Some problems in Hungarian mathematical competition. II.

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Abstract

In this work, we continue to present some interesting problems in the Transylvanian Hungarian Mathematical Competition held in 2012.

B1st Problem. Solve in $\mathbb{R}$ equation $[\log_2 x] = \sqrt{x} - 2$, where $[x]$ denotes the integer part of x.

Ferenc Kacsó

B2nd Problem. Find all real solutions of equation $7^{\log_5\left(x^2+\frac{4}{x^2}\right)} + 2\left(x+\frac{2}{x}\right)^2 = 25$.

Mihály Bencze

B3rd Problem. a) Prove that for each $z \in \mathbb{C}$ the following inequality holds:

$$\left|z^2+2z+2\right| + |z-1| + \left|z^2+z\right| \geq 3.$$

b) When does the equality hold?

Béla Bíró

B4th Problem. In triangle ABC with $AB = AC$ let I denote the incenter of the triangle. Line BI meets the circumcircle secondly in point D. Find the measures of the angles of the triangle, if $BC = ID$.

Géza Dávid

B5th Problem. a) Show that an interior point M of a triangle ABC belongs to the median from A if and only if $area\,[MAB] = area\,[MAC]$;
b) Determine that interior point M of the triangle ABC, for which

$$\frac{MA}{\sin\left(\widehat{BMC}\right)} = \frac{MB}{\sin\left(\widehat{CMA}\right)} = \frac{MC}{\sin\left(\widehat{AMB}\right)}.$$

Lajos Longáver

B6th Problem. Find the 73th digit from the end of $\underbrace{111\ldots1}_{2012 digits}{}^2$.

Anna-Mária Darvas