General relativity as geometrical approximation to a field theory of gravity

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Abstract

It is broadly believed that general relativity –a geometric theory– is fully equivalent to the field theory of a massless, self-interacting, spin-2 field. This belief is reinforced by statements in many textbooks. However, an increasing criticism to this belief has been published. To settle this old debate about the precise physical nature of gravitation, this author introduces a simple but exact argument –based in the equivalence principle– that shows that general relativity is *not equivalent to a field theory of gravity*. Subsequently, both the general relativistic Lagrangian for a particle and the Hilbert & Einstein equations are obtained as an approximation from a field theory of gravity, somehow as geometric optics can be derived from physical optics. The approximations involved in the geometrization are two: (**i**) the neglect of $T_{grav}^{\mu\nu}$ and $T_{int}^{\mu\nu}$ in the field-theoretic tensor $\Theta^{\mu\nu}$ and (**ii**) the approximation of the effective metric by the curved spacetime metric $g_{\mu\nu} = \hat{g}_{\mu\nu} + O(h_{\mu\nu}^2)$. Further discussion of this derivation and of the approximations involved is given.

Several misunderstandings about the consistency and observability of the flat spacetime theories of gravity are corrected. A detailed analysis of the fundamental differences between geometric and field-theoretic expressions reveals that all the well-known deficiencies of general relativity –including the impossibility to obtain a consistent quantum general relativity– *are direct consequences of the geometrization of the gravitational interaction*. Finally, remarks about the status of dark matter are given, from the perspective of a generalized theory of gravity.

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1 Introduction

It is broadly believed that general relativity –a geometric theory– is fully equivalent to the field theory of a massless, self-interacting, spin-2 field. This belief is reinforced by statements in many textbooks. For instance, FEYNMAN affirms [1]:

> «*It is one of the peculiar aspects of the theory of gravitation, that is has both a field interpretation and a geometrical interpretation.* [...] *the fact is that a spin-two field has this geometrical explanation* [...] *The geometrical interpretation is not really necessary or essential to physics.*»

Although ROBERT M. WALD warns [2]:

> «*It should be noted, however, that the notion of the mass and spin of a field require the presence of a flat background metric $\eta_{\mu\nu}$ which one has in the linear approximation but not in the full theory, so the statement that, in general relativity, gravity is treated as a massless spin-2 field is not one that can be given precise meaning outside the context of the linear approximation.*»

Whereas C. MISNER, K. THORNE, & J. WHEELER emphasize that the flat background is not observable in general relativity [3]:

> «*In other words, this approach to Einstein's field equation can be summarized as "curvature without curvature" or—equally well—as "flat spacetime without flat spacetime"!*»

Recently, technical criticism to THIRRING [4] and DESER [5] respective claims on the identity between general relativity and a field theory of gravity has been published [6–8]. DESER has partially answered to critics in a recent work [9], but avoided the main criticism against his claims [8]: (**i**) DESER confounds the spacetime metric $\hat{g}_{\mu\nu}$ of general relativity with the effective metric $g_{\mu\nu}$ associated to the gravitational field and (**ii**) he uses an expression for the energy-momentum tensor of the gravitational field which does not satisfy basic physical conditions as zero trace –massless graviton– and positive energy density.

To settle this old debate about the precise physical nature of gravitation, this author will introduce in the next section a simple but exact argument –based in the equivalence principle– that shows that general relativity is *not equivalent to a field theory of gravity over flat spacetime*. In section 3, general relativity is derived as a geometrical approximation to the field theory of gravity.

In this work, Greek indices run over values $0, 1, 2, 3$, whereas Latin indices run over values $1, 2, 3$, and the summation convention is used.

2 Non-equivalence between geometry and fields

For the sake of maximum pedagogical simplicity, we illustrate the non-equivalence between geometrical and field-theoretical descriptions of gravity using a simple system: a point-like non-charged particle in a curved spacetime with coordinates $\hat{x}^\beta$ and curvature being generated by external bodies.

The general relativistic Lagrangian for this particle is [2]

$$\hat{L} = -mc^2 \sqrt{\frac{\hat{v}^\mu \hat{v}^\nu}{c^2} \hat{g}_{\mu\nu}}, \tag{1}$$

where $\hat{v}^\beta \equiv \mathrm{d}\hat{x}^\beta / \mathrm{d}\hat{t}$ is the four-velocity [10].

Applying the *equivalence principle*, the general relativistic Lagrangian *strictly reduces* to the Lagrangian of the special theory of relativity for Minkowskian spacetime with coordinates x^β

$$L_{SR} = -mc^2 \sqrt{\frac{v^\mu v^\nu}{c^2} \eta_{\mu\nu}}. \tag{2}$$

This flat spacetime Lagrangian is *equivalent* to the curved spacetime Lagrangian (1) *by virtue of the equivalence principle of general relativity*.

The field-theoretic Lagrangian for a point-like non-charged particle in a flat spacetime and in presence of a gravitational field generated by external bodies is

$$L = -mc^2 \sqrt{\frac{v^\mu v^\nu}{c^2} \eta_{\mu\nu}} - \frac{1}{2} \Theta^{\mu\nu} h_{\mu\nu}, \tag{3}$$

for a field potential $h_{\mu\nu}$ and a total energy-momentum tensor $\Theta^{\mu\nu}$ [11] –as will be shown latter this total tensor includes physical components absent in general relativity–.

It is evident that the flat spacetime Lagrangians (2) and (3) are non-equivalent, except in the trivial case when the gravitational field is absent $h_{\mu\nu} = 0$. As a consequence, the so-claimed full equivalence between geometric and field-theoretic descriptions of gravitation is already broken at the Lagrangian level (1) $\not\Leftrightarrow$ (3), because (1) $\Leftrightarrow$ (2) $\not\Leftrightarrow$ (3).

We can consider more particles, electromagnetic and other interactions, the Lagrangians of the fields, heat effects, and other complexities, but the conclusion will remain: *the geometric expressions are not equivalent to field-theoretic expressions over flat spacetime*.

Since the equivalence principle when applied to the kinetic part of (3) gives (1), we must wait corrections to general relativity becoming from the interacting term in (3). We will show in the next section what approximations must be introduced in order to derive general relativity from a field theory of gravity; i.e., to derive geometrical gravity from physical gravity.

3 General relativity from a field theory of gravity

Once shown, in the previous section, that the geometrical description provided by general relativity is not fully equivalent to the physical description provided by a field theory of gravity, we will show now how the general relativistic Lagrangian (1) can be obtained under a well-defined and physically admissible set of approximations.

We start from the field-theoretic Lagrangian (3) and notice that the gravitational interaction depends on a total energy-momentum tensor $\Theta^{\mu\nu}$ [11]. This tensor is the source of the gravitational field, but is *not* the $\hat{T}^{\mu\nu}$ that we find in the HILBERT & EINSTEIN metric equations $\hat{G}^{\mu\nu} = (8\pi G/c^4)\hat{T}^{\mu\nu}$ [2, 12]. The relation with $\hat{T}^{\mu\nu}$ will be discussed in the section 4.

The total energy-momentum tensor $\Theta^{\mu\nu}$ [11] contains the matter energy-momentum tensor $T^{\mu\nu}$ plus an interacting term, and a term due to the own gravitational field [8]

$$\Theta^{\mu\nu} \equiv T^{\mu\nu} + T^{\mu\nu}_{int} + T^{\mu\nu}_{grav}. \tag{4}$$

Notice that $T^{\mu\nu}_{grav}$ is a true tensor in the field-theoretic approach and that $T^{00}_{grav} \geq 0$; both properties are merely a consequence of the fact that the gravitational field is here a physical system, as the electromagnetic field, and carries energy and momentum. As we will show, these reasonable physical properties are missing in general relativity, which –contrary to myth– does not describe the physical gravitational field and its self-interaction. Effectively, *the first approximation involved in the derivation of general relativity* is $\Theta^{\mu\nu} = T^{\mu\nu} + O_h(h_{\mu\nu})$. The precise meaning of $O_h(h_{\mu\nu})$ and higher-orders will be discussed in the section 4.

For the point-like non-charged particle, $T^{\mu\nu} \equiv m\Gamma v^\mu v^\nu$, with $\Gamma \equiv c/\sqrt{v^\mu v^\nu \eta_{\mu\nu}}$ being the kinetic time-dilation factor. Substituting all back into (3) we obtain

$$L = -\frac{mc^2}{\Gamma}\left(1 + \frac{\Gamma^2 v^\mu v^\nu}{2c^2} h_{\mu\nu}\right) + O_h(h^2_{\mu\nu}). \tag{5}$$

Using the identity $1 + A = \sqrt{1 + 2A + A^2}$, the Lagrangian can be rewritten as

$$L = -\frac{mc^2}{\Gamma}\sqrt{1 + \frac{\Gamma^2 v^\mu v^\nu}{c^2} h_{\mu\nu} + O_h(h^2_{\mu\nu})}. \tag{6}$$

This can be finally expressed in the pseudo-geometric form

$$L = -mc^2\sqrt{\frac{v^\mu v^\nu}{c^2} g_{\mu\nu} + O_h(h^2_{\mu\nu})}, \tag{7}$$

introducing an *effective metric* $g_{\mu\nu} \equiv \eta_{\mu\nu} + h_{\mu\nu}$. This effective metric, obtained from the potential $h_{\mu\nu}$ associated to the gravitational field, does not have any fundamental geometrical meaning in the field-theoretic approach. Indeed, the *physical spacetime metric* continues being $\eta_{\mu\nu}$ in this approach.

This effective metric can be used as a shorthand for simplifying expressions. It is particularly interesting that the role of $g_{\mu\nu}$ in the field-theoretic approach, where the physical spacetime is flat and the curved spacetime associated to the effective metric is non-observable, is analogue to the corresponding role of the non-observable background $\eta_{\mu\nu}$ in general relativity when the physical spacetime is not flat.

As remarked in the introduction, many authors confound this effective metric $g_{\mu\nu}$ with the metric $\hat{g}_{\mu\nu}$ of general relativity. In fact, those authors who incorrectly claim equivalence between field-theoretic and geometrical pictures do not differentiate between flat spacetime physical quantities and curved spacetime quantities [1, 3–5, 9]. YURIJ V. BARYSHEV has computed the traces for both metrics and obtains [8]

$$g_{\mu\nu} g^{\mu\nu} = 4 + 2h^{\lambda}{}_{\lambda} + O_h(h^2_{\mu\nu}) \tag{8}$$

versus

$$\hat{g}_{\mu\nu}\hat{g}^{\mu\nu} = 4. \tag{9}$$

BARYSHEV's calculation implies that $\hat{g}_{\mu\nu}$ is exactly equivalent to $g_{\mu\nu}$ only in the trivial case when the gravitational field is absent: $h_{\mu\nu} = 0$. This consequence could be advanced from a purely physical reasoning: the physical metric $\hat{g}_{\mu\nu}$ of general relativity can be equivalent to the effective metric of the field-theoretic approach only when $g_{\mu\nu}$ coincides with the corresponding physical metric $\eta_{\mu\nu}$ over which both the mass and spin of the gravitational field are defined –recall the quote by WALD reproduced in the introduction–. This analysis of the relationship between $\hat{g}_{\mu\nu}$ and $g_{\mu\nu}$ coincides with the result obtained in the section 2 for the corresponding Lagrangians (1) and (3).

For obtaining the general relativistic Lagrangian, we must first introduce the next change of variables

$$\eta_{\mu\nu} \mathrm{d}x^{\mu} \mathrm{d}x^{\nu} = \hat{g}_{\mu\nu} \mathrm{d}\hat{x}^{\mu} \mathrm{d}\hat{x}^{\nu}. \tag{10}$$

Dividing both sides by $(\mathrm{d}t)^2$ and using the definition of the *effective metric*

$$g_{\mu\nu} v^{\mu} v^{\nu} = \hat{g}_{\mu\nu} \hat{v}^{\mu} \hat{v}^{\nu} \left(1 + \frac{h_{\lambda\xi}}{\eta_{\mu\nu}} \frac{\partial x^{\lambda}}{\partial x^{\mu}} \frac{\partial x^{\xi}}{\partial x^{\nu}}\right) \left(\frac{\mathrm{d}\hat{t}}{\mathrm{d}t}\right)^2. \tag{11}$$

Substituting this identity into the pseudo-geometric Lagrangian (7) and taking into account gravitational time-dilation effects, we finally obtain the general relativistic Lagrangian (1) plus field-theoretic corrections

$$\hat{L} = -mc^2 \sqrt{\frac{\hat{v}^{\mu}\hat{v}^{\nu}}{c^2} \hat{g}_{\mu\nu}} + O_h(h^2_{\mu\nu}). \tag{12}$$

Summarizing, the general relativistic Lagrangian (1) can be obtained as an approximation to the field-theoretic Lagrangian (3), somehow as geometric optics can be derived from physical optics. The pair of approximations involved in the derivation of the general relativistic Lagrangian are: (**i**) the neglect of $T^{\mu\nu}_{grav}$ and $T^{\mu\nu}_{int}$ in the field-theoretic tensor $\Theta^{\mu\nu}$ and (**ii**) the approximation of the effective metric by the curved spacetime metric $g_{\mu\nu} \simeq \hat{g}_{\mu\nu}$. Further discussion of this derivation and of the approximations involved is given in the section 4.

A similar procedure can be used for the derivation of the HILBERT & EINSTEIN metric equations [12] of general relativity

$$\hat{R}^{\mu\nu} = \frac{8\pi G}{c^4} \left(\hat{T}^{\mu\nu} - \frac{1}{2} \hat{g}^{\mu\nu} \hat{T}^{\lambda}{}_{\lambda}\right). \tag{13}$$

We begin with the non-linear field equations [8]

$$\Box h^{\mu\nu} = \frac{16\pi G \delta^3}{c^4} \left(\Theta^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} \Theta^{\lambda}{}_{\lambda}\right), \tag{14}$$

where $\Box \equiv \eta^{\alpha\beta}\partial^2/\partial x^\alpha \partial x^\beta$ is the flat-spacetime D'Alembertian operator and δ^3 the Dirac Delta, and again approximate the total energy-momentum tensor [11] by that for matter alone $\Theta^{\mu\nu} = T^{\mu\nu} + O_h(h_{\mu\nu})$

$$\Box h^{\mu\nu} = \frac{16\pi G\delta^3}{c^4}\left(T^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}T^\lambda{}_\lambda\right) + O_h(h_{\mu\nu}^2). \tag{15}$$

Next introduce the *covariant effective metric* $g^{\mu\nu} \equiv \eta^{\mu\nu} + h^{\mu\nu}$

$$\Box h^{\mu\nu} = \frac{16\pi G\delta^3}{c^4}\left(T^{\mu\nu} - \frac{1}{2}g^{\mu\nu}T^\lambda{}_\lambda\right) + O_h(h_{\mu\nu}^2). \tag{16}$$

Now, using the change of variables (10), and the definition of the flat-spacetime D'Alembertian operator $\Box \equiv \eta^{\alpha\beta}\partial^2/\partial x^\alpha \partial x^\beta$, the left-hand-side of (16) can be modified as follows

$$\eta^{\alpha\beta}\frac{\partial^2 h^{\mu\nu}}{\partial x^\alpha \partial x^\beta} = \left(\eta^{\alpha\beta}\frac{\partial \hat{x}^\lambda}{\partial x^\alpha}\frac{\partial \hat{x}^\xi}{\partial x^\beta}\right)\frac{\partial^2 \hat{h}^{\mu\nu}}{\partial \hat{x}^\lambda \partial \hat{x}^\xi} = \hat{g}^{\lambda\xi}\frac{\partial^2 \hat{h}^{\mu\nu}}{\partial \hat{x}^\lambda \partial \hat{x}^\xi}, \tag{17}$$

where the definition of the inverse spacetime metric $\hat{g}^{\lambda\xi}$ is evident. This can be further rewritten using the Ricci tensor $\hat{R}^{\mu\nu}$, which satisfies [13]

$$\hat{g}^{\lambda\xi}\frac{\partial^2 \hat{h}^{\mu\nu}}{\partial \hat{x}^\lambda \partial \hat{x}^\xi} = 2\hat{R}^{\mu\nu} + O_h(h_{\mu\nu}^2). \tag{18}$$

To continue with the derivation, the main term that we must consider in the right-hand-side of (16) is $T^{\mu\nu}$, because the scalar $T^\lambda{}_\lambda \equiv g_{\lambda\xi}T^{\lambda\xi} + O(h_{\lambda\xi})$. This main term transforms as

$$m\Gamma v^\mu v^\nu = \frac{\Gamma}{\hat{\Gamma}}\left(\frac{\partial x^\mu}{\partial \hat{x}^\alpha}\frac{\partial x^\nu}{\partial \hat{x}^\beta}\right)m\hat{\Gamma}\hat{v}^\alpha\hat{v}^\beta\left(\frac{\mathrm{d}\hat{t}}{\mathrm{d}t}\right)^2, \tag{19}$$

where $\hat{\Gamma} \equiv c/\sqrt{\hat{v}^\lambda\hat{v}^\xi\hat{g}_{\lambda\xi}}$ is the time-dilation factor in coordinates $\hat{x}^\xi$.

Approximating the effective metric by the curved spacetime metric $g_{\mu\nu} = \hat{g}_{\mu\nu} + O(h_{\mu\nu}^2)$; using the definition of $\hat{h}_{\mu\nu}$ and (10) for obtaining $(\partial x^\mu/\partial \hat{x}^\alpha)(\partial x^\nu/\partial \hat{x}^\beta) = \delta_{\alpha\mu}\delta_{\beta\nu} + O_h(h_{\mu\nu})$; using (11) for obtaining $\mathrm{d}\hat{t}/\mathrm{d}t = \Gamma/\hat{\Gamma} + O_h(h_{\mu\nu})$; and introducing (**??**), (**??**), and (19) into (16) gives finally the Hilbert & Einstein equations of general relativity (13) plus field-theoretic corrections

$$\hat{R}^{\mu\nu} = \frac{8\pi G}{c^4}\left(\hat{T}^{\mu\nu} - \frac{1}{2}\hat{g}^{\mu\nu}\hat{T}^\lambda{}_\lambda\right) + O_h(h_{\mu\nu}^2). \tag{20}$$

It is important to remark that the factor δ^3 has been introduced into the definition of $\hat{T}^{\mu\nu}$, according to usual standards in general relativity literature. Nevertheless, now $\hat{T}^{\mu\nu}$ is not the energy-momentum tensor of matter, as incorrectly stated [1–3], but its energy-momentum *density* tensor [11].

Summarizing, the Hilbert & Einstein metric equations [12] of general relativity (13) can be obtained as an approximation to the field-theoretic gravitational equations (14), somehow as geometric optics can be derived from physical optics. The approximations involved in the derivation of the Hilbert & Einstein equations are again: (**i**) the neglect of $T^{\mu\nu}_{grav}$ and $T^{\mu\nu}_{int}$ in the field-theoretic tensor $\Theta^{\mu\nu}$ and (**ii**) the approximation of the effective metric by the curved spacetime metric $g_{\mu\nu} \simeq \hat{g}_{\mu\nu}$. Further discussion of this derivation and of the approximations involved is given in the next section.

4 Final remarks

Notice that the change of variables (10) contains at least a term of order $O(\hat{h}_{\mu\nu})$ in the right-hand-side, but is of order $O(h^0_{\mu\nu})$ in the left-hand-side. This means that field-theoretic corrections to the general relativistic Lagrangian (1) and to the HILBERT & EINSTEIN equations (13) will be mapped to higher-order terms in the deviation $\hat{h}_{\mu\nu}$ from flatness.

As would be inferred from the discussion in the previous section, the notation $O_h(h^2_{\mu\nu})$ means that field terms involving quadratic, cubic, and higher-powers of the gravitational potential $h_{\mu\nu}$ are being neglected in the expressions. Nevertheless, 'mixed' terms as $v^i v^j h_{ij}$, involving products of the gravitational potential and velocity, and 'kinetic' terms as $(v^i v^j \eta_{ij})^2$ –both terms being of order $O(h^2_{\mu\nu})$ when (v^2/c^2) is typically of $O(h_{00})$– are retained together with other terms of higher-order.

That is, the derived general relativistic expressions are valid beyond the linear regime, but are not completely equivalent to a full nonlinear field theory of gravity because are missing terms of $O_h(h^2_{\mu\nu})$ and superior. As will be explained below, precisely the absence of such field-theoretic terms is related to several well-known deficiencies of general relativity, including the impossibility to obtain a consistent quantum general relativity.

All the experimental basis of general relativity [14] is automatically satisfied by the field theory of gravity, with the higher-order field-theoretic corrections being actually undetectable by observations or experiments, although could be checked in a near future [8].

Evidently, the flat spacetime associated to $\eta_{\mu\nu}$ is unobservable in general relativity because $\hat{g}_{\mu\nu}$ is the physical metric corresponding to coordinates $\hat{x}^\beta$. However, many authors [3, 5] confound these coordinates with the x^β used in the field-theoretic approach (10) –or what is the same, confound $\hat{g}_{\mu\nu}$ with $g_{\mu\nu}$– and misguidedly claim that the flat spacetime associated to the field-theoretic approach is not observable, when it is so observable as the spacetime used in electromagnetism, for instance. It must be emphasized that the curved spacetime associated to $g_{\mu\nu}$ is *unobservable in a field theory of gravity* by analogous reasoning.

Specially exaggerated is a recent STRAUMANN's statement affirming that the «*flat Minkowski spacetime becomes a kind of unobservable ether*» [15]; his analogy is an exaggeration because the hypothesis of the ether was disproved by early 20th century experiments that lead to the development of the special theory of relativity, whereas no experiment disproves the existence of flat spacetimes with coordinates x^β. What is more, a flat spacetime is one of the key elements involved in many experimental results, including the celebrated high-precision tests of quantum electrodynamics.

MISNER, THORNE, & WHEELER state that a flat spacetime theory of gravity –what they call «*tensor theory*»– is internally inconsistent and that «*steps to repair this inconsistency in the theory lead inexorably to general relativity*» [3]. However, their «*tensor theory*» is not the correct field theory of gravity, but an inconsistent mixture of an equation of motion –see their eq. 2 in Box 7.1–, which is essentially equivalent to the geodesic equation of motion of general relativity, plus a set of 'field' equations –their eq. 4 in Box 7.1– which are not the genuine equations (14) of the field theory of gravity.

Unsurprisingly, the trio of authors obtain inconsistencies with such mixture of geometric and 'field' equations [3], but their criticism of flat spacetime theories does *not apply to the field theory of gravity discussed here*. Concretely, their main argument in the section «*H. Self-Inconsistency of the Theory*» does not apply to this field theory of gravity because the field equations (14) do not require $(\partial/\partial x^\nu)T^{\mu\nu} = 0$, unlike their eq. 4 in Box 7.1.

Using the EINSTEIN tensor $\hat{G}^{\mu\nu} \equiv \hat{R}^{\mu\nu} - (1/2)\hat{g}^{\mu\nu}\hat{R}^{\lambda}{}_{\lambda}$, the HILBERT & EINSTEIN equations (13) can be written in a well-known alternative form

$$\hat{G}^{\mu\nu} = \frac{8\pi G}{c^4}\hat{T}^{\mu\nu}. \tag{21}$$

This mathematically elegant and concise form is rather popular in the literature [2,3,5,15,16]; however, «*this is not the most useful form for actual calculations*» [14], reason for which the above equations are re-written in the so-called «*relaxed form*» [14]. We present here a different form, for the sake of comparison with the equations of the field theory of gravity. This form is as follows

$$\Box\hat{\chi}^{\mu\nu} = \frac{16\pi G}{c^4}\left(\hat{T}^{\mu\nu} + \hat{t}^{\mu\nu}\right), \tag{22}$$

with $\hat{\chi}^{\mu\nu} \equiv \hat{h}^{\mu\nu} - (1/2)\eta^{\mu\nu}\hat{h}^{\lambda}{}_{\lambda}$ and

$$\hat{t}^{\mu\nu} \equiv (c^4/8\pi G)(\hat{G}^{\mu\nu[1]} - \hat{G}^{\mu\nu}), \tag{23}$$

where $\hat{G}^{\mu\nu[1]}$ is the linearized EINSTEIN tensor.

The geometric constraint $\nabla_\nu\hat{G}^{\mu\nu} = 0$, where ∇_ν denotes a covariant partial derivative, reduces to an ordinary partial derivative for the linearized EINSTEIN tensor $(\partial/\partial x^\nu)\hat{G}^{\mu\nu[1]} = 0$, which implies

$$\frac{\partial}{\partial x^\nu}\left(\hat{T}^{\mu\nu} + \hat{t}^{\mu\nu}\right) = 0. \tag{24}$$

Equations (22), (23), and (24) look as the *a priori* waited expressions for a self-interacting gravitational 'field', gravitational 'tensor', and 'conservation' law, respectively. Nevertheless, this is incorrect and arising from a misguided analysis based in the appearances. The true meaning of (22), (23), and (24) is completely different and given next.

For a detailed comparison with the field theory of gravity, we must first write the field-theoretic equations (14) in the alternative form

$$\Box\chi^{\mu\nu} = \frac{16\pi G}{c^4}\left(\breve{T}^{\mu\nu} + \breve{T}^{\mu\nu}_{grav} + \breve{T}^{\mu\nu}_{int}\right), \tag{25}$$

introducing a 'relaxed' potential $\chi^{\mu\nu} \equiv h^{\mu\nu} - (1/2)\eta^{\mu\nu}h^{\lambda}{}_{\lambda}$ and the densities $\breve{T}^{\mu\nu} = \delta^3 T^{\mu\nu}$, $\breve{T}^{\mu\nu}_{grav} = \delta^3 T^{\mu\nu}_{grav}$, and $\breve{T}^{\mu\nu}_{int} = \delta^3 T^{\mu\nu}_{int}$. The equations (25) verify the identity

$$\frac{\partial}{\partial x^\nu}\left(\breve{T}^{\mu\nu} + \breve{T}^{\mu\nu}_{grav} + \breve{T}^{\mu\nu}_{int}\right) = 0. \tag{26}$$

The fundamental differences between geometric and field expressions are the following. First, the D'Alembertian operator $\Box$ is defined in the physical spacetime of (25), whereas this same $\Box$ is operating over a fictitious spacetime in the geometrical (22). A consequence is that the geometrical equations cannot be used to study the physical propagation of gravitational waves.

Second, $\chi^{\mu\nu}$ is associated to physical gravitational fields –carrying energy and momentum– which are responsible for gravitational forces generalizing the Newtonian and Poincaré ones [8], whereas $\hat{\chi}^{\mu\nu}$ is representing a metric deviation from flatness. This metric deviation lacks any physical meaning beyond the merely geometrical and can be done to vanish –via an adequate change of coordinates as (10)–; derivatives of this metric deviation cannot be related to gravitational forces, as a consequence.

Third, the right-hand-sides of (22) and (24) contain the energy-momentum density [11] tensor for matter plus a new object $\hat{t}^{\mu\nu}$. What is its interpretation? During a long time, physicists waited that this $\hat{t}^{\mu\nu}$, or a similar one, could be finally identified with the energy-momentum density [11] 'tensor' of the own gravitational 'field'. However, as emphasized above, general relativity is a metric theory and, as a natural consequence, $\hat{t}^{\mu\nu}$ is a *pseudo-tensor* with only a pure geometrical meaning (23), in complete agreement with the lack of true gravitational field due to the equivalence principle. This pseudo-tensor represents quadratic and higher-order deviations of the EINSTEIN tensor from its linearized version. In striking contrast, the right-hand-sides of (25) and (26) contain a *true tensor* $\check{T}^{\mu\nu}_{grav}$, giving the energy-momentum density [11] of the own gravitational field [8].

Fourth, (24) is not a true conservation law, in part due to the pseudo-tensorial character of $\hat{t}^{\mu\nu}$ and, in part, because the flat spacetime is fictitious in general relativity. The geometrical identity (24) predicts that energy is lost in the standard FRIEDMAN cosmology, for instance; a unlikely but otherwise waited result, because the energy associated to the metric expansion of space cannot be absorbed by any gravitational degree of freedom due to the absence of field. However, (26) is a true conservation law, with the same status that field-theoretic conservation laws used in the standard model of particle physics.

This detailed comparison of geometric and field-theoretic expressions reveals us that all the well-known deficiencies of general relativity –spacetime singularities, no unification with rest of interactions, gravitational pseudo-tensor, absence of conservation laws, impossibility to obtain a consistent quantum gravity theory...– *are direct consequences of the geometrization of the gravitational interaction*. Effectively, if we eliminate $\check{T}^{\mu\nu}_{grav}$ during geometrization, we cannot wait to identify any of the remaining geometrical objects with the physical energy-momentum density [11] of the gravitational field, for instance; neither we can wait to obtain a consistent quantum gravity theory by quantizing a gravitational field which is nowhere in the resulting general relativity! The formulation of a consistent and complete theory of quantum gravity is possible but will be left for a future work.

Finally, it is worth to mention that, although the geometric equations of general relativity have been here derived as a well-defined approximation to a physical theory of gravity, there exist tricky ways to use general relativity beyond its true scope. For instance, if we generalize the equations (21) to

$$\hat{G}^{\mu\nu} = \frac{8\pi G}{c^4}\left(\hat{T}^{\mu\nu} + \hat{T}^{\mu\nu}_{DM} + \hat{T}^{\mu\nu}_{DE}\right), \tag{27}$$

with $\hat{T}^{\mu\nu}_{DM}$ and $\hat{T}^{\mu\nu}_{DE}$ associated to new hypothetical forms of matter and energy –dark matter and dark energy, respectively–, then we can continue to use the equations, at least up to certain limits.

A detailed study of the dark matter term in (27) was recently done, with the following main conclusions [17]: (**i**) dark matter is a fictitious distribution of matter, what explains why all its direct searches in laboratory experiments are giving null results; (**ii**) general relativity more dark matter cannot explain recentest astrophysical observations explained, however, by a generalized theory of gravity without any need for dark matter; (**iii**) the generalized equations can be casted into ordinary form when a fictitious distribution of dark matter is added to the real mass, explaining the partial empirical success of the tandem general relativity plus dark matter; and (**iv**) from our definition of the dark matter term $\hat{T}^{\mu\nu}_{DM}$, we obtain the main properties traditionally attributed to it, in excellent agreement with dark matter literature.

A detailed investigation of the dark energy term in (27), with objectives similar to those of the above study, is currently in the schedule of this author.

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[10] Four velocities are often confounded with four proper velocities in the literature.

[11] More correctly energy-momentum-stress tensor. It is also usual in the relativistic literature to confound the energy-momentum-stress tensor with its density. In this work the 00-component of this tensor has dimensions of energy, which is consistent.

[12] «*Metric equations*» is here preferred over the traditional «*field equations*», because neither $\hat{g}_{\mu\nu}$ nor $\hat{h}_{\mu\nu}$ can be considered fields in the usual physical sense in classical electrodynamics and field theory of gravity. Moreover, it is historically shown that the equations were first proposed by DAVID HILBERT

[13] The field theory of gravity automatically satisfies the de Donder or harmonic coordinates condition $\partial h^{\alpha\beta}/\partial x^{\beta} = 0$. By simplicity, we assume that the curved spacetime coordinates introduced by (10) are one where the harmonic condition still holds. Once the equations (13) of general relativity are obtained, it is straightforward to apply a new change of variables to other general coordinates.

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[16] Specially when complemented with a special system of units so that $G = c^4$.

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