A different model of the cosmological constant and Einstein curvature tensor in relation to dark energy

J. P. Baugher*
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The meaning and existence of the cosmological constant Λ has come to the forefront of physics as a dark energy that could be responsible for an accelerating expansion of the universe, as well as having an extremely large magnitude as predicted by quantum field theory. This presents the most challenging physics problems known today. In this work I ask questions of a simple equivalency substituted into the Einstein field equation and demonstrate that this results in a repulsive Newtonian gravity that can be explained in terms of a large cosmological constant as well as a proposed path for dark matter.

I. INTRODUCTION

In 1916, Einstein introduced his general theory of relativity as a geometrical theory of gravity [1] resulting in the Einstein field equation (EFE),

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}. \tag{1}$$

It has been well documented and studied that the EFE did not predict a stable static universe, as it was theorized to be at the time [2]. The equation, however, did accurately predict gravitational redshift, magnitudes of gravitational lensing and account for Mercury's precessing orbit, which the Newtonian equation could not. In order to manufacture an equation that could account for a static universe, but still be empirically accurate, it is often stated that Einstein ad hoc threw in another constant Λ which is known as the cosmological constant (CC). This would have been placed back into the EFE with the metric $g_{\mu\nu}$ as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = G_{\mu\nu}. \tag{2}$$

Once it was discovered that the universe actually appeared to be in a decelerating expansion mode, Einstein quickly removed the Λ term. Today, though, there is empirical evidence that a very small magnitude CC exists, but some quantum field theorists estimate it as being over 120 orders of magnitude smaller than their calculations, "probably the worst theoretical prediction in the history of physics" [2]. In addition, it is now thought that the effective observed value, Λ_{eff}, requires an extremely high level of arbitrary fine tuning "for no good reason" and is a "cosmologist's worst nightmare come true" [3]. This transformation from a minor but rich interest exploded (5000 papers submitted to date [4]) near the end of the past millennium due to a startling simultaneous discovery of positive acceleration from two teams [5, 6].

The source of this unforeseen positive acceleration has come to be known as dark energy. The lack of progress in explaining this phenomena led to the creation of a Dark Energy Task Force in 2006 which stated in a report [7]:

> "Most experts believe that nothing short of a revolution in our understanding of fundamental physics will be required to achieve a full understanding of the cosmic acceleration."

This dark energy is currently expected to contribute over 73.4% [8] of the mass-energy of the universe, and there is no sound logical theory for what it is. Consider that this leaves some type of mysterious never-observed particle known as dark matter to contribute another 22.2%, leaving only 4.4% for the normal matter we are familiar with.

In light of such shocking statements and the CC problem, one may find some amusement that over 100 years after relativity settled the question of how the planets could move through a highly stiff medium of the vacuum, we are back again pondering even more troubling mysteries of it. It might be forgivable if one would think that after this amount of time, that the way forward may require a conceptual approach to our founding assumptions rather than complicating matters further by tacking on new dimensions, fields and particles. This is fraught, however, not only with the difficulty of the equations, but also contending with the historical concepts of gravity. Considering though the vast gulf that already exists between general relativity, quantum field theory, dark energy and dark matter, it is at a minimum food for thought and so we proceed with a preliminary assessment of an alternative theory.

II. THEORY: GAUGE INVARIANCE OF THE EFE?

Allowing that Einstein's derivation for his field equation (1) is correct, suppose one were to at that point disregard any physical meaning of the CC. It would be trivial for them to rewrite the CC as the sum of two tensors such that

$$g_{\mu\nu}\Lambda = \Xi_{\mu\nu} + \Pi_{\mu\nu} \tag{3}$$

* The author can be contacted at baugher.3@wright.edu

and

$$\Xi_{\mu\nu} = g_{\mu\nu}\Lambda - \Pi_{\mu\nu}. \quad (4)$$

Then letting Einstein's $G_{\mu\nu} = \Xi_{\mu\nu}$ so that

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = g_{\mu\nu}\Lambda - \Pi_{\mu\nu}. \quad (5)$$

This being a conceptual investigation and given the unnaturally large theoretical magnitude of the CC from quantum field theory, the purpose of this paper is to ask:

- is it possible for there to be a physical interpretation for this Λ such that it possesses the total quantum vacuum energy (states)? Could $\Pi_{\mu\nu}$ in Eq.(5) simply be interpreted as the remaining unoccupied quantum vacuum mass-energy states?
- What historical concepts of gravity would this run in opposition to?

In the investigation process of an article, it is incumbent upon a researcher to consult peer reviewed sources to use as a foundation. This presents a problem in any new concepts of the interface of quantum field theory and general relativity. One must not only be able to cite the source, but would also have to possess the same expertise in a dizzying array of fields in order to re-interpret their conclusions. As this is much more ability than a new researcher possesses, I am instead left to present here arguments and a framework for a theory to be answered by others or until I develop the necessary skills.

III. ARBITRARINESS OF THE EQUATIONS

Although the CC has a checkered history with being included in the field equation, its origins are well known.

The Einstein curvature tensor is sometimes stated as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R.$$

In speaking of the gravitational field in the absence of matter, Einstein derived that this tensor must become zero. Since I leave the basic derivation intact, I point out his footnote pertaining to the following quote [9], and his conclusion:

> "It must be pointed out that there is a minimum of arbitrariness in the choice of these equations. For besides $G_{\mu\nu}$ there is no tensor of second rank which is formed from the $g_{\mu\nu}$ and its derivatives, contains no derivations higher than second, and is linear in these derivatives.*
>
> These equations, which proceed, by the method of pure mathematics, from the requirement of the general theory of relativity, give us, in combination with the equations of motion ([equation number removed]), to a first approximation Newton's law of attraction, and to a second approximation the explanation of the motion of the perihelion of the planet Mercury discovered by Leverrier (as it remains after corrections for perturbation have been made). These facts must, in my opinion, be taken as a convincing proof of the correctness of the theory."

The footnote being:

> "Properly speaking, this can be affirmed only of the tensor
>
> $$G_{\mu\nu} + \lambda g_{\mu\nu} g^{\alpha\beta} G_{\alpha\beta},$$
>
> where λ is a constant. If, however, we set this tensor = 0, we come back again to the equation $G_{\mu\nu} = 0$."

This footnote is a direct contradiction to the insinuation that Einstein made up the CC once it was required to explain a static universe. It has the same meaning as a constant of integration, i.e. the equations aren't unique. Its existence came about in the original rigorous derivation. It would seem that, strictly speaking, it is also proper to state that if

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 0, \quad (6)$$

then

$$\left.\begin{aligned} G_{\mu\nu} &= 0 \\ \lambda g_{\mu\nu} g^{\alpha\beta} G_{\alpha\beta} &= 0 \end{aligned}\right\} \quad (7)$$

or

$$\left.\begin{aligned} -G_{\mu\nu} &= \lambda g_{\mu\nu} g^{\alpha\beta} G_{\alpha\beta} \\ \lambda g_{\mu\nu} g^{\alpha\beta} G_{\alpha\beta} &\neq 0. \end{aligned}\right\} \quad (8)$$

Thus Eqs. (7) or (8) correspond to Eq. (5), leaving all units and dimensions intact. We must assume, for now without proof, that any transformations one equation is capable of, so too the other. The main benefit of these equations are that in Eq. (7), the CC can be zero, but in Eq. (8) we have more freedom to allow the CC to become as large as needed, providing Eq. (5) stands.

While evidence may exist that Einstein considered this previously, I am unable to find any references in the more well known papers although an exhaustive search has not yet been performed. I consider it unlikely since it wasn't until approximately 12 years after Einstein's death when Zel'dovich proposed the connection between quantum fluctuations and the CC [10]. In addition, to put it simply, there was only one quantity to work with in the early 1900s (ponderable mass-energy) and it of course gave him the correct answer. Considering Einstein's well known aversion to quantum mechanics being a complete description of reality [11], I find it likely that considerations of an alternate view of general relativity by him would be noteworthy in physics history. Interestingly, Zel'dovich also stated [10]

> "We now turn to a different aspect of the situation, namely to the close connection between the question of Λ and the theory of elementary particles. The very first attempts of quantizing the electromagnetic field led to the paradoxical conclusion that vacuum energy has infinite density. Vacuum was thus defined as the lowest energy state of the considered system whose characteristics are given by Maxwell's equations. The particles –in this case photons - are elementary excitations of the system."

This of course, leads us into the question of quantum vacuum energy.

IV. QUANTUM VACUUM ENERGY

It is customary [12] to describe the upper limit on the quantum vacuum energy by integrating the energy from each mode of a quantum harmonic oscillator from zero up to the Planck length. Using

$$\rho_{vac} = \frac{E}{V} = \frac{1}{V}\sum_k \frac{1}{2}\hbar\omega_k = \frac{\hbar}{2\pi^2 c^3}\int_0^{\omega_{max}} \omega^3 d\omega = \frac{\hbar}{8pi^2 c^3}\omega_{max}^4 \tag{9}$$

gives

$$\rho_{vac}^{Planck} \approx (10^{19}\text{GeV})^4 \approx 10^{114}\text{erg/cm}^3. \tag{10}$$

It is often then stated that this is approximately 10^{120} greater than observed. The magnitudes, however, are not always agreed upon and even called naive [3]. Experiments showing the effect of the Casimir force have been presented as a proof that the vacuum energy is "real", but this too is controversial [13]. If physicists do not agree on the physical reality of something, this should give someone pause as to accepting that a concept has wide spread consensus on the meaning.

I do not feign to be an expert in particle physics or quantum field theory, but for the development of our model, however, we could agree that the energy calculated for a point in space-time from Eq. (9) is a *potential* energy described as ρ_{vac} within $g_{\mu\nu}\Lambda$. Technically this equation is only for the zero-point energies and does not seem to include those above T=0, so it would be reasonable that all modes need to be included. Those modes which are occupied are what we would consider the energy as described by Einstein in $G_{\mu\nu}$, and then $\Pi_{\mu\nu}$ from Eq. (5) represents the remaining unoccupied modes. Thus, as a start, gravity could be considered as either due to mass-energy or the unoccupied modes of quantum field theory. We might suppose that if zero-point energy modes are occupied at all points in space-time then they would be subtracted equally from $\Pi_{\mu\nu}$ and $g_{\mu\nu}\Lambda$.

It is worth remarking here that it is well known that quantum field theory allows for negative energy densities [14]. Coupling this with the bizarre but effective notion of renormalization due to infinite energies of electron-positron pairs, the finite energy limits of a symmetric but opposite mass energy-momentum tensor has great appeal.

However, while the original hypothesis is simple, the implications for current theories that aren't fleshed out well in and of themselves [15] [16] [17] are too complicated to yet consider in this original formulation. We will have to wait for this as a future project. Therefore we do not yet elaborate into the Friedmann equations or the mass-energy percentages of the universe.

V. NEWTONIAN APPROXIMATION

Einstein utilized the form of an ideal fluid equation as a model for physical reality and to derive the simplification to the Newtonian potential. Since the derivation from Eq. (8) is algebraically trivial to a Newtonian approximation, I leave this to the Appendix.

In [2] we are presented with a change from the normal Newtonian potential equation

$$\vec{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{\vec{r}}, \tag{11}$$

to one with the CC incorporated for a spherical mass M as

$$\vec{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{\vec{r}} + \frac{\Lambda c^2 r}{3}\hat{\vec{r}}. \tag{12}$$

The controversy of the magnitude of the CC and dark energy surrounds this equation. As can be seen, the magnitude of Λ corresponds to a repulsive force that opposes Newtonian attractive gravity. The authors ask what mechanism could suppress the large energy of the quantum vacuum yet empirically demonstrate a repulsive type force on large scales?

The differences between Eq. (A26),

$$\vec{g} = -\nabla\Phi = -\frac{\Lambda_{\text{vac}} c^2 r}{6}\hat{\vec{r}} + \frac{G\rho_{\text{res}} V}{r^2}\hat{\vec{r}},$$

derived in the appendix, to Eq. (12) are formulaically trivial. The physical differences between Eq. (11) and (A26) are indistinguishable (at least locally). As a visual aid, I provide Fig. (1), where if one multiplies both by a -1, the top Euclidean scalar field represents Eq. (11) and bottom represents Eq. (A26). The two dimensional gradients are equivalent.

From a classical consideration of gauge invariance, how does one know for certain that the concept of attractive

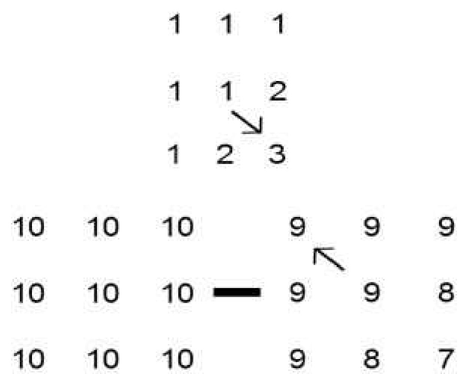

FIG. 1. Equivalent Gradients of Two Dimensional Euclidean Scalar Fields

gravity is correct? Other types of alternative theories seems to involve some type of mechanical pushing particle. Although this concept is not the same, it is interesting to note in spite of the success of relativity, even as late as the 1960s the esteemed Nobel physicist Richard Feynman stated in lectures that he had no success in proving that a particulate version (from neutrinos) of gravity was correct [18]. Considering the original thinking he is renown for, it should give one pause as to what would cause him to doubt attractive gravity.

The major initial problem with Eq. (A26) is that in spite of the equation being locally the same, once there is enough distance between galaxies, what would keep them from just flying apart away from each other? *The answer appears to be, empirically, nothing. This is the exact problem of the phenomenon of dark energy.*

One must also realize, that the only currently accepted logic behind the cause for the accelerating expansion are the equations themselves. The CC is now held up as this cause. However, prior to the verified empirical evidence the same theoretical physicists would have stated that it was not possible. This is not a conspiracy, it is just a reflection of that fact that experimental cosmology has a habit of rudely bringing in empirical evidence that proves our long held concepts are not only incomplete, but probably incorrect.

The opinions of Newton [19] and Mach [20] could be considered but are used to the point of being cliché in alternative theories. While Eq. (5) could conform with relativity and Newtonian gravity and at the same time accept a large value for the CC, it is also striking the number of pre-relativistic concepts it may be able to help explain if one were to investigate history further. For the moment, we consider that it might be more appropriate if others were to point this out. Our understanding of particles moving along geodesics requires more study.

VI. DARK MATTER

Dark matter is a hypothesized particle that only interacts gravitationally with baryonic matter and light [21]. It is thought to exist, at five times the amount of normal baryonic matter due to the high rotation speeds of galaxies and amount of gravitational lensing through them. In addition, it does not fit into the Standard Model. Despite the cosmological principle, in over two decades of searching,no dark matter particle has ever been detected. If Newtonian attractive gravity and general relativity are correct then there must be a very large amount of invisible matter within these galaxies and moving through us currently.

However, if one were to raise the value of $g_{\mu\nu}\Lambda$ or lower the value of $\Pi_{\mu\nu}$ in

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = G_{\mu\nu} = g_{\mu\nu}\Lambda - \Pi_{\mu\nu},$$

then the curvature of space-time at that point would increase. If one were to be able to do this, but break the equality of the equation by not affecting $G_{\mu\nu}$, then there would be more curvature of space-time than the mass-energy present would allow for. This gives a possible path to an explanation of dark matter and also may reinforce the notion that perhaps gravity is a quantum fluctuation phenomenon of the vacuum, and only has the appearance locally of being tied to baryonic mass-energy. Thus no dark matter would ever be located.

VII. DISCUSSION

Hypothetically, suppose someone would come to you with an equation, and state that the in spite of philosophical questions, it works amazingly well at predicting empirical data. However, there is this nasty problem that the trivial constant in the equation should be zero, and so due to the accuracy of the equation their first inclination is to treat the constant as a fudge factor to match the empirical data and thus doing so becomes ingrained into their mindset. Much later on other calculations tell them that this constant actually should be massively large. In the world of field theory, my first inclination is to consider gauge theory and think that they probably just had the equation backwards to begin with. While this instinct may not prove correct, I ask the reader to keep this in mind.

The redefined Newtonian equation (A26) is not locally distinguishable from Eq. (12). Occam's Razor dictates that given the choice between two equivalent equations, one should choose that which requires fewer unnatural explanations. If you were to assume that attractive gravity is a more natural explanation, perhaps a reexamination of the history of field theory [22] would be in order. It is the everyday appearance of attractive gravity that has brought us understanding and acceptance in modern physics. In my opinion, humans are better suited at discerning differences rather than magnitudes, such that physical theories based on human perception tend to obstruct progress and end poorly.

Given the current state of theoretical physics, what is in the harm of asking questions of a simple equivalency? While the idea may be found to have no merit, this does not preclude its presence in the body of knowledge.

VIII. CONCLUSION

We have shown that there is a potential substitution from an alternate but equal assumption within Einstein's derivation of the field equations. This substitution would allow use of a large cosmological constant from quantum field theory, result in a locally indistinguishable Newtonian gravitational potential, providing a more natural path for explanations of dark energy and dark matter and have ramifications on the understanding of field theory.

Appendix A: Derivation of a cosmically repulsive Newtonian gravity

Einstein utilized the stress energy tensor for an ideal fluid [9],

$$T^{\mu\nu} = \left(\rho + \frac{p}{c^2}\right)\mu^{\mu}\mu^{\nu} - p\eta^{\mu\nu}, \tag{A1}$$

in his derivation of GR which is

$$T^{\mu\nu} = \begin{vmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{vmatrix}.$$

However, it is known from the special relativity theory (SRT) that only tensors of the metric

$$g^{\mu\nu} = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \tag{A2}$$

are Lorentzian invariant. Although the form for an ideal fluid is correct, Lorentzian invariance demands that the actual stress energy tensor must be

$$T^{\mu\nu} = \begin{vmatrix} -\rho c^2 & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{vmatrix}$$

thus giving the strange but well known equation of state $-\rho c^2 = p$.

1. One dimensional observer

To begin our argument, let us examine the observational conditions of a one-dimensional observer, not outside an ideal fluid, but as a part of the fluid itself. Looking at Fig. 2, the horizontal axis represents a singu-

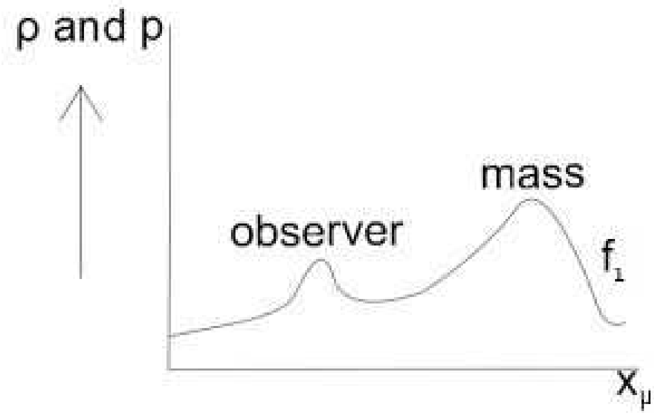

FIG. 2. Observer Within Ideal Fluid

lar dimension of x_μ and the vertical is a function of $f_1(x_\mu) = \rho c^2 = p$ (technically $\frac{p}{3}$ for a single dimension). For the observer in the fluid, let us consider that they are in the vicinity of a higher density mass so that the region under the left most $\frac{\partial f_1}{\partial x_{mu}} = 0$ is the observer, and the region under the right most $\frac{\partial f_1}{\partial x_{mu}} = 0$ is an observed mass.

We can see that the observer measures both energy and pressure with respect to themselves in agreement with the stress energy tensor of an ideal fluid with an equation of state $\rho c^2 = p$ or $\frac{\partial \rho}{\partial p} > 0$, but which is not Lorentzian invariant.

2. Alternate One-Dimensional Observer

The term on the left side of the SRT expression of

$$ds^2 = -dX_1^2 + dX_2^2 + dX_3^2 + dX_4^2 \tag{A3}$$

can only be determined by physical spatial and temporal measurements. Einstein states [9],

> "These can no longer be dependent on the orientation and the state of motion of the "local" system of co-ordinates, for ds^2 is a quantity ascertainable by rod-clock measurement of point events infinitely proximate in space-time, and defined independently of any particular choice of co-ordinates."

Note that Eq. (A2) is derived from Eq. (A3).

Consider also that ρ and p within Eq. (A1) only have meanings in reference to some defined zero value from which a measurement can be taken. This poses a problem for an observer within a fluid. *If the fluid is boundless, from where should the measurements be in reference to?* If we suppose that instead of a defined zero point, they discover that they have access to regions from which they can obtain a maximal value, then this may serve as a point of reference. Thus all measurements may be made so that

$$C_\rho - \rho c^2 = C_{\mathrm{p}} - p. \tag{A4}$$

The observer, however, must reconcile with the Minkowski metric requirement of Eq. (A3). This requirement (and the fact that quantum physics implies their reference point has maximal mass-energy) tells the observer that if they are a creature within a fluid, and therefore made up of said same fluid, then they must be a *decrease* in the energy density of the fluid so that their measurements reflect

$$C_\rho - \rho c^2 = -C_{\mathrm{p}} + p. \tag{A5}$$

While this may go against their natural senses, they also understand that they are better adapted at determining differences rather than magnitudes and the senses make for poor physical theory foundations. As in Fig. 3 we

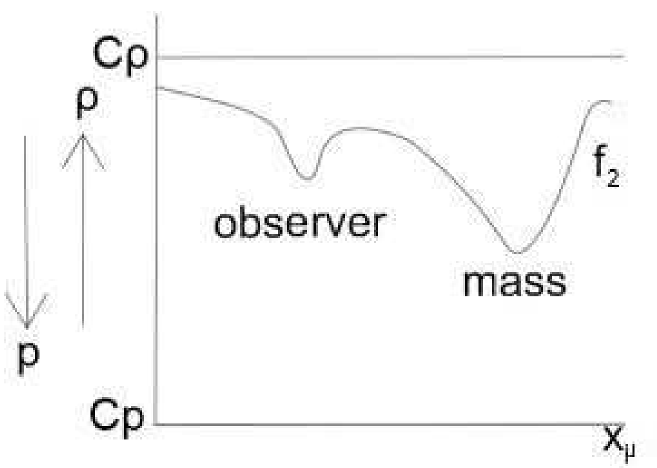

FIG. 3. Alternate Observer Within Ideal Fluid

see that all masses that this observer examines are also decreases in energy density.

If we consider their observational conditions compared to the first observer, we see that if

$$\frac{\partial f_1}{\partial x_\mu} = -\frac{\partial f_2}{\partial x_\mu} \tag{A6}$$

and

$$\int f_1 \partial x_\mu = \int (C - f_2)\partial x_\mu, \tag{A7}$$

then both will make the same quantitative measurements in their respective vicinities.

3. Alternate Energy Momentum Tensor

From Fig. 3, it follows that

$$C_\rho - \rho c^2 = -C_{\mathrm{p}} + p. \tag{A8}$$

Note that this is only a method for facilitating a graphical understanding, as it would be equally as correct to state

$$-C_\rho + \rho c^2 = C_{\mathrm{p}} - p.$$

Let us assume that the constants C_ρ and C_{p} are the maximum energy density and pressure of the vacuum but that *both* can be denoted by $\Lambda\kappa$. The equation of state for this vacuum fluid is

$$\Lambda\kappa - \rho c^2 = -\Lambda\kappa + p, \tag{A9}$$

where κ is a constant determined from [9]. Rearranging and pulling the explicit variables out of κ gives

$$\Lambda\frac{c^4}{8\pi G} - \rho c^2 = -\Lambda\frac{c^4}{8\pi G} + p. \tag{A10}$$

We now rename ρ and p as

$$\rho = \rho_{\mathrm{m}}$$

and

$$p = p_{\mathrm{m}}$$

where the "m" subscript denotes "measurable" (to the observer's senses). Separating the ρ and p terms, it follows that

$$\Lambda\frac{c^4}{8\pi G} - \rho_{\mathrm{m}}c^2 = \rho_{\mathrm{res}}c^2 \tag{A11}$$

and

$$-\Lambda\frac{c^4}{8\pi G} + p_{\mathrm{m}} = -p_{\mathrm{res}}, \tag{A12}$$

where the "res" subscript denotes "residual". Rearranging terms and equating,

$$\rho_{\mathrm{m}}c^2 = -p_{\mathrm{m}} = \Lambda\frac{c^4}{8\pi G} - \rho_{\mathrm{res}}c^2 = -\Lambda\frac{c^4}{8\pi G} + p_{\mathrm{res}} \tag{A13}$$

or

$$-\rho_{\mathrm{m}}c^2 = p_{\mathrm{m}} = -\Lambda\frac{c^4}{8\pi G} - \left(-\rho_{\mathrm{res}}c^2\right) = \Lambda\frac{c^4}{8\pi G} - p_{\mathrm{res}}. \tag{A14}$$

Substituting this into the EFE gives

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} = \frac{8\pi G}{c^4}T'_{\mu\nu}, \tag{A15}$$

where $T'_{\mu\nu}$ denotes a stress-energy tensor of constant vacuum mass-energy reduced by a stress-energy tensor of residual mass-energy. That is to say

$$\frac{8\pi G}{c^4}T'_{\mu\nu} = -g_{\mu\nu}\Lambda + \frac{8\pi G}{c^4}T^{\text{res}}_{\mu\nu}, \tag{A16}$$

$$= -\begin{vmatrix} \Lambda_{\text{vac}} & 0 & 0 & 0 \\ 0 & -\Lambda_{\text{vac}} & 0 & 0 \\ 0 & 0 & -\Lambda_{\text{vac}} & 0 \\ 0 & 0 & 0 & -\Lambda_{\text{vac}} \end{vmatrix} + \left(\frac{8\pi G}{c^4}\right)\begin{vmatrix} \rho_{\text{res}}c^2 & 0 & 0 & 0 \\ 0 & -p_{\text{res}} & 0 & 0 \\ 0 & 0 & -p_{\text{res}} & 0 \\ 0 & 0 & 0 & -p_{\text{res}} \end{vmatrix}$$

or

$$= \left(\frac{8\pi G}{c^4}\right)\begin{vmatrix} -\Lambda_{\text{vac}}\left(\frac{c^4}{8\pi G}\right) + \rho_{\text{res}}c^2 & 0 & 0 & 0 \\ 0 & \Lambda_{\text{vac}}\left(\frac{c^4}{8\pi G}\right) - p_{\text{res}} & 0 & 0 \\ 0 & 0 & \Lambda_{\text{vac}}\left(\frac{c^4}{8\pi G}\right) - p_{\text{res}} & 0 \\ 0 & 0 & 0 & \Lambda_{\text{vac}}\left(\frac{c^4}{8\pi G}\right) - p_{\text{res}} \end{vmatrix}.$$

Thus from Eq. (A16), the EFE is locally equivalent for both observers *if the first observer with a zero reference point decides that his fluid is also Lorentzian invariant* whereas the second has no choice. I refer to this as naturally Lorentz invariant as it could be argued that Minkowski flat space is a property of it, and not vice versa.

4. Repulsive gravity

Noting that there are two forms of the metric $\mathbf{g} = diag(-1,1,1,1)$ and $diag(-1,-1,-1,1)$, Einstein states [9]

> "If in addition we suppose the gravitational field to be a quasistatic field, by confining ourselves to the case where the motion of the matter generating the gravitational field is but slow (in comparison with the velocity of the propagation of light), we may neglect on the right-hand side differentiations with respect to the time in comparison with those with respect to the space coordinates, so that we have
>
> $$\frac{d^2x_\tau}{dt^2} = -\frac{1}{2}\frac{\partial g_{44}}{\partial x_\tau}(\tau = 1,2,3) \tag{A17}$$
>
> This is the equation of motion of the material point according to Newton's theory, in which $\frac{1}{2}g_{44}$ plays the part of the gravitational potential. What is remarkable in this result is that the component g_{44} of the fundamental tensor alone defines, to a first approximation, the motion of the material point.
>
> We now turn to the field equations
>
> $$\left.\begin{bmatrix} \frac{\partial}{\partial x_\alpha}\Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha} = -\kappa T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T, \\ \sqrt{-g} = 1. \end{bmatrix}\right\} \tag{A18}$$
>
> Here we have to take into consideration that the energy-tensor of "matter" is almost exclusively defined by the density of matter in the narrower sense, i.e. by the second term of the right-hand side of $[T^{\alpha\beta} = -g^{\alpha\beta}p + \rho\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}]$ [...]. If we form the approximation in question, all the components vanish with the one exception of $T_{44} = \rho = T$. On the left hand side of [(A18)] the second term is a small quantity of second order; the first yields, to the approximation in question,
>
> $$\frac{\partial}{\partial x_1}[\mu\nu,1] + \frac{\partial}{\partial x_2}[\mu\nu,2] + \frac{\partial}{\partial x_3}[\mu\nu,3] - \frac{\partial}{\partial x_4}[\mu\nu,4].$$
>
> For $\mu = \nu = 4$, this gives, with the omission of terms differentiated with respect to time,
>
> $$-\frac{1}{2}\left(\frac{\partial^2 g_{44}}{\partial x_1^2} + \frac{\partial^2 g_{44}}{\partial x_2^2} + \frac{\partial^2 g_{44}}{\partial x_3^2}\right) = -\frac{1}{2}\nabla^2 g_{44}$$
>
> The last of equations [(A18)] thus yields
>
> $$\nabla^2 g_{44} = \kappa\rho \tag{A19}$$
>
> The equations [(A17)] and [(A19)] together are equivalent to Newton's law of gravitation."

The Laplacian operator ∇^2 is derived from the Fundamental Theorem of Calculus. It follows from classical invariant gauge theory that human perception cannot tell the magnitude of the scalar(s) within g_{44}. Thus we may make a direct algebraic substitution of the difference of two large scalars provided the difference is equivalent to the original.

Following [2] beginning on pg. 185 we substitute measured mass $\rho_{\rm m}$ into the dust energy-momentum tensor giving

$$T_{\mu\nu} = \rho\mu_\mu\mu_\nu = \rho_{\rm m}\mu_\mu\mu_\nu. \tag{A20}$$

Substituting from Eq. (A10),

$$\rho_{\rm m}\mu_\mu\mu_\nu = -\kappa\left[\Lambda_{\rm vac}\left(\frac{c^2}{8\pi G}\right) - \rho_{\rm res}\right]\mu_\mu\mu_\nu, \tag{A21}$$

leading to

$$\frac{1}{2}\delta^{ij}\partial_i\partial_j h_{00} = \frac{1}{2}\kappa\left[\Lambda_{\rm vac}\left(\frac{c^2}{8\pi G}\right) - \rho_{\rm res}\right]c^2. \tag{A22}$$

This gives the equivalent Poisson equation of

$$\nabla^2\Phi = \frac{\Lambda_{\rm vac}c^2}{2} - 4\pi G\rho_{\rm res} = 4\pi G\rho_m, \tag{A23}$$

simplified to

$$= 4\pi G\left(\frac{\Lambda_{vac}c^2}{8\pi G} - \rho_{res}\right).$$

Deriving back through Gauss' Law gives

$$\vec{g} = -\nabla\Phi = -\frac{GM}{r^2}\hat{\vec{r}} = -\frac{G\left[\Lambda_{\rm vac}\left(\frac{c^2}{8\pi G}\right) - \rho_{\rm res}\right]V}{r^2}\hat{\vec{r}}, \tag{A24}$$

and

$$= -\frac{GV\left[\Lambda_{\rm vac}\left(\frac{c^2}{8\pi G}\right)\right]}{r^2}\hat{\vec{r}} + \frac{GV\rho_{\rm res}}{r^2}\hat{\vec{r}}. \tag{A25}$$

Note that this equation is for a *point mass*. The units of V for the volumeless second term cancels when combined with ρ_{res}, but the units for V in the first cancel with the radius r^2 in the denominator. This gives

$$= -\frac{\Lambda_{\rm vac}c^2 r}{6}\hat{\vec{r}} + \frac{G\rho_{\rm res}V}{r^2}\hat{\vec{r}}. \tag{A26}$$

The denominator in the first term does not match that derived within our reference [2], although the difference is trivial in relation to the numerator. As for now I am uncertain whether the assumptions for the derivation of κ are still valid or if I have made a simple mathematical error. The unit vector in the first term is defined as pointing towards "mass". As this mass vanishes, the unit vector loses its meaning but the whole equation could be considered as cosmically repulsive.

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[19] R. Bentley and I. Newton, *The Works of Richard Bentley*, Vol. 3 (F. Macpherson, 1838) "It is inconceivable that inanimate brute matter should, without the mediation of something else, which is not material, operate upon and affect other matter without mutual contact, as it must be, if gravitation, in the sense of Epicurus, be essential and inherent in it. And this is one reason why I desired that you would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent faculty of thinking, can ever fall into it. Gravity must be caused by an agent acting constantly according to certain laws; but whether this agent be material or immaterial, I have left to the consideration of my readers.".

[20] E. Mach, *The science of mechanics: A critical and historical account of its development* (Open Court, 1919) "If, however, we so interpret it that we come into conflict with our experience, our interpretation is simply wrong." "The general experience cannot be constructed from the particular case given to us. We must, on the contrary, wait until such an experience presents itself. Perhaps when our physico-astronomical knowledge has been extended, it will be offered somewhere in the celestial space, where more violent and complicated motions take place than in our environment."
"Also when we speak of the attractions or repulsions of bodies, it is not necessary to think of any hidden causes of the motions produced. We signalize by the term attraction merely an actually existing resemblance between events determined by conditions of motions and the results of our volitional impulses.".

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