Virtual Particle Interpretation of Quantum Mechanics - a non-dualistic model of QM with a natural probability interpretation

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Abstract

An interpretation of non-relativistic quantum mechanics is presented in the spirit of Erwin Madelung's hydrodynamic formulation of QM [1] and Louis de Broglie's and David Bohm's pilot wave models [2, 3]. The aims of the approach are as follows: *1) to have a clear ontology for QM, 2) to describe QM in a causal way, 3) to get rid of the wave-particle dualism in pilot wave theories, 4) to provide a theoretical framework for describing creation and annihilation of particles, and 5) to provide a possible connection between particle QM and virtual particles in QFT.* These goals are achieved, if the wave function is replaced by a fluid of so called *virtual* particles. It is also assumed that in this fluid of virtual particles exist a few *real* particles and that only these real particles can be directly observed. This has relevance for the measurement problem in QM and it is found that quantum probabilities arise in a very natural way from the structure of the theory. The model presented here is very similar to a recent computational model of quantum physics [4] and recent Bohmian models of QFT [5, 6].

Keywords: Pilot wave theory, Bohmian mechanics, Ontological interpretation of Quantum mechanics

1 Introduction

Louis de Broglie [2] and David Bohm [3] have shown that it is possible to give an interpretation of non-relativistic quantum mechanics with a clearly defined ontology and continuous particle trajectories. The models of de Broglie and Bohm, although developed independently, are essentially the same, that is: there is a wave function that evolves according to the Schrödinger equation and guides the particles along continuous tracks. This theory, which we call the pilot wave theory in this paper, produces the same predictions as the standard interpretation of QM. The pilot wave theory has many virtues, but it has also been criticized, among other things, for having a kind of dualistic ontology, i.e. needing both waves and particles to describe the world. Also, it has been difficult to extend the theory to describe the creation and annihilation of particles, although recently some progress towards that goal has been made [5, 6].

The interpretation, which we shall present in this paper removes the problem of ontological dualism by removing the status of the wave function as a fundamental building block of the theory. The wave function is replaced by a fluid of particles, which can exist in two different states: *virtual* and *real*. The number of real particles is assumed to be finite, whereas the virtual particles form a continuum, and thus the bulk of the fluid. All the particles in the fluid follow the trajectories given by the pilot wave theory. The density of the (mostly virtual) particles at some point gives the probability density of finding a *real* particle at that point at the time of measurement.

Since there is no wave function, the particles must somehow guide themselves as a fluid. The equations for such a quantum fluid motion were first introduced by Erwin Madelung in his hydrodynamic formulation of quantum mechanics [1]. The theory presented in this paper can be understood as Madelung's theory supplemented by some minimal additional definitions and assumptions needed to make it really work – in answering the measuring problem of QM, for example.

In the following sections we will see in more detail how the non-relativistic single particle quantum mechanics is handled in this model. After the detailed description of the single particle case, the generalization to the N-particle case will be shortly explained.

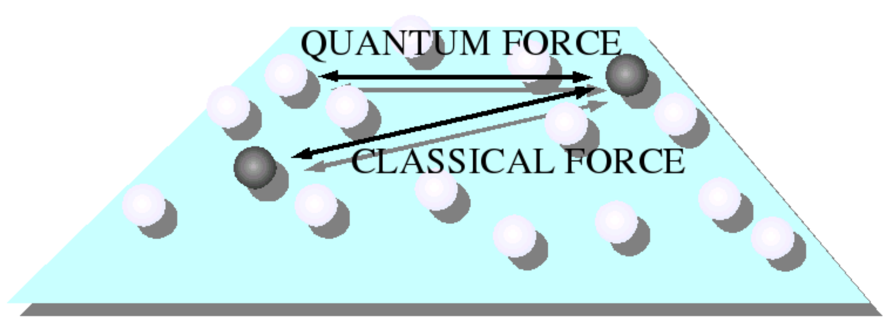

Figure 1. Schematic view of the interactions between "real" particles (dark gray) and "virtual" particles (light blue).

2 Ontology of the model

It is assumed that there is a background space, where the particles move. The particles can have mass and electric charge. Spin is not considered. The particles can be in two different states that we call here virtual and real. In addition to the background space and particles, there can also be external potential fields. These are essentially classical. The number N of real particles in a given volume is assumed to be finite. The number N_0 of virtual particles in a given volume is expected to be much larger, essentially they can be thought of as forming a continuum characterized by a fluid density $\rho(\mathbf{x}, t)$, and, consequently, a number density $N_0\rho(\mathbf{x}, t)$.

3 Interactions

Only the real particles can be observed directly. Otherwise all the particles, virtual or real, follow continuous tracks that are the same as those given by the de Broglie–Bohm pilot wave theory. Also the velocities are assumed to be the same as in the pilot wave theory. How this is possible without explicitly assuming the existence of the wave function will be shown in the next section. It will also be shown that the force affecting the particles can be understood as the sum of classical forces caused by the real particles and quantum forces caused by the fluid of virtual particles (Figure 1).

4 Equations for single particle case

The single particle case here means that there is only one real particle in the system. The number of virtual particles can be assumed to be practically infinite. The velocity of the particles, virtual or real, is given by:

$$\mathbf{v} = \frac{1}{m}\nabla S, \tag{1}$$

where $S(\mathbf{x}, t)$ is simply the phase (including the factor $\hbar$) of the wave function Ψ. There are two equations that govern the evolution of the system:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\frac{1}{m}\rho\nabla S) \tag{2}$$

$$\frac{\partial S}{\partial t} = -\frac{(\nabla S)^2}{2m} - V + \frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{3}$$

These are known as the hydrodynamic equations of QM and they are equivalent to the Schrödinger equation given that some additional constraints on the solutions $\rho(\mathbf{x}, t)$ and $S(\mathbf{x}, t)$ are imposed [7]. The last term in (3) is known as the quantum potential:

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{4}$$

Now, we would like to give up using the phase S (since it is derived from the wave function) and, instead of it, take the velocity field $\mathbf{v}$ as the second variable. We get the following two equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho\mathbf{v}) \tag{5}$$

$$m\frac{\partial \mathbf{v}}{\partial t} = -\nabla(\frac{1}{2}m\mathbf{v}^2 + V + Q) \tag{6}$$

and the constraint $\oint m\mathbf{v}\cdot d\mathbf{l} = 2\pi n,\ n \in \mathbb{Z}$, where the integration is along any closed smooth path along which $\mathbf{v}$ is well defined. Equation (5) can be interpreted as the conservation of probability, which is the usual standpoint taken in hydrodynamic models of QM, or as the conservation or matter, which is actually more suitable for the interpretation presented in this paper. Equation (6) takes a more familiar form, if the time derivative of $\mathbf{v}$ is taken along the particle trajectory:

$$m\frac{d\mathbf{v}}{dt} = -\nabla(V + Q) \tag{7}$$

or

$$m\mathbf{a} = \mathbf{F}_{Classical} + \mathbf{F}_{Quantum}, \tag{8}$$

where $\mathbf{F}_{Classical} = -\nabla V$ is the classical force and $\mathbf{F}_{Quantum} = -\nabla Q$ is a quantum force caused by the local variations of ρ.

5 Interpretation of quantum probability in the single particle case

In QM it is customary to interpret the square of the absolute value of the wave function $\rho = |\Psi(\mathbf{x},t)|^2$ to be the probability density of finding a particle at the location $\mathbf{x}$, when its position is measured at the time t. This law is held to be true in the standard interpretations of QM as well as in the pilot wave approaches of de Broglie and Bohm, and it is usually taken to be a basic fundamental feature of Quantum Mechanics.

In the standard interpretations, no further causal factors are sought to explain this probability law, whereas in the pilot wave models the probability law arises simply from our ignorance of the true particle positions. Thus, in the pilot wave models, we have to *assume* that the probability density is equal to $|\Psi(\mathbf{x},t_0)|^2$ initially. Once this assumption is made, the guidance condition ensures that the probability density will agree with the value $|\Psi(\mathbf{x},t)|^2$ at any later time t.

In the model presented in this paper, the situation is, however, slightly different. The probability density is again given by the density $\rho(\mathbf{x},t)$, but it has a very natural explanation: Since both the real and virtual particles follow the same equations of motion and their only difference is that the real particle can be detected and the virtual particles cannot, we simply assume that the single real particle can be any one among all the particles forming the quantum fluid with number density $N_0\rho(\mathbf{x},t)$. Thus, thinking by purely classical definitions of probability, the fluid density $\rho(\mathbf{x},t)$ is the only natural candidate for the probability density of the real particle. This situation is in no way different from finding one fish that has swallowed a gold coin in a big swarm of fishes swimming in the ocean and described by a "fish density" $\rho_{\text{fish}}(\mathbf{x},t)$, or spotting one ringed bird flying in a big flock of other birds without a ring.

6 Generalization to N-particle case

The generalization to the N-particle case, (where N is the number of real particles) simply requires the single particle wave function to be replaced by the N-particle wave function. This wave function can then be replaced by the N-particle density and velocity fields, analogously to the single particle case. In this case, the single particle is replaced by an N-particle multiplet: there will be one real, detectable, N-multiplet moving in a continuous "fluid" of virtual, undetectable N-multiplets. Since the fluid of N-multiplets has dimension $3N$, the term *hyperfluid* is suggested to describe it.

It can be shown that in some cases, the N-particle case reduces to many separate one particle cases. Such is the case for N free particles. An analogous situation exists in superfluids. Actually, a superfluid, such as He II, is a real world working model of the virtual particle fluid presented in this paper.

7 Particle creation and annihilation

The second remaining problem of the pilot wave theory, *vis à vis* the difficulty of handling cases where particles are created or destroyed, can also be solved, at least conceptually, using the virtual particle interpretation. All we need is some kind of a process of energy exchange between real and virtual particles.

The following gives a qualitative picture of such a situation: Let us assume, for example, that in the beginning there is a real electron–positron pair, i.e. two active particles and a sea of passive virtual particles, of various kinds, all around them. The two active particles would collide with sufficient energy and transfer their energy to a sea of passive muon–anti-muon pairs. (This could exist without difficulty since it would not be observable at low energies). Now, a passive muon–anti-muon pair would receive the energy of the electrons and become active, so that we would end up with a real muon and a real anti-muon speeding in opposite directions from the point of impact.

However, at this point it must be emphasized that the process of energy exchange described in the above example cannot be directly described using the formalism presented in the earlier sections of this paper, but the model would have to be extended or modified somehow (e.g. requiring the use of Fock Space). The reason for this is that particle creation and annihilation cannot be described by the non-relativistic particle QM. Instead one should use a QFT version of the present theory, or some other clever formalism. A Bohmian version of QFT sharing similarities with the model presented here and allowing particle creation and annihilation, has been described in [5], so the possibility of devising such a formalism also within the virtual particle scheme presented in this paper, doesn't appear to be totally unrealistic.

In case such a detailed model of particle creation and annihilation can be produced, it could perhaps also be extended to model the Hawking radiation close to a black hole. Related to this, we can also assume that the virtual particles, or the quantum potential, have a small rest mass. This might provide an answer to the mystery of dark mass and dark energy in cosmology.

8 Discussion

The interpretation presented in this paper is in many ways similar to other hydrodynamic or stochastic models [1]-[12]. The differences in the approaches usually concern such things as the role of the wave function, the nature of the particles (provided they exist at all), and the form of the particle trajectories. Many approaches assume very irregular particle trajectories or fluid motions to explain the emergence of quantum probabilities [8, 9]. Why this is not necessary was explained in section 5, where the quantum probability law was found to be a natural consequence of the formalism.

Madelung's original hydrodynamic formulation of QM [1] can be seen as a conceptual starting point for this work, and the two approaches are quite similar[1] in spirit. Madelung's paper, however, leaves open many questions relevant for the measurement problem in QM. These questions can be answered within the framework of the proposed interpretation – to a large degree thanks to the features it inherits from the pilot wave theory.

The virtual particle interpretation can be seen to be analogous to the many worlds formalism. The difference is that the multitude of worlds is here replaced by the multitude of the virtual particles (or virtual N-particle systems). The fact that hydrodynamical equations [14] appear also in the decoherent histories approach may have some significance. The situation can be expected to be similar also in some of the spontaneous localization models.

Since the theory presented here is non-relativistic, it would be interesting to study what the requirement of Lorentz invariance would bring into the model. Another interesting topic would be to investigate the possibility of formulating a Machian [13] version of this theory and include gravity in it. The connection of Feynman paths to the particle paths in various hydrodynamical models is also worth investigating.

9 Conclusions

The model presented here, apart from the more speculative ideas of sections 6 and 7, is a well defined theory. The main motivation behind it was to solve the problem of wave-particle dualism present in the de Broglie–Bohm pilot wave theory, while retaining many of the desirable features of that approach. One of the outcomes of this approach was a surprisingly simple explanation for quantum probabilities. Since the equations are the same as in the hydrodynamic

[1]Among the more recent approaches, the closest in semblance to the model presented in this paper is, quite surprisingly, a model introduced by Courtney Lopreore and Robert Wyatt for numerical computations in Quantum Physics and Chemistry [4] (The existence of such a method actually came as a big surprise to the writer). Comparing an *interpretation* of QM with a *computational method* is challenging, but at least the animations created by the computational research group give an idea of the flow of a virtual particle fluid.

formulation of QM and the pilot wave theory, and the predictions the same as those given by standard QM, the theory presented in this paper should be viewed rather as a model, or, as the title suggests, as an interpretation of quantum mechanics than as a new theory. However, by providing a new perspective and giving new insights, this model can serve as a basis for new theoretical developments.

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