YARKOVSKY EFFECT IN MODIFIED PHOTOGRAVITATIONAL 3-BODIES PROBLEM

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Abstract: Here is presented a generalization of photogravitational restricted 3-bodies problem to the case of influence of Yarkovsky effect or YORP-effect, which are known as reason of additional infinitesimal acceleration of a small bodies in the space (due to anisotropic re-emission of absorbed energy from the sun, other stellar sources).

Asteroid is supposed to move under the influence of gravitational forces from 2 massive bodies (*which are rotating around their common centre of masses on Kepler's trajectories*), as well under the influence of pressure of light from both the primaries.

Analyzing the ODE system of motion, we explore the existense of libration points for a small body (asteroid) in the case when the 2-nd primary is *non-oblate* spheroid.

In such a case, it is proved the existence of maximally *256* different *non-planar* equilibrium points in modified photogravitational restricted 3-bodies problem when we take into consideration even a small *Yarkovsky effect* (*YORP-effect*).

AMS Subject Classification: 70F15, 70F07

Key Words: Yarkovsky effect, YORP-effect, photogravitational restricted three body problem, stability, equilibrium points, libration points, oblateness

1. Introduction.

The Yarkovsky effect is a force acting on a rotating body in space caused by the anisotropic emission of thermal photons, which carry momentum [1]. It is usually considered in relation to meteoroids or small asteroids (*about 10 cm to 10 km in diameter*), as its influence is most significant for these bodies. Such a force is produced by the way an asteroid absorbs energy from the sun and re-radiates it into space as heat by anisotropic way.

In fact, there exists a disbalance of momentum when asteroid at first absorbs the light, radiating from the sun, but then asteroid re-radiates the heat. Such a disbalance is caused by the rotating of asteroid during period of warming as well as it is caused by the anisotropic cooling of surface & inner layers; the processes above depend on anisotropic heat transfer in the inner layers of asteroid.

During thousands of years such a disbalance forms a negligible, but important additional acceleration for a small bodies, so-called Yarkovsky effect. Thus, Yarkovsky effect is small but very important effect in celestial mechanics as well as in calculating of a proper orbits of asteroids & other small bodies.

Besides, Yarkovsky effect *is not predictable* (*it could be only observed & measured by astronomical methods*); the main reason is unpredictable character of the rotating of small bodies [2-3], even in the case when there is no any collision between them.

If regime of the rotating of asteroid is changing, we could observe a generalization of Yarkovsky effect, i.e. the Yarkovsky–O'Keefe–Radzievskii–Paddack effect [2].

YORP effect for short, is a second-order variation on the Yarkovsky effect which changes the rotation rate of a small body (*such as an asteroid*). The term was coined by Dr. David P. Rubincam in 2000. Note also that the YORP effect is zero for a rotating ellipsoid if there are no irregularities in surface temperature or albedo [2].

2. Equations of motion.

Let us consider the system of ordinary differential equations for photogravitational restricted 3-bodies problem, at given initial conditions [4-5].

In according with [5], we consider three bodies of masses m_1, m_2 and m such that $m_1 > m_2$ and m is an infinitesimal mass. The two primaries m_1 and m_2 are sources of radiation; q_1 and q_2 are factors characterizing the radiation effects of the two primaries respectively. We assume that m_2 is an *oblate* spheroid. The effect of *oblateness* is denoted by the factor A_2. Let r_i ($i = 1, 2$) be the distances between the centre of mass of the bodies m_1 and m_2 and the centre of mass of body m [5].

Now, the unit of mass is chosen so that the sum of the masses of finite bodies is equal to 1. We suppose that $m_1 = 1 - \mu$ and $m_2 = \mu$, where μ is the ratio of the mass of the smaller primary to the total mass of the primaries and $0 \le \mu \le ½$. The unit of distance is taken as the distance between the primaries. The unit of time is chosen so that the gravitational constant is equal to 1 [5].

The three dimensional restricted 3-bodies problem, with an *oblate* primary m_2 and both primaries radiating, could be presented in barycentric rotating co-ordinate system by the equations of motion below [5-6]:

$$\begin{aligned} \ddot{x} - 2n\dot{y} &= \frac{\partial \Omega}{\partial x}, \\ \ddot{y} + 2n\dot{x} &= \frac{\partial \Omega}{\partial y}, \\ \ddot{z} &= \frac{\partial \Omega}{\partial z}, \end{aligned} \qquad (2.1)$$

$$\Omega = \frac{n^2}{2}\left(x^2 + y^2\right) + \frac{q_1(1-\mu)}{r_1} + \frac{q_2\mu}{r_2}\left[1 + \frac{A_2}{2r_2^2}\cdot\left(1 - \frac{3z^2}{r_2^2}\right)\right], \qquad (2.2)$$

- where

$$n^2 = 1 + \frac{3}{2} A_2 ,$$

- is the angular velocity of the rotating coordinate system and A_2 - is the *oblateness* coefficient. Here

$$A_2 = \frac{AE^2 - AP^2}{5R^2} ,$$

- where *AE* is the equatorial radius, *AP* is the polar radius and *R* is the distance between primaries. Besides, we should note that

$$r_1^2 = (x + \mu)^2 + y^2 + z^2 ,$$

$$r_2^2 = (x - 1 + \mu)^2 + y^2 + z^2 ,$$

- are the distances of infinitesimal mass from the primaries.

We neglect the relativistic Poynting-Robertson effect which may be treated as a perturbation for cosmic dust (*or for small particles, less than 1 cm in diameter*), see Chernikov [7], as well as we neglect the effect of variable masses of 3-bodies [8].

3. Modified equations of motion (YORP-effect).

Modified equations of motion (2.1) for the three dimensional restricted 3-bodies problem, with an *oblate* primary m_2, both primaries radiating, and the infinitesimal mass *m* under the influence of *YORP-effect*, should be presented in barycentric rotating co-ordinate system in the form below:

$$
\begin{aligned}
\ddot{x} - 2n\,\dot{y} &= \frac{\partial \Omega}{\partial x} + Y_x(t)\ , \\
\ddot{y} + 2n\,\dot{x} &= \frac{\partial \Omega}{\partial y} + Y_y(t)\ , \\
\ddot{z} \qquad\quad &= \frac{\partial \Omega}{\partial z} + Y_z(t)\ ,
\end{aligned} \tag{3.1}
$$

- where $Y_x(t)$, $Y_y(t)$, $Y_z(t)$ – are the projecting of *YORP-effect* acceleration $\boldsymbol{Y}(t)$ on the appropriate axis *Ox*, *Oy*, *Oz*.

4. Location of Equilibrium points.

The location of equilibrium points for system (3.1) in general is given by conditions:

$$
\begin{gathered}
\ddot{x} = \ddot{y} = \ddot{z} = \dot{x} = \dot{y} = 0\,, \\
\frac{\partial \Omega}{\partial x} = -Y_x(t)\ , \quad \frac{\partial \Omega}{\partial y} = -Y_y(t)\ , \quad \frac{\partial \Omega}{\partial z} = -Y_z(t)\ .
\end{gathered} \tag{4.1}
$$

Let us consider the case when the effect of *oblateness* is absent, $A_2 = 0$ ($\Rightarrow n = 1$), see the appropriate expression:

$$
n^2 = 1 + \frac{3}{2} A_2
$$

It means a significant simplifying of expression (2.2) in the system of equalities (4.1):

$$-Y_x = n^2 \cdot x - \frac{q_1 (1-\mu)\cdot(x+\mu)}{\left((x+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} - \frac{q_2 \mu \cdot (x-1+\mu)}{\left((x-1+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} ,$$

$$-Y_y = n^2 \cdot y - \frac{q_1 (1-\mu)\cdot y}{\left((x+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} - \frac{q_2 \mu \cdot y}{\left((x-1+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} ,$$

$$-Y_z = - \frac{q_1 (1-\mu)\cdot z}{\left((x+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} - \frac{q_2 \mu \cdot z}{\left((x-1+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} .$$

Besides, we assume all equations (4.1) to be *a united system* of algebraic equations. That's why we substitute an expression for z from 3-rd equation above to the 2-nd & the 1-st equation:

$$z \cdot Y_x = z \cdot \left(-n^2 \cdot x + \frac{q_1 (1-\mu)}{\left((x+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} \right) + Y_z \cdot (x-1+\mu) ,$$

$$-z \cdot Y_y = y \cdot \left(z \cdot n^2 - Y_z\right), \quad \Rightarrow \quad Y_z \cdot y = z \cdot \left(n^2 \cdot y + Y_y\right) , \qquad (4.2)$$

$$Y_z = z \cdot \left(\frac{q_1 (1-\mu)}{\left((x+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} + \frac{q_2 \mu}{\left((x-1+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} \right) .$$

Moreover, we obtain from the 3-d equation of system (4.2) that *planar* equilibrium points exist only if $\{Y_z = 0,\ z = 0\}$ simultaneously. But the case $Y_z = 0$ – is very rare, specific condition for asteroid, which has unpredictable character of the regime of rotating during a flight through the space [2]; the same is obtained for the case $y = 0$.

Therefore we will consider only *non-planar* equilibrium points $z, y \neq 0$. So, we obtain from the 1-st & 3-d equations of system (4.2):

$$\frac{q_1(1-\mu)}{\left((x+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} = Y_x + n^2 \cdot x - Y_z \cdot \frac{(x-1+\mu)}{z},$$

$$z = Y_z \cdot \frac{y}{\left(n^2 \cdot y + Y_y\right)}, \quad y \neq -\frac{Y_y}{n^2}, \tag{4.3}$$

$$\frac{q_2 \mu}{\left((x-1+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} = -Y_x - n^2 \cdot x + Y_z \cdot \frac{(x+\mu)}{z} \quad .$$

Hence, we finally obtain the system of algebraic equations for meanings of $\{x, y\}$, which determine the location of equilibrium points (4.1):

$$\begin{cases} \dfrac{q_1(1-\mu)}{\left((x+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} = Y_x + n^2 \cdot x - Y_z \cdot \dfrac{(x-1+\mu)}{z}, \\ \dfrac{q_2 \mu}{\left((x-1+\mu)^2 + y^2 + z^2\right)^{\frac{3}{2}}} = -Y_x - n^2 \cdot x + Y_z \cdot \dfrac{(x+\mu)}{z}, \end{cases} \tag{4.4}$$

- where (n = *1*)

$$z = Y_z \cdot \frac{y}{\left(n^2 \cdot y + Y_y\right)}, \quad y \neq -\frac{Y_y}{n^2} \quad .$$

The last system (4.4) could be presented as below ($n = 1$):

$$\begin{cases} \dfrac{q_1^2(1-\mu)^2\cdot\left(n^2\cdot y+Y_y\right)^6\cdot y^2}{\left(\left((x+\mu)^2+y^2\right)\cdot\left(n^2\cdot y+Y_y\right)^2+Y_z^2\cdot y^2\right)^3} = \left(y\cdot\left(Y_x+n^2\cdot x\right) - (x-1+\mu)\cdot\left(n^2\cdot y+Y_y\right)\right)^2, \\ \\ \dfrac{q_2^2\mu^2\cdot\left(n^2\cdot y+Y_y\right)^6\cdot y^2}{\left(\left((x-1+\mu)^2+y^2\right)\cdot\left(n^2\cdot y+Y_y\right)^2+Y_z^2\cdot y^2\right)^3} = \left(-y\cdot\left(Y_x+n^2\cdot x\right) + (x+\mu)\cdot\left(n^2\cdot y+Y_y\right)\right)^2, \end{cases}$$

- where the maximal polynomial order of equations is equal to 16 x 16 = 256: indeed, the order of 1-st polynomial equation is equal to 16 (*in regard to variables x,y*); the order of 2-nd polynomial equation is also equal to 16 (*in regard to x,y*).

So, (4.4) is the polynomial system of equations of *256-th* order which has maximally *256* different roots, we should especially note that each of them strongly depends on various parameters $\{ \mu, q_1, q_2 ; Y_x, Y_y, Y_z \}$. Such a system of polynomial equations could be solved only by numerical methods (*in general, it is valid for polynomial equation of order* > *5*).

Besides, analysing the equations of system (4.2), we should note that a case of *Yarkovsky effect is negligible* determines the existence of *quasi-planar* equilibrium points in which conditions $\{Y_z \to 0, z \to 0\}$ are valid *simultaneously*.

The strongest simplifying of system (4.4) is possible when *Yarkovsky effect is zero*, $Y_x = Y_y = Y_z = 0$. In such a case, it has been proved the existence of maximally *9* different equilibrium points $\{L_1, \ldots, L_9\}$ in photogravitational restricted 3-bodies problem [9].

5. Conclusion.

It has been proved the existence of maximally *256* different *non-planar* equilibrium points z, $y \neq 0$ in modified photogravitational restricted 3-bodies problem when we take into consideration even a small *Yarkovsky effect* (*YORP-effect*). This result is different both from classical restricted 3-bodies problem and photogravitational restricted 3-bodies problem.

Stability of such a points is an open problem in celestial mechanics for the case of non-zero *YORP-effect* (*the Yarkovsky–O'Keefe–Radzievskii–Paddack effect*) [2].

This model may be applied to examine the dynamic behaviour of small rotating objects such as meteoroids or small asteroids (*about 10 cm to 10 km in diameter*).

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