GRAVITY AS CENTRIPETAL FORCE

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Abstract

The earth's gravity can be viewed as the centripetal force resulting from the coupling of its rotational and orbital motions. These motions were (presumably) set in train in the early universe, when the solar system formed from a spinning nebula. The original angular momentum of the nebula has (apparently) been largely conserved during the subsequent evolution of the solar system. In an idealized (frictionless) system, involving the circulation of an object around a centre at constant velocity, a centripetal force is exerted by the object towards the centre; in the case of the coupled rotational-orbital motion of the earth, the centripetal force would be directed towards its own centre. The law of conservation of angular momentum requires that the circulation continue *ad infinitum* in the absence of an external force. The gravity apparently experienced by objects approaching the earth, and the variation of gravity with latitude are explicable by extending the above ideas. (The estimated value of the above centripetal acceleration on the earth is 1.4×10^2 ms^{-2}, which compares reasonably with the observed value of g of 9.8 ms^{-2}, considering the approximations employed.)

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Gravity is that unique natural phenomenon that is ever-present and – at least on Earth – ubiquitous. It is considered to be the weakest of the four fundamental forces of nature, which hold the entire universe together in a grand cosmic web of interactions. Once the status of gravity as a 'fundamental force' is accepted, any question as to its origins acquires a philosophic tinge, as it is entangled with the ultimate question about the origin of the universe itself. Be that as it may, the fascination of gravity – like that of its cousin, magnetism – lies in its remote, 'non-contact' nature. Intriguingly also, despite its relative weakness, it is believed to act across the vast expanses of the universe, quite literally spanning hundreds of millions of miles.[1-3]

Although there are several theories of gravity, Sir Isaac Newton's Universal Law of Gravitation (ULG, 1687) is widely accepted.[1b, 2d] The ULG (eqn. 1), apart from being a mathematical description, also states that the force of gravitational attraction (F) exists between any two objects, in direct proportion to the product of their masses (m_1 and m_2), and in inverse proportion to the square of the distance between them (r). (G is the gravitational constant with a value of 6.7×10^{-11} $\mathrm{Nm^2kg^{-2}}$) Thus, despite the severity of the distance criterion, the heavenly bodies apparently act upon each other because of their enormous masses.

$$F = (Gm_1m_2)/r^2 \qquad \text{eqn. 1}$$

Attempts to experimentally verify eqn. 1 date back to the late 1700's, and essentially involve the determination of the constant G.[3] These are based on the measurement of the minute torsion that is induced by the putative gravitational attraction between two metallic spheres of known mass in a suitably designed apparatus ('the Cavendish balance').

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*This paper is based on elementary concepts that are to be found in most standard textbooks of physics (*e.g.* refs. 1 and 2). Any undefined symbols have the normally accepted meaning.

However, because of the infinitesimal magnitude of *G,* the incursion of spurious effects, particularly friction and air turbulence, can never really be ruled out. (These problems have indeed been ingeniously addressed in the recent versions of the experiment;[3] however, it remains that there may well be a fundamental limit to the accuracy with which *G* can be measured.)

In view of the above uncertainties, it is not surprising that alternative explanations for gravity have been sought.[3] Explored herein is an apparently novel explanation for gravity that is also based on elementary and well-established principles of classical mechanics.[1, 2]

Gravity as centripetal force. A rather straightforward alternative possibility is that terrestrial gravity is the centripetal force that arises from both the rotation of the earth around its own axis, as also its orbital motion around the sun. Both these circular motions must be accompanied by a centripetal force that is directed towards the centre of the earth (for rotation) and towards the sun (for orbital motion). A centripetal force (CF) accompanies the motion of an object along a circular path, and derives from a centripetal acceleration (CA), both of which are directed towards the centre of the circle being traversed (*vide infra* for a detailed discussion).[1c, 2a] Eqns. 2 and 3 relate the magnitudes of CA and CF in terms of the mass of the object (m), its circular velocity (v) and the radius (r) of the circular path. (Rigorously, a negative sign is to be included to indicate the inward direction of CA and CF, but is omitted here for simplicity.)

$$\mathrm{CA} = v^2/r \qquad \text{eqn. 2}$$

$$\mathrm{CF} = m(\mathrm{CA}) = mv^2/r \qquad \text{eqn. 3}$$

Furthermore, the rotational and orbital motions of the earth cannot be treated independently, but must be 'coupled' to each other (as both occur simultaneously). Essentially, this implies that the velocity of the earth's rotation is, effectively, much greater than is believed, as discussed below.

The rotational and orbital centripetal forces, [$(\mathrm{CF})_{rot}$] and [$(\mathrm{CF})_{orb}$] respectively, would derive from corresponding accelerations, [$(\mathrm{CA})_{rot}$] and [$(\mathrm{CA})_{orb}$]. Normally, [$(\mathrm{CA})_{rot}$] and [$(\mathrm{CA})_{orb}$], calculated on the basis of eqn. 2, are both far less (by several orders of magnitude) than the observed value of the acceleration due to gravity (g). Thus, with v_{rot} = 4.7 x 10^2 ms^{-1} (earth's rotational velocity), v_{orb} = 3 x 10^4 ms^{-1} (earth's orbital velocity), r_{rot} = 6.4 x 10^6 m (earth's radius), and r_{orb} = 1.5 x 10^{11} m (earth's distance from the sun),[4] $[(CA)_{rot}]$ and $[(CA)_{orb}]$ would be 3.5 x 10^{-2} ms^{-2} and 6.0 x 10^{-3} ms^{-2} respectively. The observed value of g,[4] however, is 9.8 ms^{-2}.

'Coupling' of orbital and rotational motions. As the orbital and rotational motions of the earth occur simultaneously, the effective path of the earth around the sun would be a resultant of these motions. Although the orbital path of the centre of the earth defines a circle (more exactly, an ellipse), this is not true of any other point on the earth's surface. The motion of a point at the earth's equator, for example, would define an asymmetrical sinusoid as shown in Fig. 1 (p. 14). This is a result of the fact that $v_{orb} > v_{rot}$, so that any point on the earth's surface is effectively moving in the same direction as the centre of the earth at all times. (The tilt of the earth's axis to the orbital plane is ignored in this treatment for simplicity, which also assumes a circular orbital path.)

The effective velocity of the above sinusoidal motion of a point on the earth's surface will not be constant. It would be greatest when v_{orb} and v_{rot} act in the same direction (when the point faces away from the sun, 'outward' traverse), least when they are opposed (when the point faces towards the sun, 'inward' traverse), and intermediate otherwise. However, the traverse would also be correspondingly shorter on the 'inward' part than on the 'outward'. In general, these variations in the sinusoidal velocity would be dwarfed by its overwhelming dependence on v_{orb} rather than v_{rot}, as discussed further below.

Also, the sinusoidal trajectory implies the existence of a centripetal force, which is directed at every point towards the centre of the earth. This is because the trajectory followed by the point coincides with the rotation of the earth around its centre, which itself moves along the orbital path. The above centripetal force at the particular point on the earth's surface may be estimated as described further below.

In fact, the motion of any point on the earth's surface would define a sinusoid. However, the motion of a point on the earth's surface away from the equator, *i.e.* towards the poles, would be shorter relative to a point on the equator. Consequently, the velocity of the motion would be lower (as a shorter path is traversed in the same time period).

Interestingly, however, it can be shown that the centripetal force at a point towards the poles is greater than at the equator. This is because the decrease in the sinusoidal velocity is much less than that in the earth's 'radius', on going from the equator towards the poles. Interestingly, this would also explain the observed greater value of the acceleration due to gravity (g), at northern and southern latitudes relative to the equator.[1c, 2d] (*cf.* 'Appendix' for a detailed discussion. 'Radius' is defined here as the shortest distance from the point on the earth's surface and its axis, and the centripetal force would be directed along this distance towards the axis.)

The magnitude of the centripetal acceleration. This may be approached as follows. Firstly, the length of the trajectory may be estimated from the fact that the time for any point on the earth to successively intersect the orbital path defines half a day (12 h). (During this period, the earth travels a distance of ~1.3 x 10^9 m.) As seen in Fig. 1 (p. 14), the length of the path traversed by the point on the earth's surface (ℓ) closely approximates this distance, and may be assumed to be equal to it. This implies that the velocity of the sinusoidal motion (v_s) is similar to v_{orb}. (Strictly, however, ℓ and v_s are underestimated by the fact that ℓ is 'circumferential' relative to the orbital path.)

Secondly, the radius corresponding to this centripetal force is assumed to be approximately the same as the earth's radius, for the following reason. As seen in Fig. 1, the curve at every point on the trajectory is essentially defined by the resultant of the earth's rotation and its motion along the orbital path; the centre of the earth, therefore, acts as a moving pivot. In such a case, apparently, the overall centripetal force would be directed towards the pivot. [Thus, the resulting centripetal acceleration $(CA)_g$ may be estimated as: $(CA)_g \sim (v_s^2/r_{rot}) \sim (v_{orb}^2/r_{rot})$.]

On the basis of the above arguments, and eqn. 2, the centripetal acceleration experienced by a point at the earth's surface [$(CA)_g$] would be ~1.4 x 10^2 ms^{-2}. This compares reasonably with the observed value of g (9.8 ms^{-2}), bearing in mind the approximations employed. (Thus, g is overestimated by just over an order of magnitude). This apparently implies that the radius of the sinusoidal arc (constituting the trajectory, *cf.* Fig. 1, p. 14) is correspondingly underestimated by r_{rot}. This is understandable, as the actual radius of the sinusoidal arc is indeed expected to extend beyond the earth's centre [apparently by a factor of ~14, as indicated by the value of $(CA)_g$ calculated above].

All earth-bound objects would be subject to this centripetal force deriving from [$(CA)_g$]. Also, work done against this force would be manifested as potential energy. Thus, an object raised from the earth's surface would possess a higher potential energy and would fall back to the ground if not held back. This would appear to be 'gravity'.

It is also noteworthy that, at a constant circular velocity (v) the centripetal acceleration decreases with increasing distance from the centre (r, eqn. 2). This explains the decrease in gravity that is experienced by an object leaving the earth, with increasing distance from it. (The circular velocity of the object would, at the most, be what it acquired at the earth's surface, in the absence of additional propulsion.)

Origins of gravity: conservation of angular momentum. The above treatment of terrestrial gravity apparently 'shifts the burden' to the putative coupling of rotational and orbital motions. It is noteworthy, however, that the existence of these is a consequence of the law of conservation of angular momentum (LCAM), by which a body continues to move in a defined circular path unless it is acted upon by an external force.[1c, 2c] An explanation more far reaching in time would involve the origins of the solar system, and indeed, the universe itself. It is believed that the solar system evolved from a spinning nebula, which gradually congealed into the planets and their satellites, which also retained the original (nebular) angular momentum in the process.[5, 6]

On this basis, the observed terrestrial gravity would be a direct consequence of the LCAM and the concept of centripetal acceleration. Intriguingly, also, the LCAM requires the absence of an external force (or torque), which implies the absence of the traditional gravitational field (eqn. 1). Thus, the traditional view of gravity apparently 'inverts' cause and effect.*

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*It is noteworthy that the direction of the earth's rotation corresponds to its orbital motion, *i.e.* both are counterclockwise as viewed from a plane above its north pole and parallel to the orbital plane (*cf.* Fig. 1, p. 14). Thus, the angular momentum pseudovectors of both the motions point 'north'. This would clearly facilitate their coupling, and possibly the conservation of the momenta as well.

On the nature of the centripetal force: principles of circular motion reassessed. As mentioned above, an object moving in a circular path experiences a centripetal force that is directed towards the centre of the circle.[1c, 2a,c] (Interestingly, the corresponding centripetal acceleration affects the direction but not the magnitude of the velocity, which is a vector.) Generally, the object would be in remote physical contact with the centre, say *via* a tether. The orbiting of the earth around the sun represents a special case, wherein there is no physical contact between the centre (the sun) and the object (the earth).

In the earth's case, it is traditionally believed that the centripetal force is provided by gravity. However, as argued above the earth has been set into motion along a circular path by past events of unknown origin, and the LCAM requires that it continue to follow the circular path in the absence of an external force. This motion must be accompanied by a centripetal acceleration and force.

The centripetal force is, of course, to be distinguished from a centrifugal force, which is directed away from the centre, and manifests itself in two ways. Firstly, the centrifugal force is the inertia experienced by the object at the start of the circular motion. This will disappear once the circular motion is stabilized at a constant velocity (v); however, an increase in v will again induce a centrifugal force, although with a decrease in v the inertial force would be directed towards the centre (thus being centripetal rather than centrifugal).

Secondly, the centrifugal force may be viewed as the (equal and opposite) counterpart of the centripetal force, but one which is exerted by the centre of the circular path on the moving object. (This is required by Newton's third law of motion,[1a, 2b] and is clarified further with the help of eqn. 4 below.) In the above example, these forces would be exerted through the tether. However, this requires that the moving object be in physical contact with the centre, so does not apply to the earth's orbit around the sun.*

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*The above variants of the centrifugal force have been termed 'reactive' and 'fictitious' respectively; the first exists relative to an inertial reference frame and the second relative to a rotating reference frame.

It is particularly important to note that the centripetal force is a property of the object undergoing the circular motion, and not one that is imposed upon it (by the tether). This is because an object moving with a positive force will exert the force upon another object that may be in contact with it. On this basis, the (positive) centripetal force is exerted by the moving object on the centre. Thus, the notion that the centripetal force is an external force exerted by the centre upon the moving object is invalid.

$$\mathrm{CA} = v^2/r = [(2\pi r)/t]^2/r = 4\pi^2 r/t^2 \qquad \text{eqn. 4}$$

It is also noteworthy that the centripetal acceleration (CA) is directly proportional to the distance from the centre of the circular motion (r); this is seen by expressing the circular velocity (v) in terms of the circumference ($2\pi r$) and the time taken for completing one circular orbit (t; *cf.* eqns. 2 and 4). (In this analysis, v is proportional to r, but is constant at a fixed value of r.)

This implies that the centripetal force (say) in a rotating disc is greater at the periphery than at intermediate distances from the centre; and in the case of an object tethered to a centre and rotating around it, the object would exert a centripetal force on the tether. If the tether is rigid, it would exert an equal and opposite centrifugal force on the object (as all parts of the tether would experience a smaller centripetal force than the object at the periphery).*

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*The inertial centrifugal force has important implications for the functioning of centrifuges (used for separating macromolecules, etc.). This force develops during the approach to the final rotational velocity and is the basis of centrifuge action: more massive molecules possess greater inertia, *i.e.* experience a lower centripetal force initially, so remain farther from the centre. However, once the final velocity becomes constant (over the entire sample), the centripetal force would take over, which would tend to reverse the 'density gradient' that developed during the centrifugal phase. Apparently, this is generally mistaken as a diffusion effect. (Intriguingly, the final centripetal force would be greater for the more massive molecules lodged at the periphery!)

Clearly, therefore, the view that a tether maintains an object along a circular path is strictly not valid. Whilst a tether would (generally) be needed to get the object into circular motion, <u>in principle</u>, it would be dispensable after the circular motion has been stabilized at a constant velocity.

Misconceptions. Once an object is set into circular motion at constant velocity, there are only two forces that need to be considered: centripetal, exerted by the object upon the centre, and centrifugal, which is equal and opposite to the centripetal, and is exerted upon the object by the centre. These would be exerted *via* a rigid tether.

Misconceptions about these principles of circular motion can arise if a flexible tether is used. In such cases, the fact that the tether becomes taut during the circular motion creates the illusion that the tether is 'pulling at' the object undergoing circular motion. However, the tautness would be the result of the <u>centrifugal</u> force that is likely resulting from increasing circular velocity, *i.e.* a continuous inertial force. If the circular motion occurs at constant velocity, the tether should not be taut.*

A perfect practical demonstration of the above principles, however, would be almost impossible under normal terrestrial conditions, essentially because of interference from the earth's gravity and friction (involving both the contraption and air). The analysis of circular motion presented above is idealized in ignoring friction, as this facilitates the application of the LCAM.

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*In the case of the earth's orbital and rotational motions, friction is presumed absent; therefore, no (further) input of energy would be required to maintain these motions. In a conventional contraption, however, friction would be unavoidable; the resulting energy losses would have to be compensated for the circular motion to continue: this input usually occurs from the centre *via* the tether. Also, in the case of the earth, assuming that no energy leaves the system, no perpetual motion is involved, *i.e.* even in the absence of conventional gravity. (Thus, the energy in the system is conserved.)

The above caveat applies particularly to the case of a rigid tether, as the proposed explanation of the centripetal force implies that the object undergoing circular motion ‘leans on’ the centre *via* the tether. This, of course, cannot apply to the case of a flexible tether, or in the absence of a tether altogether. (A force may be present without acting upon an object: *e.g.* an accelerating spacecraft; clearly, Newton’s third law of motion[1a, 2b] is only applicable to the case of two objects in mutual contact!)*

Furthermore, the above analysis based on the LCAM implies that the original ‘centripetal energy’ that gave rise to the circular motion remains ‘trapped’ in the system. (This would be the kinetic energy of the rotating mass, equal to $mv^2/2$.) <u>Note, again, that this is not the energy being imparted by the centre to the rotating mass *via* a tether: rather, it is the energy possessed by the rotating mass itself</u>.

Examples. The simplest example of the above principles is a spinning top: once it is set spinning, and in the absence of friction, it should continue spinning for ever (by the LCAM). The analogous example of a spinning hollow ball is perhaps more apt, as the surface of the ball is not in contact with the centre.

These, however, represent cases of a rigid object spinning on its own axis. A man-made analog of the orbital motion of the earth around the sun is apparently unknown. Accordingly, it is contrary to normal phenomenal experience, and hence appears implausible. The construction of such an analog of an object moving in a circular path without any contact with the centre of the circle, would be stymied by friction and gravity, due to which constant velocity (and momentum) cannot be maintained.* (But *cf.* ‘Appendix’ for a discussion of orbital motion around the earth.)

However, that an object can follow a circular path at constant velocity by virtue of the LCAM, is not without a trace of experiential support. Consider, for instance, sitting in a vehicle moving in a (horizontal) circular path at constant velocity: an object tossed into the air inside the vehicle returns to the same spot in the vehicle, and does not necessarily fly off at a tangent. This implies that the object retains its angular momentum even when airborne, *i.e.* when not in physical contact with the moving vehicle.

Indeed, the most convincing example of the above ideas is the fact that, on jumping up from the ground, we land at the same spot on the earth’s surface: our angular momentum was conserved during the process!

Origin of the gravity apparently experienced by external objects. If the earth's gravity is a centripetal force, it would apparently manifest itself only in objects that are terrestrially attached. On this basis, the gravity (apparently) experienced by objects external to the earth (*e.g.* a spacecraft) needs to be explained.

Consider a spacecraft approaching the earth from outer space, *i.e.* reentering the earth's 'gravitational field'. Interestingly, such a spacecraft would be moving in tandem with the earth's rotational and orbital movements, so it would be generating its own centripetal force: this would appear to be 'gravity'. (Clearly, an external object cannot approach the moving earth without being propelled towards it, *i.e.* being directed by a force of its own.) The apparent increase in gravity as the spacecraft approaches closer to the earth would be due to the fact that their movements would be more similar.*

The gravity supposedly experienced by a stray object from space, *e.g.* a meteorite, would be due to its accidentally possessing the orbital (and, possibly, the rotational) characteristics of the earth – as it must, if it is to collide with the earth. (For a discussion of permanent orbits, see the 'Appendix'.)

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*Note that the CA of the spacecraft would be lower than at the earth's surface, if its (tangential) velocity is the same as at the earth's surface (*cf.* eqn. 2). However, a spacecraft 'hovering' over a particular point on the earth, would possess a greater CA than at the earth (*cf.* eqn. 4).

APPENDIX

Explanation for the variation of gravity with latitude.[1c, 2d] As was noted, a point on the earth's surface will trace out a sinusoidal path, because of the earth's rotational and orbital motions. However, the resulting arc will be larger for a point at the equator than at another latitude (*i.e.* towards either of the poles). This is because the (shortest) distance from a point on the earth's surface to its axis is greatest at the equator. Let this distance be r_e and r_x, at the equator and another latitude respectively (*cf.* Fig. 2, p. 15).

The arcs of the sinusoidal paths may be considered to be parts of corresponding (hypothetical) circles, of radii R_e and R_x (corresponding to r_e and r_x above). Note that R_e and R_x bear a similar and constant relationship to r_e and r_x respectively (eqn. 5).

Also, the velocity resulting from the sinusoidal traverse (v_e or v_x) would be proportional to the radius of the arc traversed, R_e or R_x respectively (eqns. 6 and 7), t being the time taken to traverse either of the hypothetical circles; and the corresponding centripetal accelerations, $[(CA)_e]$ and $[(CA)_x]$, would each be related to the square of the respective velocity [eqns. 8 and 9; *cf.* eqn. 2, noting that CA was estimated above as (v_{orb}^2/r_{rot})].

$$(R_e - R_x) = (r_e - r_x) = a \qquad \text{eqn. 5}$$

$$v_e = 2\pi R_e/t \qquad \text{eqn. 6}$$

$$v_x = 2\pi R_x/t \qquad \text{eqn. 7}$$

$$[(CA)_e] = v_e^2/r_e = 4\pi^2 R_e^2/t^2 r_e \qquad \text{eqn. 8}$$

$$[(CA)_x] = v_x^2/r_x = 4\pi^2 R_x^2/t^2 r_x \qquad \text{eqn. 9}$$

$$R_e^2/r_e < R_x^2/r_x \qquad \text{eqn. 10}$$

$$R_e^2/R_x^2 < r_e/r_x \qquad \text{eqn. 11}$$

$$(R_x+a)^2/R_x^2 < (r_x + a)/r_x \qquad \text{eqn. 12}$$

$$(R_x^2+2aR_x + a^2)/R_x^2 < (r_x + a)/r_x \qquad \text{eqn. 13}$$

$$1 + (2a/R_x) + (a^2/R_x^2) < 1 + (a/r_x) \qquad \text{eqn. 14}$$

$$(2/R_x) + a/R_x^2 < (1/r_x) \qquad \text{eqn. 15}$$

$$[2 + (a/R_x)] < (R_x/r_x) \qquad \text{eqn. 16}$$

The condition for gravity being greater at a latitude other than the equator,[1c, 2d] *i.e.* $[(CA)_e] < [(CA)_x]$, is given by eqn. 10, which follows from eqns. 8 and 9. The proof of the inequality in eqn. 10 is as follows. Cross multiplication of eqn. 10 leads to eqn. 11, which with eqn. 5, leads to eqn. 12. Expansion of eqn. 12 leads to eqns. 13 and 14, which simplifies to eqn. 15. This, upon cross-multiplication, yields final eqn. 16.

Eqn. 16 is valid for the following reasons. Firstly, eqn. 5 implies that the maximum value of a would be equal to r_e, corresponding to the condition $r_x = 0$, which would obtain at the poles. Secondly, $R_x >> a$, as R_x is the radius of an arc which has nearly the same curvature as the orbital path: thus, R_x is of the order of 10^{11} m, whereas $a < 10^7$ m (*vide supra*). By the same token, $R_x >> r_x$, noting also that $r_x \leq r_e$. Therefore, the left hand side of eqn. 16 would take on values between 2 and 3, whereas the right hand side is of the order of 10^4. The validity of eqn. 16 thus proves the validity of the (original) key inequality in eqn. 10.

Furthermore, the usual explanation for this variation in gravity is based on a 'fictitious centrifugal force' at the equator. However, in a system of constant angular momentum and velocity, the centripetal and centrifugal forces would be equal and opposite at any given point (as argued above): this would apply at any latitude in the earth's case. (The only valid centrifugal force would be that exerted by the earth itself on an object on its surface, which would be the 'equal and opposite reaction' to the centripetal force exerted by the object on the earth.)

Permanent orbits around the earth. By the LCAM,[1c, 2c] an object in an orbit around the earth would remain in orbit unless acted upon by an external torque. Although, in principle, the orbital motion can occur at any distance from the earth, atmospheric friction would make it impossible at low altitudes (assuming that the object is not powered). An earth-bound object can be placed in orbit, therefore, only at very high altitudes (beyond the atmosphere). For this, of course, the object would need to overcome the centripetal force ('gravity') at the earth's surface; thus, it would need to have a source of power that gives it a thrust that exceeds the acceleration due to gravity, and also puts it into orbit at a certain angular velocity. Thenceforth, <u>in principle</u> (by the LCAM), the power source would not be necessary. (In practice, however, it would be, to effect corrections relative to deviations in the earth's orbital path.)

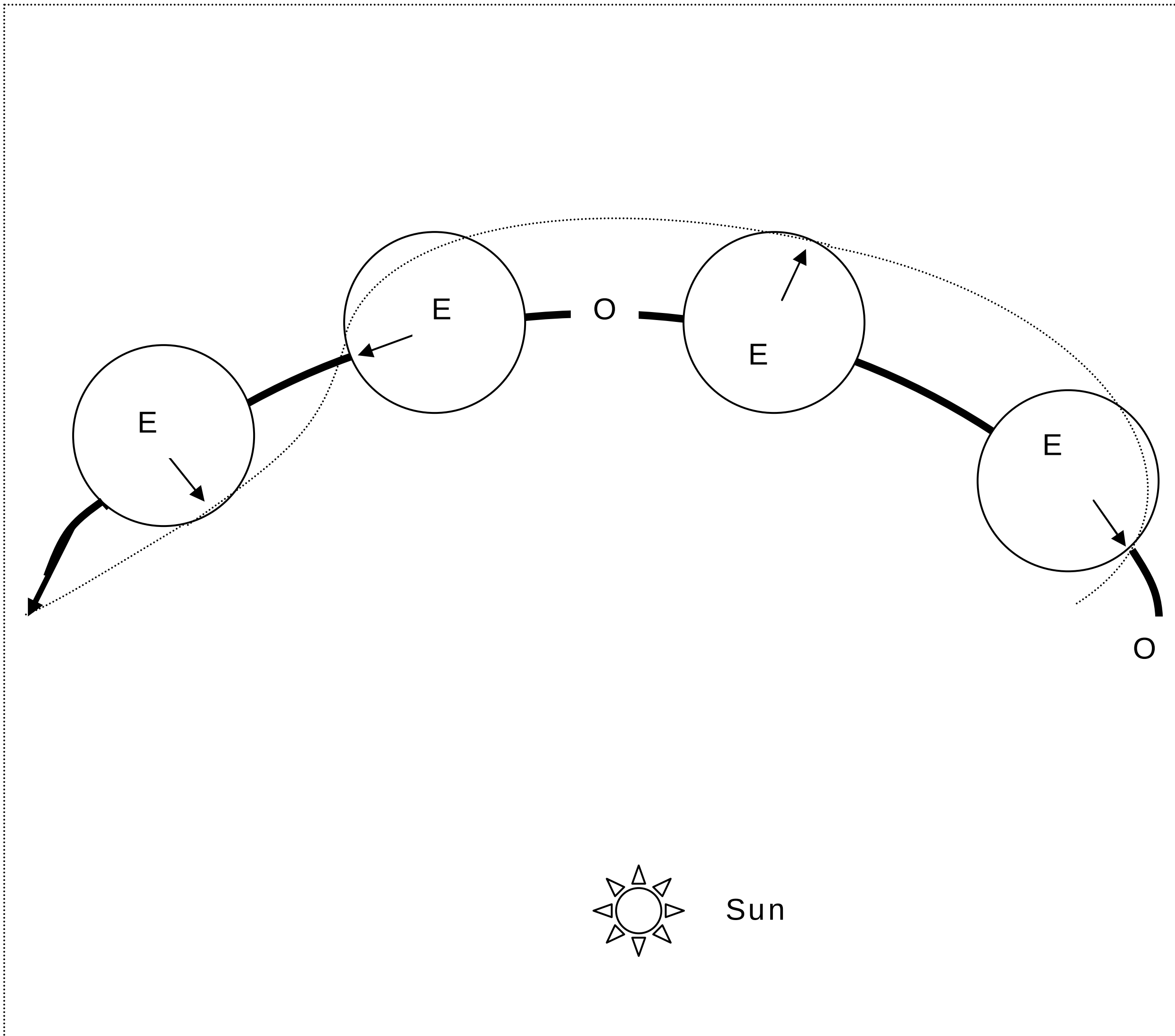

Fig. 1. Depiction of the sinusoidal trajectory of a point on the earth's surface, which results from the coupling of the earth's rotational and orbital motions. 'E' is the earth and 'O' the orbital path around the sun (shown in the foreground); the arrow indicates the position of the point, and the dotted line the sinusoidal trajectory of the point. The successive positions of the earth as shown represent a six-hour period. The earth's centre (not shown) thus acts as a moving pivot. The view presented is from a plane parallel to the plane of the orbital path and above the earth's north pole. The earth is thus moving from right to left as shown. (The depiction is not to scale.)

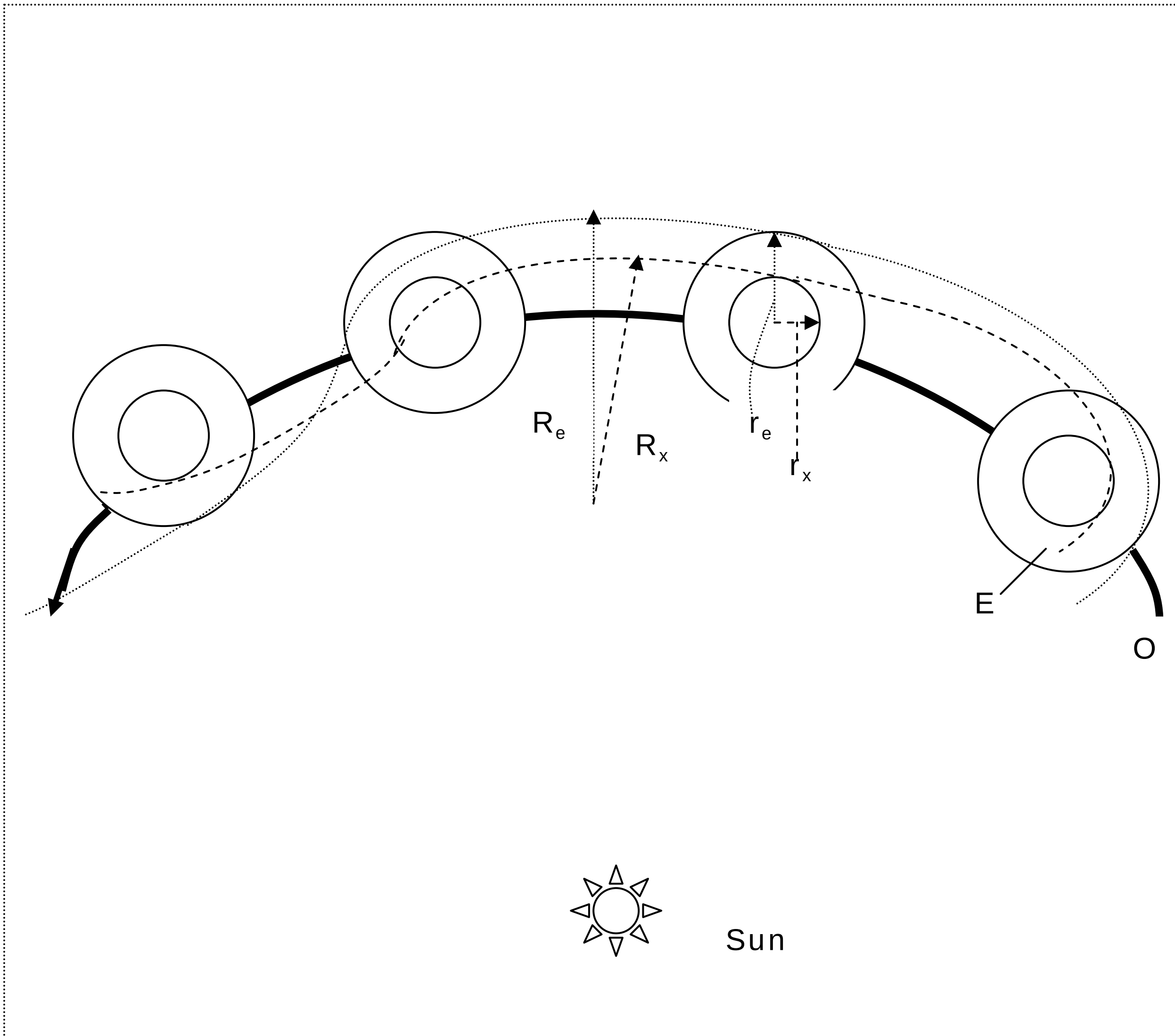

Fig. 2. Depiction of the relative sinusoidal trajectories of points on the earth's surface (*cf.* Fig. 1), at the equator (dotted line) and a northern latitude (dashed line). The corresponding 'radii' of the earth are indicated as r_e and r_x. R_e and R_x represent the radii of the (hypothetical) circles that could be formed from the arcs corresponding to the above two trajectories (as shown, other symbols as in Fig. 1). The centripetal acceleration at the equator would be lower than at other latitudes, in accord with the observed difference in gravity.

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