

# Power generator based on atmospheric pressure and gravity

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## Abstract

This is a study of a relatively simple system that seems to provide an energy output greater than the visible energy input thanks to the extraction of energy from the surrounding atmosphere. This is a theoretical study, no experiments with real systems of this type has been performed by the author. The idea of this system is the possibility of using hydrostatic pressure to obtain compressed air to be used in various ways, however the source of energy is not the water pressure but the atmospheric pressure.

## 1 Description

The system is composed of two cylinders, 1 and 2, with cylinder 1 having a larger diameter than cylinder 2. The two cylinders have pistons rigidly connected so that the discs in the cylinders move always together.

Cylinder 1 has an input valve 1 from which water at a high pressure comes in. This water comes from a tank placed at an height  $H$  above the median line of the cylinders.

Cylinder 2 has an output valve 2 from which water is pumped back into the tank.

Between the two cylinders there is a valve 3 that allows the water to be transferred from cylinder 1 to cylinder 2 during the return of the piston.

The remaining part of cylinder 1 is filled with air initially at atmospheric pressure. This air can exit from cylinder 1 by means of valve 4 and go to a device that converts it into another form of energy.

The system is surrounded by air at the atmospheric pressure which pushes the disc 2 from the outside. The external air can enter into cylinder 1 by means of valve 5 when the piston is retracted.

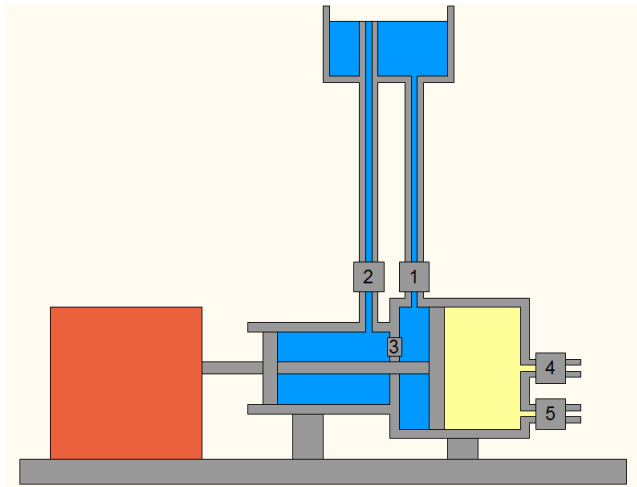


Figure 1: System

Finally there is an external motor that can push or pull the pistons when it is needed

## 2 Cycle

### Initial state

At the initial state the piston is at its leftmost position, cylinder 2 is full of water, left part of cylinder 1 is full of water just to ensure continuity of action, and the compression chamber is at its maximum volume, filled with air at the atmospheric pressure. Valves 1, 2, 3, 4 and 5 are all closed.

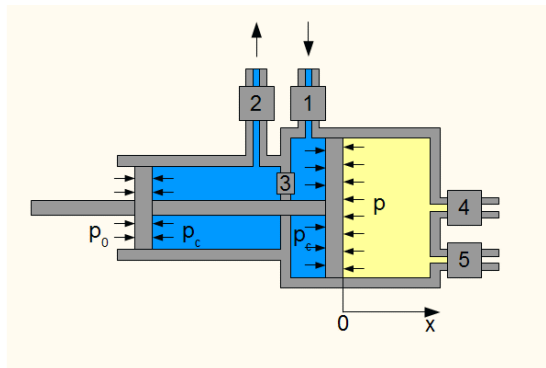


Figure 2: Basic system

### Active phase

From the initial state, valves 1 and 2 are suddenly opened, then the water inside the cylinders acquires pressure, the same pressure in both cylinders, but disc 1 is larger than disc 2 so there is a net force that pushes the piston towards right. The piston starts moving towards right and so it compresses the air. At the same time disc 2 pushes water from cylinder 2 up into the tank. The piston accelerates, but the pressure of the air in cylinder 1 increases, so the piston reaches a maximum speed, then starts to decelerate and eventually stops. This is the point of maximum compression. At this point valve 4 is opened and the compressed air is allowed to exit into the conduit, expanding and providing work on to an external device.

### Push phase

Starting from the end of the active phase, valve 3 and 5 are still closed and valves 2 and 4 are still open. Valve 1 is closed to stop water entering in cylinder 1. An external motor starts pushing the piston towards right to finish pumping water from cylinder 2 into the tank. At the same time disc 1 helps the air to exit, however the air has already exited for the major part because of the expansion into the conduit. This phase requires a work  $W_{mp}$  from the motor. When the piston has pushed all the water from cylinder 2 back into the tank the motor stops and valves 2 and 4 are closed.

### Return phase

Starting from the end of the push phase now valves 1, 2 and 4 remain closed, while valves 3 and 5 are opened, and the motor starts pulling the piston back. Then new air enters into the compression chamber through valve 5 with atmospheric pressure, and the water from cylinder 1 passes into cylinder 2 through valve 3. This phase requires little work by the motor because the water has to pass from a cylinder to another at the same height, so the only energy expended is that for energy losses in the water flow. When the piston has returned to the initial position the motor stops, valves 3 and 5 are closed and the system is again in the initial state, and the amount of water in the tank hasn't changed. The energy expended by the motor in this phase is called  $W_{mr}$  and is small.

## 3 Model

The height of water in the tank is considered constant during the cycle because the tank's horizontal section is supposed to be much larger than the cylinder's section. The corresponding hydrostatic pressure is then constant:

$$p_1 = \rho g H \tag{1}$$

The total pressure of the water in the cylinders is given by:

$x$	position of the right face of the piston	variable
$v$	speed of the piston, equal to $\dot{x}$	variable
$A_1$	area of the the right disc of the piston	constant
$A_2$	area of the left disc of the piston	constant
$m$	total mass of the piston	constant
$p_0$	atmospheric pressure	constant
$p_1$	hydrostatic pressure due to the tank, equals to $\rho g H$	constant
$p_c$	hydrostatic pressure in the cylinders, equals to $p_0 + p_1$	constant
$p$	pressure of air in compression chamber	variable
$V$	volume of compression chamber	variable
$V_0$	maximum volume of compression chamber, equals to $A_1 b$	constant
$g$	gravity constant	constant
$\rho$	water density	constant
$H$	external water height	constant
$b$	maximum length of the compression chamber	constant
$s$	length of the push section	variable
$x_m$	distance traveled by the piston in the active phase	variable
$\xi$	compression ratio, equals to $x_m/b$	variable

Table 1: Model's variables and constants

$$p_c = p_0 + p_1 \quad (2)$$

The actual pressure exerted by the water inside the cylinders on the two discs is supposed to be equal to  $p_c$  also during the motion of the piston.

This is the strongest hypothesis of this model, because when the water pushes the piston, the piston accelerates and so the effective pressure of the water may be different from the static one. Furthermore the left disc moves against the water pressure while the right disc moves in the same direction of the water pressure. However the water from the left cylinder is pushed away and goes up into the tank, while the water in the right cylinder comes down to occupy the space left by the disc that moves to the right. In static conditions the water pressure on the discs would be  $p_c$  so we can suppose that the piston moves slowly enough.

Notice that the area of disc 1 subject to the pressure of the air is slightly larger than the area subject to the pressure of the water because of the piston's shaft, this has to be taken into account into a more detailed model.

Anyway a complete analysis of this system would require an hydrodynamic model of the fluid inside the cylinder, and such model requires to know the exact shape of the internal walls and the valves.

The volume of the compression chamber is given by:

$$V = V_0 - A_1 x = V_0 \left(1 - \frac{x}{b}\right) \quad (3)$$

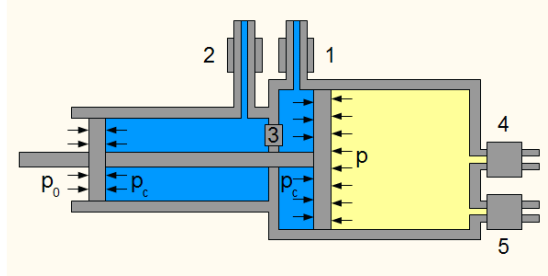


Figure 3: Active phase

where  $V_0 = A_1 b$  is the initial volume, corresponding to  $x = 0$ . The position  $x$  is always between 0 and  $b$ .

The expression of the pressure of the air as a function of the volume of air depends on the kind of compression. The process can be adiabatic, isothermal or an intermediate type. However the pressure of the air will increase as the volume decreases, and it will be a function of the piston's position.

## 4 Active phase

During the active phase, the equation of motion of the piston along the  $x$  axis is:

$$p_0 A_2 - (p_1 + p_0) A_2 + (p_1 + p_0) A_1 - p A_1 = m \ddot{x} \quad (4)$$

so simplifying:

$$p_1 (A_1 - A_2) + p_0 A_1 - p A_1 = m \ddot{x} \quad (5)$$

or:

$$m \ddot{x} + p A_1 = F_a \quad (6)$$

with:

$$F_a = p_0 A_1 + p_1 (A_1 - A_2) \quad (7)$$

The force  $F_a$  is originated by the difference in the piston's areas and from the atmospheric pressure exerted on disc 2 but not on disc 1.

Notice that the piston starts moving towards right even if the pressure of the tank  $p_1$  is lower than the atmospheric pressure  $p_0$  because the total pressure of the water in the cylinders is  $p_c = p_0 + p_1$  which is greater than  $p_0$  as long as  $p_1$  is positive.

In the compression phase the air receives mechanical work from the piston. Let's call  $w_c$  the total work done on the internal air during the time interval

$[0, t]$ . To calculate this work we need to know the pressure of the air for each intermediate value of volume. We have:

$$w_c = \int_0^t p dV \quad (8)$$

or:

$$w_c = \int_0^t p \dot{x} A_1 d\tau \quad (9)$$

so:

$$w_c = A_1 \int_0^t p \dot{x} d\tau \quad (10)$$

Notice that the pressure  $p$  is a function of the position  $x$  and not of the time.

From the differential equation for the motion of the piston we can find the speed of the piston as a function of the piston's position. We multiply the equation by the speed  $\dot{x}$ :

$$m \dot{x} \ddot{x} + p A_1 \dot{x} = F_a \dot{x} \quad (11)$$

then integrate from the initial time to the current time:

$$m \int_0^t \dot{x} \ddot{x} d\tau + A_1 \int_0^t p \dot{x} d\tau = F_a \int_0^t \dot{x} d\tau \quad (12)$$

Having the conditions  $x(0) = 0$  and  $\dot{x}(0) = 0$ , we find:

$$\int_0^t \dot{x} \ddot{x} d\tau = \frac{\dot{x}^2}{2} \quad (13)$$

$$\int_0^t \dot{x} d\tau = x \quad (14)$$

and for the second integral we have:

$$\int_0^t p \dot{x} d\tau = \frac{1}{A_1} \int_0^t p A_1 \dot{x} d\tau = \frac{w_a}{A_1} \quad (15)$$

So we have:

$$\frac{1}{2} m v^2 + w_a = F_a x \quad (16)$$

To find the point where the piston stops and starts moving backwards, we use the equation of the piston's speed. Putting the piston's speed to zero we obtain:

$$w_c(x_m) = F_a x_m \quad (17)$$

where  $w_c$  is the function that gives the work done on the air as a function of  $x$ . To determine  $x_m$  we need the exact form of the function  $w_c$ , which depends on the type of compression. It is useful to introduce the normalized piston's end point:

$$\xi = \frac{x_m}{b} \quad (18)$$

which is an dimensionless variable, with a value between 0 and 1, which can be called compression factor.

The total work done on the air from the beginning of the active phase until the piston stops at the point of maximum compression is then:

$$W_c = F_a x_m \quad (19)$$

or:

$$W_c = [p_0 A_1 + p_1 (A_1 - A_2)] x_m \quad (20)$$

It is convenient to normalize it respect to the energy  $p_0 V_0$ :

$$y_c = \frac{W_a}{p_0 V_0} \quad (21)$$

so:

$$y_c = \frac{p_0 A_1 + p_1 (A_1 - A_2)}{p_0 V_0} x_m \quad (22)$$

which after some passages becomes:

$$y_c = \left[ 1 + \frac{p_1}{p_0} \left( 1 - \frac{A_2}{A_1} \right) \right] \xi \quad (23)$$

During the active phase the right cylinder receives a volume of water  $V_1 = A_1 x_m$  from the tank, and the left cylinder pumps a volume of water  $V_2 = A_2 x_m$  back into the tank. The volume  $V_1$  of water was entirely into the left cylinder at the beginning of the cycle, so, when the piston stops at  $x_m$ , the volume of water that remains inside the left cylinder is:

$$V_p = A_1 x_m - A_2 x_m = (A_1 - A_2) x_m \quad (24)$$

During the active phase the external air does a positive work  $W_{aa}$  on the piston because atmospheric pressure pushes disc 2 from the outside towards right:

$$W_{aa} = p_0 A_2 x_m \quad (25)$$

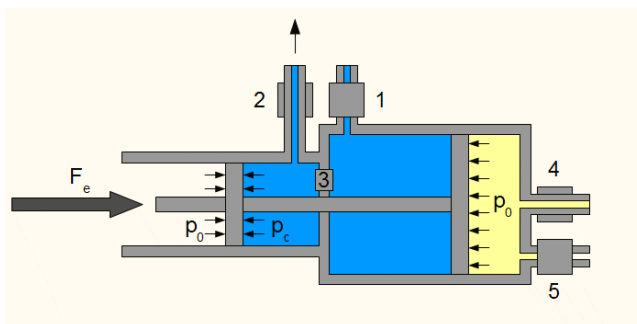


Figure 4: Push phase

## 5 Push phase

The push phase starts at the end of the active phase. Cylinder 2 has to pump a volume of water  $V_p$  and to push this water back into the tank, disc 2 has to move for an additional distance  $s$  that is given by:

$$s = \frac{V_p}{A_2} = \left( \frac{A_1}{A_2} - 1 \right) x_m \quad (26)$$

In the push phase the water pressure in cylinder 1 is zero because valve 1 has been closed, and the pressure of the air in the compression chamber is the atmospheric pressure  $p_0$  because the air has already expanded, out of the compression chamber, and returned to its original pressure. The equation of motion of the piston during the push phase is then:

$$m\ddot{x} = F_e + p_0 A_2 - p_c A_2 - p_0 A_1 \quad (27)$$

where  $F_e$  is the external force applied by the motor on the shaft. The force  $F_e$  doesn't have to be constant, in fact the motor should push with a variable force in such a way that the piston arrives at the end of the run with zero speed. So inserting  $p_c = p_0 + p_1$  we have:

$$m\ddot{x} = F_e - p_0 A_1 - p_1 A_2 \quad (28)$$

and then:

$$m\ddot{x} + F_p = F_e \quad (29)$$

with:

$$F_p = p_0 A_1 + p_1 A_2 \quad (30)$$

Let's take the origin of time at the beginning of the push phase, and let's take the origin of  $x$  at the initial position of the push phase. Let  $t_p$  be the time required to complete the push phase and let's suppose that the external



force pushes in such a way that the piston stops at the final point. Then we have  $x(0) = 0$ ,  $\dot{x}(0) = 0$ ,  $x(t_p) = s$ ,  $\dot{x}(t_p) = 0$ . From the equation of motion, multiplying by the speed, we have:

$$m\dot{x}\ddot{x} + F_p\dot{x} = F_e\dot{x} \quad (31)$$

now integrating from 0 to  $t$  and using the initial conditions:

$$\frac{1}{2}m\dot{x}^2 + F_px = \int_0^t F_e\dot{x}d\tau \quad (32)$$

The last integral is the work done by the external force  $F_e$  during the interval  $[0, t]$ . At the instant  $t_p$  the speed must be zero and the position must be  $s$  so the total work of the external force in the push phase is:

$$W_{mp} = \int_0^{t_p} F_e\dot{x}d\tau = F_ps \quad (33)$$

This is the value of work whatever the variation of the force  $F_e$ , as long as  $F_e$  causes the piston to arrive at distance  $s$  and stop there. So we have found:

$$W_{mp} = (p_0A_1 + p_1A_2) \left( \frac{A_1}{A_2} - 1 \right) x_m \quad (34)$$

The normalized work of the motor in the push phase is then:

$$y_p = \frac{W_p}{p_0V_0} = \left( 1 + \frac{p_1}{p_0} \frac{A_2}{A_1} \right) \left( \frac{A_1}{A_2} - 1 \right) \xi \quad (35)$$

During the push phase the external air does a work on the piston because atmospheric pressure pushes disc 2 from the outside towards right and atmospheric pressure pushes disc 1 from the outside towards left. The work done by the atmosphere during the push phase is then:

$$W_{ap} = p_0A_2s - p_0A_1s = p_0s(A_2 - A_1) \quad (36)$$

which is negative because  $A_2 < A_1$ . Inserting the expression of  $s$  in terms of  $x_m$ :

$$W_{ap} = p_0 \left( \frac{A_1}{A_2} - 1 \right) (A_2 - A_1)x_m \quad (37)$$

## 6 Return phase

The return phase starts at the end of the push phase, with the piston at its rightmost position. Valves 1, 2, and 4 are closed, valve 3 and 5 are open. Since valves 1 and 2 are closed the water in the cylinders has zero pressure. The external air enters in the compression chamber with atmospheric pressure through valve 5, so disc 1 receives a force  $p_0A_1$  towards left, while disc 2 receives a force  $p_0A_2$  towards right. Then the piston starts moving towards left and

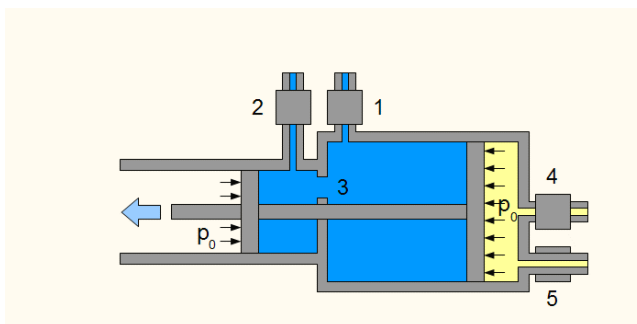


Figure 5: Return phase

accelerates. The motor now has to do a negative work to brake the piston and let it arrive at the leftmost position with zero speed. The water flows from cylinder 1 to cylinder 2.

At the end of the return phase the piston is at its leftmost position, which is the start position for another cycle, and the volume of water  $A_1 x_m$  has passed into cylinder 2.

During the return phase the external air does a work that is balanced by the motor and losses in water flow. If  $W_{mr}$  is the work done by the motor in the return phase, and  $W_{ar}$  is the work done by the external air in the return phase then we must have:

$$W_{mr} = -W_{ar} \quad (38)$$

The work done by the atmosphere during the return phase is:

$$W_{ar} = p_0 A_1 (x_m + s) - p_0 A_2 (x_m + s) \quad (39)$$

and so:

$$W_{ar} = p_0 (A_1 - A_2) \frac{A_1}{A_2} x_m \quad (40)$$

## 7 Compressed air energy

In the compression phase the air goes through a thermodynamic process: it receives mechanical work from the piston, and it can also exchange some heat with the cylinder. At the end of the compression the internal energy of the air in the compression chamber may have changed. If  $W_a$  is the total work done by the piston on the air, and  $Q$  is the heat given by the air to the cylinder, then the change of internal energy of the air is:

$$\Delta U = W_a - Q \quad (41)$$

## Isothermal process

In the isothermal case the temperature of the air doesn't change, and since the internal energy of an ideal gas depends only on the amount of gas and the temperature, the internal energy does not change, which means  $Q = W_a$ . This means that the work done on the air goes entirely lost in heat that the air gives to the cylinder. In this case to return to its original volume and pressure the air has to absorb the same amount of heat but this is not possible in practice.

## Adiabatic process

In the adiabatic case the air does not give any heat to the cylinder, and so its internal energy increases, and  $\Delta U = W_a$ , this means that the temperature of the air rises. The air can then do some work to an external device before returning to the initial pressure and temperature. Ideally the compressed air would do a work equal to the work received by the piston, but of course there are some heat losses so the work done by the compressed air will be a little lower than  $W_a$ .

## 8 Energy balance

After a cycle the water has returned in the same initial position so the energy of water hasn't changed. Instead the compressed air has been extracted from the system taking away an energy  $W_c$ . The motor has done a total work  $W_m = W_{mp} + W_{mr}$  on the system and the atmosphere has done a total work  $W_a = W_{aa} + W_{ap} + W_{ar}$  on the system. Then the energy balance of the system in a cycle should be:

$$W_c = W_m + W_a \quad (42)$$

Since the atmospheric pressure is constant, the energy that we must provide to the system to complete a cycle is just the work  $W_m$  so the net energy that we obtain from this system in a cycle is:

$$W_n = W_c - W_m = W_a \quad (43)$$

So in theory the system provides always a positive net energy at each cycle, and this energy is equal to the work done on the system by the atmosphere in a cycle.

Notice that during the return phase the motor works as a brake and probably the energy  $-W_{mr}$  cannot be converted back into electric energy, so perhaps a better estimation of the net energy is  $W_c - W_{mp}$  which is lower.

In practice the energy acquired by the air during the compression must also be converted into electric energy by means of a turbofan or another system and this conversion has losses too.

Using the expressions of the energies found before, the total work done by the atmosphere is:

$$W_a = p_0 A_2 x_m + p_0 (A_2 - A_1) \left( \frac{A_1}{A_2} - 1 \right) x_m \quad (44)$$

$$+ p_0 (A_1 - A_2) \frac{A_1}{A_2} x_m = p_0 A_1 x_m = p_0 V_0 \xi \quad (45)$$

and the total work done by the motor is:

$$W_m = (p_0 A_1 + p_1 A_2) \left( \frac{A_1}{A_2} - 1 \right) x_m - p_0 (A_1 - A_2) \frac{A_1}{A_2} x_m \quad (46)$$

$$= p_1 (A_1 - A_2) x_m \quad (47)$$

and so:

$$W_m + W_a = [p_0 A_1 + p_1 (A_1 - A_2)] x_m = W_c \quad (48)$$

so the expressions obtained from the model satisfy the energy balance. It is convenient to define the following parameters:

$$\alpha = \frac{A_2}{A_1} \quad (49)$$

$$\beta = \frac{p_1}{p_0} \quad (50)$$

The parameter  $\alpha$  is always between 0 and 1, while the parameter  $\beta$  is just positive. With these parameters the expressions of the energies normalized respect to  $p_0 V_0$  are:

$$y_{aa} = \alpha \xi \quad (51)$$

$$y_{ap} = (\alpha - 1) \left( \frac{1}{\alpha} - 1 \right) \xi \quad (52)$$

$$y_{ar} = \left( \frac{1}{\alpha} - 1 \right) \xi \quad (53)$$

$$y_{mp} = (1 + \alpha \beta) \left( \frac{1}{\alpha} - 1 \right) \xi \quad (54)$$

$$y_{mr} = \left( 1 - \frac{1}{\alpha} \right) \xi \quad (55)$$

$$y_a = \xi \quad (56)$$

$$y_c = [1 + \beta(1 - \alpha)] \xi \quad (57)$$

$$y_m = \beta(1 - \alpha) \xi \quad (58)$$

$$y_n = \xi \quad (59)$$

## 9 Volume constraint

As we have seen, after reaching the end point  $x_m$ , the piston has to advance for another length  $s$  to finish pushing the water from cylinder 2 into the tank, so the piston's final position, and the end of the push phase, respect to the initial position is:

$$x_f = x_m + s = \frac{A_1}{A_2} x_m \quad (60)$$

The length of the compression chamber  $b$  must be greater than or equal to this value or else the right disc would hit the right wall, so it must be:

$$\frac{A_1}{A_2} x_m < b \quad (61)$$

which is:

$$\xi < \alpha \quad (62)$$

Since  $\xi$  increases with the pressure  $p_1$ , if we fix the ratio of  $A_1$  and  $A_2$  this puts an upper limit to the pressure  $p_1$ , and then an upper limit for  $\xi$  to allow the piston's complete run. The useful interval of the parameters  $\alpha$  and  $\beta$  is then restricted. Notice however that this constraint appears just because we have supposed that the discs of the piston are fixed to each other. The water that remains in cylinder 2 could be pumped back into the tank by another subsystem, the work done to pump it back would be almost the same but the disc in cylinder 1 would not have to advance more, and so this constraint would be avoided. The formulas for the motor work however would be different.

## 10 Adiabatic end point

In the adiabatic case the pressure is related to the volume by the following equation:

$$pV^\gamma = \text{const} \quad (63)$$

then we have:

$$p_0 V_0^\gamma = pV^\gamma \quad (64)$$

and so, using the expression of the volume, we have:

$$p = \frac{p_0}{\left(1 - \frac{x}{b}\right)^\gamma} \quad (65)$$

Inserting this in the integral that gives the work done on the air, we find that the work done on the air by the piston during the active phase is:

$$w_a = \frac{p_0 V_0}{\gamma - 1} [(1 - \xi)^{-\gamma+1} - 1] \quad (66)$$

Then the end point is determined by the equation:

$$\frac{p_0 V_0}{\gamma - 1} [(1 - \xi)^{-\gamma+1} - 1] = F_a b \xi \quad (67)$$

so:

$$(1 - \xi)^{-\gamma+1} - 1 = (\gamma - 1) \frac{F_a b}{p_0 V_0} \xi \quad (68)$$

but:

$$\frac{F_a b}{p_0 V_0} = \frac{p_0 A_1 + p_1 (A_1 - A_2)}{p_0 A_1} = 1 + \beta(1 - \alpha) \quad (69)$$

so the equation is:

$$(1 - \xi)^{-\gamma+1} - 1 = (\gamma - 1)[1 + \beta(1 - \alpha)]\xi \quad (70)$$

which is the intersection of the function:

$$f(\xi) = \frac{1}{(1 - \xi)^{\gamma-1}} - 1 \quad (71)$$

with the straight line:

$$g(\xi) = K\xi \quad (72)$$

with:

$$K = (\gamma - 1)[1 + \beta(1 - \alpha)] \quad (73)$$

The function  $f(\xi)$  tends to infinite for  $\xi$  tending to 1 from the left, and its value is zero for  $\xi = 0$ . However:

$$f'(\xi) = \frac{\gamma - 1}{(1 - \xi)^\gamma} \quad (74)$$

$$f'(0) = \gamma - 1 \quad (75)$$

then the intersection of the curve of this function with the straight line  $K\xi$  exists in the interval  $[0,1)$  if and only if  $K > \gamma - 1$  but this is true and so there is always an intersection. This equation cannot be solved in closed form but its solution can be calculated numerically, for example with the Newton's method. It is clear that  $\xi$  increases as  $\beta$  increases and as  $\alpha$  decreases. In appendix there is a table that reports the value of  $\xi$  corresponding to various values of  $\alpha$  and  $\beta$ .

Once that the final compression is known the air temperature can be calculated supposing that the air follows the law of ideal gas, from which it comes:

$$\frac{T_f}{T_0} = \frac{1}{(1 - \xi)^{\gamma-1}} \quad (76)$$

## 11 Example

Let's suppose that  $\beta = 2$ , i.e. the pressure of water from the tank is twice the atmospheric pressure. To achieve this pressure the height  $H$  must be about 20 meters, since the hydrostatic pressure of water is about 1 atmosphere for 10 meters of water. Using fresh water with density  $1 \text{ kg/dm}^3$  we have:

$$H = \frac{p_1}{\rho g} = \frac{2 * 101325 \text{ Pa}}{1000 \text{ kg/m}^3 * 9.81 \text{ m/s}^2} = 22.07 \text{ m} \quad (77)$$

Let's put  $\alpha = 0.6$  which means  $A_2 = 0.6A_1$ . From this we calculate the compression factor  $\xi$  which comes out 0.58, and so it satisfies the condition  $\xi < \alpha$ . Then the normalized energies are:

$$y_a = (1 + 2 * (1 - 0.6)) * 0.58 = 1.044 \quad (78)$$

$$y_m = 2 * (1 - 0.6) * 0.58 = 0.464 \quad (79)$$

$$y_n = 0.58 \quad (80)$$

Let's suppose that  $A_1 = 1 \text{ m}^2$  and  $b = 1 \text{ m}$ , then the initial volume  $V_0$  is  $1 \text{ m}^3$ , then:

$$p_0 V_0 = 101325 \text{ Pa} * 1 \text{ m}^3 = 101325 \text{ J} \quad (81)$$

and so:

$$W_a = 101325 * 1.044 \text{ J} = 105783 \text{ J} \quad (82)$$

$$W_m = 101325 * 0.464 \text{ J} = 47014 \text{ J} \quad (83)$$

$$W_n = 101325 * 0.58 \text{ J} = 58768 \text{ J} \quad (84)$$

Assuming that the cycle happens in 2 s, which is quite a high speed for such big pistons, the useful power, not counting the losses, would be about 29 kW.

## 12 Calculated tables

In the following table the combinations that give  $\xi < \alpha$  are not allowed by this system.

$\alpha$	$\beta$	$\xi$	$y_c$	$y_m$	$y_n$	$\eta$	$p_m$
0.10	0.10	0.12	0.13	0.01	0.12	12.11	1.19
0.10	0.50	0.42	0.61	0.19	0.42	3.22	2.15
0.10	1.00	0.62	1.17	0.56	0.62	2.11	3.85
0.10	2.00	0.80	2.23	1.43	0.80	1.56	9.33
0.10	5.00	0.94	5.17	4.23	0.94	1.22	50.50

0.10	10.00	0.98	9.81	8.83	0.98	1.11	265.26
0.10	20.00	1.00	18.91	17.92	1.00	1.06	1841.53
0.20	0.10	0.10	0.11	0.01	0.10	13.50	1.17
0.20	0.50	0.39	0.54	0.16	0.39	3.50	1.99
0.20	1.00	0.58	1.05	0.47	0.58	2.25	3.42
0.20	2.00	0.77	2.00	1.23	0.77	1.63	7.84
0.20	5.00	0.93	4.64	3.71	0.93	1.25	39.33
0.20	10.00	0.98	8.79	7.82	0.98	1.13	195.92
0.20	20.00	0.99	16.90	15.90	0.99	1.06	1308.82
0.30	0.10	0.09	0.10	0.01	0.09	15.29	1.15
0.30	0.50	0.35	0.48	0.12	0.35	3.86	1.85
0.30	1.00	0.55	0.93	0.38	0.55	2.43	3.02
0.30	2.00	0.74	1.77	1.03	0.74	1.71	6.52
0.30	5.00	0.91	4.10	3.19	0.91	1.29	29.95
0.30	10.00	0.97	7.77	6.80	0.97	1.14	140.45
0.30	20.00	0.99	14.88	13.89	0.99	1.07	901.47
0.40	0.10	0.08	0.09	0.00	0.08	17.67	1.12
0.40	0.50	0.32	0.41	0.10	0.32	4.33	1.71
0.40	1.00	0.50	0.80	0.30	0.50	2.67	2.65
0.40	2.00	0.70	1.54	0.84	0.70	1.83	5.34
0.40	5.00	0.89	3.56	2.67	0.89	1.33	22.22
0.40	10.00	0.96	6.73	5.77	0.96	1.17	97.14
0.40	20.00	0.99	12.86	11.87	0.99	1.08	575.59
0.50	0.10	0.07	0.07	0.00	0.07	21.00	1.10
0.50	0.50	0.28	0.35	0.07	0.28	5.00	1.57
0.50	1.00	0.45	0.67	0.22	0.45	3.00	2.31
0.50	2.00	0.65	1.30	0.65	0.65	2.00	4.31
0.50	5.00	0.86	3.02	2.15	0.86	1.40	15.96
0.50	10.00	0.95	5.69	4.74	0.95	1.20	63.67
0.50	20.00	0.98	10.83	9.85	0.98	1.10	352.09
0.60	0.10	0.05	0.06	0.00	0.05	26.00	1.08
0.60	0.50	0.23	0.28	0.05	0.23	6.00	1.45
0.60	1.00	0.39	0.54	0.16	0.39	3.50	1.99
0.60	2.00	0.58	1.05	0.47	0.58	2.25	3.42
0.60	5.00	0.82	2.46	1.64	0.82	1.50	11.00
0.60	10.00	0.93	4.64	3.71	0.93	1.25	39.33
0.60	20.00	0.98	8.79	7.82	0.98	1.13	195.92
0.70	0.10	0.04	0.04	0.00	0.04	34.33	1.06
0.70	0.50	0.18	0.21	0.03	0.18	7.67	1.33
0.70	1.00	0.32	0.41	0.10	0.32	4.33	1.71
0.70	2.00	0.50	0.80	0.30	0.50	2.67	2.65
0.70	5.00	0.75	1.89	1.13	0.75	1.67	7.16
0.70	10.00	0.89	3.56	2.67	0.89	1.33	22.22



0.70	20.00	0.96	6.73	5.77	0.96	1.17	97.14
0.80	0.10	0.03	0.03	0.00	0.03	51.00	1.04
0.80	0.50	0.13	0.14	0.01	0.13	11.00	1.21
0.80	1.00	0.23	0.28	0.05	0.23	6.00	1.45
0.80	2.00	0.39	0.54	0.16	0.39	3.50	1.99
0.80	5.00	0.65	1.30	0.65	0.65	2.00	4.31
0.80	10.00	0.82	2.46	1.64	0.82	1.50	11.00
0.80	20.00	0.93	4.64	3.71	0.93	1.25	39.33
0.90	0.10	0.01	0.01	0.00	0.01	101.00	1.02
0.90	0.50	0.07	0.07	0.00	0.07	21.00	1.10
0.90	1.00	0.13	0.14	0.01	0.13	11.00	1.21
0.90	2.00	0.23	0.28	0.05	0.23	6.00	1.45
0.90	5.00	0.45	0.67	0.22	0.45	3.00	2.31
0.90	10.00	0.65	1.30	0.65	0.65	2.00	4.31
0.90	20.00	0.82	2.46	1.64	0.82	1.50	11.00
0.95	0.10	0.01	0.01	0.00	0.01	201.00	1.01
0.95	0.50	0.03	0.04	0.00	0.03	41.00	1.05
0.95	1.00	0.07	0.07	0.00	0.07	21.00	1.10
0.95	2.00	0.13	0.14	0.01	0.13	11.00	1.21
0.95	5.00	0.28	0.35	0.07	0.28	5.00	1.57
0.95	10.00	0.45	0.67	0.22	0.45	3.00	2.31
0.95	20.00	0.65	1.30	0.65	0.65	2.00	4.31
0.96	0.10	0.01	0.01	0.00	0.01	251.00	1.01
0.96	0.50	0.03	0.03	0.00	0.03	51.00	1.04
0.96	1.00	0.05	0.06	0.00	0.05	26.00	1.08
0.96	2.00	0.10	0.11	0.01	0.10	13.50	1.17
0.96	5.00	0.23	0.28	0.05	0.23	6.00	1.45
0.96	10.00	0.39	0.54	0.16	0.39	3.50	1.99
0.96	20.00	0.58	1.05	0.47	0.58	2.25	3.42
0.97	0.10	0.00	0.00	0.00	0.00	334.33	1.01
0.97	0.50	0.02	0.02	0.00	0.02	67.67	1.03
0.97	1.00	0.04	0.04	0.00	0.04	34.33	1.06
0.97	2.00	0.08	0.09	0.00	0.08	17.67	1.12
0.97	5.00	0.18	0.21	0.03	0.18	7.67	1.33
0.97	10.00	0.32	0.41	0.10	0.32	4.33	1.71
0.97	20.00	0.50	0.80	0.30	0.50	2.67	2.65
0.98	0.10	0.00	0.00	0.00	0.00	501.00	1.00
0.98	0.50	0.01	0.01	0.00	0.01	101.00	1.02
0.98	1.00	0.03	0.03	0.00	0.03	51.00	1.04
0.98	2.00	0.05	0.06	0.00	0.05	26.00	1.08
0.98	5.00	0.13	0.14	0.01	0.13	11.00	1.21
0.98	10.00	0.23	0.28	0.05	0.23	6.00	1.45
0.98	20.00	0.39	0.54	0.16	0.39	3.50	1.99

0.99	0.10	0.00	0.00	0.00	0.00	1001.00	1.00
0.99	0.50	0.01	0.01	0.00	0.01	201.00	1.01
0.99	1.00	0.01	0.01	0.00	0.01	101.00	1.02
0.99	2.00	0.03	0.03	0.00	0.03	51.00	1.04
0.99	5.00	0.07	0.07	0.00	0.07	21.00	1.10
0.99	10.00	0.13	0.14	0.01	0.13	11.00	1.21
0.99	20.00	0.23	0.28	0.05	0.23	6.00	1.45

The following table gives the final temperature of the compressed air for various values of the compression factor, considering an initial temperature of 15°C or 25°C.

$\xi$	$T_f/T_0$	$t_f$ with $t_0 = 15^\circ\text{C}$	$t_f$ with $t_0 = 25^\circ\text{C}$
0.1	1.043	27.40	37.83
0.2	1.093	41.90	52.84
0.3	1.153	59.19	70.72
0.4	1.227	80.32	92.59
0.5	1.320	107.07	120.26
0.6	1.443	142.56	156.99
0.7	1.619	193.26	209.45
0.8	1.904	275.39	294.42
0.9	2.512	450.65	475.77
0.95	3.314	681.91	715.05
0.99	6.310	1544.95	1608.05

### 13 Other solutions

Instead of pumping the water by pushing the piston we could use a different way to bring the remaining water back into the tank. For example we could let the water flow into another smaller cylinder and then push the piston of that cylinder. This would save some energy because the smaller cylinder doesn't have to win the atmospheric pressure on an area  $A_1$ . In this way the constraint imposed by the geometry of the piston inside the cylinders would be avoided, and the main cylinder could work at a higher water pressure, producing more power. The work needed to bring the water back into the tank is at least:

$$W_b = p_1 V_p = p_1 (A_1 - A_2) x_m = p_1 V_0 \left(1 - \frac{A_2}{A_1}\right) \xi \quad (85)$$

### 14 Conclusions

In this system the energy acquired by the compressed air is greater than the work done by the motor. The net useful energy is given by the work of the

atmospheric pressure. The theoretical net energy that can be obtained is equal to the total work done by the atmosphere on the system in a cycle, and is given by  $p_0V_0\xi$ . The compression factor  $\xi$  can be increased by increasing the height of the water column, but this particular system has a geometrical limit that imposes the constraint  $\xi < \alpha$ . This constraint could be avoided with a different system of pumping the remaining water back into the tank.

In a real system the energy of the compressed air would be lower because of the heat losses, and it also has to be converted into electric energy by means of a turbofan connected to an alternator, or another electromechanical system. The efficiency of this conversion is crucial to obtain a net energy from this machine. The compressed air produced by this machine has a relative low pressure.

If we had a way to extract all the energy from the low-pressure compressed air then this machine would be a valid power generator. The problem then is to find a way to convert the energy of an amount of compressed air with relatively low pressure into another form of mechanical energy and then into electric energy.

The author is available for collaborations with people who want to test the theory here described. To contact the author send an email to [mail@softwaresphere.com](mailto:mail@softwaresphere.com).

## **Disclaimer**

This study has been developed in a total independent way. In 2011 I read the very little information published on the Web about a system called Hydro+, but the websites and the papers available were absolutely insufficient to achieve an understanding of that machine. After almost one year of, not uninterrupted, study and efforts I have devised this system that gives an explanation in terms of the energy of the atmospheric pressure, not because of the hydrostatic pressure, so every reference to Hydro+ is excluded. This is an independent and separate work done by Dr. Ing. Alessandro Forcella which cannot be considered based on any other existing system.