The theory of the relativity of electromotive space-time

---Discussion and development of perfection of special relativity

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Abstract: The theory of the relativity of electromotive space-time is to study the transform of time and space of two electromotive inertial reference systems (with inertial movement and electric potential difference). It is a fundamental theory of theoretic physics based on the experimental facts of Einstein's special relativity and the inversion proportional law of Coulomb's force. It founded new physical space-time conception, i.e., space-time is composed of quaternion space and time and proposed some new basic concepts of physics, such as limit electric potential, quaternion potential etc., revealing the inherent relationship between electric potential—velocity and time—space. This paper discussed in detail the process of establishment of complex variable electromotive space-time relativity and quaternion electromotive space-time relativity as well as their various conversion forms. The paper is one on the basic theory of the author's serial papers.

Keyword: theory of the relativity of electromotive space-time; theory of complex variable electromotive space-time relativity; quaternion electromotive space-time relativity; theory of the relativity of electric potential; special relativity; limit of electric potential; complex velocity; complex electric potential; quaternion velocity; quaternion electric potential; quaternion

The two pillars of modern physics are quantum mechanics and the theory of relativity which were proven through numerous experiments. However, on the fundamental understandings, such as the "actuality" of physics, there are profound contradictions between quantum mechanics and general relativity. So much so that Einstein convinced that quantum physics is an incomplete theory, pursued a unified field theory. Dirac also believed that in the future there will be an improved quantum mechanics, which would make it return to the determinism and prove that Einstein's view is correct. Though, this can only be achieved by giving up some basic ideas [1]. For the past century, however, many attempts to unite quantum mechanics and general relativity have not been successful.

It is well-known that the Dirac equation of quantum mechanics was built upon the relationship between energy and momentum of special relativity. The general relativity was also developed through advancement of the basic hypotheses of the special theory of relativity. If there are unsolvable conflicts between quantum mechanics and general relativity, the original cause may be related to special relativity. However, special relativity has been well proven through abundant experiments, and its correctness is sufficiently confirmed. Hence one cannot help but to doubt the completeness of special relativity. In another word, our current understanding of time, space, and momentum may not be complete. To discover what the incompleteness of special relativity is and to establish more complete space-time relativity are of great significance for deepening our understanding on the basic physical concepts such as space-time, substance and motion, resolving a series of basic problems in current physics and accelerating rapid development of physics.

1. The relativity of complex variable electromotive space-time

It is well-known that Einstein's special theory of relativity is based on two basic assumptions about inertial motion, that is

1. Relativity principle of motion: physical law has the same form in any inertial system;
2. Assumption of invariable light velocity: in any inertial system, the light velocity in vacuum is a constant.

According to these two assumptions, the famous Lorentz transform equation [2] can be derived.

$$X' = \gamma(X - V_X t) \quad (1)$$

$$Y' = Y \quad (2)$$

$$Z' = Z \quad (3)$$

$$t' = \gamma\left(t - \frac{V_X}{{C_0}^2}X\right) \quad (4)$$

$$\gamma = \frac{1}{\sqrt{1-\frac{{V_X}^2}{{C_0}^2}}} \quad (5)$$

Where $V_X$is the motion velocity along x-axis of inertial system $F'(X', Y', Z', t')$ relative to the inertial system $F(X, Y, Z, t)$. C_0 is the velocity of light.

Upon analyzing the basic assumptions of the special relativity, a question will be asked: is our understanding of motion complete? Is there another physical reference system except the inertial reference system, is the same the physical law in each reference system when it is under different states? Can this be tested through experiments? Does the related physical quantity have a limit? And would such limit lead to the relativistic effect of time and space?

After deepened study, It is found that equipotential bodies are of such physical reference systems.

Based on Coulomb's inverse propotional square law and its experiments [3], we know that within a confined conductor of any shape, the electric potential at any point is the same regardless of how many electric charges its surface carries. In addition, the interior electric field strength E is zero. Thus, an ideal experiment can be carried out: there are two identical metal confined carriages A and B, and they are steady relative to each other and are insulated from each other with only carriage B being grounded. Suppose that a pole of an ultra high voltage static electric generator is connected to B, the potential of the ground is zero, the other end of the ultra high static electric generator is connected to the carriage A. Once the generator starts running, continues to charge the carriage A with electrical charges (either positive or negative), the electric potential on the surface and in the interior of the carriage increasingly increases with it, when the surface electric charges reach Q; the interior and surface potential is ϕ and the same everywhere. At the same time, the electric field intensity is zero. Therefore for the people in the carriage A, it is impossible to know the magnitude of electric potential the positive or the negative through any experiment. That is the same feeling of the observer in the carriage B (where the electric potential is zero), even though there is a very high electric potential difference between them. This is the same as the feeling of the observer for the reference system of two phase even velocity linear motion. Therefore, we can put forward two assumptions of equipotential reference system:

3. Relative electric potential principle: physical law has the same form in any electric potential reference system;

4. Hypothesis of electric potential limit: there exists a potential difference limit Φ_0 in either reference systems of electric potential.

By comparison of the relative principle of electric potential, the assumption of electric potential limit as well as the basic assumption of the special relativity, it can be found that their forms are very similar. Is it possible to derive a symmetrical relativity from special relativity based on the assumption, that is the theory of electric potential relativity? Although in modern physics, electric potential and velocity are two completely different concepts, based on the similarity between assumptions1 and 2, and assumptions 3 and 4, a question must be asked: is there an innate connection between velocity and potential? More precisely, is there some inherent relationship between potential and velocity? It is obvious that scalar electrical potential cannot be converted into the real vector velocity of an object in our three-dimensional space, because the electrical potential of a conductive object does not have corresponding relation with its mechanical movement in any way. Therefore, for the two physical quantities to be related, new physical concepts of imaginary movement and complex movement must be introduced.

To investigate this problem, suppose at random point P in the space with equipotential ϕ in carriage A there is the following equation:

Where Φ_0 is the potential limit, ϕ is the potential difference between two reference systems, and β is the ratio between potential difference and potential limit.

$$\beta = \frac{\phi}{\Phi_0}$$

Multiply both the numerator and denominator of equation 6 with imaginary speed of light, iC_0, where, $i = \sqrt{-1}$ i.e.

$$\beta = \frac{\frac{\phi i C_0}{\Phi_0}}{iC_0} \tag{6}$$

The numerator on the right side of equation 6 is an imaginary number with the dimension of velocity, represented by $V_\phi i$ whose sign (positive or negative) is arbitrarily defined. In view of complex number, $V_\phi i$ is called imaginary motion or imaginary velocity, whose modulus isV_ϕ. That is:

$$\frac{\phi}{\Phi_0} = \frac{V_\phi}{C_0}$$

$$V_\phi i = \frac{\phi C_0}{\Phi_0} i = K\phi \tag{7}$$

Here, $K = \frac{C_0}{\Phi_0} i$, is an imaginary constant, and is the electromotive conversion factor.

From this it can be seen, every point in the equipotential carriage A is in the same imaginary dynamic state $V_\phi i$. At the same time, every point is also in the same state of real motion (real state) V_X. They are two completely different states of motion. This is the exact purpose of introducing imaginary factor i. To further investigate its space-time relationship, we must extend our conception motion and space-time from the real number domain to the complex number domain. The more general motion can be abstractly understood as the motion state of complex plane. If a coordinate reference system possesses both real and imaginary motion, then we call it as complex motion state. The reference system situated at complex motion state is called as complex electromotive inertial reference system; it has both equipotential and even velocity linear motion. The theory to describe this space-time relation of such motion is called relativity of complex variable electromotive space-time.

Although in mathematics, planar space can be describe using complex number and vectors, one will see that the physics fundamentals are more precisely described using complex number. Motion is abstractly as different states of the complex space. Because the module of the complex number or vector are equal when describing the same subject, therefore according to the actual case, the equation of complex number(the module of state –velocity) can be converted into the equation of vector motion. Vector is used here as supplementary physics quantity to help our understanding and derivation. Hence, to accommodate both the understanding of precise expression of physical concepts and the ease of narration, it is defined here that any complex nouns or vector noun combinations, such as complex velocity and complex displacement, or imaginary velocity and imaginary displacement, their physical meanings represents their state of motion and position of the state respectively.

As shown in Figure 1, suppose there are two complex coordinate systems of reference FOX and F'O'X' in the same complex plane, their virtual axis F and F', real axis X and X' are parallel each other, suppose FOX is the observing reference system and in steady state, i.e., the virtual velocity is zero(electrical potential is zero) and real velocity is also zero), the real velocity is also zero, while F'O'X' is the observed system and in complex motion state relative to FOX its complex velocity is V_w, its complex velocity is V_w, its virtual velocity is V_ϕ, and real velocity is V_X.

$$V_w = V_X + V_\phi i \tag{8}$$

The module of complex velocity is: $|V_w| = \sqrt{{V_X}^2 + {V_\phi}^2}$ (9)

Where, $V_\phi = \frac{\phi C_0}{\Phi_0}$ (10)

Because the complex space and two-dimensional vector space corresponds to each other, so for the ease of derivation and expression, here the module of complex number vector is introduced. If the complex coordinate system FOX 和F′O′X′ is regarded as Cartesian coordination system represented by $\boldsymbol{FOX}$ and $\boldsymbol{F'O'X'}$, therefore $\boldsymbol{F'O'X'}$ is moving relative to the system $\boldsymbol{FOX}$. Suppose the velocity vector is $\mathbf{V}_\theta$ and $\mathbf{V}_\theta$ is equal to the module of complex velocity.

Its direction is the direction of positive direction of axis X_1, and is called complex modulus velocity. Its components in the coordinate system $\boldsymbol{FOX}$ are $\mathbf{V}_X$ and $\mathbf{V}_\phi$. Hence, there is:

$$\mathbf{V}_\theta = \mathbf{V}_X + \mathbf{V}_\phi \tag{11}$$

$$V_\theta = |\mathbf{V}_\theta| = |V_w| \tag{12}$$

Suppose at a random planar point P_0 in the complex coordinate system FOX, the magnitude of the corresponding module $|R_{\mathbf{w}}|$ **is** equals to the magnitude of displacement vector R_p. Therefore, there is $|\mathbf{R_p}| = |R_{\mathbf{w}}|$, where the direction of $\mathbf{R_p}$ is the original direction point of the coordinate ***FOX***. Its components in the coordinate system ***FOX*** are **X** and **F**, and are called displacement vectors of module.

$$\mathbf{R_p} = \mathbf{X} + \mathbf{F} \quad (13)$$

Hence, their dot products are

$$\mathbf{R_p} \cdot \mathbf{V}_\theta = XV_X + FV_\phi \quad (14)$$

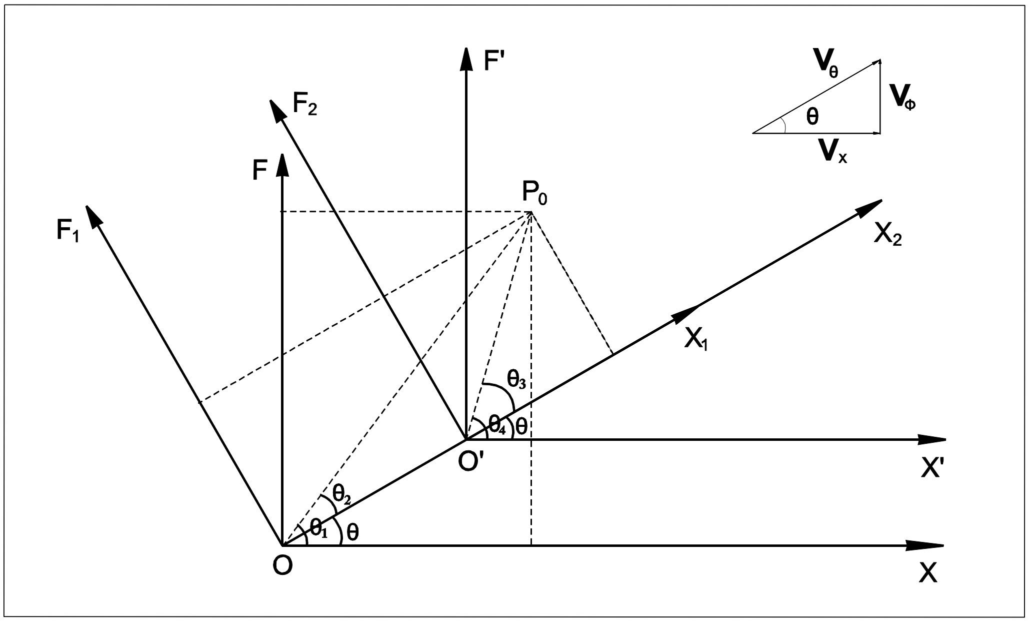

Figure 1

Since real number electric potential can be converted into virtual velocity, then by symmetry principle, real velocity can be converted into virtual potential $\phi_X i$. Therefore, $\phi_X i = V_X \frac{1}{K} = -\frac{V_X \Phi_0}{C_0} i$, where real potential ϕ and virtual potential ϕ_X together forms complex potential ϕ_w:

$$\phi_w = \phi + \phi_X i \quad (15)$$

The module of complex electric potential is:

$$|\phi_w| = \sqrt{\phi^2 + {\phi_X}^2} \quad (16)$$

Where, $\phi_X = -\frac{V_X \Phi_0}{C_0}$ (17)

Multiplying both sides of the equation (15) with K, and comparing with equation 8, we get:

$$\phi_w K = V_w \quad (18)$$

This shows that complex velocity and the complex potential are interconversible, the reference system of electric potential is a type of complex electromotive inertial reference system. To take one step further, the above four basic assumptions will be combined into two basic assumptions of complex electromotive space time relativity. Since complex number cannot be compared in magnitude, but their module can, hence we have:

5. The relativity of complex electromotive space-time: physical law has the same form in any complex electromotive inertial reference system;

6. Assumption of complex electromotive time –space limit: there is a limit in the module of complex velocity in any electromotive inertial reference system, it is equal to the light velocity C0 in vacuum, or complex potential module has limit Φ_0 (to be examined by experiment. Its value is about the Planck's voltage, where $\Phi_0 = 1.04295 \times 10^{27}$ volts)

If the inference 1 can be derived from assumption 6, the module of complex physical quantity and that of the vector physic quantity of two dimension space are equivalent.

If it can get the inference 2 from assumption 6, then the time in complex space is isotropic and is equal to the time in real space.

When $V_X = 0,\ V_w = V_\phi$, the two above hypotheses become the fundamental hypotheses of the relativity of electric potential. When $V_\phi = 0,\ V_w = V_X$, the two above hypotheses become the hypotheses of the special relativity. Hence, special relativity and relativity of electric potential are two special cases of the theory of complex electromotive relativity.

The special relativity is commonly referred to the motion of the observed system relative to the observing system along an axis and is called one dimension special relativity. In fact, such motion can have two-dimension or three-dimension form, the corresponding special relativity becomes more intricate but also more universal. There are already detailed discussions in literatures on this matter, showing in the real space, two-dimensional and three-dimensional special theory of relativity in arbitrary direction can be derived through rotation and translation of the coordinate system of one-dimensional special theory of relativity (5); or three-dimensional special theory of relativity can be derived through vector transformation of one-dimensional special theory of relativity (6, 7). Although both are different in their derivation methods, but there is a common point, that is, through the use of one-dimension special theory of relativity and appropriate mathematical approach, the higher real form of special theory of relativity can be derived. Therefore, based on the above mentioned hypotheses, there may be a transformation method that can derive the complex electromotive relativity of space-time by utilizing the special theory of relativity in combination with some transform method of complex coordinate system.

As shown in Figure 1, suppose there is a point P_0 in two dimension space. In different coordinate system, it can be represented by different coordinate parameters. In the complex coordinate system FOX, the coordinate of P_0 is represented using complex number $R_{\mathbf{w}} = X + Fi$. The complex angle is θ_1, in the coordinate system F'O'X', the coordinate of point P_0 is represented as complex number $R_{\mathbf{w}}' = X' + F'i$. Referring to two dimensional coordinate transformation of real numbers (4), and expanding it into the complex planar space, reference system F'O'X' can be obtained through three coordinate transformation from reference system FOX.

(1). In the complex coordinate system FOX, its time is t, rotating the coordinate system by θ degree counterclockwise, we get complex coordinate system F_1OX_1, whose time is t_1;

(2). Complex coordinate system F_1OX_1 is translated along axis X_1 of real number in the quantity equal to the module of the complex velocity $|V_w|$, we get complex coordinate system $F_2O'X_2$, whose time is t_2;

(3). Complex coordinate system $F_2O'X_2$ is rotated by θ degree clockwise, we get complex coordinate system $F'O'X'$, whose time is t'.

The detailed derivation steps are as follow:

A. The coordinate system FOX is related by θ degree counter clockwise, making the real axis $X_1$of the reference system F_1OX_1 pass through the origin point of the coordinate system $F'O'X'$. Point P_0 in FOX complex coordinate system is complex number W, whose complex angle is θ_1. Point P_0 in the coordinate system F_1OX_1 is complex number R_{w_1} whose complex angle is θ_2. That is,

$$R_w = X + Fi = |R_w|e^{\theta_1 i} \quad (19)$$

$$R_{w_1} = X_1 + F_1 i = |R_{w_1}|e^{\theta_2 i} \quad (20)$$

Also, $|R_w| = |R_{w_1}|$, therefore,

$$R_{w_1} = R_w\, e^{(\theta_2-\theta_1)i}$$

$$R_{w_1} = R_w\, e^{-\theta i} \quad (21)$$

$$R_{w_1} = (X + Fi)(\cos\theta - i\sin\theta) \quad (22)$$

From equations (12 and 13, we get:

$$X_1 = X\cos\theta + F\sin\theta \quad (23)$$

$$F_1 = F\cos\theta - X\sin\theta$$

B. Suppose F_1OX_1 is the static reference system, its time is t_1, $F_2O'X_2$ is the moving reference system whose time is t_2, and real axes $X_1$and X_2 are overlapping. The reference system $F_2O'X_2$ moves along the positive direction relative to F_1OX_1 in the magnitude of $|V_w|$. Since X_1 is the real number axis of F_1OX_1, and can be imagined as the real motion. Suppose when $t_1 = t_2 = 0$, origin points O and O′ are overlapping. At the same time, a lightening event occurs. According to hypothesis 6, the module of light velocity in two complex reference systems is equal in any direction and any complex angle. The module of complex velocity and the module of complex displacement of the complex coordinate system $F_2O'X_2$ are equivalent in velocity and distance in the plane of real number. So, we have:

$$|V_w| = V_\theta,\ |F_1 i| = F_1,\ |F_2 i| = F_2,\ |X_1| = X_1,\ |X_2| = X_2 \quad (24)$$

According to hypothesis 5, the forms of equations of the module of the same flashing complex velocity in both reference systems are the same.

$$|X_1|^2 + |F_1 i|^2 = |C_0 t_1|^2$$

$$|X_2|^2 + |F_2 i|^2 = |C_0 t_2|^2$$

Substituting equation (25) into the above equation,

$$X_1^{\,2} + F_1^{\,2} = (C_0 t_1)^2$$

$$X_2^{\,2} + F_2^{\,2} = (C_0 t_2)^2$$

This is the same status that the special relativity studies. Because the time is isotropic in the complex space, that is, time is not related to the rotary mathematic transformation of the coordinate, therefore $t = t_1$, $t_2 = t'$, and it can obtain:

$$X_2 = \gamma(X_1 - |V_w|t) \quad (25)$$

$$F_2 = F_1 \tag{26}$$

$$t' = \gamma\left(t - \frac{|V_w|}{C_0{}^2} X_1\right) \tag{27}$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|V_w|^2}{C_0{}^2}}}$ (28)

C. In complex coordinate reference system $F_2O'X_2$, P_0 is represented by complex number $R_{\mathbf{w}_2}$, complex angle θ_3, rotating clockwise θ degree, we get reference system $F'O'X'$. In $F'O'X'$, P_0 is represented by complex number $R_{\mathbf{w}}{}'$, complex angle θ_4:

$R_{\mathbf{w}_2} = X_2 + F_2 i = |R_{\mathbf{w}_2}| e^{\theta_3 i}$

$R_{\mathbf{w}}{}' = X' + F' i = |R_{\mathbf{w}}{}'| e^{\theta_4 i}$

And also, $|R_{\mathbf{w}}{}'| = |R_{\mathbf{w}_2}|$, hence,

$$R_{\mathbf{w}}{}' = R_{\mathbf{w}_2}\, e^{(\theta_4-\theta_3)i} = R_{\mathbf{w}_2}\, e^{\theta i} \tag{29}$$

Because $R_{\mathbf{w}_2} - R_{\mathbf{w}_1} = X_2 - X_1$, therefore,

$$R_{\mathbf{w}}{}' = (R_{\mathbf{w}_1} - (X_1 - X_2))\; e^{\theta i} \tag{30}$$

Substituting equation (21) into equation (30),

$R_{\mathbf{w}}{}' = R_{\mathbf{w}} - (X_1 - X_2)\, e^{\theta i}$

Because $V_w = |V_w| e^{\theta i}$, therefore,

$$R_{\mathbf{w}}{}' = R_{\mathbf{w}} - V_w \frac{(X_1 - X_2)}{|V_w|} \tag{31}$$

Suppose time $t_w = \frac{(X_1 - X_2)}{|V_w|}$, t_w is ratio of the module of the complex velocity and the real displacement difference at the direction of complex modular velocity it can be called the time of complex electromotive inertial systematic time, a real number. It is not the special time of a reference system. It is the time needed to solve fundamental physic problems. So we have:

$$t_w = \frac{(X_1 - X_2)}{|V_w|} \tag{32}$$

To sum up the above discussion, we obtain the basic relationship between time and space of the relativity of complex electromotive space-time space-time relativity as follows:

$$R_{\mathbf{w}}{}' = R_{\mathbf{w}} - V_w t_w \tag{33}$$

$$t' = \gamma\left(t - \frac{|V_w|}{C_0{}^2} X_1\right) \tag{34}$$

Where,, $\gamma = \frac{1}{\sqrt{1-\frac{|V_w|^2}{C_0{}^2}}}$ (35)

$$t_w = \frac{(X_1 - X_2)}{|V_w|} \tag{36}$$

$$X_2 = \gamma\left(\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_w|} - |V_w| t\right) \tag{37}$$

$$X_1 = X\cos\theta + F\sin\theta \tag{38}$$

Depending on difference purpose, the above relationship can be further expanded into different forms.

Substituting equation (32) into (33):

$$R_{\mathbf{w}}' = R_{\mathbf{w}} - \frac{V_w}{|V_w|}(X_1 - X_2) \quad (39)$$

Since $\cos\theta = \frac{V_X}{V_\theta}$, substituting $\sin\theta = \frac{V_\phi}{V_\theta}$ into (23), and from equation 14, we can get:

$$X_1 = \frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{V_\theta} \quad (40)$$

By substituting equation (40) into equation(25), we get:

$$X_2 = \gamma(\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} - |V_w|t) \quad (41)$$

And from equation(40) and equation （41）, we have:

$$X_1 - X_2 = (1-\gamma)\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t \quad (42)$$

Therefore, by substituting equation （42） into equation（39） and (40) into (34), we can obtain relational equations describing the motion in the relativity of complex of electromotive space-time:

$$R_{\mathbf{w}}' = R_{\mathbf{w}} - \frac{V_w}{|V_w|}\left((1-\gamma)\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t\right) \quad (43)$$

$$t' = \gamma\left(t - \frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{{C_0}^2}\right) \quad (44)$$

Where, $\gamma = \frac{1}{\sqrt{1 - \frac{|V_w|^2}{{C_0}^2}}}$ (45)

$$\frac{V_w}{|V_w|} = e^{\theta i} = (\cos\theta + i\sin\theta) \quad (46)$$

By decomposing (46) into equations of real number and imaginary number, we obtain:

$$X' = X - \frac{V_X}{|V_w|}\left((1-\gamma)\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t\right) \quad (47)$$

$$F'i = Fi - i\frac{V_\phi}{|V_w|}\left((1-\gamma)\frac{\mathbf{R_p} \cdot \mathbf{V_\theta}}{|V_w|} + |V_w|\gamma t\right) \quad (48)$$

Therefore when $\theta = 0$, $V_\phi = 0$, $|V_w| = V_X$, $\mathbf{R_p} \cdot \mathbf{V}_\theta = XV_X$, the form of the special relativity can be obtained:

$$X' = \gamma(X - V_X t)$$

$$t' = \gamma\left(t - \frac{V_X}{{C_0}^2}X\right)$$

$$F' = F$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{{V_X}^2}{{C_0}^2}}}$$

When $\theta = \frac{\pi}{2}$, $V_X = 0$, $|V_w| = V_\phi$, $\mathbf{R_p} \cdot \mathbf{V}_\theta = FV_\phi$, $|V_w| = \frac{C_0}{\Phi_0}\phi$, a new specific relativity can be obtained,

$$\mathrm{F}'i = \gamma\left(\mathrm{F}i - \frac{C_0}{\Phi_0}\phi i\mathrm{t}\right) \tag{49}$$

$$\mathrm{X}' = \mathrm{X} \tag{50}$$

$$\mathrm{t}' = \gamma\left(\mathrm{t} - \frac{\phi}{\Phi_0 \mathrm{C}_0}\mathrm{F}\right) \tag{51}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{\phi}{\Phi_0{}^2}^2}} \tag{52}$$

It is called relativity of the electric potential, which has the identical form of special relativity.

If dividing the both side of equation (48) by i, an equation of real number can be obtained:

$$\mathrm{F}' = \mathrm{F} - \frac{\mathrm{V}_\phi}{|\mathrm{V}_\mathrm{w}|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V}_\theta}{|\mathrm{V}_\mathrm{w}|} + |\mathrm{V}_\mathrm{w}|\gamma\mathrm{t}\right) \tag{53}$$

If V_ϕ is understood as the motion velocity V_Y at the direction of the axis X commonly perpendicular to V_Y, then equations(47), (53)and (44) become the general form of the scalar two-dimensional special relativity.

2. Quaternion relativity of electromotive space-time

The relativity of complex variant electromotive space-time describes the complex velocity motion status relationship between space-time in complex two dimensional spaces and the reference system, but its space of real number is one dimensional. In fact, our space of real number is tri-dimensional X, Y, Z. Therefore two dimensional space of the relativity of complex variant electromotive space-time must be expanded into four dimensions. In mathematics, the higher form of complex number is quaternion. Thus the relativity of complex variant electromotive space-time can be developed into quaternion relativity of electromotive space-time. Although the elements of quaternion can have many variations, they can all be converted uniformly into quaternion velocity or quaternion electric potential. Their corresponding reference system can be called reference system of equal quaternion velocity or equal quaternion electric potential.

Hence, the two basic hypotheses 5 and 6 o of the relativity of complex electromotive space-time can be further expanded into the basic hypotheses of the relativity of quaternion of electromotive space-time:

7. Relative principle of quaternion electromotive space-time: in any quaternion electromotive inertial reference system, physical laws have the same form;

8. Hypothesis of quaternion electromotive space-time limit: the module of quaternion velocity in any quaternion electromotive inertial reference system has a limitC_0, its value is equal to the velocity of light, or there is a limit Φ_0 in the module of quaternion electric potential(to be measured by experiment, its possible value is about the value of Plank's voltage, $\Phi_0 = 1.04295 \times 10^{27}$volts).

According to the basic equation (33) of the relativity of complex electromotive space-time, and expanding it into equation of real number and equation of imaginary number:

$$\mathrm{F}'i = \mathrm{F}i - \mathrm{V}_\phi i\mathrm{t}_\mathrm{w} \tag{54}$$

$$\mathrm{X}' = \mathrm{X} - \mathrm{V}_\mathrm{X}\mathrm{t}_\mathrm{w} \tag{55}$$

We noticed that equation (55)is a real scalar expression, while the fact is that the observed reference system $\mathrm{F}'\mathrm{O}'\mathrm{X}'$ moves along the real axis X of FOX with velocity $\mathbf{V_x}$, **where** $\mathbf{V_x}$ is a vector. Suppose its unit vector is $\mathbf{e}_1$, and because axes X' and X are the same direction as $\mathbf{e}_1$. Both sides of equation (55) are multiplied by $\mathbf{e}_1$, equation (56) becomes a vector equation:

$$\mathbf{X}' = \mathbf{X} - \mathbf{V}_\mathbf{x} t_w \quad (56)$$

Equations (54) and (56) describe the physical nature more objectively than the complex equation (33). Also, equation (56) can be expanded in three-dimension. If the vector $\mathbf{X}'$ and $\mathbf{X}$ in equation (56) are defined as $\mathbf{r}'$ and $\mathbf{r}$ of three dimension space X, Y, Z and X', Y', Z', corresponding velocity $\mathbf{V}_\mathbf{x}$ is defined as $\mathbf{V}_\mathbf{r}$. Also, $\mathbf{r}', \mathbf{r}$ and $\mathbf{V}_\mathbf{r}$ all have same direction. Suppose $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ **are** the unit vectors in the coordinate of three dimensional space along the axis X, Y, Z, respectively, we have:

$$\mathbf{r} = X\mathbf{e_1} + Y\mathbf{e_2} + Z\mathbf{e_3} \quad (57)$$

$$\mathbf{r}' = X'\mathbf{e_1} + Y'\mathbf{e_2} + Z'\mathbf{e_3} \quad (58)$$

$$\mathbf{V}_\mathbf{r} = V_X\mathbf{e_1} + V_Y\mathbf{e_2} + V_Z\mathbf{e_3} \quad (59)$$

In Figure 1, the physical quantities of X', X_1, X_1 and X will have corresponding changes. Suppose t_w becomes t_q, X_1 becomes R_1, X_2 becomes R_2, then equation (54) and (56) can be converted into:

$$F'i = Fi - V_\phi i t_q \quad (60)$$

$$\mathbf{r}' = \mathbf{r} - \mathbf{V}_\mathbf{r} t_q \quad (61)$$

Equation (60) and (61) forms a quaternion space system composed of three real number vectors and one imaginary number, and is called type Y_i quaternion space. Obviously it is not Hamiltonian quaternion. Expand the motion status of the reference system in the complex space into motion of the type Y_i quaternion space, that is, the reference system $F'_q(F', X', Y', Z',)$ moves with velocity V_q (V_ϕ, V_X, V_Y, V_Z) relative to reference system $F_q(F, X, Y, Z)$.Suppose R'_q, R_q are quaternion displacement coordinates at any point P_0 in the reference system F'_q of motion in Y_i type quaternion space and in the reference system F_q, and there is:

$$R_q = Fi + \mathbf{r} = Fi + X\mathbf{e_1} + Y\mathbf{e_2} + Z\mathbf{e_3} \quad (62)$$

The module of R_q is $|R_q|$, that is $|R_q| = \sqrt{F^2 + X^2 + Y^2 + Z^2}$ (63)

$$R'_q = F'i + \mathbf{r}' = F'i + X'\mathbf{e_1} + Y'\mathbf{e_2} + Z'\mathbf{e_3} \quad (64)$$

The module of R'_q is $|R'_q|$, i.e., $|R'_q| = \sqrt{F'^2 + X'^2 + Y'^2 + Z'^2}$ (65)

Type Y_i quaternion velocity V_q and its module $|V_q|$ are:

$$V_q = V_\phi i + \mathbf{V}_\mathbf{r} = V_\phi i + V_X\mathbf{e_1} + V_Y\mathbf{e_2} + V_Z\mathbf{e_3} \quad (66)$$

$$|V_q| = \sqrt{{V_\phi}^2 + {V_X}^2 + {V_Y}^2 + {V_Z}^2} \quad (67)$$

Where $i = \sqrt{-1}$

Adding equation (60)with (61), and substituting (62), (64) and (66)into it, also substituting the physical quantities of equations (33), (34), (23),(25) and (32) with corresponding type Y_i quaternion number, the basic relational space-time equation for type Y_i quaternion relativity can be obtained.

$$R_q' = R_q - V_q t_q \quad (68)$$

$$t' = \gamma\left(t - \frac{|V_q|}{{C_0}^2} r_1\right) \quad (69)$$

where， $\gamma = \frac{1}{\sqrt{1-\frac{|V_q|^2}{C_0{}^2}}}$ (70)

$R_1 = r\cos\theta + F\sin\theta$ (71)

$R_2 = \gamma\left(R_1 - |V_q|t\right)$ (72)

$t_q = \frac{(R_1 - R_2)}{|V_q|}$ (73)

Depending on the need, the above basic equation can be transformed into different variations.

Substituting equation (73) into(68), we have:

$R_q{}' = R_q - \frac{V_q}{|V_q|}(R_{\mathbf{1}} - R_{\mathbf{2}})$ (74)

Suppose there are displacement vector $\mathbf{R_p} = \mathbf{F} + \mathbf{X} + \mathbf{Y} + \mathbf{Z}$ and velocity vector $\mathbf{V}_\theta = \mathbf{V}_\phi + \mathbf{V}_X + \mathbf{V}_Y + \mathbf{V}_Z$

$V_\theta = \sqrt{V_\phi{}^2 + V_X{}^2 + V_Y{}^2 + V_Z{}^2} = |V_q|$

Then their dot product is $\mathbf{R_p} \cdot \mathbf{V}_\theta = XV_X + YV_Y + ZV_Z + FV_\phi$

Because $\cos\theta = \frac{V_r}{|V_q|}$, $\sin\theta = \frac{V_\phi}{|V_q|}$, $\mathbf{r} = X\frac{V_X}{V_r} + Y\frac{V_Y}{V_r} + Z\frac{V_Z}{V_r}$, so there is:

$r_1 = r\cos\theta + F\sin\theta = \frac{\mathbf{R_p}\cdot\mathbf{V}_\theta}{|V_q|}$

$r_2 = \gamma(\frac{\mathbf{R_p}\cdot\mathbf{V}_\theta}{|V_q|} - |V_q|t)$

$r_1 - r_2 = (1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V}_\theta}{|V_q|} + V\gamma t$

Hence the velocity equation of the relativity of type Y_i quaternion electromotive space-time is:

$R_q{}' = R_q - \frac{V_q}{|V_q|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V}_\theta}{|V_q|} + |V_q|\gamma t\right)$ (75)

$t' = \gamma\left(t - \frac{\mathbf{R_p}\cdot\mathbf{V}_\theta}{C_0{}^2}\right)$ (76)

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|V_q|^2}{C_0{}^2}}}$ (77)

When $\theta = 0$， $\mathbf{V}_\theta = \mathbf{V_r}, \mathbf{R_p} = \mathbf{r}$， $V_\phi = 0, R_q{}' = \mathbf{r}'$, $R_q = \mathbf{r}$, $\frac{V_q}{|V_q|} = \frac{\mathbf{V_r}}{|V_r|}$, then the form [5] and [6] of special relativity at any direction $\mathbf{r}(\mathbf{x},\mathbf{y},\mathbf{z})$ in tri-dimension space of the three dimension can be obtained0 [5], [6].

$\mathbf{r}' = \mathbf{r} - \frac{\mathbf{V_r}}{|\mathbf{V_r}|}((1-\gamma)\frac{\mathbf{r}\cdot\mathbf{V_r}}{V_r} + V_r\gamma t)$

$t' = \gamma\left(t - \frac{\mathbf{r}\cdot\mathbf{V_r}}{C_0{}^2}\right)$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{V_r{}^2}{C_0{}^2}}}$

Because imaginary velocity V_ϕ and real electric potential ϕ are convertible each other, and real velocity can be converted into imaginary electric potential vector, hence the expression of the velocity of the type Y_i quaternion electromotive relativity can be converted into another type of electric potential expression of quaternion electromotive relativity, and suppose this new quaternion is H-type quaternion.

Multiplying both sides of equation (75) by $(-i.)$, we get:

$$R_q{}'(-i) = R_q(-i) - \frac{V_q}{|V_q|}(-i)\left((1-\gamma)\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|} + |V_q|\gamma t\right) \quad (78)$$

Suppose H-type quaternion electric potential is ϕ_h**,** and there is:

$$\phi_h = \frac{V_q}{K} = (V_\phi i + V_X\mathbf{e_1} + V_Y\mathbf{e_2} + V_Z\mathbf{e_3})\frac{\Phi_0}{C_0}(-i)$$

Suppose $\phi = \frac{V_\phi \Phi_0}{C_0}$, $\phi_X = \frac{V_X\Phi_0}{C_0}$, $\phi_Y = \frac{V_Y\Phi_0}{C_0}$, $\phi_Z = \frac{V_Z\Phi_0}{C_0}$, so there is:

$$\phi_h = (\phi i + \phi_X\mathbf{e_1} + \phi_Y\mathbf{e_2} + \phi_Z\mathbf{e_3})(-i) \quad (79)$$

Suppose $\phi_q = \phi i + \phi_X\mathbf{e_1} + \phi_Y\mathbf{e_2} + \phi_Z\mathbf{e_3}$**,** that is , ϕ_q is the quaternion electric potential of type Y_i, we have:

$$\phi_h = (-i)\phi_q \quad (80)$$

$$|\phi_h| = |\phi_q|$$

Suppose $\mathbf{e_1}(-i) = \mathbf{i}$, $\mathbf{e_2}(-i) = \mathbf{j}$, $\mathbf{e_3}(-i) = \mathbf{k}$ (81)

Also let i, j, k be the imaginary unit vector, and let corresponding real unit vector be $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$**.** $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ are perpendicular to each other**,** so, $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are three imaginary unit vector perpendicular to each other. Substituting them into equation(79), we get:

$$\phi_h = \phi + \phi_X\mathbf{i} + \phi_Y\mathbf{j} + \phi_Z\mathbf{k} \quad (82)$$

This shows that H-type quaternion potential ϕ_h is composed of one scalar electric potential and three imaginary vectors of electric potential. For the same reason, the displacement of H-type quaternion is $R_h{}'$ and R_h**:**

$$R_h{}' = R_q{}'(-i) = F' + X'\mathbf{i} + Y'\mathbf{j} + Z'\mathbf{k} \quad (83)$$

$$R_h = R_q(-i) = F + X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k} \quad (84)$$

Multiplying $\frac{1}{K}$ with both numerator and denomanator of $\frac{V_q}{|V_q|}$, and also from the definition of ϕ_h, we get:

$$\frac{V_q}{|V_q|}(-i) = \frac{\phi_h}{|\phi_h|} \quad (85)$$

Hence $|V_q| = \frac{C_0}{\Phi_0}|\phi_h|$ (86)

Suppose, $\boldsymbol{\phi}_\theta = \frac{C_0}{\Phi_0}\mathbf{V}_\theta$ (87)

Then, $\frac{\mathbf{R_p}\cdot\mathbf{V_\theta}}{|V_q|} = \frac{\mathbf{R_p}\cdot\boldsymbol{\phi}_\theta}{|\phi_h|}$ (88)

Substituting equation(83),(84) ,(85) and (88) into the electric potential equation of the relativity of complex variable electromotive:

$$\mathrm{R_h}' = \mathrm{R_h} - \frac{\boldsymbol{\phi}_\mathbf{h}}{|\phi_\mathrm{h}|}\left((1-\gamma)\frac{\mathbf{R_p}\cdot\boldsymbol{\phi}_\theta}{|\phi_\mathrm{h}|} + \frac{C_0}{\Phi_0}|\phi_\mathrm{h}|\gamma t\right) \quad (89)$$

$$t' = \gamma\left(t - \frac{\mathbf{R_p}\cdot\boldsymbol{\phi}_\theta}{\Phi_0 C_0}\right) \quad (90)$$

Where, $\gamma = \frac{1}{\sqrt{1-\frac{|\phi_\mathrm{h}|^2}{\Phi_0{}^2}}}$, (91)

By analyzing the H-type quaternion, it was discovered that not only is it composed of one scalar and three imaginary vectors, but also the sum of square of the module of four components is equal to the square of the module of the H-type quaternion. And there is:

$$\mathbf{i^2} = \mathbf{j^2} = \mathbf{k^2} = -1 \quad (92)$$

$$\mathbf{ij} = -\mathbf{ji} = \mathbf{k},\ \mathbf{jk} = -\mathbf{kj} = \mathbf{i},\ \mathbf{ki} = -\mathbf{ik} = \mathbf{j} \quad (93)$$

H-type Quaternion is the Hamilton quaternion. According to the nature of the Hamiltonian quaternion, we have:

$$\frac{\boldsymbol{\phi}_\mathbf{h}}{|\phi_\mathrm{h}|} = e^{\theta I} \quad (94)$$

Where, $I = \frac{\phi_X\mathbf{i}+\phi_Y\mathbf{j}+\phi_Z\mathbf{k}}{\sqrt{\phi_X{}^2+\phi_Y{}^2+\phi_Z{}^2}}$ (95)

$$\theta = \arctan\left(\frac{\sqrt{\phi_X{}^2+\phi_Y{}^2+\phi_Z{}^2}}{\phi}\right) \quad (96)$$

Because the dot product Hamilton quaternion is equal to a dot product of four-dimensional vector,

$$\mathbf{R_p}\cdot\boldsymbol{\phi}_\theta = \mathrm{R_h}\cdot\phi_\mathrm{h} \quad (97)$$

Therefore, the potential space-time of the Hamiltonian quaternion electromotive time- space relativity can be expressed using Hamilton quaternion’s variable equation.

This shows that our space-time composed of two kinds of quaternion space and time. They are closely linked and are inter-convertible. The relativity of electromotive time- space reveals their intrinsic nature and their mutual relations. Also the theory unites inertial motion, electromagnetic motion, time and space. This revolution in the concept of time and space will cause a series of corresponding changes in physics, which subsequent papers will further explore.

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