Maxwell's theory of gravity and thermodynamics

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Abstract

We argue that the entropic origin of gravitation correctly reproduced general relativity and quantum mechanics, with a particular treatment, where entropic gravity can be viewed as Maxwell's theory of gravity and thermodynamics. And this application will give us more detailed knowledge on the origin of gravity.
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1 Introduction

Maxwell's theory of electromagnetism is not just a model specified by electromagnetic phenomena, However, it is a method of thinking to unify two distinct theories into a single super-theory, Verlinde's argument that gravity can be viewed as an emergent phenomenon driven by the second law of thermodynamics suggest a deep connection between gravity and thermodynamics, and the separation of Verlinde's view of gravity from Jacobson's view of the Einstein field equation will make the research on black hole thermodynamics by Bekenstein and Hawking looks like a subsidiary result of doing quantum field theory in a space time with horizon, but Verlinde's assumptions does not fully explain how to work with microscopic systems within the framework of entropic gravity.

So, in this proposal that Entropic gravity can be modeled as Maxwell's theory of gravity and thermodynamics, we will present a further explanations to see how both macroscopic and microscopic cases should be handled with Entropic gravity.

2 Gravity and Thermodynamics

An entropic force acting in a system is a phenomenological and an effective macroscopic force resulting from the entire system's statistical tendency to increase its entropy, A standard example of an entropic force is the elasticity of a freely-jointed polymer molecule. If the molecule is pulled into an extended configuration, the system has an increased amount of predictability. But randomly coiled configurations are overwhelmingly more probable (they have greater entropy). This results in the chain eventually returning (through diffusion) to such a configuration. To the macroscopic observer, the precise origin of the microscopic forces that drive the motion is irrelevant. The observer simply sees the polymer contract into a state of higher entropy.

Thus, maximizing the chain entropy means reducing the distance between its two free ends, and therefore we will going to define the area -in which entropic force acts- as an entropic field, where the entropic field intensity H is defined as the entropy gradient :

$$F = T\nabla S, \quad H = \nabla S \tag{1}$$

To determine the entropic force, one considers the micro-canonical ensemble $\Omega(E + Fx, x)$ takes into account the total energy E and the energy Fx that was put into the system by pulling the polymer out of its equilibrium position and imposes that the entropy is extremal. This gives

$$H = \frac{d}{dx} S(E + fx, x) = 0 \tag{2}$$

This equation can be viewed as a defining property of the entropic field,

$$\frac{1}{V}\int H \cdot dA = \frac{1}{V}\int S(E + Fx, x)\, dx = 0 \tag{3}$$

Then we can define the divergence of the entropic field as :

$$\nabla \cdot H = 0 \tag{4}$$

Entropic forces also occur in the physics of gases, where they generate the pressure of an ideal gas (the energy of which depends only on its temperature, not its volume), entropic force acts adiabatically and the energy increases because we have done work on the gas and no energy can be transferred by heating or cooling by the surroundings. The increase in the energy causes the temperature to increase. Hence, both the decrease in the volume and the increase in the temperature cause the pressure to increase more. If we relax the condition that the change be adiabatic and allow the system to interact with its surroundings, we find in general that the heat flow causes a change in the entropy of the system. If, in addition, there are mass flows across the system boundaries, the total entropy of the system will also change due to this convected flow. the entropy change of an ideal gas due to a change in temperature is $dS = \frac{C_V}{T} dT$,and from heat equation we get :

$$\frac{\partial S}{\partial t} = \alpha \frac{\partial^2 S}{\partial x^2} \tag{5}$$

Therefore we can define the curl of the entropic field as :

$$\nabla \times H = \frac{1}{\alpha} \frac{\partial S}{\partial t} \tag{6}$$

Motivated by Bekenstein's entropy formula, Verlinde proposed that gravity can be described using the holographic principle. The entropy in accord with the holographic picture is stored on holographic screens and the space is emergent between two such screens, He postulated that the change of entropy associated with the information on the boundary equals (We assume units in which $\hbar = c = k_B = 1$) :

$$\Delta S = 2\pi m \Delta x \tag{7}$$

Where ΔS is the increase in the entropy of a holographic screen due to the approaching particle which has the mass m and is located at the distance Δx from the screen. Assuming that the holographic principle holds, the total number of bits N is proportional to the area A :

$$N = \frac{A}{G} \tag{8}$$

If the energy is divided evenly over the bits N, Then the temperature is determined by the equipartition rule and from the equivalence principle the energy represents the mass M that would emerge in the part of space enclosed by the screen,

$$E = \frac{1}{2} NT, \quad E = M \tag{9}$$

Next one uses the postulate (7) for the change of entropy to determine the force along with the fact that the area of the holographic screen is $4\pi R^2$, one obtains Newtons law of gravity,

$$F = G \frac{Mm}{R^2} \tag{10}$$

If gravity acts as an entropic force, Hence, the entropy is maximized by the gravitational field g that acts along the holographic directions :

$$g = \eta \frac{\partial S}{\partial t} \tag{11}$$

where η is constant, and the gravitational field g is proportional to the entropy accumulation with time. And therefore the time varying of the entropic field creates a gravitational field and this phenomenon is described by the equation :

$$\nabla \times g = -\eta \frac{\partial H}{\partial t} \tag{12}$$

It appears that the entropic field is a function of primary field $B = \eta H$. Thus from equation (4) and the vector identity that $\nabla \cdot (\nabla \times \mathcal{A}) = 0$ we are going to define the entropic field in terms of the vector potential :

$$B = \nabla \times \mathcal{A} \tag{13}$$

now we obtain the main principle of Entropic gravity : inertia is a consequence of the fact that a particle in rest will stay in rest because there is no entropy gradient. Given this fact it is natural to introduce the Newton potential Φ and the vector potential using the acceleration a :

$$a = -\nabla \Phi - \frac{\partial \mathcal{A}}{\partial t} \tag{14}$$

To determine the gravitational force by using virtual displacements, and calculating the associated change in energy, Consider a static matter distribution ρ completely contained within a volume enclosed by a holographic screen $\mathcal{S}$ specified by Φ= constant, The Unruh temperature $T = \frac{a}{2\pi}$ may be written in terms of the normal derivative of the potentials,

$$T = \frac{1}{2\pi} \hat{n} \cdot (\nabla \Phi + \frac{\partial \mathcal{A}}{\partial t}) \tag{15}$$

via equipartition and holography

$$E = \frac{1}{2} \int_{\mathcal{S}} T \, dN, \quad E = M \tag{16}$$

where the number of bits on the holographic screen is given by the postulate (8), we find that :

$$\int_{\mathcal{S}} (\nabla \Phi + \frac{\partial \mathcal{A}}{\partial t}) \cdot \, dA = 4\pi G M \tag{17}$$

which satisfies the field equation :

$$\nabla \cdot g = -4\pi G \rho \tag{18}$$

we conclude that the equations(4),(6),(12),(18) are the Maxwell equations of Entropic gravity.

In order for the entropic and gravitational fields to satisfy special relativity, The entropic/gravitational four-potential can be defined as:

$$A^{\alpha} = (\Phi, \mathcal{A}) \tag{19}$$

And the entropic/gravitational tensor is the exterior derivative of the differential 1-form A_α

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{20}$$

3 General relativity with Entropic gravity

Inspired by the close ties between Minkowskis four-dimensional space time and Maxwells unification of gravity and thermodynamics we are going to try unifying the description of gravity in general relativity and entropic gravity in a theory of five dimensions, specifically, following Kaluza-Klein theory we can successfully obtain the field equations of both general relativity and entropic gravity from a single five-dimensional theory.

The Einstein equations in five dimensions with no five-dimensional energy-momentum tensor are :

$$\hat{G}_{ab} = 0 \tag{21}$$

or, equivalently :

$$\hat{R}_{ab} = 0 \tag{22}$$

Where $\hat{G}_{ab}$ is the Einstein tensor,and $\hat{R}_{ab}$ is the five-dimensional Ricci tensor, The five-dimensional Ricci tensor and Christoffel symbols are defined in terms of the metric exactly as in four dimensions :

$$\begin{aligned} \hat{R}_{ab} &= \partial_c \hat{\Gamma}^c_{ab} - \partial_b \hat{\Gamma}^c_{ac} + \hat{\Gamma}^c_{ab}\hat{\Gamma}^d_{cd} - \hat{\Gamma}^c_{ad}\hat{\Gamma}^d_{bc} \\ \hat{\Gamma}^c_{ab} &= \frac{1}{2}\hat{g}^{cd}(\partial_a \hat{g}_{db} + \partial_b \hat{g}_{da} - \partial_d \hat{g}_{ab}) \end{aligned} \tag{23}$$

Everything now depends on ones choice for the form of the five-dimensional metric. In general, one identifies the $\alpha\beta$-part of $\hat{g}_{ab}$ with $g_{\alpha\beta}$ (the four-dimensional metric tensor), the $\alpha 4$-part with A_α (the entropic/gravitational four-potential), and the 44-part with ϕ (a scalar field). A convenient way to parametrize things is as follows:

$$\hat{g}_{ab} = \begin{pmatrix} g_{\alpha\beta} + \kappa^2\phi^2 A_\alpha A_\beta & \kappa\phi^2 A_\alpha \\ \kappa\phi^2 A_\beta & \phi^2 \end{pmatrix} \tag{24}$$

where we have scaled the entropic/gravitational four-potential A_α by a constant κ. If one then applies the key feature of Kaluzas theory (the cylinder condition), which means dropping all derivatives with respect to the fifth coordinate, then one finds, using the metric (24) and the definitions (23) that the $\alpha\beta$-, $\alpha 4$-, and 44-components of the five-dimensional field equation (22) reduce respectively to the following field equations in four dimensions :

$$G_{\alpha\beta} = \frac{\kappa^2\phi^2}{2}T_{\alpha\beta} - \frac{1}{\phi}[\nabla_\alpha(\partial_\beta\phi) - g_{\alpha\beta}\partial_\alpha\partial^\alpha\phi], \tag{25}$$

$$\nabla^\alpha F_{\alpha\beta} = -3\frac{\partial^\alpha\phi}{\phi}F_{\alpha\beta}, \quad \partial_\alpha\partial^\alpha\phi = \frac{\kappa^2\phi^3}{4}F_{\alpha\beta}F^{\alpha\beta} \tag{26}$$

where $F_{\alpha\beta}$ is the entropic/gravitational filed tensor. And the last two equations are just the Einstein and Maxwell equations if the scalar field ϕ is constant throughout space time,

$$G_{\alpha\beta} = 8\pi G\phi^2 T_{\alpha\beta}, \quad \nabla^\alpha F_{\alpha\beta} = 0 \tag{27}$$

where we have identified the scaling parameter κ in terms of the gravitational constant G (in four dimensions) by :

$$\kappa \equiv 4\sqrt{\pi G} \tag{28}$$

4 Quantum mechanics with Entropic gravity

Let us consider a system that consists of the fixed mass M at origin and the test particle m at x, we will describe this system in terms of a holographic screen that is known to contain the particle m. The entropy of this screen depends on the total energy E of the screen and on the macroscopic parameter x that describes the position of the particle :

$$\frac{1}{\eta}\nabla \times \mathcal{A} = \frac{d}{dx}S(E,x) = \frac{d}{dx}\log\Omega(E,x) \tag{29}$$

where $\Omega(E,x)$ is the number of the micro-states on the screen that are associated with the potential $\mathcal{A}$. If we introduce a generalized interaction between the particle m and the entropic field with the help of the potential $\mathcal{A}$, then the lagrangian of this particle should be ,

$$\mathcal{L} = \frac{1}{2}mv^2 + mv\mathcal{A} \tag{30}$$

And the classical particle momentum is :

$$p = \frac{\partial \mathcal{L}}{\partial v} = mv + m\mathcal{A}, \quad v = \frac{1}{m}(p - m\mathcal{A}) \tag{31}$$

Therefore the Hamiltonian operator of a particle in a potential energy $V(x)$:

$$H = \frac{1}{2}mv^2 + V(x), \quad \hat{H} = \frac{1}{2m}(\hat{p} - m\mathcal{A})^2 + V(x) \tag{32}$$

replacing the classical momentum p by the differential operator $\hat{p} = \frac{\hbar}{i}\nabla$. The wave function $\psi(x,t)$ of the particle is then described by the Schrodinger equation,

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,t) \tag{33}$$

finally, we find that Entropic gravity does not contradict the standard quantum mechanical description of the particle in a gravitational potential .

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