THE PROTON-ELECTRON MASS RATIO, THE FINE-STRUCTURE CONSTANT, THE ELECTRON G-FACTOR, AND VON KLITZING'S CONSTANT.

ABSTRACT.

In this paper, we shall show how the proton-electron mass ratio is determined by the values of α, the fine-structure constant, g_e, the electron g-factor, and R_K, von Klitzing's constant.

In his book, *The Accidental Universe*, Paul Davies[1] gives an equation for the average 'life-span' of a main sequence star of ~0.841 $M_\odot$ (i.e., 0.841 solar masses[2]). This age, he says, is given by:

$$t_* = \frac{h}{m_p c^2}\frac{\hbar c}{G m_p^2}\left[\frac{\alpha^2}{100}\left(\frac{m_p}{m_e}\right)^2\right] = 1.34 \times 10^{16}\ \mathrm{s}\,. \qquad (1)^3$$

Unfortunately, this is only 424,620,376.71 years, which is spectacularly wrong for the (hydrogen fusing) life span of such a star[4]! However, if instead of multiplying the square of the proton-electron mass ratio by $\alpha^2/100$, we multiply by $\alpha/100$, we obtain:

$$t_* = 7.46277551 \times 10^{14} \times 246.0271 = 1.836045 \times 10^{17}\ \mathrm{s}\,. \qquad (2)$$

[1]Davies, PCW (1982), *The Accidental Universe*, Cambridge: CUP, pp.52, 55.

[2]Of local stars, ε-Eridani has a mass of 0.82 ± 0.02 $M_\odot$ and HD17925's [a K-type 25.76 light years away] is 0.85 ± 0.02 $M_\odot$; see: Gonzalez, G, Carlson, MK & Tobin, RW (2010), 'Parent stars of extrasolar planets – X. Lithium abundances and *v* sin *i* revisited,' ***Monthly Notices of the Royal Astronomical Society*** **403(3):1368-80**, April 2010, DOI: 10.1111/j.1365-2966.2009.16195.x; available online at: http://arxiv.org/pdf/0912.1621v1.pdf, Table 3, p.1376.

[3]Equation 2.28, p.55; mass derived from Equations 2.17-18, p.52.

[4]See: Paczynski, B (1970), 'Evolution of Single Stars. I. Stellar Evolution from Main Sequence to White Dwarf or Carbon Ignition,' ***Acta Astronomica*** **20(2):47-58**, BibCode: 1970AcA….20…47P, online at: http://articles.adsabs.harvard.edu/cgi-bin/nph-iarticle_query?1970AcA....20...47P&data_type=PDF_HIGH&whole_paper=YES&type=PRINTER&filetype=.pdf (where Paczynski notes, p.57, that stars with masses of < ~3.5 $M_\odot$ 'never burn carbon', and thus the reaction ${}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{16}_{8}\mathrm{O} + \gamma$ does not take place in them);
Franck, S, Block, A, Bloh, W, Bounama, C, Garrido, I & Schellnhuber, H-J (2001), 'Planetary habitability: is Earth commonplace in the Milky Way?', ***Naturwissenschaften*** **88(10):416-26**, 23rd August 2001, DOI: 10.1007/s001140100257, online at: http://garage.physics.iastate.edu/astro250/francknatureg2001.pdf, Figure 8 and accompanying text, p.422

This is 5.818 × 10^9 years[5], so we are now doing rather better, even if this is still only about half of the time we need for our purposes here, defined by the weak anthropic principle[6], which in turn is based on RH Dicke's assessment of the length of time taken by stellar nucleosynthesis to forge the vital chemical elements, such as carbon, nitrogen and oxygen, needed for organic chemistry and biology, and thus for our existence as observers, work which, in its turn, depended on that of Fred Hoyle, the Burbages, and WA Fowler[7].

Given that the age of the Earth, and of the rest of the Solar System, is an estimated 4.6 billion years[8], and the age of the Universe, according to the Wilkinson Microwave Anisotropy Probe (WMAP) 7 year results, is 13.75 ± 0.13 Gyr,[9] what we might call the 'seed star' (or stars) that produced the C, N, O and other elements that later formed our biosphere had to have had the time to be born, to live and die, between ~13.75 billion years ago and 4.6 billion years ago, a total of 9.15 billion years. The earliest that such a star could have

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[5]Using 3,600 s × 24 hours × 365.25 days.

[6]The *locus classicus* for the definitions of the weak and strong anthropic principles is Carter, B, 'Large number coincidences and the anthropic principle in cosmology,' in Longair, MS, ed (1974), *Confrontation of cosmological theories with observational data; proceedings of the [IAU] Symposium [63], Krakow, Poland, September 10-12, 1973*, Dordrecht, Netherlands: D Reidel & Co, BibCode: 1974IAUS…63…291C, pp.291-8, online at: http://articles.adsabs.harvard.edu/cgi-bin/nph-iarticle_query?1974IAUS...63..291C&data_type=PDF_HIGH&whole_paper=YES&type=PRINTER&filetype=.pdf, see esp. pp.292-3. Penrose, R (1990), *The Emperor's New Mind. Concerning Computers, Minds, and the Laws of Physics*, London: Vintage, pp.560-2, describes both the weak and strong anthropic principles, as does, in more extended form, idem (2005), *The Road to Reality. A Complete Guide to the Laws of the Universe*, London: Vintage, pp.757-62. He summarises the two principles in the text to Fig. 28.13 on p.759, thus:

> **'Anthropic principle: (A) weak form. Sentient beings must find themselves in a spatio-temporal location in the universe, at which the conditions are suitable for sentient life.**
> **(B) strong form. Rather than considering just one universe we consider an ensemble of possible universes, among which the fundamental constants of Nature may vary. Sentient beings must find themselves to be located in a universe where the constants of Nature (in addition to the spatio-temporal location) are congenial.'**

Others, such as Rees (Rees, MJ [2002], *Our Cosmic Habitat*, Princeton, NJ: Princeton University Press) and Tegmark (Tegmark, M [1998], 'Is the "theory of everything" merely the ultimate ensemble theory?', ***Annals of Physics*** **270(1):1-51**, 20th November 1998, DOI: 10.1006/aphy.1998.5855, online at: http://arxiv.org/pdf/gr-qc/9704009v2.pdf; idem [2007], 'The Mathematical Universe,' ***Foundations of Physics*** **38(2):101-50**, DOI: 10.1007/s.10701-007-9186-9, online at: http://arxiv.org/pdf/0704.0646v2.pdf), argue that possible, or actually existing parallel universes (in a 'Multiverse') should be considered that have different laws of physics to our universe, as well as different values for the physical constants.

[7]Dicke, RH (1961), 'Dirac's cosmology and Mach's Principle,' ***Nature*** **192:440-1**, 4th November 1961, DOI: 10.1038/192440a0, abstract and refs online at: http://www.nature.com/nature/journal/v192/n4801/abs/192440a0.html; Burbidge, EM, Burbidge, GR, Fowler, WA & Hoyle, F (1957), 'Synthesis of the Elements in Stars,' ***Reviews of Modern Physics*** **29(4):547-650**, DOI: 10.1103/RevModPhys.29.547, BibCode: 1957RvMP…29.547.

[8]Turney, C (2006), *Bones, Rocks and Stars. The Science of When Things Happened*, Houndmills, Basingstoke, Hants.: Macmillan, pp.155-8; Patterson, C (1956), 'Age of meteorites and the Earth,' ***Geochimica et Cosmochimica Acta*** **10(4):230-7**, October 1956, DOI: 10.1016/0016-7037(56)90036-9.

[9]See: http://lambda.gsfc.nasa.gov/product/map/current/best_params.cfm.

come into existence would have been a 100 million years after the Big Bang[10], making its life-span 9.05 billion years. Clearly, what is needed, then, is an adjustment to Davies' factor that produces the needed longer life-span (taking it as read that the larger the larger the mass, the shorter the life-span[11]). In fact, if we make the bracketed multiplying factor in Equation 1:

$$\frac{\alpha}{64}\left(\frac{m_p}{m_e}\right)^2 = 384.4173\,, \tag{3}$$

then we arrive at a value for t_* of ~9.091 billion years (let us not quibble over the odd 40 million or so!).

What, then, of Paczynski's limit of 3.5 $M_\odot$ below which stars cannot achieve carbon fusion, producing oxygen, in their terminal phases? If Strobel's table is correct[12], these stars live for about 370 million years or so. This is actually good news for our argument here, because during the course of our 'seed star's' ~9 billion year 'life', there will have been time for 24 (in round figures) life-spans of the shorter-lived and more massive stars, which produce oxygen (the triple alpha process[13] produces the carbon-12 and -14 which is the main building block for life). The early universe is believed to have contained a generation of massive so-called 'Population III' stars, all now long-extinct,

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[10]Larson, B & Bromm, V (2009), 'The First Stars in the Universe,' ***Scientific American***, 19th January 2009, http://www.scientificamerican.com/article.cfm?id=the-first-stars-in-the-un&print=true.

[11]Strobel, N (2001), 'Stellar Evolution. Mass Dependence,' http://www.astronomynotes.com/evolutn/s2.htm, has a useful table of solar mass versus age and spectral type, given below:

star mass, $M_\odot$	time (years)	Spectral type
60	3 million	O3
30	11 million	O7
10	32 million	B4
3	370 million	A5
1.5	3 billion	F5
1	10 billion	G2 (Sun)
0.1	1000s billions	M7

[12]Fernandes, J, Lebreton, Y, Baglin, A & Morel, P (1998), 'Fundamental stellar parameters for nearby visual binary stars: *η Cas*, *ξ Boo*, 70 *Oph* and 85 *Peg*. Helium abundance, age and mixing length parameter for low mass stars,' ***Astronomy and Astrophysics*, 338:455-64**, October 1998, BibCode: 1998A&A…338..455F; online at:
http://articles.adsabs.harvard.edu/cgi-bin/nph-iarticle_query?1998A%26A...338..455F&data_type=PDF_HIGH&whole_paper=YES&type=PRINTER&filetype=.pdf demonstrate just how hard it is to measure the ages of stars. The figures they give in Table 3, p.461 (see also Fig. 6, p.463, showing age against metallicity) for the ages of η Cassiopiae A & B of 4 ± 2 Gyr; for ξ Bootes A & B of 2 ± 2 Gyr (i.e., they think the stars could be new borns); and for 70 Ophiuci A & B of 3 ± 2 Gyr. They were unable to work out the age of the remaining biary star system, 85 Pegasi A & B.

[13]Wiescher, M, Regan, P & Aprahamian, A (2002), 'Nuclear astrophysics: a new era,' ***Physics World*** February 2002, pp.33-8, online at:
http://personal.ph.surrey.ac.uk/~phs1pr/papers/nucastr_phys_world_reprint.pdf, describe the process (p.34) and the related carbon-12 nuclear resonance discovered by Fred Hoyle (p.36 and Fig. 2a, p.35).

which furnished the heavy elements which were to form the later generations of Population II and I stars[14], including our own Sun, which is Population I.

Equation (1), as re-written, can be expressed in the following way:

$$t_* = \frac{\alpha\alpha_G^{-1} t_{Cp}}{64}\left(\frac{m_p}{m_e}\right)^2 . \tag{4}$$

Here, t_{Cp} is the Compton period of the proton (the reciprocal of its Compton frequency, ν_{Cp}), and α_G is the gravitational coupling constant. We may now obtain:

$$\pm\sqrt{(t_* \nu_{Cp} \alpha_G)} = \frac{\pm\alpha^{½}}{8}\left(\frac{m_p}{m_e}\right) = \pm 19.6065625 . \tag{5}$$

This an interesting result, and worth spelling out: for the equation says that plus or minus the square root of the product of age of our 'seed star', the Compton frequency of the proton and the gravitational coupling constant is equal to plus or minus the fundamental charge, divided by 8, times the proton-electron mass ratio.

The seed star life-span $t_* \times \nu_{Cp} = 6.509339 \times 10^{40}$, and its square root is ~2.55×10^{20}. This compares with a value for α_G^{-1} of 1.6933×10^{38}.

These figures are very close in value, and coincide with another, the square root of Eddington's number, the number of protons and electrons in the observable universe[15], N_{Edd}, which astrophysicists and astronomers now prefer to treat as a round figure, ~10^{80} (see Barrow [2003][16], pp.85-6; this is an approximation, as we shall see below). Paul Davies (op.cit., p.80) notes that this number is equal to the ratio of the proton life-time, as estimated by GUTs[17],

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[14]For a brief definition and description, see: http://hyperphysics.phy-astr.gsu.edu/hbase/starlog/pop12.html.

[15]Eddington, AS (1939), *The Philosophy of Physical Science*, Tarner Lectures 1938, Cambridge: CUP, Chapter XI, p.170, available online at: http://www.questia.com/PM.qst?a=o&d=62000178. He computed the number by multiplying 136 (which he believed, mistakenly, was the value of α^{-1}) by 2^{256}, a number which Bertrand Russell was astonished he had computed himself! (See: Chadrasekhar, S [1983], *Eddington. The most distinguished astrophysicist of his time*, Cambridge: CUP, p.3.)

[16]Barrow, J (2003), *The Constants of Nature. From Alpha to Omega*, London: Vintage.

[17]The Georgi-Glashow SU(5) GUT postulates a proton decay after 10^{31} years; a Minimal Supersymmetric SO(10) GUT has a proton decay-time of 2.97×10^{33} years (see: Goh, HS, Mohapatra, RN, Nasri, S & Siew-Phang, N [2004], 'Proton Decay in a Minimal SUSY SO(10) Model for Neutrino Mixings,' ***Physics Letters B*** **587(1-2):105-16**, 6th May 2004, DOI: 10.1016/physletb.2004.02.063, online at: http://arxiv.org/pdf/hep-ph/0311330v2.pdf, p.105). The Pati-Salam Model (Pati, J & Salam, A [1974], 'Lepton number as the fourth "color" [*sic*],' ***Physical Review D*** **10(1):275-89**, 1st July 1974, DOI: 10.1103/PhysRevD.10.275, online at: http://hcpl.knu.ac.kr/lhc/PhysicsAnalysis/wprime_reference/theory/PhysRevD.10.275.pdf) argues for the following 3-body proton decay modes (p.286):

and the Planck time[18]. There is a explanation for this, if the reasoning in n.17 is followed. We find that:

$$t_{p(\mathrm{D})} = t_{Cp}\left(\frac{m_{\mathrm{X}}}{m_p}\right)^3 = 1.952 \times 10^{49}\ \text{years}\,. \tag{6}$$

The age of the universe[19] is, as we have seen, some 13.75 billion years, so $t_{p(\mathrm{D})}$ is 1.41963×10^{39} times longer than the present cosmic age. This is $8.8381\alpha_G^{-1}$. We can thus write:

$$t_{p(\mathrm{D})} = 8.8381 t_0 \frac{\hbar c}{\mathrm{G}m_p^2} = 8.8381 t_0 \frac{M_{\mathrm{PL}}^2}{m_p^2}\,. \tag{7}$$

In fact, $t_0 = 2\mathrm{G}M/c^3$, where M, the mass of the observable universe, is given by:

$$\begin{aligned} p &\rightarrow 3\nu + \pi^+ \\ &\rightarrow 4\nu + e^+ \text{ or } 4\nu + \mu^+ \\ &\rightarrow 4\nu + \mu^+ + e^+ + e^-, \text{ etc.} \end{aligned}$$

As for for the decay time, this equals, Davies tells us (op.cit., p.80) on $t_{Cp}(m_x/m_p)^4$, where m_x is the mass of the GUT gauge boson, the 'X' particle. In their 'basic model', Pati and Salam tell us (p.278, Table 1 on p.279), the X particle has a mass of $m_x = 1/\alpha^2 G_F$, where G_F is the Fermi coupling constant, whose dimensions are energy per cubic metre (Feynman, RP & Gell-Mann, M [1958], 'The Theory of the Fermi Interaction,' ***Phys Rev*** **109(1):193-8**, 1st January 1958, online at: http://authors.library.caltech.edu/3514/1/FEYpr58.pdf, p.195), but is more usual to treat it as energy, and divide by the factor $(\hbar c)^3$ to obtain the Fermi coupling constant, $G_F/(\hbar c)^3 = 1.166364 \times 10^{-5}\ \mathrm{GeV}^{-2} = 4.54366 \times 10^{14}\ \mathrm{J}^{-2}$, which can then be multipled by c^4 to give $3.6702 \times 10^{48}\ \mathrm{kg}^{-2}$ (see: http://physics.nist.gov/cgi-bin/cuu/Value?gf|search_for=atomnuc!). In this form, it is also equal to $2^{½}/8(g_w/m_w)^2$, where g_w is the coupling constant of the weak interaction ($\alpha_w = g_w^2/4\pi = 1/30$, see: http://www2.warwick.ac.uk/fac/sci/physics/teach/module_home/px435/weak.pdf, and m_w is the mass of the $W^{\pm}$ boson [see: http://en.wikipedia.org/wiki/Fermi's_interaction]). The square root of this is $1.918 \times 10^{24}\ \mathrm{kg}^{-1}$, and its reciprocal is $\sim 5.22 \times 10^{-25}$ kg. Multiplying this by $\alpha^{-2} = 18{,}778.86504 = 9.802568 \times 10^{-21}$ kg, so we have a value for m_x, which turns out to be equal to $1/\alpha^2(G_F c^4)^{½}$. Now applying Davies' equation for the decay time, we get $(m_x/m_p)^4 = (1.118 \times 10^{27})^4 = 1.56 \times 10^{108}$ times $t_{Cp} = 2.18 \times 10^{77}$ years, which seems rather excessive, but consistent with the estimate given by Pati and Salam (p.278) of m_x being in the range $10^4 - 10^5$ GeV (the figure above is equal to 8.81×10^{-4} J, which is $\sim 5.5 \times 10^6$ GeV. Is there any way to rescue Davies again? To obtain a value for the putative proton decay time, $t_{p(\mathrm{D})}$ more consistent with other GUTs and with the (largely negative) experimental findings, it is only necessary to replace the '4' in the indexical term by a '3'.

[18]According to Kajita, T (2011), 'Results from massive underground detectors on solar and atmospheric neutrino studies and proton decay searches,' ***Journal of Physics: Conference Series*** **308(1):012003**, DOI: 10.1088/1742-6596/308/1/012003, online at: http://iopscience.iop.org/1742-6596/308/1/012003/pdf/1742-6596_308_1_012003.pdf, 'There has been no candidate event in the signal box. The 90% [confidence level] lower limit on the lifetime of [the] proton for this decay mode [$p \rightarrow e^+\pi^0$] is 1.01×10^{34} years. …There has been no candidate event for [the $p \rightarrow \bar{\nu}K^+$ decay mode]. The limit of the proton lifetime for this decay mode is 3.3×10^{33} years' (pp.6-7, PDF pagination).

[19]The age of the universe is related to t_{Cp} by $t_0 = t_{Cp}(\hbar c/2\pi \mathrm{G} m_p m_e)$, but only at the present epoch, another instance of the weak anthropic principle at work.

$$\mathrm{M} = \frac{M_{\mathrm{PL}}^4}{m_p^2 m_e} = 8.8085585 \times 10^{52}\ \mathrm{kg}\,,$$

(8)

which gives us a value for t_0 of 13.827 × 10^9 years, well within the margin of error of the WMAP figure (see above). Thus (7) becomes:

$$t_{p(\mathrm{D})} = 17.672\,\frac{\mathrm{G}M_{\mathrm{PL}}^6}{m_p^4 m_e c^3}\,.$$

(9)

The square root of Eddington's number is given by:

$$\sqrt{N_{\mathrm{Edd}}} = \frac{e^2}{4\pi\varepsilon_0 \mathrm{G} m_p m_e} = \frac{\alpha\hbar c}{\mathrm{G} m_p m_e} = 2.2688 \times 10^{39}\,.$$

(10)[20]

This gives a value for N_{Edd} itself of 5.1475 × 10^{78}, which relates to the mass of the observable universe, M, by:

$$\mathrm{M} = \sqrt{N_{\mathrm{Edd}}}\,\frac{M_{\mathrm{PL}}^2}{\alpha m_p}\,.$$

(11)

We have also seen that α_G^{-1} is 1.6933 × 10^{38} and the value of $t_* \times v_{Cp}$ = 7.03593 × 10^{40}. Dividing N_{Edd} by $N_* = \alpha_G^{-3/2}$, the maximum[21] number of protons in a star (according to Davies, op.cit., p.51), gives an estimate for the total number of stars in the observable universe, 2.336 × 10^{21}. If $\alpha_G^{-3/2} m_p$ = 3.68552 × 10^{30} kg (~1.853 $M_\odot$), the star has a life-span (see n.11) of < 3 billion years. If, instead, we use $\alpha_G^{-1.492419126}$ = 1.13056 × 10^{57}, we obtain a (proton) mass for the star of 1.891 × 10^{30} kg = 1 $M_\odot$, and a life-span of ~10 billion years, based on a nuclear

[20]See: Lloyd, S (2002), 'Computational capacity of the universe,' ***Phys Rev Letters* 88(23):237901**, 24th May 2002, DOI: 10.1103/PhysRevLett.88.237901, online at: http://arxiv.org/pdf/quant-ph/0110141.pdf, pp.8-9 (arxiv. pagination). As Lloyd tells us, 'Three quarters of a century ago, Eddington noted that two large numbers that characterize our universe happen to be approximately equal. In particular, the ratio between the electromagnetic force by which a proton attracts an electron and the gravitational force by which a proton attracts an electron is $\alpha = e^2/\mathrm{G}m_p m_e \approx 10^{40}$. Similarly, the ratio between the size of the universe and the classical size of an electron is $\beta = ct/(e^2/m_e c^2) \approx 10^{40}$. … A third large number, the square root of the number of baryons in the universe, $\gamma = \sqrt{(\rho\rho c^3 t^3/m_p)}$ is also ≈ 10^{40}. This is not a coincidence given the values of α and β: $\alpha\beta \approx \gamma^2$ in a universe near its critical density $\rho_c \approx 1/\mathrm{G}t^2$. The astute reader may have noted that the number of operations that can have been performed by the universe is approximately equal to the Eddington-Dirac large number cubed.'
[21]This is not quite accurate, as can be seen from the main text. Stars as large as 60 $M_\odot$ are not unknown, just very short-lived (with life-spans of 3 million years, see n.11.

fusion energy release of 10% of the total available proton rest-mass (Davies, op.cit., p.54)[22].

If the proton life-span is given by Equation (9), and the average life-span of the seed stars that gave birth to our own Solar System, and to ourselves, made, as Carl Sagan[23] told us, of star dust, is given by Equation (4), then their ratio, $t_{p(\mathrm{D})}/t_*$ is:

$$1{,}131.08\,\frac{\alpha_{\mathrm{G}}\mathrm{GM}_{\mathrm{PL}}^{6}}{\alpha hc}\left(\frac{m_e}{m_p^5}\right) = 8.831\alpha_{\mathrm{G}}^{-1}\,. \tag{12}$$

The number 1,131.08/8.831 = 128.080625 is close to α^{-1} at TESLA energies[24], another fascinating result.

There are another set of 'coincidences', a whole set of them, involving the proton-electron mass ratio. They involve e, Euler's number, π and $|g_e|$, the absolute value of the electron g-factor (usually given as a negative number), which, in turn is, as is self-evident, $|1 + a_e/s|$, where a_e is the electron's magnetic moment anomaly, 0.00115965218076, and s is the electron's spin quantum number, ±½. These 'coincidences' (if that is what they are) are:

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[22]If we do the calculation, $0.01\alpha_{\mathrm{G}}^{-1.492419126}m_pc^2 = 1.699546 \times 10^{46}$ J = $\mathrm{E}_{\mathrm{S(TOT)}}$, the total energy produced during the Sun's hydrogen burning. If the Sun's power output over its entire life-time is given by:

$$L(t) \cong L_0\left[1 + 0.3\left(1 - \frac{t}{t_0}\right)\right]^{-1},$$

i.e., the time-variable solar luminosity, $L(t)$, where t_0 is the current age of the Sun, ~4.6 Gyr, and t is time, measured from the origin of the Solar System (equation taken from Vokrouhlicky, D, Broz, M, Morbidelli, A, Bottke, WF, Nesvorny, D, Lazzaro, D & Rivkin, AS [2006], 'Yarkovsky footprints in the Eos family,' ***Icarus*** **182(1):92-117**, May 2006, DOI: 10.1016/j.icarus.2005.12.011, online at: http://www.boulder.swri.edu/~bottke/Reprints/Vok_2006_Icarus_182_92_Yark_Footprints_Eos_Family.pdf, p.108), then $\mathrm{E}_{\mathrm{S(TOT)}}$ divided by the Sun's total hydrogen fusion life-span of ~10^{10} years = 3.15576×10^{17} s, gives its *mean* power output (luminosity, $L_\odot$) of 5.38554×10^{28} W, or 5.38554×10^{16} TW. It is *currently* 3.939×10^{26} W (including both photonic and neutrino radiation) but this will continue to increase with time, see: http://en.wikipedia.org/wiki/Solar_luminosity; Schröder, K-P & Connon Smith, R (2008), 'Distant future of the Sun and Earth revisited,' ***MNRAS*** **386(1):155-63**, May 2008, DOI: 10.1111/j.1365-2966.2008.13022.x, online at: http://arxiv.org/pdf/0801.4031.pdf, who tell us that, according to the 'ZAMS' (Zero Age Main Sequence) model, the Sun began its life with only 70% of its current luminosity (p.1 [arxiv. pagination]), and 'The present Sun is increasing its average luminosity at a rate of 1% in every 110 million years, or 10% over the next billion years' (p.3).

[23]Sagan, C (1985), *Cosmos*, New York: Ballantine (Random House), book of 1980 US Public Broadcasting System (PBS) TV documentary series, *Cosmos: A (sic) Personal Voyage*.

[24]See: Jegerlehner, F (2001), 'The Effective Fine Structure Constant at TESLA Energies,' http://arxiv.org/pdf/hep-ph/0105283v1.pdf, pp.1-2, 4 (& Figure 1 on p.4), 5, 9, Figures 4 & 5, p.10, 12, Table 2, p.14. The abstract has a table showing the value of α^{-1} at 100 GeV (≅ m_zc^2, where m_z is the mass of the Z^0 boson), which is 128.79 ± 0.054. At 300 GeV, it is 127.334 ± 0.054. The average of those two figures is 128.062, which should be the value of α^{-1} at 200 GeV. 246 GeV is the 'electro-weak' energy scale, 246 GeV, the vacuum expectation value of the Higgs field. The ATLAS experiment at the LHC at CERN has shown evidence within the 95% Confidence Level of the light Higgs boson in the range 115-131 GeV, and the CMS Experiment has produced a similar range, with the same CL, of 115-127 GeV, with strong hints of a particle at 124 GeV. See: http://cms.web.cern.ch/news/cms-higgs-boson-search-results-2010-2011-data-samples.

$$\frac{m_p}{m_e} \approx 6\pi^5 = 1836.118109\,;$$

$$\frac{m_n}{m_e} \approx \frac{e\pi^2}{2\alpha} = 1838.22599\,;$$

$$\frac{m_p}{m_e} \approx \frac{e\pi^2}{\alpha|g_e|} = 1836.096756\,;$$

$$\alpha \approx \frac{e}{6\pi^3|g_e|} = 0.0072972677067\,.$$

(13a-d)

These coincidences are quite well known[25], and the fourth of them explains the first when substituted into the third (here m_n is the mass of the neutron). We would expect α to be related to g_e in any event, as we will explain shortly.

Not so well known, we may suspect, is this 'coincidence':

$$\frac{m_p}{m_e} \approx \frac{4\pi\alpha|g_e|}{j^4} = \frac{g_L^2|g_e|}{j^4} = 1836.1515687\,.$$

(14)

Here, j is the theoretical coupling constant of quantum electrodynamics[26], which Feynman (1985, op.cit., p.91) tells us has the value ± 0.1, and $g_L = (4\pi\alpha)^{½}$.[27]

[25]At least, $6\pi^5$ is; see: http://physics.stackexchange.com/questions/8373/dimensionless-constants-in-physics; for numerical coincidences generally, see: McKean, HP (2003), 'Some Mathematical Coincidences,' http://silverdialogues.fas.nyu.edu/docs/CP/303/mckean.pdf; http://en.wikipedia.org/wiki/Mathematical_coincidence. 'Ganzfeld', in: http://www.physicsforums.com/showpost.php?s=2e71e9243ddea80973f9996f87fb5e88&p=1027775&postcount=261, 'The $6\pi^5$ ratio for the proton-electron mass ratio was also independently published in an item by me in Nature (July 7th 1983 I think the date was)...which the editor rather scathingly titled, 'The Temptations Of Numerology'. The same stuff was also published in 1984 in 'Speculations In Science And Technology'. In that article, I also included a number of other fascinating 'coincidences'. For example...the combined mass of the particles of the meson octet is 3.14006 times the proton mass. Pretty close to pi. Amazingly, the combined mass of the particles in the baryon octet is around 9.8 times the proton mass. Close to pi squared.'

[26]The term 'coupling constant' seems to be somewhat ambiguous, and is used differently by different authorities. For example, some will tell the student that α, the fine-structure constant, is the coupling constant of the electromagnetic interaction; others that it is $\pm\alpha^{½}$. See: Gabrielse, G, Hanneke, D, Kinoshita, T, Nio, M & Odom, B (2006), 'New Determination of the Fine Structure Constant from the Electron *g* Value and QED,' ***Phys Rev Lett*** **97:030802**, 17th July 2006, DOI: 10.1103/Phys.RevLett.97.030802, online at: http://collargroup.uchicago.edu/pdf/phys-rev_2006-1.pdf. Feynman, RP (1985), *QED – The Strange Theory of Light and Matter*, London: Penguin, pp.91, 120-1, 125-30, 143, calls *j* the 'junction number' and specifies its value as ± 0.1.

[27]See: Pati & Salam, op.cit., p.279.

Equation (14)'s approximation is ~99.99994% of the 2010 CODATA value of m_p/m_e.

Is this 'just' a coincidence, like sin 666° = cos 216° (= 6 × 6 × 6°) = $-\phi/2$, where ϕ is the golden ratio, which gives the 'Number of the Beast' of the Book of Revelation (Rev.13:18)? The electron's magnetic moment anomaly, a_e, is, according to Aoyama, *et al* (2005)[28], and standard textbooks[29] on the subject, the sum of a power series, of ever smaller numbers, being powers of α/π multipled by constants, the first one in the series, the largest, being the number the QED pioneer Julian Schwinger (1918-94) discovered[30], ½(α/π) = α/2π = 0.001161409733. The fine-structure constant itself, as usually expressed, incorporates π, but can be written as:

$$\alpha = \frac{e^2 \mu_0 c}{4\pi\hbar} = \frac{e^2 Z_0}{2h}\,, \tag{15}$$

where Z_0 is the characteristic impedance of the vacuum, 376.730310461 Ω. Thus, our approximation becomes:

$$\frac{m_p}{m_e} \approx \frac{e^2 Z_0 |g_e|}{\hbar j^4} = 1836.151554361\,. \tag{16}$$

In fact, we find we find that:

$$Z_0 \approx \frac{j^4 |g_e| R_{\mathrm{K}}}{8\pi(1 + 2a_e)}\left(\frac{m_p}{m_e}\right) = 376.7310453743\,\Omega\,. \tag{17}$$

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[28]Aoyama, T, Hayaka, M, Kinoshita, T & Nio, M (2006), 'Automated Calculation Scheme for α^n Contributions of QED to Lepton g – 2: Generating Renormalized Amplitudes for Diagrams without Lepton Loops,' ***Nuclear Physics B* 740(1-2)138-80**, 17th April, 2006, DOI: 10.1016/j.nuclphysb.2006.01.040, online at: http://arxiv.org/pdf/hep-ph/0512288.pdf, inform us that the 'pure QED contribution' to a_e =

$$A_1^{(2)}\left(\frac{\alpha}{\pi}\right) + A_1^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + A_1^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + A_1^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + \cdots,$$

with $A_1^{(2)}$ = 0.5; $A_1^{(4)}$ = –0.328478965…; $A_1^{(6)}$ = 1.181241456…; and $A_1^{(8)}$ = –1.7283(35) (see: p.3, arxiv. pagination).

[29]For example, Zeidler, E (2008), *Quantum Field Theory II. Quantum Electrodynamics. A Bridge Between Mathematicians and Physicists*, Berlin: Springer-Verlag.

[30]Schwinger, J (1948), 'On Quantum Electrodynamics and the Magnetic Moment of the Electron,' ***Phys Rev* 73:416-9**, online at: http://www.fisicateorica.me/repositorio/howto/artigoshistoricosordemcronologica/1948b%20SCHWINGER%201948B%20First%20theoretical%20calculation%20of%20g-2%20for%20the%20electron.pdf, p.417.

Re-arranging, this gives:

$$\frac{m_p}{m_e} \approx \frac{8\pi Z_0(1 + 2a_e)}{j^4|g_e|R_\mathrm{K}} = 1836.14909055\,. \tag{18}$$

Here, R_K is von Klitzing's constant, h/e^2 = 25,812.8074434 Ω and a_e is the electron magnetic moment anomaly, 0.00115965218076 (CODATA 2010 values). Obviously, $|g_e|/(1 + a_e) \cong 2$, so:

$$\frac{m_p}{m_e} \approx \frac{4\pi Z_0}{j^4 R_\mathrm{K}} = 1834.02470233\,. \tag{19}$$

This multiplied by Schwinger's constant (α/2π), +1 yields:

$$\frac{m_p}{m_e} \approx \frac{4\pi\left[1 + \left(\frac{\alpha}{2\pi}\right)\right] Z_0}{j^4 R_\mathrm{K}} = 1836.154777471\,. \tag{20}$$

This is 1.00000114643 of the CODATA 2010 value, or $1 + [0.99999998502 \times 10^{-3}(\alpha/2\pi)]$. If we give this (seemingly arbitrary) constant a symbol, say ξ, then we may write:

$$\frac{4\pi\left[1 + \left(\frac{\alpha}{2\pi}\right)\right] Z_0}{j^4 \xi R_\mathrm{K}} = 1836.15267245\,, \tag{21}$$

which, of course, is exact.

The reasons for the coincidences listed and outlined here are unclear to the present author. There can be no doubt, however, that the explanation lies in an as yet to be formulated GUT, whether supersymmetric or otherwise, and ultimately, with a unified field theory of the forces of nature, or 'TOE'. *Per nunc*, however, the (somewhat unsatisfactory) explanation must be that the relationships depend on the operation of the strong anthropic principle – because if they were any different, which in the set or ensemble of possible universes they could be – then life, and therefore intelligent life, and observers, would be impossible.