THE CONTROLLED REFRACTIVE INDEX WARP DRIVE

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Abstract

When a space-time *warp bubble* is moving at velocity (*i.e.* v > c), Doppler shifted photons with energy tending to infinity, approach from the direction of motion. Event horizons form on the leading and trailing walls of the bubble. It is demonstrated herein that these phenomena mathematically arise from the application of shift-only space-time solutions; derived retrospectively from the geometrical interpretation of General Relativity. Moreover, it shall be demonstrated that these phenomena do not manifest in a properly formulated Polarizable Vacuum warp drive solution, IFF the refractive index within the space-time bubble is precisely controlled. This formalism is consistent within the analog gravity framework of the Polarizable Vacuum Model, [1-6] representing a variable speed of light interpretation of General Relativity.

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1 THE POLARIZABLE VACUUM METHOD

It is typical when working with General Relativity (GR) that the metric signature be $-+++$ and the convention $(c = G = 1)$ be used. However, in Special Relativity (SR) the opposite signature is typically applied; both are utilised herein. The flat space-time line element found in SR and the GR theory of gravity may be expressed as,

$$ds^2 = -dt_o^2 + dx_o^2 + dy_o^2 + dz_o^2 \tag{1}$$

In the Polarizable Vacuum (PV) Model of gravity, [1] a variable refractive index is utilized to represent the variable speed of light, (VSL), such that the speed of light is, $1/K_{PV}$. Where, K_{PV} is the refractive index of the Polarizable Vacuum. It is a variable that is dependent upon the relative density of matter and energy at any given set of coordinates. Therefore, it is typically represented by a scalar function of the gravitational potentials as seen by an observer at a different relative potential, *i.e., having a different value of* K_{PV}. The *local* value for all observers is, $K_{PV} = 1$, and is consistent with SR. This distinction can cause some confusion. It is important to understand that variations in the speed of light, $1/K_{PV}$ are not measurable locally.

In PV theory, rulers and clocks are variable. They physically change with respect to the relative potentials in a gravitational field. The relative potentials act as a scaling factor from the Planck scale and upwards, controlling the length of a ruler and the rate of a clock in contra-variance. This occurs because Force is invariant with respect to changes in K_{PV}. However, length, time, energy density and other relative measures are not. A ruler will physically contract or expand as determined by the equilibrium condition that exists between the energy density of matter and the energy density of the PV in which it is immersed, [7] such that rulers and clocks become *renormalized* by the local value of K_{PV}. Therefore, the relative value of K_{PV} can only be observed by a distant observer at a different relative potential, by measuring the gravitational frequency shift from a light source at the potential to be measured.

To find the line element in the PV as seen by an observer, the following substitutions to equation (1) should occur, $dt_o^2 = dt^2 / K_{PV}$, $dx_o^2 = K_{PV} dx^2$, $dy_o^2 = K_{PV} dy^2$, $dz_o^2 = K_{PV} dz^2$. The ensuing *isotropic* PV line element is then,

$$ds^2 = -\frac{dt^2}{K_{PV}} + K_{PV}\left(dx^2 + dy^2 + dz^2\right) \tag{2}$$

Examples of this line element in spherical coordinates include the Schwarzschild and the Reissner-Nordstrom metrics of GR; where, $K_{PV} = \left(1 - \frac{\varphi_1}{r} + \frac{\varphi_2}{r^2}\right)^{-1}$. In cylindrical polar coordinates, the solutions are also consistent with GR. [5, 6]

Substituting the coordinate velocities, $dx/dt = 0,\ dy/dt = 0,\ dz/dt = \mathrm{v}_z$, results in the ratio of proper time to coordinate time, as is typically found in SR.

$$-\left(\frac{ds}{dt}\right)^2 = \left(\frac{d\tau}{dt}\right)^2 = \frac{1 - K_{PV}^2 \mathrm{v}_z^2}{K_{PV}} \tag{3}$$

From this it is evident that the coordinate time is related to the proper time by $dt^2 = K_{PV} d\tau^2$, and the coordinate length is related to proper length by $dz^2 = dz_o^2 / K_{PV}$. If K_{PV} is a constant at all coordinates, then space-time is defined to be *flat*, regardless of its value. However, since K_{PV} is an arbitrary function of the coordinates, it is instructive to examine its effect upon the line element.

2 THE SHIFT-ONLY DILEMMA

First, let $v_s f(r_s)$, be an arbitrary velocity vector field in the z direction; where the *shape function,* $f(r_s)$ determines the shape and boundaries of the field. Then, let the refractive index K_{PV} within $f(r_s)$ be a function of the coordinate velocity in the z direction. Such that,

$$K_{PV} = \left(1 - \frac{v_s}{\mathrm{v}_z} f(r_s)\right) \tag{4}$$

Equation (2) is an isometric line element, and when (4) is substituted into (3), the resulting line element is that of the Alcubierre *warp drive* line element, [8] modified by a scaling factor, $1/K_{PV}$.

$$\left(\frac{ds}{dt}\right)^2 = \frac{-1+\mathrm{v}_z^2+\left(v_s f\left(r_s\right)\right)^2 - 2v_s f\left(r_s\right)\mathrm{v}_z}{\left(1-\frac{v_s}{\mathrm{v}_z}f\left(r_s\right)\right)} \tag{5}$$

$$\rightarrow ds^2 = \frac{-\left(1-\left(v_s f\left(r_s\right)\right)^2\right)dt^2 - 2v_s f\left(r_s\right)dtdz + dz^2}{\left(1-\frac{v_s}{\mathrm{v}_z}f\left(r_s\right)\right)}$$

$$\rightarrow ds^2 = \frac{-dt^2 + \left(dz - v_s f\left(r_s\right)dt\right)^2}{\left(1-\frac{v_s}{\mathrm{v}_z}f\left(r_s\right)\right)}$$

Without the scaling factor, this is identical to Alcubierre's equation, in two dimensions. It is representative of a family of *shift-only* solutions in GR. The scaling factor is what occurs when a contra variant Lapse function is included in the line element.

Note: due to the scaling factor, this line element [Eq. (5)] is not in the shift-only family of solutions in GR.

The root-cause of problems with shift-only solutions is that they can only achieve $d\tau = dt$ by forcing the refractive index to equal zero, $K_{PV} = 0$; analogous to an ON/OFF switch, there is no adjustment. In Alcubierre's equation, one can adjust the value of v_s; however, the solution is constrained by $\left(1-\frac{v_s}{\mathrm{v}_z}f\left(r_s\right)\right) = 0$ or the desired effect will not be achieved. This fact has been completely ignored in the scientific literature, along with the scale of rulers and the rate of clocks that depend upon the value of the refractive index, as evidenced by the Schwarzschild metric. This is primarily due to the geometrical interpretation given to GR, as opposed to the physical interpretation afforded the PV Model.

In shift-only solutions, particularly the solutions found by Alcubierre, Natario and others, [8-10] it is implicitly assumed that the moving volume of space-time, has the same relative refractive index at the center, as exists outside the bubble in normal space-time; yet, it is moving at an arbitrarily high velocity. In Natario's work for example, the vacuum is assumed to behave like an incompressible fluid, contradicting Schwarzschild widely accepted solution with respect to the properties of space-time. Shift-only solutions may be a useful mathematical convenience in fluid mechanics, however, this assumption appears to be the root-cause of some serious, anomalous, and unrealistic results. Such as, infinite energy, blue Doppler shifted light rays and event horizons that form behind and ahead of the space-time bubble. [9-10] This has been shown to exist in these examples of shift-only solutions. It shall be shown here that these issues are not present in a properly formulated PV solution.

Shift-only solutions start from the non-physical premise that the space-time bubble is already moving. In Alcubierre's equation for example; if the bubble is at rest, space-time is not warped. The bubble moves itself by contracting space-time in the forward wall, and expanding it at the rear wall. What Schwarzschild's metric illustrates is that space-time contracts, *(or expands),* dependent upon its mass-energy content. Removing mass-energy will cause space-time to expand. Therefore, to set the bubble in motion, mass-energy would need to be displaced from the rear wall of the bubble, to the forward wall. The flow of this *momentum is from back to front*. Momentum will be conserved through the center of mass of the system, and the bubble will remain stationary in the absence of any external action. A ship inside the space-time bubble may be in free-fall toward the front wall, but then the bubble will also be in free-fall toward the ship; *i.e., the center of mass will not move if the bubble is not already in motion.*

Before moving forward it is worth mentioning that the modified Alcubierre equation, (5) is more realistic than the original, only in that it includes a scaling factor that restores the *physical* effects that gravitation has upon matter, as evidenced by the Schwarzschild solution. Examining equation (5), notice that when, $K_{PV} \rightarrow 0$, the length of the line element becomes immeasurable, $ds \rightarrow \infty$. In the PV method, this result would be properly interpreted as *the ruler has expanded to immeasurable length.* It is preferred then, that the refractive index should be maintained between,

$$1 \geq K_{PV} > 0 \tag{6}$$

3 GRAVITATIONAL RED SHIFT, BLUE SHIFT AND DOPPLER SHIFT

The frequency shift of light rays between a source and an observer must be determined in both directions, when approaching with a non-zero relative velocity, where each is immersed in a space-time with a different value of K_{PV}, the Doppler shift can be found by following the standard textbook derivation. [11]

A source in the reference frame where it is at rest, emits N waves in a proper time, Δt_S. The proper time transforms to the time measured by the receiver as, $\Delta t_S = \Delta t_R / \gamma$. The Relativistic Doppler equation in a VSL vacuum is then,

$$f'(K_{PV}) = \frac{1/K_{PV}}{1/K_{PV} - \mathrm{v}} \frac{N}{\Delta t_R} = \frac{1}{1-\mathrm{v}\,K_{PV}} \frac{N}{\Delta t_R} \tag{7}$$

Substituting equation (3) as, $\Delta t_R = \Delta t_S \cdot \gamma = \Delta t_S / \sqrt{\dfrac{1-K_{PV}^2 \mathrm{v}_z^2}{K_{PV}}}$, and the proper frequency, $1/\Delta t_S = f_o$. Equation (7) becomes,

$$f'(K_{PV}) = \frac{1}{1-\mathrm{v}\,K_{PV}} \sqrt{\frac{1-\mathrm{v}^2 K_{PV}^2}{K_{PV}}} f_o = \sqrt{\frac{1+\mathrm{v}\,K_{PV}}{1-\mathrm{v}\,K_{PV}}} \frac{f_o}{\sqrt{K_{PV}}} \tag{8}$$

If they are moving away from each other, the opposite sign of the velocity is used. When the velocity in equation (8) is zero, what remains is a gravitational frequency shift, from which the value of K_{PV} may be determined by the distant observer.

If the value of K_{PV} is controlled such that, $d\tau = dt$, then $f'(K_{PV})$ is linear and finite for all finite velocities.

$$f' = \frac{1}{1-\mathrm{v}\,K_{PV}} f_o = \frac{2\,\mathrm{v}}{2\,\mathrm{v} - \sqrt{1+4\,\mathrm{v}^2} + 1} f_o \tag{9}$$

The frequency varies linearly with the velocity such that, as $\mathrm{v} \to \infty$, $f' \to 2\,\mathrm{v}\,f_o$ The result is a finite blue shift for all observers and velocities, including velocities faster than light. Since the path of light rays is not interrupted and its energy remains finite, no event horizons form at any speed.

4 THE CONTROLLED REFRACTIVE INDEX WARP DRIVE

A properly formulated PV solution to equation (3) is perhaps more useful. Next, equation (3) is solved to achieve the desired effect, $d\tau = dt$ at all velocities, by first solving the following Quadratic equation for K_{PV} as a function of the coordinate velocity, v.

$$1 = \frac{1 - K_{PV}^2 \, v^2}{K_{PV}} \rightarrow K_{PV}^2 \, v^2 + K_{PV} - 1 = 0$$
$$\therefore \tag{10}$$
$$K_{PV}(v) = \frac{\sqrt{1 + 4v^2} - 1}{2v^2}$$

Note that velocities ranging from, $0 \le |v| < \infty$ result in a smooth function of the refractive index from, $1 \ge K_{PV} > 0$, as shown in Figure 1.

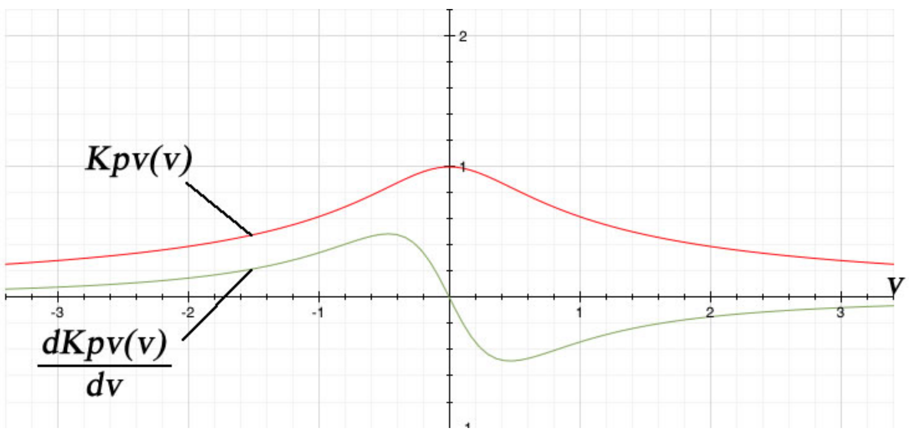

Figure 1. Controlled Refractive Index: *Plot of equation (10) and its derivative with respect to the velocity. The derivative peaks, at around 0.46c. For very large velocities, the incremental change in Kpv is relatively small, compared to the initial change required to break the speed of light.*

Controlling the value of $K_{PV}(v)$ using equation (10) will eliminate time dilation and Lorentz contraction for any arbitrary value of v, *when using conventional methods of propulsion*. The value of $K_{PV}(v)$ is maintained such that it may be used to compensate for the effects of SR, and thereby achieve $d\tau = dt$, at any arbitrary velocity. However, the method by which this is achieved is a significant distinction. Unlike Alcubierre's equation, in the Controlled Refractive Index Warp Drive, *it is not a requirement for the ship or the bubble to be in free-fall.*

Equation (10) is incomplete however, in that dependence on the coordinates relative to the ship is also required. Therefore, it is necessary to have a *shape function,* $f(r_s)$ that inflates when K_{PV} is reduced, such that;

$$f(r_s) = 1,\ 0 \le r_s < R$$
$$f(r_s) = 0,\ r_s \ge R \tag{11}$$

where the size of the bubble and its contents are scaled accordingly, as viewed by the distant observer.

$$r_s^2 = \frac{\left(x - x_s\right)^2 + \left(y - y_s\right)^2 + \left(z - z_s\right)^2}{K_{PV}}$$
$$R = \frac{R_o}{\sqrt{K_{PV}}} \tag{12}$$

The coordinates, $\left(x_s,\ y_s,\ z_s\right)$ are the position of the field generator centered within the space-time bubble, and carried on board a ship. R_o approximates the proper size of the ship at rest. It is only necessary to constrain the value of R_o to enclose the volume of the ship, and to maintain uniformity throughout to avoid tidal forces that could cause damage to the ship *as it is is inflated by the field*. Beyond the boundaries of the ship is of no concern because it has no effect on the ship. The Controlled Refractive Index Warp Drive is quite different from other warp drive equations in this regard. The warp bubble is inflated from the inside-out, and *the ship is inflated with it, but it does not propel the ship.*

Combining (10) and (11), the result is the Controlled Refractive Index Warp Drive equation,

$$K_{PV}(\mathrm{v}(t), r_s) = \frac{\sqrt{1 + 4\left(\mathrm{v}(t) f(r_s)\right)^2} - 1}{2\left(\mathrm{v}(t) f(r_s)\right)^2} \tag{13}$$

Where, $\mathrm{v}(t)$ is the motion of the center of mass as observed by the distant observer at rest, and acceleration is driven by a conventional propulsion device.

The shape function may be altered slightly to produce Newtonian gravitational fields within the bubble that will provide acceleration to offset the acceleration of the engines and provide inertial dampening. To see just how slightly, calculate the gradient derivative of the field, restore the proper units and renormalize $K_{PV} = 1$ for observers being carried along, inside the bubble,

$$c^2 \frac{\partial K_{PV}(\mathrm{v}(t), r_s)}{\partial r_s} = \mathrm{a}(t, r_s) \tag{14}$$

Therefore, a very small gradient in the refractive index is enough to generate significant acceleration within the ship.

Note: that this will not propel the ship in any way. It will only generate a Newtonian gravitational field between the field generator and the ship it is attached to. Thus, providing inertial compensation at high accelerations and a comfortable 1g environment for the passengers on board.

5 CONCLUSION

Shift-only space-times can only achieve $d\tau = dt$, by requiring the refractive index to go to zero, $K_{PV} \to 0$. This assumption results in unrealistic anomalies. Such as, infinitely blue shifted light and event horizons. This family of equations is borrowed from fluid mechanics and are mathematically equivalent solutions to the field equations of GR. However, they cannot be realized physically because they completely ignore the effects of gravity on matter. It is assumed that matter, as well as space-time, is incompressible. This blatantly rejects physical evidence to the contrary, supported by the Schwarzschild metric under the PV Model interpretation.

It may be concluded that in the Controlled Refractive Index Warp Drive, when equation (13) is a solution to equation (2), there will be no time dilation or length contraction. The refractive index is maintained to negate the effects of Special Relativity at any finite velocity, as viewed by a distant observer at rest. Under these circumstances, the Doppler shift is linear and finite for finite velocities, and the ship is not causally disconnected from the rest of the universe. Therefore, there are no event horizons or infinitely blue shifted light rays in front of, or behind the ship. Two observers can approach or separate from each other at any arbitrary velocity.

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