirac distribution; and *odd-n* terms have the alternative $\pm$ sign and *even-n* terms only have $+$ sign. Since $x = \exp\left(\pm E/kT\right)$, we have $\log x = \log[\exp\left(\pm E/kT\right)]$. This is same format as $\alpha^{1/2} = \log[\exp\left(\pm\phi/K\pi\right)]$.

The spin $\frac{n}{2}$ distinguishes between fermions ($n = odd$) and bosons ($n = even$). An elemental charge in [esu] is given by (4.1)

$$\mathbf{e} = \pm\left(\alpha\hbar c\right)^{1/2} = \pm\sqrt{\alpha}\cdot\mathbf{q}_{\mathrm{P}} = \pm\left(\frac{\hbar}{2}\frac{c}{2}\right)^{1/2}\frac{M}{K\Phi}\left[\frac{1\cdot3\cdot5\cdots}{2\cdot4\cdot6\cdots}\right]^2 \tag{4.5}$$

The fermion with half-integral spin $\hbar/2$ is based on $\mathbf{e}\cdot\mathbf{e}$ interaction

$$\frac{\mathbf{e}^2}{c} = \pm\alpha\hbar = \pm\frac{\hbar}{2}\left(1\cdot3\cdot5\cdots\right)\sqrt{\alpha}\frac{M(2n+1)!!}{K\Phi[(2n)!!]^2} = \pm\frac{\hbar}{2}\left(1\cdot3\cdot5\cdots\right)C \tag{4.6}$$

(4.6) shows that a particle with charge $\mathbf{e}^{\pm}$ defined by $\alpha^{1/2}$ in (4.1) is a spin-half Fermion. Pauli proposed, "*The spin value 1/2 is discriminated through the possibility of a definite charge density, and the spin values 0 and 1 are discriminated through the possibility of defining a definite energy density.*" [63] This is confirmed by calculating the proton-electron mass ratio $\beta \sim 1836$, where the Dirac monopole g is the major contributor for the mass/energy of a proton. The Dirac magnetic monopole g obeys $2\mathbf{e}g = \hbar c$, which is a boson in scalar field, and has never been found experimentally [108,109]

$$g = \frac{1}{2}\frac{\mathbf{q}_\mathrm{P}}{\sqrt{\alpha}} = \pm\mathbf{q}_\mathrm{P}\left[\frac{(2n)!!}{(2n+1)!!}\right]^2 \frac{K}{M\phi} = \pm(\hbar c)^{1/2}\frac{K}{M\phi}\left[\frac{2\cdot4\cdot6\cdots}{1\cdot3\cdot5\cdots}\right]^2 \tag{4.7}$$

It has [esu] dimension but is not an electric charge, $\mathbf{q}_\mathrm{P} = (\hbar c)^{1/2}$ is a Planck charge. Therefore, $\alpha^{+1/2}$ is related to electric charge and $\alpha^{-1/2}$ is related to magnetism (e.g. $R_{xy} = \frac{\pm Z_0}{2\alpha_\nu^{1/2}\alpha^{1/2}}$ in I/FQHE). The magnetic coupling is $\beta_m = \frac{g^2}{\hbar c} = \frac{1}{4\alpha} = \frac{\sigma_0}{G_0} = 34.259$. In $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, the strong-electroweak coupling is $\alpha_{swe} = m_{\nu_e}/m_{\nu_\mu} \approx 34.26^{-1}$. [33] From (4.5)~(4.7), Fermions and Bosons can be easily discerned by flipping the *even* and *odd* consecutive sequences, $\Phi = 1/\phi$ and $\log e = 1/\ln 10$. The link between Spin-Statistics and *odd*/*even* numbers in the α math formula may be what Pauli was looking for. [63]

4-2 The Electron g-factor

The electron magnetic moment is $\mu_e = \frac{g_e}{2}\frac{e\cdot s}{m_e}$ where e, s and m_e are the electron's charge, spin, and mass, $|g_e| = 2$ given by the Dirac equation $(i\partial_\mu - eA_\mu)\gamma^\mu\psi = m\psi$ in 1928 is not accurate. In 1941, Pauli added a term $a_e\frac{e}{2m_e}\sigma_{\mu\nu}F^{\mu\nu}\psi$ to the Dirac equation and redefined $|g_e| = 2(1 + a_e)$ where a_e is the electron anomaly ($g_e < 0$) calculated by the perturbation theory of QED. [63] Pauli thought that the value of fine structure constant should determine the QED perturbation calculation of a_e, not *vice versa*.

The Harvard group QED α calculation, based on the measurement of $g/2$, reached 0.70 ppb in 2006, allowing $5.91 < H < 6.16$ in $K(J,H) = 1 + \frac{1^H}{J+}\frac{3^H}{J+}\frac{5^H}{J+}\frac{7^H}{J+}\cdots$. [127, 128] In 2008, $\alpha^{-1}_{\mathrm{HV08}} = 137.035999084\,(51)$, $u_r = 0.37$ ppb, [130] which is matched by *one* of the BO-Interferometer experiments $\alpha^{-1}_{\mathrm{BO10}} = 137.035999037\,(91)$, $u_r = 0.66$ ppb in 2010. [89] Then, we have $H = \frac{37+3}{3+3}$ and $\alpha^{-1}_{6.666} = 137.035999048781$; or $H = \frac{2}{9}\pi(e + e^{-1})^2 = (4\pi)\sin^2\theta_w\cosh^2(\pm1)$ and $\alpha^{-1}_{6.64828} = 137.03599907395$ to match the CODATA-2010 $\alpha^{-1}_{2010} = 137.035999074\,(44)$. (4.1) can easily fit with new incoming data, however, those so-called *most reliable* QED calculations *perpetually* change.[22] [131] Current values of the fine structure constant were calculated using supercomputer numeral fitting ("*the hand of God*"). This *best measured* $g_e/2$ from a single-electron in a magnetic Penning trap yields two real roots (4-red in **Fig. 4.1**) when calculating the electron (positron) anomaly $a_e = (|g_e| - 2)/2$ up to x^5 ($x = \alpha/\pi = 1/430.511$) [130]

$$\begin{aligned} a_e^{QED} &= \boldsymbol{c}_2(\tfrac{\alpha}{\pi}) + \boldsymbol{c}_4(\tfrac{\alpha}{\pi})^2 + \boldsymbol{c}_6(\tfrac{\alpha}{\pi})^3 + \boldsymbol{c}_8(\tfrac{\alpha}{\pi})^4 + \boldsymbol{c}_{10}(\tfrac{\alpha}{\pi})^5 \\ &\quad + \cdots + a_{\mu\tau} + a_{\mathrm{hadronic}} + a_{\mathrm{weak}} \end{aligned} \tag{4.8}$$

where $\boldsymbol{c}_2 = 0.5$, $\boldsymbol{c}_4 = -0.328478965$, $\boldsymbol{c}_6 = 1.181241456$, $\boldsymbol{c}_8 = -1.7283(35)$ and estimated $\boldsymbol{c}_{10} = 0.0\,(4.6)$ are built by Schwinger-Sommerfield-Remiddi-Kinoshita. [132–135] QED calculation claim reached the sub-ppb level ($< 10^{-12}$), however, $a_{\mu\tau} = A_2(\frac{m_e}{m_\mu}) + A_2(\frac{m_e}{m_\tau}) + A_3(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau})\cdots$ used in (4.8) are poorly measured lepton mass ratios.[23] Using (4.8), only the *odd*-powers ($1, 3, 5$) with the alternate sign have a solitary solution for $(\frac{\alpha}{\pi})$ (**Fig. 4.1**), which needs $3.8 > \boldsymbol{c}_{10} > 0.002046$ for $(\alpha/\pi)^5$ (12672 Feynman diagrams). There is no limitation for the fine structure constant in **Fig. 4.1**

[22] The latest reported values by Kinoshita are $\boldsymbol{c}_8 = -1.9108(25)$, $\boldsymbol{c}_{10} = 4.364(733)$, $\alpha^{-1}_{K10} = 137.035999132(9)(6)(33)$ on Nov. 2 (2011)

[23] e.g., $\frac{m_e}{m_\mu} = \frac{1}{3477.48(57)}$ and $\frac{m_e}{m_\tau} = \frac{1}{3477.48(57)}$. $A_2^{(4)}(\frac{m_e}{m_\mu}) = 5.19738771(12)\times10^{-7}$ and $A_2^{(4)}(\frac{m_e}{m_\tau}) = 1.83763(60)\times10^{-9}$; $A_2^{(6)}(\frac{m_e}{m_\mu}) = -7.37394158(28)\times10^{-6}$ and $A_2^{(46)}(\frac{m_e}{m_\tau}) = -6.5819(19)\times10^{-8}$. [129]

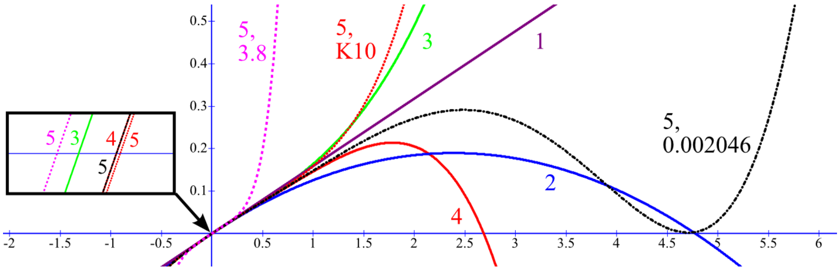

Fig. 4.1: Perturbation theory calculation of the fine structure constant by (4.8) with two solutions in 4-red. Enlargement shows that there is no limitation.

According to Feynman and Dyson, Feynman diagrams are for $\mathbf{e}^2, \mathbf{e}^4, \cdots, \mathbf{e}^{2n}$ (i.e., $\alpha^n = (\frac{\mathbf{e}^2}{\hbar c})^n = (\frac{1}{137})^n$) but not $\frac{\mathbf{e}^2}{\pi}, \frac{\mathbf{e}^4}{\pi^2}, \cdots, \frac{\mathbf{e}^{2n}}{\pi^n}$ (i.e., $(\frac{\alpha}{\pi})^n = (\frac{2\mathbf{e}^2}{hc})^n = (\frac{1}{430})^n$). [52,136] It looks strange in Feynman diagrams ($e/\sqrt{\pi} \Rightarrow \sqrt{\alpha/\pi}$ instead of $e \Rightarrow \sqrt{\alpha}$). Schwinger originally proposed $a_e = (\frac{1}{2\pi})\alpha \propto \frac{1}{137}$ but not $(\frac{1}{2})(\frac{\alpha}{\pi}) \propto \frac{1}{430}$. [132] According to Sommerfeld, $\frac{\alpha}{2\pi} = \frac{E_e}{E_\phi}$ is the energy ratio of interactions between the electron and the photon, where $E_\mathrm{e} = \frac{\mathbf{e_1 e_2}}{\lambda_c}$ is the energy needed to bring two electrons from infinity to a distance of λ_c against their electrostatic repulsion, and $E_\phi = \frac{hc}{\lambda_c}$ is the energy of a single photon with a wavelength λ_c. The α yielded by (4.8) on the base of $\frac{\alpha}{\pi}$ needs a basic theoretical explanation. [137] Feynman even called the "*renormalization*" of QED as "*a dippy process*", which "*is not mathematically legitimate*," and "*It's one of the* greatest *damn mysteries of physics: a* magic number *that comes to us with no understanding by man.*". [52] The α value obtained by the g-2 measurement is an experimentally dependent value, not a theoretical value. Therefore, QED can not answer the question of "Why 137?"

Recently, many people try to calculate the electron anomaly without the perturbation theory. We also get a simple approximate formula based on Schwinger theory

$$\begin{aligned} a_e &= \frac{\alpha}{2\pi}\frac{\ln^3\pi}{\tanh^{-1}(\frac{7}{9})\cdot\zeta^2(3)} = \frac{\alpha}{2\pi}\frac{\ln\pi}{\tanh^{-1}(\cos^2\theta_w)}\frac{\ln^2\pi}{\zeta^2(3)} \\ &= \frac{\alpha}{\pi}\left(\frac{\ln\pi}{\ln 8}\right)\left(\frac{\ln\pi}{\zeta(3)}\right)^2 = 0.00115965217523 \end{aligned} \tag{4.9}$$

To test if $\alpha = 0.0072973524999966932$ from (4.1) can create an understandable perturbation formula, a_e is built as a quintic function in the α^n alternating series, to fit $a_e^{\mathrm{HV08}} = \underline{0.00115965218073}\,(28)$ and $a_e^{2010} = \underline{0.00115965218076}(27)$ up to 0.15 ppb, [130] then multiplying $\ln e = 1$

$$\begin{aligned} a_e &= \sum_{n=1}^{5}(-1)^{n-1}\,c_{2n}\alpha^n = \frac{1}{2\pi}\alpha^1 - \frac{1}{3\pi^2}\alpha^2 + \frac{1}{3\pi}\alpha^3 - \frac{1}{\pi^2}\alpha^4 + \frac{\pi-2}{2}\alpha^5 \\ &= \ln e\cdot\sum_{n=1}^{5}(-1)^{n-1}\,c_{2n}\alpha^n = \ln\left(\frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2}+\frac{\mathbf{2}}{\mathbf{3}})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2}+\frac{\mathbf{1}}{\mathbf{3}})]}\cdot\frac{\exp[E(0)\alpha^{\mathbf{5}}]}{\exp[E(1)\alpha^{\mathbf{5}}]}\right) \\ &= \sum_{n=1}^{4}(-1)^{n-1}\,c_{2n}\alpha^n + \mathcal{O}_e(\alpha^5) = \ln\left(\frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2}+\frac{\mathbf{2}}{\mathbf{3}})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2}+\frac{\mathbf{1}}{\mathbf{3}})]}\cdot e^{\mathcal{O}_e}\right) \\ &= \underline{0.0011596521}6891 + 1.181\times10^{-11} = \underline{0.0011596521807}2 \end{aligned} \tag{4.10}$$

where $\mathcal{O}_e(\alpha^5) = 1.181\times10^{-11}$ is within range of the *Lamb shift* and *Hyperfine splitting*, and both are sensitive to the *magnetic field*. (4.10) does not oppose QED and returns to its principle $a_e = \sum(-1)^{n-1}\,c_{2n}(\frac{1}{137})^n$. The $c_2 = \frac{1}{2\pi}$ in (4.8) by Schwinger is kept, the rest follow the same pattern ($c_2 \sim c_8$ involves the electroweak coupling which will be discussed in the next section).[24] (4.10) is the equivalent of a virtual sphere with an additional spin magnetic moment $R = \frac{\delta\mu}{\mu} = \frac{\alpha}{2\pi} = \frac{r_e}{\lambda_c^e} = \frac{\mathbf{e}^2}{hc}$ (**Fig. 4.2**), and can be expressed as[25]

$$\begin{aligned} a_e(\alpha_0) &= R - 2\frac{V}{L} + \frac{LV}{R} - S^2 + (\frac{\pi}{2} - 1)L^5 \\ &= R - \frac{4}{3}R^2 + \frac{8\pi^2}{3}R^3 - (4\pi)^2R^4 + (\frac{\pi}{2} - 1)(2\pi)^5R^5 \end{aligned} \tag{4.11}$$

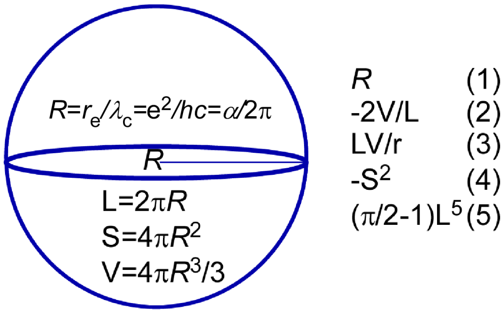

Fig. 4.2: A virtual spherical sketch in (4.11)

In (4.10), the physical definition of α^n is governed by the Rydberg constant ($1\,[\mathrm{Ry}] = hcR_\infty = \frac{1}{2}m_ec^2\alpha^2 = 13.60569253\,(30)\,[\mathrm{eV}]$)

$$4\pi R_\infty \equiv \frac{\alpha}{a_0} \equiv \frac{\alpha^2}{\lambda\!\!\!^-_c} \equiv \frac{\alpha^3}{r_e} \equiv \left(\frac{\alpha^4}{a_0 r_e}\right)^{\frac{1}{2}} \equiv \left(\frac{\alpha^5}{\lambda\!\!\!^-_c r_e}\right)^{\frac{1}{2}} \equiv \left(\frac{\alpha^6}{a_0\lambda\!\!\!^-_c r_e}\right)^{\frac{1}{3}} \tag{4.12}$$

where $a_0 = \mathbf{e}^2/m_ec^2\alpha^2$ is the Bohr radius, $\lambda\!\!\!^-_c = \lambda_c/2\pi = \hbar/m_ec = \mathbf{e}^2/m_ec^2\alpha$ is the reduced Compton wavelength, and $r_e = \mathbf{e}^2/m_ec^2$ is a classical electron radius. (4.10) can be expressed as a *nested* series of α in the first order

$$a_e(\alpha) = \frac{1}{2\pi}\alpha\left(1 - \frac{2}{3\pi}\alpha\left(1 - \pi\alpha\left(1 - \frac{3}{\pi}\alpha\left(1 - \pi^{3/2}\alpha\right)\right)\right)\right) \tag{4.13}$$

in a coupling of the classical electron radius and Compton wavelength (r_e/λ_c)

$$a_e\left(\frac{r_e}{\lambda_c}\right) = \frac{r_e}{\lambda_c}\left(1 - \frac{4}{3}\frac{r_e}{\lambda_c}\left(1 - 2\pi^2\frac{r_e}{\lambda_c}\left(1 - 6\frac{r_e}{\lambda_c}\left(1 - 2\pi^{5/2}\frac{r_e}{\lambda_c}\right)\right)\right)\right) \tag{4.14}$$

This extremely high precision data matching in (4.10)-(4.14) is a strong justification of the theoretical α yielded by (4.1). From $\alpha^2 \equiv r_e/a_0$, $\alpha \equiv r_e/\lambda\!\!\!^-_c \equiv \lambda\!\!\!^-_c/a_0$ and $\mu_s \equiv \mu_B g_e/2$,

[24] $c_4 = \frac{1}{3\pi^2}$ was considered Relativistic Symmetry without the vacuum polarization. [138] $c_2 = 1/2\Gamma^2(1/2)$; $c_4 = 1/18\zeta(2)$; $c_6 = 1/3\Gamma^2(1/2)$; $c_8 = 1/6\zeta(2)$; $c_{10} = \frac{\pi}{2} - 1 = E(0) - E(1)$, where $E(k)$ is a complete elliptic integral of the second kind (note $c_{10} \simeq \frac{1}{3}(e-1) \cong \frac{2}{3}(4-\pi) \cong \frac{\ln\pi}{2} \cong \frac{9}{5\pi} \cong \frac{2\pi}{11}$). In (10), we may also express $\mathcal{O}_e(\alpha) = \frac{\alpha^5}{2} + \frac{3}{\pi}\frac{\alpha^4}{\beta} = 1.18213\times10^{-11}$, which is within the range of *Lamb shift* $\propto \alpha^5 = 2.069\times10^{-11}$ and *Hyperfine splitting* $\propto \alpha^4/\beta = 1.544\times10^{-12}$, and both are very sensitive to the magnetic field. We also get $\mathcal{O}_\mu(\alpha) = \frac{m_\mu}{m_e}\frac{10\beta\alpha^5}{3(2+\pi\ln 2)} \approx \frac{5\beta\alpha^4}{2+\pi\ln 2}$, so $a_\mu(\alpha_0) = 0.001165920797$ to match with $a_\mu^{2010} = 0.00116592091(63)$

[25] From (2.35), $\frac{\pi}{2} - 1 = \sum_{n=1}^{\infty}\frac{n!}{(2n+1)!!} = \frac{1}{3} + \frac{1}{3}\cdot\frac{2}{5} + \frac{1}{3}\cdot\frac{2}{5}\cdot\frac{3}{7} + \frac{1}{3}\cdot\frac{2}{5}\cdot\frac{3}{7}\cdot\frac{4}{9}\cdots = \frac{1}{3}\left(1 + \frac{2}{5}\left(1 + \frac{3}{7}\left(1 + \frac{4}{9}\left(1 + \cdots\right)\right)\right)\right)$, $\frac{8\pi^2}{3} = 2^4\zeta(2) = 16\sum_{k=1}^{\infty}\frac{1}{k^2}$ and $(4\pi)^2 = 3\cdot2^5\zeta(2) = 96\sum_{k=1}^{\infty}\frac{1}{k^2}$

Bohr magneton $\mu_B \equiv (\hbar/2)(\mathbf{e}/m_e)$ is working with α on the reciprocity in **Fig. 4.3**, where the measurable reduced Compton wavelength serves as the unity.

$$\begin{aligned}
\frac{2\mu_B}{c\cdot\mathbf{e}}\frac{1}{a_0} &= \alpha = \left(\frac{\log e^{\pm 1}}{2K\Phi}\frac{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7\cdots}{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdots}\right)^2 \\
\frac{2\mu_B}{c\cdot\mathbf{e}}\frac{1}{\lambda\!\!\!^{-}_c} &= \alpha\cdot\alpha^{-1} = 1 \qquad (4.15)\\
\frac{2\mu_B}{c\cdot\mathbf{e}}\frac{1}{r_e} &= \frac{1}{\alpha} = \left(\frac{2K\Phi}{\log e^{\pm 1}}\frac{2\cdot 2\cdot 4\cdot 4\cdot 6\cdot 6\cdots}{1\cdot 3\cdot 3\cdot 5\cdot 5\cdot 7\cdots}\right)^2
\end{aligned}$$

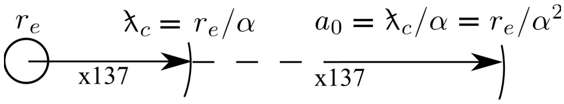

Fig. 4.3: Bohr magneton $\mu_B \equiv (\hbar/2)(\mathbf{e}/m_e)$ is working with α on the reciprocity

The electron g-factor $g_e/2 = 1+a_e$ can be presented as a finite alternative power series of α^n using the *gamma* function to fit CODATA-2010

$$\begin{aligned}
\frac{|g_e|}{2} &= \mathbf{1} + \frac{\alpha^1}{2\pi} - \frac{\alpha^2}{3\pi^2} + \frac{\alpha^3}{3\pi} - \frac{\alpha^4}{\pi^2} + \frac{\pi-2}{2}\alpha^5 = \ln\left(e^{\mathbf{1}}\cdot\frac{\exp[\frac{\alpha^3}{2\pi}(\alpha^{-2}+\mathbf{\frac{2}{3}})]}{\exp[\frac{\alpha^2}{\pi^2}(\alpha^{+2}+\mathbf{\frac{1}{3}})]}\frac{e^{E(0)\alpha^5}}{e^{E(1)\alpha^5}}\right) \qquad (4.16)\\
&= \mathbf{1} + \frac{\alpha^1}{\Gamma(3)\Gamma^2(\frac{1}{2})} - \frac{\Gamma(3)\alpha^2}{\Gamma(4)\Gamma^4(\frac{1}{2})} + \frac{\Gamma(3)\alpha^3}{\Gamma(4)\Gamma^2(\frac{1}{2})} - \frac{\alpha^4}{\Gamma^4(\frac{1}{2})} + \left(\frac{\Gamma^2(\frac{1}{2})}{\Gamma(3)} - 1\right)\alpha^5
\end{aligned}$$

4-3 Electroweak and Chirality $e^{-i\pi}\alpha^{1/2}$

The β^- decay $(\mathbf{n}^0 \to \mathbf{p}^+ + \mathbf{e}_L^- + \bar{\boldsymbol{\nu}}_e^R)$ involves the weak interaction. The electroweak interactions have been merged by $SU(2)_L \otimes U(1)_Y$ with the coupling of charge $\mathbf{Q}$, isospin $\mathbf{I}_\mathrm{W}$ and hyper-charge $\mathbf{Y}_\mathrm{W}$, linked to the Fermi coupling constant $G_F \cong \sqrt{2}\mathbf{g}^2/8m_\mathrm{w}^2$, by defining $\boldsymbol{e} = \mathbf{g}\sin\theta_w = \mathbf{g}'\cos\theta_w$ ($\mathbf{g} \sim \mathrm{W}^\pm$ and $\mathbf{g}' \sim \mathrm{Z}^0$),[26] and the Weinberg angle $\cos\theta_w = m_\mathrm{w}/m_\mathrm{z}$ or $\tan\theta_w = \mathbf{g}'/\mathbf{g}$ have to be measured experimentally. [139–141] The pair production of $\mathrm{W}^\pm$ bosons in electron-positron annihilation shows $\alpha_\mathrm{W}^{-1} \cong 128.952(49)$. [142] We get

$$\begin{aligned}
\alpha_\mathrm{W}^{1/2} &= \frac{\pm\log F}{\Phi\pi}(1-\tfrac{2}{9}\alpha) = \frac{\pm\log F}{\Phi\pi}[1-\tfrac{2}{9}(\frac{\pm\log e}{\Phi\pi})^2] = \frac{\log F^{\pm 1}}{\Phi\pi}(1-\frac{1}{9}\frac{R_K}{Z_0}) \qquad (4.17)\\
&= \pm\frac{\log F}{\Phi\pi}(1-\alpha\cdot\sin^2\theta_w) = \pm\frac{\log F}{\Phi\pi}\{1-(\frac{\pm\log e}{\Phi\pi})^2[1-(\frac{m_\mathrm{w}}{m_\mathrm{z}})^2]\}\\
&= \frac{\log F^{\pm 1}}{\Phi\pi}(1+Q_dQ_u\alpha) = \alpha_{\mathrm{W}'}^{1/2}(1-\alpha\sin^2\theta_w) = \pm 0.0880614
\end{aligned}$$

and $\alpha_\mathrm{W}^{-1} = 128.9519596$, where Fransén-Robinson constant $F = \int_0^\infty \frac{dt}{\Gamma(t)} = \lim_{\alpha\to 0}\alpha\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n)} = \lim_{\alpha\to 0}\alpha E_{\alpha,0}(1) = 2.807772420$ [27] as the continued gamma function integral for the time interval $\triangle t \to 0$, vs. $e = \sum_{n=0}^{\infty}\frac{1}{\Gamma(n)} = 2.71828$ as the infinite sum of the discrete gamma function (see the footnote 14). (4.17) further approves the correctness of the fine structure constant formula based on $\alpha^{1/2} \approx \pm\frac{\log e}{\Phi\pi}$.

[26] $G_F = 1.166364(5)\times 10^{-5}[\mathrm{GeV}^{-2}]$, $\mathbf{g}$, $\mathbf{g}'$, $\alpha_{em} = \frac{\boldsymbol{e}^2}{4\pi} = \frac{1}{137}$ and $\boldsymbol{e} = 0.302822119$ are in the natural unit. The one-loop *beta*-function in QED is $\beta(e) = \frac{\boldsymbol{e}^3}{12\pi^2}$ or $\beta(\alpha) = \frac{2\alpha^2}{3\pi}$, then $c_4\alpha^2 = \frac{\alpha^2}{3\pi^2} = \frac{\beta_1(\alpha)}{2\pi}$, it is *proportional* to the energy scale.

[27] The Mittag-Leffler function $E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\alpha n+\beta)} = \frac{1}{2\pi i}\oint_C \frac{t^{\alpha-\beta}e^t}{t^\alpha - z}dt$ is the contour starts and ends at $-\infty$ and circles around the singularities of the integrand (**Fig. 2.6**), for the Continuous Time Random Walk-CTRW. [143]

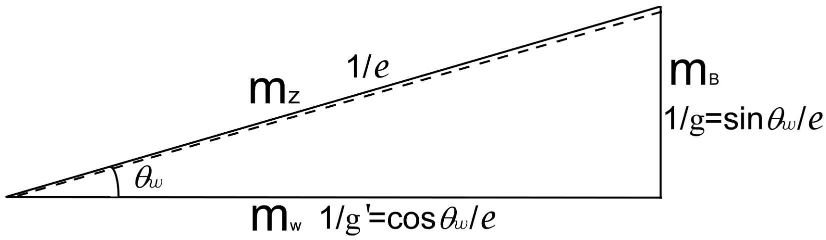

Fig. 4.4: The charge and mass relationship in the electroweak interactions $SU(2)_L \otimes U(1)_Y$

The anomalous electron will couple with the weak force in the nucleus as the *electroweak coupling* (per Pythagorean theorem in **Fig. 4.4**). In the $a_e(\alpha)$ equation (4.10), $c_2 = 1/2\pi$ is originally given by Schwinger, [132] $c_4 = 1/3\pi^2$, $c_6 = 1/3\pi$ and $c_8 = 1/\pi^2$ follow a similar pattern. Here we propose that the *electroweak coupling* may be directly linked to α by $c_2 \sim c_8$ (or $c_2\alpha^1 \sim c_8\alpha^4$) in the electron anomaly formula (4.10)

$$\begin{aligned} \eta &= \frac{\alpha}{\sin^2\theta_w} = \frac{\mathbf{g}^2}{4\pi} = \frac{c_2 \cdot c_8}{c_6 \cdot c_4} \cdot \alpha = \frac{\mathbf{9}}{\mathbf{2}}\alpha = \frac{\alpha}{0.2222\overline{2}} = \frac{1}{30.452} \\ \eta' &= \frac{\alpha}{\cos^2\theta_w} = \frac{\mathbf{g}'^2}{4\pi} = \frac{c_2 \cdot c_8 \cdot \alpha}{c_2 \cdot c_8 - c_6 \cdot c_4} = \frac{\alpha}{0.7777\overline{7}} = \frac{1}{106.583} \end{aligned} \tag{4.18}$$

i.e., $\eta_{\mathrm{W}} = \alpha_{\mathrm{W}}/\sin^2\theta_w = 28.656$ and $\eta'_{\mathrm{W}} = \alpha_{\mathrm{W}}/\cos^2\theta_w = 100.296$.[28] So, $G_F \cong \eta\pi/\sqrt{2}m_{\mathrm{W}}^2 = 9\pi\alpha/\sqrt{8}m_{\mathrm{W}}^2$. (4.18) yields a reciprocal sum formula $\alpha^{-1} = \eta^{-1} + \eta'^{-1}$ similar to the type of the reduced mass, the equivalent parallel resistance/serial capacitors, or the geometrical optics.[29] Compare it to a concave mirror formula $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ (neglecting aberrations). However, the only possible solution for $\{\alpha, \eta, \eta'\}$ is $R = \infty$, i.e., a flat mirror (no space-time curvature or the general relativistic correlation for the quantum theory). From (4.1),

$$\begin{aligned} \eta^{-1} \quad &+ \quad \eta'^{-1} &= \quad& \alpha^{-1} \\ \left(\sqrt{\mathbf{2}} \cdot \frac{K\Phi\pi}{\log e^3}\right)^2 &+ \left(\sqrt{\mathbf{7}} \cdot \frac{K\Phi\pi}{\log e^3}\right)^2 &= \quad& \left(\frac{K\Phi\pi}{\log e}\right)^2 \\ \left(\frac{K\pi\sin\theta_w}{\log e^\phi}\right)^2 &+ \left(\frac{K\pi\cos\theta_w}{\log e^\phi}\right)^2 &= \quad& \left(\frac{K\Phi\pi}{\log e}\right)^2 \end{aligned} \tag{4.19}$$

i.e., $30.4524444 + 106.5835557 = 137.0360003$ or $\frac{1}{0.032838} + \frac{1}{0.0093823} = \frac{1}{0.007297352}$. It obeys Pythagoras Theorem $A^2 + B^2 = C^2$ or the trigonometric identity $\sin^2\theta_w + \cos^2\theta_w = 1$, and fits with Glashow-Weinberg-Salam theory. It is the angle by which spontaneous symmetry breaking rotates the original W^0 and B^0 vector boson plane, producing as a result the Z^0 boson ($m_{\mathrm{Z}}^2 = m_{\mathrm{W}}^2 + m_{\mathrm{B}}^2$), and the γ photon.

$$\begin{pmatrix} \gamma \\ \mathrm{Z}^0 \end{pmatrix} = \begin{pmatrix} \cos\theta_w & \sin\theta_w \\ -\sin\theta_w & \cos\theta_w \end{pmatrix} \begin{pmatrix} \mathrm{B}^0 \\ \mathrm{W}^0 \end{pmatrix} \tag{4.20}$$

In this way, the weak-mixing angle is also linked to the $a_e(\alpha)$ equation (4.10)

$$\begin{aligned} \sin^2\theta_w &= \frac{\alpha}{\eta} = \frac{c_6\alpha^3}{c_2\alpha^1}\frac{c_4\alpha^2}{c_8\alpha^4} = \frac{c_6}{c_2}\frac{c_4}{c_8} = \frac{2\pi}{3\pi}\frac{\pi^2}{3\pi^2} \\ &= s_{\mathrm{W}}^2 = 1 - \left(\frac{m_{\mathrm{W}}}{m_{\mathrm{Z}}}\right)^2 = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9} = 0.2222\overline{2} \end{aligned} \tag{4.21}$$

[28] GUT's expersions are $\alpha_1 = \frac{5}{3}\frac{\alpha}{\cos^2\theta_w} \sim \frac{1}{63.95}$; $\alpha_2 = \frac{\alpha}{\sin^2\theta_w} \sim \frac{1}{30.45}$; $\alpha_3 = \alpha_s \sim \frac{1}{8.54}$ and running.

[29] They all involve $\sum_i^n \frac{1}{X_i} = \frac{1}{X_{eq}}$, e.g., Gauss thin lens/concave mirror formula $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$

where 2/9 is expressed as the electric charge of u-quarks 2/3 times d-quarks 1/3. The values of $\frac{1}{3}$ and $\frac{2}{3}$ had appeared in the electron anomaly in (4.10). In the Feynman diagram of the β^- decay (**Fig. 4.5**(a)), $\mathrm{W}^- \rightarrow \mathbf{e}_L^- + \bar{\boldsymbol{\nu}}_{eR}$, the $d_{-\frac{1}{3}}$ quark changes into a $u_{\frac{2}{3}}$ quark, also changes flavor. (4.21) works with CODATA-2010 $\sin^2\theta_w = 0.2223\,(21)$ with $u_r = 9.5 \times 10^{-3}$, and requires further experimental confirmation.

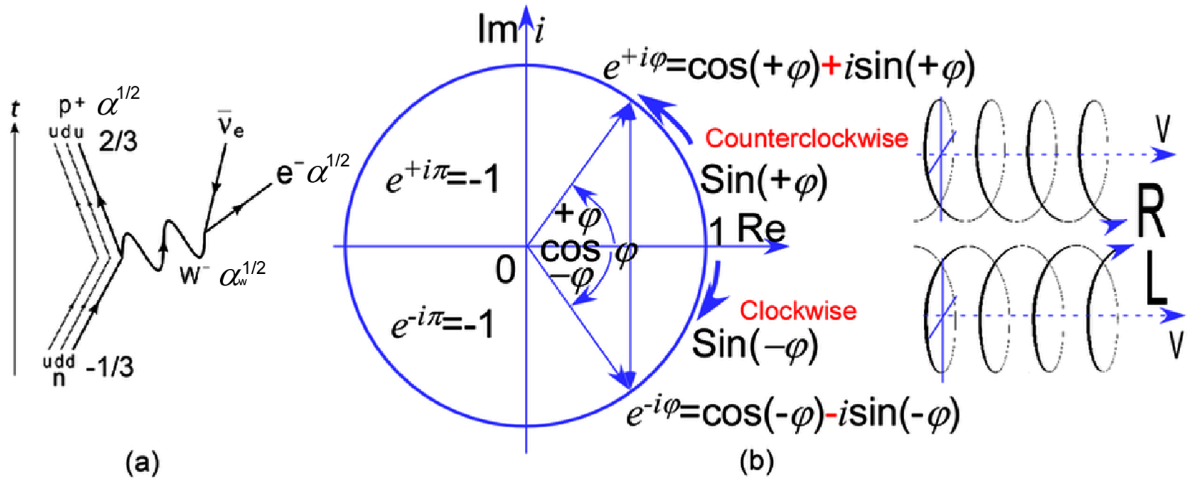

Fig. 4.5: (a) Weak Feynman diagram of β^-decay ($\alpha^{1/2}$and $\alpha_{\mathrm{W}}^{1/2}$); (b) Reformed Euler identity and the right- and left-handed helix.

In the β^- decay $\left(\mathbf{n}^0 \rightarrow \mathbf{p}^+ + \mathbf{e}_L^- + \bar{\boldsymbol{\nu}}_{eR}\right)$, a neutron emits a proton, and a *left*-handed electron with a *right*-handed electron anti-neutrino. Chirality projection operators are $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ where the gamma matrices $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, satisfy $P_+ + P_- = 1$; $P_+^2 = P_+$; $P_-^2 = P_-$; $P_+P_- = P_-P_+ = 0$. Where does this hand $\psi_L^R = \frac{1}{2}(1 \pm \gamma_5)\psi$ originally come from? According to Salam, the physical picture is that "*a photon right circular polarised is 2 neutrinos traveling together and a photon left circular polarised is 2 antineutrinos traveling together.*" [141] In the Euler identity $e^{\pm i\pi} = \phi - \Phi = -1$, regardless of $i\pi$ having $+$ or $-$ sign, $e^{\pm i\pi} \equiv -1$. In the unit-circle (**Fig. 5**(b)), however, two-types of helicities $H = \pm 1$ represent the different rotational directions ("$-$" clockwise and "$+$" counter-clockwise). The "$-$" clockwise spin becomes a 3D left-handed helix rotating around the electron traveling vector (out of paper), and the negatively charged electron has the alternate series for $1/e$. From $e^{\pm i\pi} = -1$, since $\bar{\boldsymbol{\nu}}_{eR}$ takes $e^{+i\pi}$, $\alpha^{1/2}$ in weak interaction has the format of $e^{-i\pi}\alpha^{1/2}$, due to Landau CP conservation. [144] From (4.1)

$$\begin{aligned} \left|\alpha^{1/2}\right| &\equiv e^{-i\pi}\left[\mp\frac{M\phi}{K\pi}\right] \equiv e^{-i\pi}\log e^{\mp 1/K\Phi\pi} \\ &= -\log e^{\mp 1/K\Phi\pi} = \log e^{\eta/K\Phi\pi} \end{aligned} \tag{4.22}$$

where $\eta = \pm 1$ represents the intrinsic parity of the particle. Parity violation theory in weak interactions was confirmed by ^{60}Co experiments. [145] However, the cause of the parity violation is not theoretically grounded. It is noticed that (4.22) temporally changes $e^{\pm 1/K\Phi\pi}$ to $e^{\mp 1/K\Phi\pi}$, i.e., the reverse statistic. Since this could only happen with the electronic antineutrino of *right*-handed helicity, the parity violation may only be found in the weak interaction. It also explains why there is no CP violation for the strong interaction in QCD. **Fig. 4.6** shows that a positron (red) emerging from an electron-positron pair seems to change charge to the electron in a very short time.

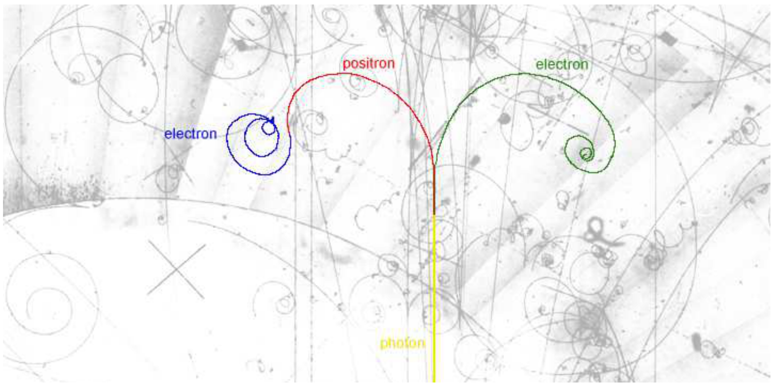

Fig. 4.6: A positron (red) emerging from an electron-positron pair, produced by a gamma ray, curves round through about 180 degrees. Then it seems to change charge: it begins to curve in the opposite direction (blue).

Not only the electron is *left*-handed, in fact, all leptons and quarks found in Nature are *left*-handed, and their anti-particles are *right*-handed.

$$\begin{array}{lcccc} quarks & \begin{pmatrix} \frac{+2}{3}e \\ \frac{-1}{3}e \end{pmatrix} & \begin{pmatrix} u \\ d \end{pmatrix}_L & \begin{pmatrix} c \\ s \end{pmatrix}_L & \begin{pmatrix} t \\ b \end{pmatrix}_L \\ leptons & \begin{pmatrix} -e \\ 0 \end{pmatrix} & \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L & \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}_L & \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}_L \end{array} \tag{4.23}$$

This must be due to the spacetime properties of the electromagnetic wave discussed in section (2-1). The electromagnetic field must obey the *right*-handed rule, and oppositely, the particle properties must obey the *left*-handed rule. The fundamental natural philosophy is based on the principle of symmetry/asymmetry.[30] We have derived the fine structure constant formula based on this principle. That is why and how we have the beautiful universe.

4-4 Topology of Quark Charge and Confinement

There are two mysteries surrounding quark charge in the strong interaction: (a) triplets and (b) confinement. [146–150] They involve geometric topology in the form of a vector bundle. A non-trivial *fiber bundle* (e.g., the Möbius strip) is a line bundle over the 1-sphere S^1. The electromagnetic wave in 4D space-time can be polarized and twisted to form a Möbius strip in the particle generation, with a reverse process as the particle annihilates (**Fig. 4.7**). This is a physical picture for the conversion between the photon and particle.

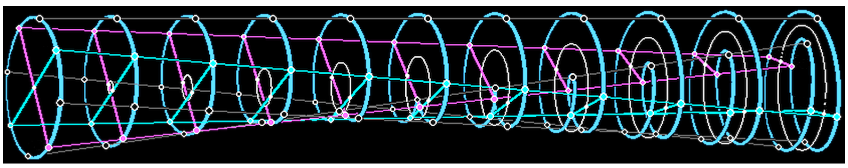

Fig. 4.7: The electromagnetic wave can be polarized into a Möbius strip. Notice the spin-1 ($h/2\pi$) changing to spin-$\frac{1}{2}$ ($h/4\pi$), and the cardioid section in the middle.

[30] Antimatter are producible in any environment with a sufficiently high temperature, and the matter/antimatter are balance existed at Big Bang.

The topological illustration of the electric charge is linked to the Möbius strip, either as a twisted cylinder or a lemniscate belt, depending on the length of the strip. There are two types of Möbius strips depending on the direction of the half-twist: clockwise and counterclockwise. It is therefore *chiral*, i.e., *"handed"*. The Möbius strip is related to the *fiber-bundle* and the "*Zitterbewegung*" mathematically. The term "*gauge*" is more often used in Particle physics. Since its surface is not orientable, a round trip must take two cycles (4π) to return to the starting point. This verifies the particles with spin-1/2 ($\hbar/2 = h/4\pi$).

In cylindrical polar coordinates (r, θ, h), an unbounded Möbius strip is

$$\log(r)\tan(\tfrac{\theta}{2}) = h \tag{4.24}$$

Let $h = \alpha^{1/2} = \log(e^{\pm 1/K\Phi\pi})$ in (4.1) and at $\theta = \pm\pi/2$, we naturally get

$$\log(r) = \log(e^{\pm 1/K\Phi\pi}) \tag{4.25}$$

i.e. $r_{\pm\pi/2} = e^{\pm 1/K\Phi\pi}$. In $\mathbf{R}^3 - t$ (4D), a Möbius strip can be presented as a group of parametric equations with $\alpha^{1/2}$ (or $\alpha_{\mathrm{W}}^{1/2}$ in high energy scale)

$$\begin{aligned} x(u,v,t) &= \left(\alpha^{1/2} \pm \tfrac{u}{2}\cos\left(\tfrac{v}{2}\right)\right)\cos(v + \omega_0 t \pm 2n\pi) \\ y(u,v,t) &= \left(\alpha^{1/2} \pm \tfrac{u}{2}\cos\left(\tfrac{v}{2}\right)\right)\sin(v + \omega_0 t \pm 2n\pi) \\ z(u,v,t) &= \tfrac{u}{2}\sin\left(\tfrac{v}{2} + \omega_0 t \pm 2n\pi\right) \end{aligned} \tag{4.26}$$

where $-\alpha^{1/2} \le u \le +\alpha^{1/2}$, $0 \le v \le 2\pi$, ω_0=constant, and $0 < t < \infty$; "+" for *right*-handed or "–" for *left*-handed. This creates two types of live-rotating Möbius strips of width $\alpha^{1/2}$, whose immobile central circle lies on the $x - y$ plane and is centered at (0,0,0) with radius $\alpha^{1/2}$. The angle parameter u runs around the strip while v moves from one edge to the other. (4.26) is a *torus* in 4D space-time when $\omega_0 t$ is turning. However, charge is timeless as $\sin(\theta) = \sin(2n\pi \pm \theta)$,[31] so it can be plotted in 3D as **Fig. 4.8**. Therefore, a particle with linear speed αc, charge e $= (\alpha\hbar c)^{1/2}$ and spin $\hbar/2$ can be illustrated as a *left*-handed Möbius strip. Obviously, its mirror image is *right*-handed.

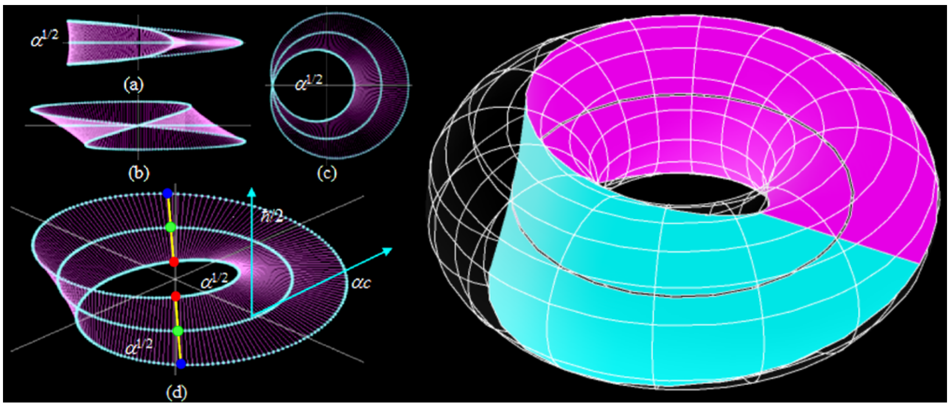

Fig. 4.8: The $\alpha^{1/2}$ illustration of equation (4.26) in 3D: (a) XOZ, (b) YOZ, (c) XOY, (d) Isometric view shows R-G-B flipped on the edge, and 4D torus.

[31] Mach's principle that a rotation leaves no trace at all, which also works for the creation of mass.

If we cut this Möbius strip at 1/3 width of $\alpha^{1/2}$, we get two conjoined rings in **Fig. 4.9**: one immobile ring in the middle equal to -1/3 and another equal to +2/3 in a limited range motion (the sign alternates in each cycle of the Möbius strip, so $(-1)^2 = +1$).

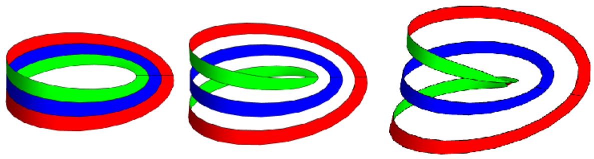

Fig. 4.9: Cutting 1/3 of a Möbius strip into triplet charge: one immobile middle ring (blue) and another confinement ring.

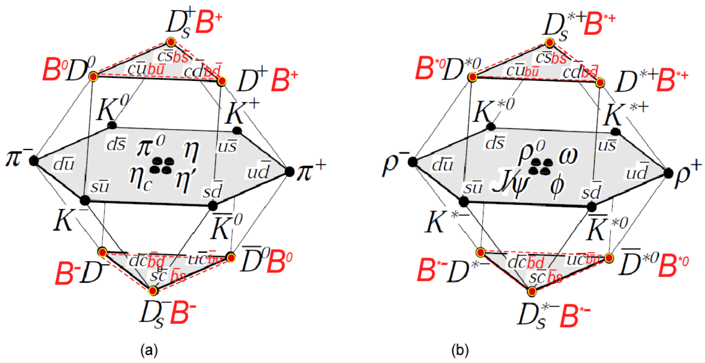

Fig. 4.10: Quark-antiquark pair of u, d, s, c, b makes Mesons in extended $SU(3)^*_c = (|u\rangle, |d\rangle, |s\rangle) \oplus (|c\rangle, |b\rangle, |t\rangle)$ (**a**) spin-0; (**b**) spin-1.

This is similar to the quark charge, and the conjoined or paradromic rings also illustrate why there are no free quarks (color confinement-asymptotic freedom).

In the group theory, $SU(3)_c = (|u\rangle, |d\rangle, |s\rangle)$ organizes mesons in octets: $\mathbf{3} \otimes \overline{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}$, and baryons in octets or decuplets: $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10}_S \oplus \mathbf{8}_M \oplus \mathbf{8}_M \oplus \mathbf{1}_A$. The algebraic sum of two spin-1/2 becomes spin-0 or 1. This model fits the Mesons (**Fig. 4.10**(a)-(b)) as a valence quark-antiquark pair (e.g., $\pi^+ = \bar{d} + u \Rightarrow \frac{1}{3} + \frac{2}{3} = +1$; $\pi^- = d + \bar{u} \Rightarrow -\frac{1}{3} - \frac{2}{3} = -1$; $\pi^0 = \left(d\bar{d} \pm u\bar{u}\right)/\sqrt{2} \Rightarrow 0$). These pions form the unstable transformation center of Mesons, Baryons and Leptons. In weak interactions, heavier Mesons and Baryons decay to pions, then pions decay to Leptons or γ-rays. The decay rate from a vector meson to leptons is proposed by van Royen-Weisskopf [151]

$$\Gamma_\gamma = 16\pi(\alpha_{em}^2/M_V)^2 |\sum_i a_i Q_i|^2 |\psi(0)|^2 \tag{4.27}$$

From (4.1), (4.27) can be linked to the *vacuum* conductivity in a beautiful square format

$$\begin{aligned} \Gamma_\gamma &= \frac{G_0}{\sigma_0}(\alpha_{em} \cdot \frac{\boldsymbol{e}}{M_V})^2 |\sum_i a_i Q_i|^2 |\psi(0)|^2 \\ &= \left[\frac{2}{\pi}\frac{\log e^{\pm 1}}{K\Phi}\right]^2 \left[\frac{\alpha_{em}\boldsymbol{e}}{M_V}\right]^2 |\sum_i a_i Q_i|^2 |\psi(0)|^2 \end{aligned} \tag{4.28}$$

where $\alpha_{em} = \boldsymbol{e}^2/4\pi = 1/137$, M_V is the mass of the vector meson, Q is the charge of the quarks, and $\psi(0)$ is the wavefunction for the two quarks to overlap each other. (4.28) shows that the meson decay involves the *vacuum/quantum* conductivity ratio $G_0/\sigma_0 = 4\alpha$ and the charge/mass ratio $\boldsymbol{e}/M_V$. Note that b quarks make B-mesons and t quarks only decay to W-bosons and a b quark.

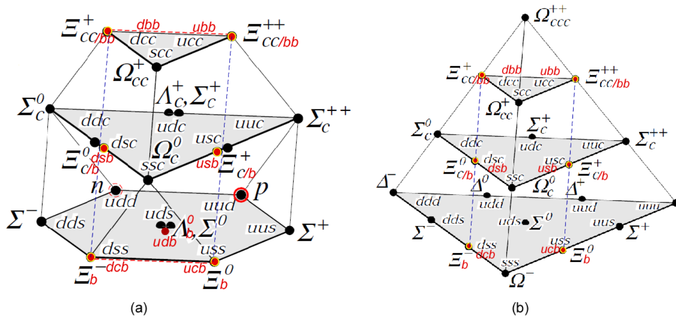

Fig. 4.11: Combinations of three u, d, s, c, b quarks make baryons in extended $SU(3)^*_c$ (**a**) spin-1/2 octet where the stable p and *quasi-stable* n make atoms; (**b**) spin-3/2 decuplet. Note b quark makes $\Lambda \sim \Xi$ Baryons.

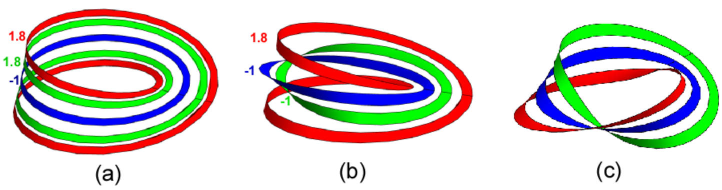

Fig. 4.12: Cutting the Möbius strip into three paradromic rings (a) $\mathbf{p}^+ - uud$ $2 \times (2/3) - 1/3 = 1$, (b) $\mathbf{n}^0 - ddu$ $2 \times (-1/3) + 2/3 = 0$, (c) Δ^--ddd $3 \times (-1/3) = -1$ or Δ^{++}-uuu $3 \times (2/3) = 2$.

Baryons in **Fig. 4.11**(a)-(b) are made by three quarks with spin-1/2 or 3/2, and their electric charge can be $0, \pm 1, 2$ (e.g., proton as $uud \Rightarrow 2 \times (2/3) - 1/3 = 1$ and neutron as $ddu \Rightarrow 2 \times (-1/3) + 2/3 = 0$). They can also be illustrated by cutting the Möbius strip into three paradromic rings ($2 \times (2/3) - 1/3$ or $2 \times (-1/3) + 2/3$) in **Fig. 4.12**(a)-(c)). Note that only **Fig. 4.12**(a) is a regularly cut Möbius strip symbolizing a stable proton, and (b)-(c) are made of composite Möbius strips, symbolizing the unstable particles.

4-5 Proton and Neutron *g*-factors

The particle's g-factor is given as $g_i = 2\mu_i/\mu_N$. The quark model was proposed after the g-factors confirmation of proton $g_p = 5.585$ and neutron $g_n = -3.826$, which indicates they are composite nucleons made by sub-particles. Quark model $SU(6) = (|u\rangle, |d\rangle, |s\rangle) \otimes (|\frac{1}{2}\rangle, |-\frac{1}{2}\rangle)$ only gives a scalar explanation for the ratio of neutron and proton magnetic moments [147]

$$\frac{\mu_p}{\mu_n} = \frac{g_p}{g_n} = \frac{\langle\psi_p|\mu|\psi_p\rangle}{\langle\psi_n|\mu|\psi_n\rangle} = \frac{\langle P, \frac{1}{2}|Q\sigma_3|P, \frac{1}{2}\rangle}{\langle N, \frac{1}{2}|Q\sigma_3|N, \frac{1}{2}\rangle} = \frac{36/36}{-24/36} = -\frac{3}{2} \tag{4.29}$$

In $SU(6)$ $\{\mathbf{6}\otimes\mathbf{6}\otimes\mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_M \oplus \mathbf{70}_M \oplus \mathbf{20}_A\}$, $\frac{\mu_p}{\mu_n} = \frac{\mathbf{33}}{\mathbf{12}} = -2.75$ is also discussed. [147]

In CODATA-2010, many experimental data point to the same number

$$
\begin{aligned}
\frac{\mu_p}{\mu_n} &= \frac{1.410606743(33)\times 10^{-26}[\mathrm{JT}^{-1}]}{-0.96623647(23)\times 10^{-26}[\mathrm{JT}^{-1}]} \\
= \frac{\gamma_p}{\gamma_n} &= \frac{2.675222005(63)\times 10^{8}[\mathrm{s}^{-1}\mathrm{T}^{-1}]}{-1.83247179(43)\times 10^{8}[\mathrm{s}^{-1}\mathrm{T}^{-1}]} \\
= \frac{\gamma_p/2\pi}{\gamma_n/2\pi} &= \frac{42.5774806(10)[\mathrm{MHz}\cdot\mathrm{T}^{-1}]}{-29.1646943(69)[\mathrm{MHz}\cdot\mathrm{T}^{-1}]} = \\
\frac{\mu_p/\mu_B}{\mu_n/\mu_B} = \frac{g_p/2\beta}{g_n/2\beta} &= \frac{1.521032210(12)\times 10^{-3}}{-1.04187563(25)\times 10^{-3}} = \\
\frac{g_p}{g_n} = \frac{2\mu_p/\mu_N}{2\mu_n/\mu_N} &= \frac{\mathbf{5.585694713}(46)}{-\mathbf{3.82608545}(90)} = \\
\frac{g_p/2}{g_n/2} = \frac{\mu_p/\mu_N}{\mu_n/\mu_N} &= \frac{\mathbf{2.792847356}(23)}{-\mathbf{1.91304272}(45)} \\
&= -\mathbf{1.45989806}(34)
\end{aligned} \tag{4.30}
$$

Since the g-factor of an electron with a whole Möbius strip is about $|g_e|/2 = 1 + a_e$, we assume that the $g/2$ of those immobile ring or rings is also close to ± 1, and the $g/2$-factor of a cut confinement ring with about two cycles is close to 1.8<2 (**Fig. 4.12**(a)-(b)). In fact, all lepton g_i values are *negative* (g_e = -2.00231930436153(53); g_μ = -2.0023318418(13) and $g_\tau = -2.008$), which is often *ignored* (noted in section (4-2)). The *negative* sign means that the particle has a tendency to align anti-parallel to a magnetic field. The neutron $g_n/2 = -1.91304272(45)$ is also negative, and only the proton $g_p/2 = 2.792847356(23)$ is a *strange* positive. From this simple quark toy-model of $\mathbf{p}^+(uud)$ and $\mathbf{n}^0(udd)$ in **Fig. 4.12**(a)-(b), [146, 148] we get [147]

$$
\begin{aligned}
|\mu_p| &= +\tfrac{2}{3}\mu_u + \tfrac{2}{3}\mu_u - \tfrac{1}{3}\mu_d = \tfrac{1}{3}(4\mu_u - 1\mu_d) = \tfrac{|g_p|}{2}\cdot|\mu_0| \\
|\mu_n| &= +\tfrac{2}{3}\mu_u - \tfrac{1}{3}\mu_d - \tfrac{1}{3}\mu_d = \tfrac{1}{3}(2\mu_u - 2\mu_d) = \tfrac{|g_n|}{2}\cdot|\mu_0|
\end{aligned} \tag{4.31}
$$

where μ_p is the same as $SU(6)$ and μ_n uses the same concept. Let $|\mu_0| \equiv 1$, (4.31) yields

$$
\begin{aligned}
\mu_u/|\mu_0| = g_u/2 &= +1.836325\underline{5994} \cong +2 - \tfrac{6}{\pi}\alpha^{1/2} \underline{-\tfrac{\sqrt{7}}{\pi}\alpha^{3/2}} \\
\mu_d/|\mu_0| = g_d/2 &= -1.033238\underline{8094} \cong -1 - \tfrac{11}{9\pi}\alpha^{1/2} \underline{-\tfrac{2\pi}{7}\alpha^{5/2}}
\end{aligned} \tag{4.32}
$$

where $\sqrt{7}$ appears in the electroweak coupling (4.19). The first and <u>second</u> correlation is proportional to $\alpha^{n+1/2}$. The $\alpha^{n+1/2}$ series used here for QCD has been ignored by QED as a divergent perturbation series. In the strong-electroweak coupling, the baryon anomaly is $a_B(\alpha^{n+1/2}) = \sum (-1)^{n+1} c_n \alpha^{n+1/2}$. From (4.31) and (4.32)

$$
\begin{aligned}
\frac{|g_p|}{2} &= \mathbf{3} + \tfrac{1}{\pi}[\tfrac{\mathbf{1}}{\mathbf{3}}(1+\tfrac{\mathbf{2}}{\mathbf{9}}) - \mathbf{2^3}]\alpha^{\mathbf{1}/2} \underline{-\tfrac{\mathbf{2}}{\mathbf{3}}\cdot\tfrac{2\sqrt{7}}{\pi}\alpha^{\mathbf{3}/2} + \tfrac{\mathbf{1}}{\mathbf{3}}\cdot\tfrac{2\pi}{7}\alpha^{\mathbf{5}/2}} \\
&= \mathbf{3} + [\tfrac{\mathbf{11}}{\mathbf{27}} - \mathbf{8}]\tfrac{\alpha^{\mathbf{1}/2}}{\pi} \underline{+\mathcal{O}_p} = \ln\left(e^{\mathbf{3}}\cdot\frac{\exp[\frac{\mathbf{1}}{\mathbf{3}}(\frac{11}{9\pi}\alpha^{1/2}+\frac{\mathbf{2}\pi}{7}\alpha^{5/2})]}{\exp[\frac{\mathbf{1}}{\pi}(8\alpha^{1/2}+\frac{4\sqrt{7}}{3}\alpha^{3/2})]}\right) \\
&= \mathbf{2.792\underline{847578}} \\
\frac{|g_n|}{2} &= \mathbf{2} + \tfrac{1}{\pi}[\tfrac{\mathbf{2}}{\mathbf{3}}(1+\tfrac{\mathbf{2}}{\mathbf{9}}) - \mathbf{2^2}]\alpha^{\mathbf{1}/2} \underline{-\tfrac{\mathbf{1}}{\mathbf{3}}\cdot\tfrac{2\sqrt{7}}{\pi}\alpha^{\mathbf{3}/2} + \tfrac{\mathbf{2}}{\mathbf{3}}\cdot\tfrac{2\pi}{7}\alpha^{\mathbf{5}/2}} \\
&= \mathbf{2} + [\tfrac{\mathbf{22}}{\mathbf{27}} - \mathbf{4}]\tfrac{\alpha^{\mathbf{1}/2}}{\pi} \underline{+\mathcal{O}_n} = \ln\left(e^{\mathbf{2}}\cdot\frac{\exp[\frac{\mathbf{2}}{\mathbf{3}}(\frac{11}{9\pi}\alpha^{1/2}+\frac{\mathbf{2}\pi}{7}\alpha^{5/2})]}{\exp[\frac{\mathbf{1}}{\mathbf{2}\pi}(8\alpha^{1/2}+\frac{4\sqrt{7}}{3}\alpha^{3/2})]}\right) \\
&= \mathbf{1.913\underline{042844}}
\end{aligned} \tag{4.33}
$$

where $\alpha^{n+1/2}$ involves the *odd number* charge interaction $\mathbf{e}^1, \mathbf{e}^3, \mathbf{e}^5, \cdots$ (e.g., $p^+(uud)$ and $n(udd)$ in three charged quarks, plus W^- and $\mathbf{e}^-$ in the neutron decay as **Fig. 4.5**(a)).

It is noted that $\mathbf{\frac{1}{3}}$ and $\mathbf{\frac{2}{3}}$ appeared in (4.10), (4.16), (4.21), (4.31) and (4.33), also $\mathbf{\frac{2}{9}}$ in (4.17), and (4.21). From (4.32) and (4.33)

$$\begin{aligned}
\begin{bmatrix} \frac{|g_p|}{2} \\ \frac{|g_n|}{2} \end{bmatrix} &= \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \left\{ \begin{bmatrix} \frac{11}{27} \\ \frac{22}{27} \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \end{bmatrix} \right\} \frac{1}{\pi}\alpha^{\mathbf{1}/2} \underline{- \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix} \frac{2\sqrt{7}}{\pi}\alpha^{\mathbf{3}/2} + \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \frac{2\pi}{7}\alpha^{\mathbf{5}/2}} \\
&= \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \frac{\mathbf{1}}{\pi} \cdot \left\{ \left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{3}} \cdot \frac{\mathbf{2}}{\mathbf{3}} \right) \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} - \begin{bmatrix} 2^3 \\ 2^2 \end{bmatrix} \right\} \cdot \alpha^{\mathbf{1}/2} \underline{+ \begin{bmatrix} \mathcal{O}_p \\ \mathcal{O}_n \end{bmatrix}} \\
&= \ln \left(\exp \begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot \frac{\exp\left\{ \frac{\mathbf{11}}{9\pi} \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \alpha^{\mathbf{1}/2} \right\}}{\exp\left\{ \frac{\mathbf{1}}{\pi} \begin{bmatrix} 8 \\ 4 \end{bmatrix} \alpha^{1/2} \right\}} \cdot \underline{\exp \begin{bmatrix} \mathcal{O}_p \\ \mathcal{O}_n \end{bmatrix}} \right) = \begin{bmatrix} \mathbf{2.7928\underline{47578}} \\ \mathbf{1.9130\underline{42844}} \end{bmatrix}
\end{aligned} \tag{4.34}$$

There is $(\mathbf{3}, \mathbf{8})$ for the stable proton and $(\mathbf{2}, \mathbf{4})$ for the decaying free neutron in (4.34). It may be related to the gluons represented by the **8** Gell-Mann matrices. (4.34) can be physically expressed as

$$\begin{aligned}
\begin{bmatrix} \frac{|g_p|}{2} \\ \frac{|g_n|}{2} \end{bmatrix} &= \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \frac{\mathbf{e}}{\pi \mathbf{q}_\text{p}} \left\{ \sin^2\theta_w \begin{bmatrix} \frac{11}{6} \\ \frac{11}{3} \end{bmatrix} - \begin{bmatrix} 2^3 \\ 2^2 \end{bmatrix} \right\} \underline{\mp \mathbf{2} \begin{bmatrix} Q_u \\ Q_d \end{bmatrix} \frac{\sqrt{7}}{\pi \mathbf{q}_\text{p}} \frac{\bar{\lambda}_c}{a_0} \mp \mathbf{1} \begin{bmatrix} Q_d \\ Q_u \end{bmatrix} \frac{\tan^2\theta_w}{\mathbf{q}_\text{p}/\pi} \frac{r_e}{a_0}} \\
&= \ln \left(\frac{\exp \begin{bmatrix} 3 \\ 2 \end{bmatrix} \cdot \exp\left\{ \frac{\mathbf{e}}{\pi \mathbf{q}_\text{p}} \left(\sin^2\theta_w \begin{bmatrix} \frac{11}{6} \\ \frac{11}{3} \end{bmatrix} - \begin{bmatrix} 2^3 \\ 2^2 \end{bmatrix} \right) \right\}}{\exp\left\{ \underline{\pm \mathbf{2} \begin{bmatrix} Q_u \\ Q_d \end{bmatrix} \frac{\sqrt{7}}{\pi \mathbf{q}_\text{p}} \frac{\bar{\lambda}_c}{a_0} \pm \mathbf{1} \begin{bmatrix} Q_d \\ Q_u \end{bmatrix} \tan^2\theta_w \frac{\pi}{\mathbf{q}_\text{p}} \frac{r_e}{a_0}} \right\}} \right) = \begin{bmatrix} \mathbf{2.7928\underline{47578}} \\ \mathbf{1.9130\underline{42844}} \end{bmatrix}
\end{aligned} \tag{4.35}$$

where the quark charges are $Q_u = +\frac{\mathbf{2}}{\mathbf{3}}\mathbf{e}$ and $Q_d = -\frac{\mathbf{1}}{\mathbf{3}}\mathbf{e}$. (4.35) is comparable to the *beta*-function in QCD. Since $SU(6) = SU(3) \times SU(2)$ has the total number of generators $\mathbf{8} + \mathbf{3} = \mathbf{11}$ on $\mathbf{6}$ dimensional tensor product space, the *beta*-function of the asymptotic freedom $\beta_1(\alpha_s) = \frac{\alpha_s^2}{\pi}(\frac{-\mathbf{11}N}{\mathbf{6}} + \frac{n_f}{\mathbf{3}}) = \frac{\alpha_s^2}{3\pi}(\frac{-\mathbf{11}N}{\mathbf{2}} + n_f)$. [148, 149][32]

The toy-model of the asymptotic freedom in **Fig. 4.12**(a)-(b) requires g_p and g_n in different signs, i.e., $g_p/2 = \mathbf{2.7928}$ and $g_n/2 = -\mathbf{1.9130}$ for particle ($g_{\overline{p}}/2 = -\mathbf{2.7928}$ and $g_{\overline{n}}/2 = \mathbf{1.9130}$ for anti-particle). This gives the ratio in (4.30)

$$\frac{\mu_p}{\mu_n} = \frac{g_p/2}{g_n/2} = \frac{g_p}{g_n} = \frac{4\mu_u - 1\mu_d}{2\mu_d - 2\mu_u} = -\mathbf{1.4598\underline{98082}} \tag{4.36}$$

The g-factors of proton and neutron g_p and g_n have a different signs. Neutrons with negative g_n will decay after leaving the nuclear quark confinement.

In the atomic Periodic table, the experimental ratio $N_p/N_n = 1/1 \sim 1/1.6$. Using the toy-model of the Möbius strip, we can illustrate how $N_n > N_p$ and $N_n/N_p = 1 \sim 1.6 < \Phi$ in the heavy nucleus makes it more stable. $N_n > N_p$ means $N_d > N_u$ or $N_{-\frac{1}{3}} > N_{\frac{2}{3}}$, i.e., the number of immobile rings is more than that of the confinement rings in the cut Möbius strip in **Fig. 4.12**. The neutron's two immobile central rings act like one, and a free neutron's decay behavior is alike a boson. The nucleus is more stable if there are more immobile central rings.

[32] In the asymptotic freedom, alike QED, the one-loop *beta*-function with n_f flavors in QCD $SU(3)_c$ is $\beta_1(g) = \frac{g^3}{(4\pi)^2}(-11 + \frac{2n_f}{3})$ or $\beta_1(\alpha_s) = \frac{\alpha_s^2}{2\pi}\beta_0 = \frac{\alpha_s^2}{2\pi}(-11 + \frac{2n_f}{3}) = \frac{\alpha_s^2}{\pi}(\frac{-11N}{6} + \frac{n_f}{3})$, where $SU(3)$ give $N = 3$, $\alpha_s(Q^2) = \frac{g^2}{4\pi} = \frac{4\pi}{\beta_0 \ln[Q^2/\Lambda^2]}$ and g is a color charge, $\Lambda \approx 217\text{MeV}$ is QCD scale. If $n_f < 16$, the coupling *is inversely proportional to* the energy scale. Because of asymptotic freedom, no free quark with the asymmetry of fractional charge is found in nature.

The $g/2$-factors of Leptons and Baryons in (4.10) and (4.35) have a similar logarithmic format, and can be uniformly expressed as the information entropy

$$\begin{aligned}\begin{pmatrix}\frac{|g_e|}{2} & \frac{|g_p|}{2}\\ \frac{|g_\mu|}{2} & \frac{|g_n|}{2}\end{pmatrix} &= \begin{pmatrix}\mathbf{1.0011596521807} & \mathbf{2.792847578}\\ \mathbf{1.0011659207972} & \mathbf{1.913042844}\end{pmatrix} \\ &= \begin{pmatrix}\ln\left(e^{\mathbf{1}}\cdot\frac{\exp[\mathbf{A}(\alpha^{-2}+\frac{\mathbf{2}}{\mathbf{3}})]}{\exp[\mathbf{B}(\alpha^{+2}+\frac{\mathbf{1}}{\mathbf{3}})]}\cdot e^{\mathcal{O}_e}\right) & \ln\left(e^{\mathbf{3}}\cdot\frac{\exp(\frac{\mathbf{1}}{\mathbf{3}}\boldsymbol{A})}{\exp(1\boldsymbol{B})}\cdot e^{\mathcal{O}_p}\right)\\ \ln\left(e^{\mathbf{1}}\cdot\frac{\exp[\mathbf{A}(\alpha^{-2}+\frac{\mathbf{2}}{\mathbf{3}})]}{\exp[\mathbf{B}(\alpha^{+2}+\frac{\mathbf{1}}{\mathbf{3}})]}\cdot e^{\mathcal{O}_\mu}\right) & \ln\left(e^{\mathbf{2}}\cdot\frac{\exp(\frac{\mathbf{2}}{\mathbf{3}}\boldsymbol{A})}{\exp(\frac{1}{2}\boldsymbol{B})}\cdot e^{\mathcal{O}_n}\right)\end{pmatrix}\end{aligned} \tag{4.37}$$

where $\mathbf{A} = \frac{1}{2\pi}\alpha^3$, $\mathbf{B} = \frac{1}{\pi^2}\alpha^2$ for the Leptons and $\boldsymbol{A} = \frac{11}{9\pi}\alpha^{1/2}$, $\boldsymbol{B} = \frac{\mathbf{8}}{\pi}\alpha^{1/2}$ for the Baryons. Of course, there is a lot of theoretical work, which is beyond the focus of this paper.

4-6 Running alpha?

From $|\alpha^{1/2}| \equiv \pm\frac{M\phi}{K\pi} \equiv \frac{\log e^{\pm 1}}{K\Phi\pi} \equiv \log e^{\pm\phi/K\pi}$, the electric charge sign of electrons and positrons are given by $\log e^{\mp}$, the particle topology of electrons and positrons are linked to the *left*-handed Möbius strip and its mirror image, the proton and neutron are illustrated by the cut Möbius strip for the color charge. Particles are generated when photons become trapped in Möbius topology, and annihilated as photons are released. After all, the particle-photon conversion obeys Einstein $E = h\nu = mc^2$, which is also quantized by the fine structure constant.

It is well-known that only 5 Fermat primes {3, 5, 17, 257, 65537} satisfy $F_n = 2^{2^n} + 1$. Here we show the prime double factorial equation (2.38) only has 3 solutions {**3**, **37**, **61**}, which initialize the fermion charge quantization ($K(3, 37, 61) \approx 1$), $\alpha^{1/2}_{naked} = \log e/\Phi\pi = 1/\Phi\pi \ln 10$, associated by countless variations of $\Phi - \phi - e - \pi$ to constitute the Universe Information. For W$^{\pm}$ boson, $\alpha^{1/2}_{\mathrm{W}} = \frac{\log F^{\pm 1}}{\Phi\pi}(1 - \alpha\sin^2\theta_w)$. Euler constant e and Fransén-Robinson constant F can be unified as the limitation of the Riemann sum I_n [152]

$$I_n = \frac{1}{n}\sum_{n=0}^{\infty}\frac{1}{\Gamma(\frac{k}{n})} = \begin{cases} e=\sum\limits_{n=0}^{\infty}\frac{1}{\Gamma(n)}=2.718281828... & n=1\\ \frac{1}{\sqrt{4\pi}}+\frac{e}{2}\cdot\mathrm{erfc}(-1)=2.7865848321 & n=2\\ \Downarrow & \Downarrow\\ F=\int_0^\infty\frac{dx}{\Gamma(x)}=2.8077702420285... & n\to\infty\end{cases} \tag{4.38}$$

where the complementary error function

$$\begin{aligned}\mathrm{erfc}(x) &= \tfrac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt = 1 - \mathrm{erf}(x)\\ &= \tfrac{e^{-x^2}}{x\sqrt{\pi}}\sum_{n=0}^{\infty}(-1)^n\cdot\tfrac{(2n-1)!!}{(2x^2)^n}\\ &= 1 - \tfrac{2}{\sqrt{\pi}}(x - \tfrac{x^3}{3\cdot 1!} + \tfrac{x^5}{5\cdot 2!} - \tfrac{x^7}{7\cdot 3!} + \cdots)\end{aligned} \tag{4.39}$$

or as a continued fraction

$$\begin{aligned}\mathrm{erfc}(x) &= \tfrac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt = 1 - \mathrm{erf}(x)\\ &= \tfrac{xe^{-x^2}}{\sqrt{\pi}}\cfrac{a_1}{x^2+\cfrac{a_2}{1+\cfrac{a_3}{x^2+\cfrac{a_4}{1+\cdots}}}}\end{aligned} \tag{4.40}$$

where $a_1 = 1$, $a_m = \frac{m-1}{2}$ $(m \geq 2)$. The standard normal distribution, described by the probability density function $\phi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$, then the cumulative distribution function of the normal Gaussian distribution $\Phi(x)$ is

$$\Phi(x) \quad = \quad \int_{-\infty}^{x} \phi(t)dt = \tfrac{1}{2}\mathrm{erfc}(-\tfrac{x}{\sqrt{2}}) \tag{4.41}$$

This explains the statistic character of the electric charge at the high-energy scale.

(4.38) can make alpha running from the starting points of SM or SUSY as **Fig. 4.13** [153, 154] Experimentally, the strong coupling constant $\alpha_s(m_Z) = 0.112 \sim 121$ and world average $\alpha_s(m_Z)_{1\,loop} = 0.1171 \sim \alpha_s(m_Z)_{3\,loop} = 0.1184$. [155] It is too early to tell whether relate to $f(0) = \frac{1}{e\pi} = 0.117099$ of the $F - e$ integral in **Fig. 4.14**

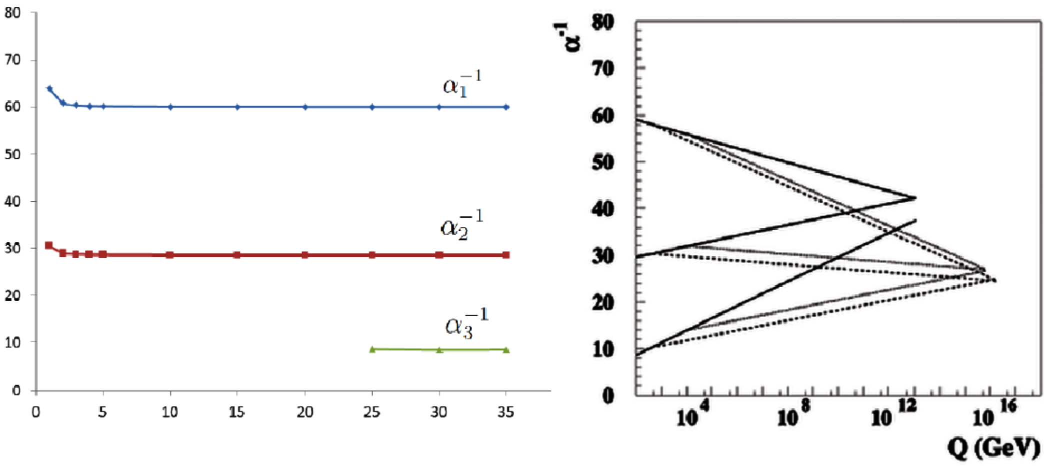

Fig. 4.13: $\alpha_1^{-1} = \frac{3}{5}\alpha^{-1}\cos^2\theta_w \sim 60$ and $\alpha_2^{-1} = \alpha^{-1}\sin^2\theta_w \sim 30$ calculated from (4.38) and $\alpha_3^{-1} = \alpha_s^{-1} \sim 8.5$ running from the starting points of SM or SUSY.

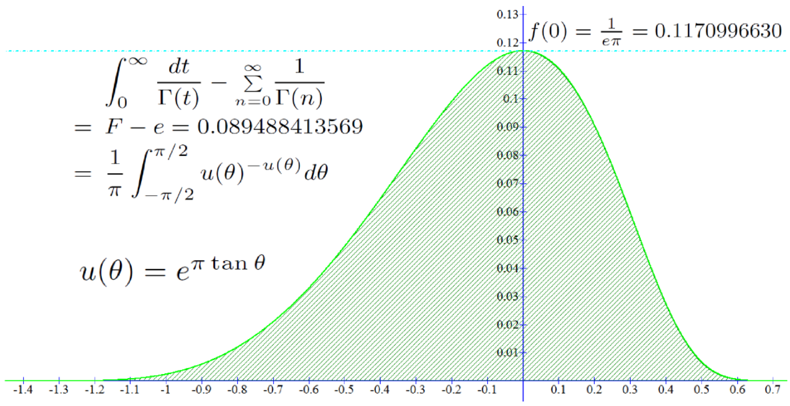

Fig. 4.14: $F - e$ integral has a maximum of $f(0) = \frac{1}{e\pi} = \frac{1}{4}(\frac{1}{2})^{3/2}(\frac{3}{2}\frac{3}{4})^{5/4}(\frac{5}{4}\frac{5}{6}\frac{7}{6}\frac{7}{8})^{9/8}\cdots =$ 0.1170996630

We present the dimensionless physical constants with the basic math constants, which is formulated as the beautiful Euler Identity $e^{\pm i\pi} + \Phi = \phi$. As Einstein opined, "*In a reasonable theory there are no (dimensionless) numbers whose values are only empirically determinable.*" [156] "*Dimensionless constants in the laws of nature, which from the purely logical point of view can just as well have different values, should not exist. To me, with* *my 'trust in God' this appears to be evident, but there will be few who are of the same opinion.*" [157]

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