2 Page Solution for Collatz conjecture

The Collatz Conjecture states that for any natural number n, if n is even, divide it by 2, if n is odd, multiply it by 3 and add 1, repeat the process indefinitely, and you reach 1 regardless of what number you start with.

Now, let x be 1 + … + 1, such that there are even number of 1's. Let y be 1 + … + 1, such that there are odd number of 1's. The natural number is a repetition of 4 different number types: even number of 2's, even number of 2's plus 1, odd number of 2's, odd number of 2's + 1, even number of 2's, and so on.

Formula:

Case 1: i = 2 + … + 2, such that there are even number of 2's.

Case 2: i = 2 + … + 2 + 1, such that there are even number of 2's + 1.

Case 3: i = 2 + … + 2, such that there are odd number of 2's.

Case 4: i = 2 + … + 2 + 1, such that there are odd number of 2's + 1.

Case 1:

$i = 2x$, $i = x$, $i = y$, $i = 3y + 1 = 3x/2 + 1$, start the formula again

$(x/2)$

Case 2:

$i = 2x + 1$, $I = 6x + 4$, $I = 3x + 2$, $i = 3x/2 + 1$, start the formula again

Case 3:

$i = 2x + 2$, $i = x + 1$, $i = 3x + 2$, $i = 3x/2 + 1$, start the formula again

Case 4:

$i = 2x – 1$, $i = 6x – 4$, $i = 3x – 2$, $i = 3x/2 - 1$, start the formula again

Whatever the starting number is, by the recursive formula, the number gets smaller and smaller.

Now, the problem is that how do we know it reaches 1? Consider the following equations.

Case 1	Case 2	Case 3	Case 4
$2x > 3x/2 + 1$	$2x + 1 > 3x/2 + 1$	$2x + 2 > 3x/2 + 1$	$2x – 1 > 3x/2 - 1$
$X > 2$	$x > 0$	$x > -2$ i.e. $x > 0$ since it has to be > 0	$x > 0$
$2x > 4$	$2x + 1 > 1$	$2x + 1 > 2$	$2x - 1 > 0$

Now, we can write a recursive function f(x) such that:

$$
\text{Let } f(x) = \begin{cases}
f\left(\frac{3x}{4} + 1\right), & x \text{ is the sum of even number of 2's and bigger than 4.} \quad x > 3x/4 + 1 \text{ if } x > 4 \\
f\left(\frac{3x}{4} + \frac{1}{4}\right), & x \text{ is the sum of even number of 2's + 1 and bigger than 1.} \quad x > 3x/4 + 1/4 \text{ if } x > 1 \\
f\left(\frac{3x}{4} - \frac{1}{2}\right), & x \text{ is the sum of odd number of 2's and bigger than 2.} \quad x > 3x/4 - 1/2 \text{ if } x > 2 \\
f\left(\frac{3x}{4} - \frac{1}{4}\right), & x \text{ is the sum of odd number of 2's + 1, and bigger than 0.} \quad x > 3x/4 - 1/4 \text{ if } x > 0 \\
1, & x = 4, 2, 1 \\
f(2), & x = 3
\end{cases}
$$

Hence, for any natural number we start with, we reach 1.