Geometry, pregeometry and beyond

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Abstract

This article explores the overall geometric manner in which human beings make sense of the world around them by means of their physical theories; in particular, in what are nowadays called pregeometric pictures of Nature. In these, the pseudo-Riemannian manifold of general relativity is considered a flawed description of spacetime and it is attempted to replace it by theoretical constructs of a different character, ontologically prior to it. However, despite its claims to the contrary, pregeometry is found to surreptitiously and unavoidably fall prey to the very mode of description it endeavours to evade, as evidenced in its all-pervading geometric understanding of the world. The question remains as to the deeper reasons for this human, geometric predilection—present, as a matter of fact, in all of physics—and as to whether it might need to be superseded in order to achieve the goals that frontier theoretical physics sets itself at the dawn of a new century: a sounder comprehension of the physical meaning of empty spacetime.

Keywords: Geometry in physics; Geometry in pregeometry; Empty spacetime; Beyond geometry.

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1 Introduction

Quantum gravity is a field of physics whose specific subject matter cannot be easily identified. In fact, one might even say that there are almost as many conceptions of quantum gravity as there are researchers of it. The task has been approached from innumerable angles, for different motivations, and with different underlying physical, mathematical, and philosophical assumptions. The tentative results are as varied.

In very general terms, however, quantum gravity attempts to unify, in a physically meaningful way, the current understanding of Nature in terms of quantum mechanics and general relativity, which is as such perceived as incongruous and fragmented. Because, according to general relativity, gravitation is associated with the geometric structure of spacetime, a common factor of this search consists in reconsidering the structure of spacetime from novel points of view.

Among the numerous existing approaches, we have found a special class rather conspicuous: those that attempt to go beyond the pseudo-Riemannian manifold of physics without assuming any other type of metric manifold for spacetime and without quantizing general relativity taken as a presupposed starting point, but rather accounting for spacetime in terms of entities, and possibly their interactions, that are ontologically prior to it. This, we believe, is a fair, *preliminary* characterization of *what is known* by the name of *pregeometry* (e.g. Monk, 1997, pp. 11–18).

Varied though pregeometric approaches can be, they all share a feature in common: their all-pervading *geometric understanding* of the world. Such a kind of understanding is by no means particular to pregeometry; it is also present in all of quantum gravity and, moreover, in all of physics. The intriguing, particularly curious case that pregeometry makes with respect to the previous assertion resides in its name and the corresponding intentions of its practitioners, i.e. it strikes us as peculiar to find geometric explanations in a field called, precisely, pre-geometry. One must therefore pose the question: does pregeometry live up to its name and matching objectives? And if not, why not and in what sense not?[1]

The clarification of the role of geometry in human physical understanding and, in particular, in pregeometry—an attempt to understand spacetime in a new light that tries to liberate itself from geometry but to no avail—constitutes the content of the present article. In Section 2, the vital role of geometry in all of physics and, in particular, in modern physics will be discussed as preparation for the material ahead. In Section 3, the traditional, as well as the present article's, motivations for the search for more novel structures for spacetime will be reviewed. In Section 4, pregeometry and some of its schemes will be analyzed. Finally, in Section 5, a seldom investigated approach to the problem of spacetime structure will be presented: the key to rethinking spacetime anew might lie not in overtly geometric or pregeometric explanations but in those that manage to go beyond geometry altogether.

2 Geometry and its role in physics

The very conceptual foundations of geometry could be laid down in the following way. Geometry and geometric thinking are built out of two basic ingredients. The first and more primitive of them is that of *geometric objects* such as, for example, point, line, arrow, polygon, sphere, cube, etc. As to the origin of geometric objects, it appears after some consideration that they must arise in no other way than through idealization by the mind of sensory perceptions. Different natural objects are readily idealized by the human mind as possessing different shapes, and thus

[1]The criticism of pregeometry to be put forward here is not based merely on a verbal objection. It consists of an objection to the activities of pregeometricians proper, given that they explicitly set out to avoid geometry—and fail. Since this goal is, indeed, depicted very well by this field's name, the criticism applies to the word "pregeometry" just as well. It is only in this sense that it is *also* a verbal objection.

they become geometric objects of human thought. This activity of abstracting form from Nature, we believe, must have had its origin in the mists of antiquity.

The second and much more advanced ingredient that geometry consists of is that of *geometric magnitudes*. As its name suggests, these magnitudes involve the expression of a quantity, for which the concept of number is a prerequisite. Assuming that the notion of counting is already available, geometric magnitudes arise as result of a process (measurement) by means of which geometric objects can be attributed different notions of *size*. For example, the geometric magnitude *length* applies only to a geometric object (e.g. line, curve) or a part of it (e.g. edge of a polygon), and its original meaning is that of counting how many times a unit of measure can be juxtaposed along the object in question. Other geometric magnitudes are: distance, area, volume, overlap, etc.

We stress that the said idealization of the natural object (and of the physical unit of measure) as a geometric object is essential here; without such a geometric abstraction, it would not be possible to know in what way successively to lay down the standard upon the measured object. In other words, the introduction of geometric magnitudes presupposes the existence of geometric objects.

Beyond the human primeval recognition of geometric form and, much later, magnitude, the rise of more sophisticated geometric ideas is recorded to have had its origin in the Neolithic Age in Babylonia and Egypt. A landmark in the development of abstract geometry, and perhaps the oldest geometric construction that rates higher in abstraction than other common uses of geometric thought is so-called Pythagoras' theorem. Incontestable evidence of knowledge of Pythagorean triples[2] and of the Pythagorean theorem can be found in the Babylonian cuneiform text Plimpton 322 and in the Yale tablet YBC 7289, which date from between 1900 and 1600 b.C. in the Late Neolithic or Early Bronze Age.

There exist records, such as the papyrus Berlin 6619 dating from around 1850 b.C., showing knowledge of Pythagorean triples by the Egyptians, too. However, no triangles are mentioned here or elsewhere. Van der Waerden (1983, p. 24) suggested that the Egyptians may have learnt about Pythagorean triples from the Babylonians. In support of this view, Boyer and Merzbach (1991) wrote: "It often is said that the ancient Egyptians were familiar with the Pythagorean theorem, but there is no hint of this in the papyri that have come down to us" (p. 17).

As far as the use of geometry is concerned, little has changed in the last 4,000 years. Physical theories make extensive use of qualitative and quantitative geometric concepts; moreover, in trying to find a physical theory that does not include them, one is certainly bound to fail: the role of geometry in all of past and present physics is ubiquitous. At first glance, it appears that especially quantitative geometric notions, such as sizes of objects and distances between them, are only introduced into theories to establish a connection with the world of experience through measurements. However, just as significant is the fact that great facilitation is achieved

[2]A Pythagorean triple consists of an ordered triple (x, y, z), where x, y, and z satisfy the Pythagorean theorem $x^2 + y^2 = z^2$.

in human thought processes by the introduction of geometric notions of both kinds. Probability, a concept in principle free from any geometric connotations, will be seen to be geometrized in this sense in Section 2.2.

An example from another scientific field will be given presently to clarify the second reason above. In the theory of coding, one is concerned with sending messages over a noisy information channel and with being able to recover the original input from a possibly distorted output. Evidently, no intrinsically geometric magnitudes are involved in this problem; however, these are readily constructed to ease analysis and understanding of the situation. A word c is thought of as a vector $\vec{c} = (c_0, c_1, \ldots, c_{n-1})$ and a distance function, called Hamming metric, is defined for any pair of words c and c' as $\max\{i \mid c_i \neq c'_i, 0 \leq i \leq n\}$. After certain assumptions about the nature of the channel, a received word is corrected searching for the codeword that is *closest* to it in the above sense, i.e. differing from a codeword in the least number of elements. As this example shows—and as will become progressively clearer—it would seem it is a trait of the human mind to seek a geometric understanding of the world.

In physical contexts,[3] there exists the more primitive concept of inner product from which three key, quantitative geometric notions emerge, namely: overlap of $|A\rangle$ and $|B\rangle$, $\langle A|B\rangle$ (with parallelism and orthogonality arising as special cases of this concept); length of $|A\rangle$, $\langle A|A\rangle^{1/2}$; and distance[4] between $|A\rangle$ and $|B\rangle$, $\langle A-B|A-B\rangle^{1/2}$. Therefore, the *inner product* has the power of introducing the most significant quantitative geometric notions into physical theories.

In what follows, the geometric way Nature is understood in modern times by means of the theories of general relativity and quantum mechanics will be investigated. It will be seen that, although the portrayal of the physical world has come a long way since the dawn of mankind, something has remained unchanged in it ever since: the everywhere-present use of geometric explanations. Although dressed up in much more sophisticated garments, the role of geometry in today's received description of the world is, just as in earlier times, nothing short of crucial. This is contrary to common belief, as expressed in Ashtekar's statement:

> [W]e can happily maintain a *schizophrenic* attitude and use the precise, *geometric picture of reality* offered by general relativity while dealing with cosmological and astrophysical phenomena, and the quantum-mechanical world of chance and intrinsic uncertainties while dealing with atomic and subatomic particles. (Ashtekar, 2005, p. 2) [Italics added]

[3] This is not necessarily so in mathematical contexts where, for example, norm and distance can arise from non-inner-product constructions such as $\|f\| = \sup_{x\in\mathbb{R}} |f(x)|$ in Banach spaces, and $d(x, y) = \min_{\text{paths}(x,y)}\{n(\text{edge})\}$ in graph theory.

[4] If the space to which $|A\rangle$ and $|B\rangle$ belong were not flat, one should then resort to a line integral to find out this distance; yet, the essence of the concept remains the same (see Section 2.1).

2.1 Geometry in general relativity

According to general relativity, spacetime is described by means of a curved, four-dimensional manifold M. But what objects characterize spacetime physically? At first glance, these would seem to be spacetime's building blocks, its points. On closer inspection, it can be noticed that it is not the points that are physically real but the measurable interval relations $\mathrm{d}s^2$ between point-events E, F, etc. Specifying now these intervals, i.e. the generalized distances between neighbouring point-events, the geometry of spacetime is fixed. This is in accordance with the earlier remarks that quantitative geometry arises from a specification of sizes of objects or distances between them.

Misner, Thorne, and Wheeler (1973, pp. 306–308) analyzed the immensity of the problem of giving the distances between a vast numerousness of point-events by crudely writing down a number for each pair. They showed how the method can be refined until all that needs to be provided is a function $\mathbf{g}(\overline{E-F}, \overline{E-F})$ capable of producing the squared interval between neighbouring events E and F. On the basis of the metric, now the distance between any two point-events in spacetime arises as the line integral, along the shortest joining curve σ, of the (infinitesimal) length of the tangent vector $\vec{T}_\sigma$ to σ at each point; $\int_\sigma \mathbf{g}^{1/2}[\vec{T}_\sigma(u), \vec{T}_\sigma(u)]\mathrm{d}u$. Hence the introduction of a metric field $\mathbf{g}(x)$ as a local concept in general relativity.

Mathematically, the metric is a two times covariant tensor field $\mathbf{g}(x)$ equivalent in every respect to an *inner product* $\langle \mid \rangle$ of vectors on the tangent space $T_P(M)$ of the spacetime manifold M at every point P. Even if one knew nothing of general relativity and were at the moment totally ignorant of the purposes of the existence of an inner product in this theory, it could be safely taken, as emphasized earlier, as a sign of the introduction of quantitative geometry into the theory. This is because the inner product is an undisputable bearer of geometric magnitudes. In the theory, moreover, geometric objects (point, geodesic, tangent plane, arrow vectors, etc.) are everywhere present.

In dealing with spacetime as a mathematical manifold M as we just did, the association is made between physical point-events E of spacetime and mathematical points P of M. This association is extremely significant and potentially perilous, and must not be taken lightly; whereas point-events are endowed with a perfectly clear physical meaning (say, a flash of lightning), the case is not so for the spacetime points P themselves. The physical reality or unreality of the latter continues to be the object of heated philosophical debate and constitutes, as it were, one of the issues underlying our research.

All the geometric aspects of spacetime are *logically* (but not physically) obtained in terms of the metric tensor $\mathbf{g}(x)$, the most important of these being, by far, the invariant interval $\mathrm{d}s^2$, given by means of the famous relation $\mathrm{d}s^2 = g_{\mu\nu}(x)\mathrm{d}x^\mu \mathrm{d}x^\nu$. The significance of this interval cannot be overemphasized: it constitutes the most basic *physical* relationship between spacetime point-events E that remains unchanged for any system of reference in any state of motion, while at the same time giving a physical basis for the metric field.

Finally, the metric field also logically defines manifold curvature, and thus gravitation acquires a *new* geometric character:[5] free particles are now thought to fall along spacetime geodesics. Therefore, it can be said with little hesitation that the use of geometric concepts is essential in general relativity.

2.2 Geometry in quantum mechanics

Imagine a plate which only transmits light polarized perpendicular to its optical axis and a beam of light, polarized at an angle θ with the above axis, to be fired at the plate such that photons arrive one at the time. How is one to understand the result that the intensity of the transmitted light is a fraction $\sin^2(\theta)$ of the total intensity in terms of *individually polarized photons*? This experimental result can be understood thinking that the polarization state $|P\rangle$ *of each photon* can be expressed as a linear superposition of two polarization states $|P_t\rangle$ and $|P_a\rangle$, one associated with transmission and the other with absorption, respectively: $|P\rangle = \langle P_t|P\rangle|P_t\rangle + \langle P_a|P\rangle|P_a\rangle$, where $\langle P_i|P\rangle$ $(i = t, a)$ denotes "how much" states $|P\rangle$ and $|P_i\rangle$ have in common. With the further requirement that $\sum_i |\langle P_i|P\rangle|^2 = 1$, one can associate $|\langle P_t|P\rangle|^2$ with the probability of an individual photon being transmitted and $|\langle P_a|P\rangle|^2$ with its being absorbed. When the number of photons is large enough, the classical result is recovered (Dirac, 1958, pp. 4–7).[6]

This physically inspired superposition principle leads directly to the suggestion that quantum-mechanical states consist of *metric vectors* or *arrows*,[7] since it holds physically that the linear combination of two states is also a state, and the overlap (cf. *inner product*) of two states gives the probability amplitude of the original state collapsing onto another. Despite being thus inspired in the natural world, however, the situations that state vectors describe evade any mechanistic interpretation (i.e. visualization), and the physical meaning of state vectors themselves continues to incite a great deal of philosophical controversy.

State vectors belong to a Hilbert space $\mathcal{H}$, whose inner-product structure is vital in order to sustain quantum theory. This is because the notion of probability wants an inner product after the geometrically minded manner in which quantum mechanics naturally developed. The formalism of quantum mechanics postulates that, concerning an observable O and a system in state $|\psi\rangle$, the probability that the degenerate eigenvalue λ_i with associated eigenvectors $|u_{i,j}\rangle$ will result is $P(\lambda_i) = \sum_{j=1}^{g_i} |\langle u_{i,j}|\psi\rangle|^2$. In words, this probability is given by the sum of the squared *lengths* of the *projections* of the original state *vector* onto the basis vectors of the target space. Thus probability, which is in principle a concept that only has to do with counting positive and negative outcomes, acquires its geometric meaning.

[5] Notice that gravitation's Newtonian description as a vectorial force was also a geometric concept, namely, a directed arrow in Euclidian space with a further notion of size (strength) attached.

[6] For an illuminating account of the experimental and now conventional theoretical foundations of quantum mechanics, see Chapters 1 and 2 of this reference.

[7] The inner product effectively turns in principle shapeless state vectors into arrows, since geometric magnitudes demand geometric objects.

In this connection, Isham (1995, pp. 13–17) explained how, contrary to the case in statistical physics, in quantum mechanics probabilities originate in Pythagoras' theorem in n dimensions. Given an initial state vector $|\psi\rangle$, the interest is in predicting what are the chances that it will turn into a certain other after measurement. Then, if all the possibly resulting state vectors are organized at right angles with each other and each at an angle θ_i with $|\psi\rangle$, it will hold that $\sum_{i=1}^{n} \cos^2(\theta_i) = 1$ by virtue of Pythagoras' theorem. Finally, the association of each term with a probability of a particular result immediately suggests itself.

It is in this manner that probability is geometrized in order to provide the human mind with geometric understanding of, in principle, geometry-unrelated observations. Pythagoras' theorem comes thus from its humbler, Neolithic uses to serve human understanding in yet another facet of Man's inquiries into Nature.

3 Rethinking spacetime

It is currently believed that the spacetime manifold of general relativity is in need of rethinking, pregeometry being one of the most drastic means to this end. Two paradigmatic motivations behind this search are the various forms of gravitational collapse predicted by general relativity and the confirmed existence of quantum-mechanical correlations.[8] In fact, Monk (1997, pp. 3–9) presented these two points as central for the search for a quantum theory of gravity; yet we believe that their relevance to this search is not necessarily guaranteed. We present our own perspective in Section 3.3.

3.1 Spacetime manifold collapse

Misner et al. (1973) called gravitational collapse "the greatest crisis in physics of all time" (p. 1196). The singularities predicted by general relativity, it is argued, are events at which the pseudo-Riemannian manifold that characterizes spacetime breaks down. This is seen as a sign that spacetime only manages to look like a continuous manifold but that, in fact, it may be better represented by a theoretical picture of a different kind. But do these spacetime singularities represent a problem, or are they simply irrelevant cases where general relativity has reached its limits of applicability, much like the Newtonian gravitational law $-(GMm/r^3)\vec{r} = m(\mathrm{d}^2\vec{r}/\mathrm{d}t^2)$ when $r = 0$? In this light, general relativity could be taken as a complete and consistent theory of spacetime, singularities being an inconsequential domain outside it.

At this point, one must not necessarily discard spacetime singularities as motivations for the search for better theories. However, such singularities may only be taken as proper motivations if one believes that they are in fact meaningful questions whose solution will have a bearing on the understanding of the newer theories.

[8]See Butterfield and Isham (2000, pp. 34–36) for a classification of motivations according to whether they are viewed from the perspective of the particle physicist or the general relativist.

This might or might not be the case. The case of Newtonian singularities, for their part, was certainly not relevant to the invention of relativistic or quantum theories. On the other hand, the problem of electromagnetic singularities was relevant to the development of a quantum-mechanical theory, as evidenced by Bohr's efforts to solve the problem of the collapse of the atom.

3.2 Failure of locality

Stapp (1975) called Bell's (1987) theorem "the most profound discovery of science" (p. 271). Indeed, another paradigmatic motivation to rethink the description of spacetime stems from the analysis of non-local, quantum-mechanical correlations, whose experimental confirmation (e.g. Aspect, Dalibard, & Roger, 1982) tells that two distant parts of an entangled system know more about each other than local, realistic premises allow.[9] Whereas some have, in response, searched for an explanation within the spacetime manifold framework,[10] others have maintained that, since non-locality is not well-integrated in the manifold notion of spacetime, space is not what it is believed to be; that, although the correlated subsystems are irreconcilably distant within the traditional manifold notion of space, they may as well present a more amenable relationship—in this respect—within a different framework in which space is not a manifold, but a structure of a different kind.[11]

However, since quantum non-locality may as well be taken as an unproblematic, specific feature of quantum mechanics, a similar criticism as above ensues. Again one cannot tell whether these correlations that defy the ideas of locality should be addressed as a troublesome feature to be accommodated, a problem to be solved, by a theory that will supersede the current ones, or whether they simply are an intrinsic, trouble-free characteristic of quantum mechanics irrelevant to any future developments. As with spacetime singularities, this criticism does not aim at denying the significance of quantum non-locality as a motivation for research on the structure of space and time, but only to put it in its proper perspective.

3.3 The physical existence of space and time

It is here put forward that a more substantial reason for investigating these matters is to try to find out whether space and time might have an own structure independently of matter and fields; in other words, whether *empty* spacetime can claim physical existence. Despite the phrase being a commonplace, the meaning of

[9] Locality forbids signals to propagate instantaneously in space, and realism is meant in the sense that systems possess values for their physical magnitudes independently of their being observed.

[10] See e.g. (Selleri, 1988, pp. vii–ix) for a list of alternatives.

[11] See Section 4.6.1 and references therein for a proposed solution of this type. In particular, Eakins and Jaroszkiewicz (2003) wrote: "If Einstein locality is synonymous with classical Lorentzian manifold structure, and if this structure is emergent, then it seems reasonable to interpret quantum correlations as a signal that there is a pre-geometric (or pre-emergent) structure underlying the conventional spacetime paradigm" (p. 17).

"empty space" or "empty spacetime" is far from trivial. Does the phrase refer to spacetime as devoid of all matter, or also devoid of *all* fields?

Due to its particular feature of affecting all free bodies in the same way, gravitation has become a geometric property of spacetime, its curvature, and is logically characterized by spacetime's metric field $\mathbf{g}(x)$. Therefore, from this perspective, gravitation is always present as a *content* of any metric spacetime. One is inescapably given with the other, even if the spacetime in question were a flat, Minkowskian one. This conception is, in fact, Einstein's:

> If we imagine the gravitational field, *i.e.* the functions g_{ik}, to be removed, there does not remain a space of the type (1) [flat], but absolutely nothing... A space of the type (1), judged from the standpoint of the general theory of relativity, is not a space without a field, but a special case of the g_{ik} field... There is no such thing as an empty space, *i.e.* a space without field. Space-time does not claim existence on its own, but only as a structural quality of the field. (Einstein, 1952b, p. 176)

As a consequence, in order to vacate spacetime, one must, along with all its matter, rid spacetime of its *quantitative geometry.* When the metric field is gone, so is the network of intervals $\mathrm{d}s^2$ logically built upon it,[12] and only spacetime's building blocks remain: its points. However, according to general relativity's diffeomorphism invariance principle, these points cannot be observed by means of any physical tests, and empty spacetime—so the argument remains—cannot claim physical existence.

Must one give up on the problem of the existence of empty spacetime at the point where a deadlock seems to have been reached? Not quite, because in the above quantum theory—so far the only branch of natural science forced to confront directly the problem of physical *existence*[13]—was not taken into account. Moreover, we propose that understanding spacetime anew demands going beyond its intrinsic geometric description *entirely*; not only the metric (source of geometric magnitudes) but also the points (geometric objects) must be left behind. For this task, the use of *non-geometric means* should be required, as well as the guidance of some *physical principle* directly relevant to the *existence of spacetime*, so that one's search would not be blind guesswork.

One the one hand, the choice to transcend geometric methods is based on the very general observation that genuine scientific explanations come about only when older concepts are explained in terms of new ones of a different kind. Spacetime, from its metric structure down to its points, is first and foremost a geometric concept; as such, it may require a more basic explication of a non-geometric kind. History teaches us that progress in the scientific world-view is achieved only when such fundamentally different explanations are given for the theoretical notions in current use (e.g. the new understanding of gravity as the curvature of spacetime).

[12] Intervals $\mathrm{d}s^2$ here refer to relations between *points.* Intervals between physical events would not go anywhere if the metric were to disappear so long as we still had a geometrodynamic clock (Ohanian, 1976, pp. 192–198)—*physically*, intervals between events are prior to the metric field.

[13] See (Isham, 1995, p. 65).

Might this be a hint that a better understanding about the nature of spacetime could be achieved by going beyond its geometric bases altogether? One the other hand, the need to supersede geometry entirely also transpires, in our view, from a look at the stalemates that physicists and philosophers, in their need for familiar geometric concepts and words, continue to relentlessly confront in their study of the ontology of space, time, and the quantum principle; most notable among these problems is the search for the physical meaning of spacetime points, state vectors, and related issues.

The choice invariably made by practically everyone—and as shall be seen next, unwittingly, by pregeometricians—is to attempt to clarify the geometric nature and structure of the spacetime manifold with more geometric concepts. The choice to explore spacetime from a non-geometric point of view favoured here is a drastic alternative and a path that hardly anyone (but for Clifford, Eddington and Wheeler; see Section 5) has attempted to walk before; and certainly one that no-one has walked to completion.

4 Pregeometry

The word "pregeometry" is evocative of positioning oneself before geometry. But what does this mean in more detail? In the Introduction, a characterization of pregeometry was given as an attempt, by design, to account for spacetime geometry resting on entities ontologically prior to it and of an essentially new character. However, pregeometry has turned out to be an incongruous enterprise. Indeed, the distinctive feature of pregeometric approaches is that they drop the assumption of spacetime being correctly described by a pseudo-Riemannian manifold; but *only to replace it with some type or other of geometric-theoretical conception.*

Nothing supports this statement more dramatically and vividly than the following quotation from Gibbs:

> Once it has been decided which properties of space-time are to be discarded and which are to remain fundamental, the next step is to choose the appropriate *geometric structures from which the pregeometry is to be built.* (Gibbs, 1996, p. 18) [Italics added]

In this use of the term, pregeometry is synonymous to pre-Riemannian-manifold physics, although not synonymous to pregeometry in the proper semantic and historic sense of the word—the sense its inventor, J. A. Wheeler, endowed it with and which its current practitioners attempt to fulfill; in terms of Wheeler's (1980) two most striking pronouncements: "a concept of pregeometry that breaks loose at the start from all mention of geometry and distance" and "...to admit distance at all is to give up on the search for pregeometry" (p. 4) (See Section 5.3).

In connection with these quotations from Wheeler and with those from Gibbs above, we also find the following remarks by Cahill and Klinger quite striking and worthy of mention:

> The need to construct a non-geometric theory to explain the time and space phenomena has been strongly argued by Wheeler, under the name of pregeometry. Gibbs has recently compiled a literature survey of such attempts. (Cahill & Klinger, 1996, p. 314)

At the reading of this passage, one is at first quite pleased and subsequently nothing but disheartened. For, whereas it is true that Wheeler vowed for a non-geometric theory of space and time and named it pregeometry, it could not be farther from the truth to say that the pregeometric theories devised so far (for example, those in Gibbs' or Monk's survey) refrain from the use of geometric concepts; to the contrary, they use them extensively, as is our purpose to show.

In order to support the above statements, an analysis of some pregeometric works will be presented shortly, in which pregeometry's modes of working will be investigated from the particular perspective of this article. What will follow is therefore not an impartial review, nor does it intend to be complete in its coverage of its subject-matter but only to display some show-pieces of it. At the risk of displeasing some authors, attempts to transcend the spacetime manifold in the way explained earlier are here classified as pregeometric, even when this designation is not used explicitly in their programmes. This way of proceeding is somewhat unfair since the following works will be criticized mainly as far as they are to be expressions of a *genuine pregeometry*. When they exist, explicit mentions of pregeometry will be highlighted; lack of any such mentions will be acknowledged as well. Broad categories of pregeometric frameworks, although possibly mutually overlapping, will be identified. One or more actual cases exemplifying each group will be included.

4.1 Discreteness

One category could have embraced most of—if not all—the works in this analysis. Without doubt, the best-favoured type of programme to deal with new ways of looking at spacetime is, in general terms, of a discrete nature. The exact meaning of "discrete" we will not attempt to specify; the simple intuitive connotation of the word as something consisting of separate, individually distinct entities (Merriam-Webster Online Dictionary) will do. A choice of this sort appears to be appealing to researchers; firstly, because having cast doubt upon the assumption of spacetime as continuous, its reverse, discreteness of some form, is normally envisaged; secondly, because quantum-theoretical considerations also tend to suggest, at least on an intuitive level, that also spacetime must be built by discrete standards.

Last but not least, many find some of the special properties of discrete structures quite promising themselves. Among other things, such structures seem to be quite well-suited to reproduce the continuous manifold in some limit, because they *naturally support geometric relations* as a built-in property. As is well-known, this was originally Riemann's realization. He went further in his remarks as to the possible need for such a type of structure:

> Now it seems that the empirical notions on which the metrical determinations of space are founded, the notion of a solid body and of a ray of light, cease to be valid for the infinitely small. We are therefore quite at liberty to suppose that the metric relations of space in the infinitely small do not conform to the hypothesis of geometry; and we ought in fact to suppose it, if we can thereby obtain a simpler explanation of phenomena.
>
> The question of the validity of the hypotheses of geometry in the infinitely small is bound up with the question of the ground of the metric relations of space. In this last question, which we may still regard as belonging to the doctrine of space, is found the application of the remark made above; that in a discrete manifoldness, the ground of its metric relations is given in the notion of it, while in a continuous manifoldness, this ground must come from outside. Either therefore the reality which underlies space must form a discrete manifoldness, or we must seek the ground of its metric relations outside it, in the binding forces which act upon it. (Riemann, 1873, p. 17)

Riemann can therefore be rightly considered the father of all discrete approaches to the study of spacetime structure. He should not be considered, however, the mentor of an authentic pregeometry since, as he himself stated, discrete manifolds naturally support geometric relations, becoming, through these relations, geometric objects themselves.[14] The absolute veracity of Riemann's statement can be witnessed in every attempt at pregeometry, not as originally designed by Wheeler, but as it of itself came to be.

4.2 Graph-theoretical pictures

Graphs are a much-favoured choice in attempts at pregeometric schemes. Following the characterization made by Wilson (1985, pp. 8–10 and 78), a graph of a quite (although perhaps not the most) general type consists of a pair $(V(G), E(G))$, where $V(G)$ is a possibly infinite set of elements called vertices and $E(G)$ is a possibly infinite family of ordered pairs, called edges, of not necessarily distinct elements of $V(G)$. Infinite sets and families of elements allow for an infinite graph which can be, however, locally-finite if each vertex has a finite number of edges incident on it. Families (instead of sets) of edges permit the appearance of the same pairs more than once, thus allowing multiple edges between the same pair of vertices; in particular, an edge consisting of a link of a single vertex to itself represents a loop.

[14] Geometric objects can appear either self-evidently, or first as general, abstract objects, which subsequently become geometric objects via the attachment of geometric magnitudes to them. For example, a "set of objects with neighbouring relations" becomes a graph, i.e. a set of *points* with joining *lines*, when a distance is prescribed on the abstract relations. In quantum mechanics, we saw that state vectors likewise become geometric objects (arrows) after the specification of their inner product.

Finally, ordered (rather than unordered) pairs allow for the existence of directed edges in the graph.

A particular reason for the popularity of graphs in pregeometry is that, along the lines of Riemann's thoughts, they naturally support a metric. It is especially noticed here that, when a metric is introduced, all edges are taken to be of the same length (usually a unit) irrespective of their actual shapes and lengths as drawn on paper or as conjured up by the imagination. This distinguishes graphs from certain types of lattices, which have a more literal interpretation, as will be seen in Section 4.3.

4.2.1 Dadić and Pisk's discrete-space structure

Dadić and Pisk (1979) went beyond the manifold assumption by representing "space as a set of abstract objects with certain relations of neighbourhood among them" (p. 346), i.e. a graph; in it, the vertices appear to correspond to space's points. This approach assumes a *metric* as naturally inherited from the graph, and a definition of dimension—based on a modification of the Hausdorff dimension—that is scale-dependent. The graph $|G\rangle$ representing spacetime is required to be an unlabelled one in its points and lines and must be characterizable by just its topological structure. Operators $b^\dagger$ and b are defined for the creation and annihilation of lines, such that $|G\rangle$ can be constructed from the vacuum-state graph $|0\rangle$ by repeated application of $b^\dagger$; this is indeed a Fock-space framework.

What led Dadić and Pisk to this choice of graph-theoretical approach? In this respect, they argued that they found the existence of some objects and the relation of neighbourhood between them to be essential in the intuitive notion of space. Based on this, they proceeded to construct their discrete-space structure. However, it can be realized that this basis is not grounded on any further physical principles.

Quantum mechanics—in particular, a Fock-space method—is used in this approach, if nothing else, as a formalism to deal with graphs, their states, and their evolution. One notices at this point that Fock spaces include a metric as part of their built-in structure, too.

As can be seen, geometric concepts are here clearly assumed; they were typified by the natural metric defined on the graph on the one hand, and that of Fock spaces on the other, and not least by a graph as a composite geometric object (vertices and edges) itself. This fact alone would make this approach a questionable case of an authentic pregeometry, although it must be acknowledged that its authors did not so argue.

4.2.2 Antonsen's random graphs

Antonsen (1992) devised a graph-theoretical approach that is statistical in nature. Space is identified with a dynamical, random graph, whose vertices appear to be associated with space points and whose links are taken to have unit *length*; time is given by the parameterization of these graphs (spaces) in a metaspace. Primary point creation and annihilation operators $a^\dagger$ and a are defined, as well as secondary corresponding notions for links, $b^\dagger$ and b; probabilities for the occurrence of any such case are either given or in principle calculable.

This approach is, according to its author, "more directly in tune with Wheeler's original ideas" (p. 8) and, in particular, similar to that of Wheeler's "law without law" (p. 9) in that the laws of Nature are viewed as being a statistical consequence of the truly random behaviour of a vast number of entities working at a deeper level. One must have reservations about this view since Antonsen is already working in a geometric context (a graph with a notion of distance), which is not what Wheeler had in mind. Moreover, there does not appear to be any principle leading Antonsen to introduce this kind of graph structure in the first place. He sometimes (pp. 9, 84) mentioned the need to assume as little as possible, perhaps overlooking the fact that not only the quantity but also the nature of the assumptions made—as few as they might be—bears as much importance.

According to Antonsen, this framework does not really assume traditional quantum mechanics although it might look like it. He claimed that the operators and the Hilbert space introduced by him are only formal notions without direct physical interpretations besides that of being able to generate a geometric structure. On the contrary, he attempted to derive quantum mechanics as a hidden-variable theory.

Antonsen is therefore the first claimant to pregeometry who calls for the use of geometric objects (a graph) and magnitudes (distance) in its construction; for this reason, his scheme is rather suspicious as an authentic expression of pregeometry.

4.2.3 Requardt's cellular networks

Requardt also went beyond the manifold assumption by means of a graph-theoretical approach. Space is associated, at a deeper level, with a graph whose nodes n_i are to be found in a certain state s_i (only differences of "charge" $s_i - s_k$ are meaningful), and its bonds b_j in a state J_{ik} that can be equal to 0 or ± 1 (vanishing or directed bonds, respectively). A law for the evolution in what he called the "clock-time" of such a graph is simply introduced (Requardt & Roy, 2001, pp. 3042–3043) on the grounds that it provides, by trial and error, some desired consequences. Requardt (2000, p. 1) then assumed that this graph evolved from a chaotic initial phase or "big bang" in the distant past, characterized by the complete absence of stable patterns, in such a way as to have reached a generally stable phase—explained by the theory—which can be associated with ordinary spacetime.

A peculiarity of this scheme is that it seeks to understand the current structure of spacetime as emergent in time,[15] i.e. as a consequence of the primordial events

[15] As is typical of arguments considering the dynamic emergence of time (also as in space*time*), it is unclear in what dynamic time the emergent time is supposed to appear. This hypothetical, deeper dynamic time, when considered at all (discrete ticks of clock time for Requardt), is nevertheless always the same kind of simple and intuitive external parameter as the notion of time it attempts to give rise to; the end result is to trade one little-understood, external-parameter time

that are believed to have given rise to it in the first place. In this respect, it could have also been classified as a cosmological scheme, although not as a quantum-cosmological one since quantum mechanics is not assumed. Regarding this issue, Requardt (1996) explained that his goal is to "identify both gravity and quantum theory as the two dominant but derived aspects of an underlying discrete and more primordial theory..." (pp. 1–2).

It is interesting to note that, in contrast to the preceding scheme, in this scheme spacetime points are not associated with the graph's vertices but with "densely entangled subclusters of *nodes* and *bonds* of the underlying network or graph." More specifically, they are cliques, "the maximal complete subgraphs or maximal subsimplices of a given graph" (Requardt & Roy, 2001, p. 3040).

A lack of guiding principles must again be criticized. Requardt's (1995) choice of approach seems to rest on "a certain suspicion in the scientific community that nature may be 'discrete' on the Planck scale" (p. 2). Therefore, he argued, cellular networks make a good choice because they are naturally discrete and can, moreover, generate complex behaviour and self-organize. However, it must be said that basing a choice on a generally held suspicion whose precise meaning is not clear may not be very cautious.

Although Requardt (1995) intended to produce a pregeometric scheme ("What we are prepared to admit is some kind of '*pregeometry*' " (p. 6)) and strove for "a substratum that does not support from the outset... geometrical structures" (p. 2), he soon showed—in total opposition to the earlier remarks—that his graphs "have a natural metric structure" (p. 15). He introduced thus a very explicit notion of *distance*. Requardt was concerned with avoiding the assumption of a continuous manifold, and in this he succeeded. However, geometric notions are not merely manifold notions, as argued earlier. This attempt at pregeometry suffers therefore from the same geometric affliction as the previous one.

4.3 Lattice pictures

At an intuitive level, (i) a lattice consists of a regular geometric arrangement of points or objects over an area or in space (Merriam-Webster Online Dictionary). From a mathematical point of view, (ii) a lattice consists of a partially ordered set $\langle L, \prec \rangle$ in which there exists an infimum and a supremum for every pair of its elements.[16]

The first notion of lattice (i) goes hand in hand with the same idea as used in Regge calculus. In it, an irregular lattice is used in the sense of an irregular mesh whose edges can have *different* lengths. For this reason, such a lattice cannot be a graph, where all links are equivalent to one another. The second notion of

for another.

[16] A lattice can also be defined as an algebra $\langle L, \wedge, \vee \rangle$, with $\wedge$ and $\vee$ subject to certain rules, rather than as a partially ordered set. It can be shown (e.g. Eric Weisstein's World of Mathematics, http://mathworld.wolfram.com/Lattice.html) that, after the identifications $a \wedge b = \inf\{a, b\}$, $a \vee b = \sup\{a, b\}$ and $a \leq b \Leftrightarrow a \wedge b = a$, these two concepts are equivalent.

lattice (ii) is, *to a degree*, connected with the approach to spacetime structure called causal sets. Although strictly speaking, a causal set is only a locally finite, partially ordered set without loops, when it also holds that there exists a unique infimum and supremum for every pair of its elements (existence of a common past and a common future), the causal set becomes a lattice in the second sense of the term. Despite the fact that they are not rigorously always a lattice, causal sets have been included here in order to also exemplify, in as close a fashion as possible, the use of a lattice of the second type in the construction of an alternative structure for spacetime.[17]

4.3.1 Simplicial quantum gravity by Lehto et al.

Lehto (1988), and Lehto, Nielsen, and Ninomiya (1986a,b) attempted the construction of a pregeometric scheme in which it was conjectured that spacetime possesses a deeper, pregeometric structure described by three dynamical variables, namely, an abstract simplicial complex ASC, its number of vertices n, and a real-valued field ℓ associated with every one-simplex (pair of vertices) Y. It is significant that, up to this point, the approach has a chance of being truly pregeometric since the fields ℓ are not (yet) to be understood as lengths of any sort, and the vertices could also be thought as (or called) elements or abstract objects (cf. *abstract* simplicial complex). However, the introduction of geometric concepts follows immediately.

Next, an abstract simplicial complex is set into a unique correspondence with a geometric simplicial complex GSC, a scheme of vertices and geometric simplices. By piecing together these geometric simplices, a piecewise linear space can be built so that, by construction, it admits a triangulation. Furthermore, the crucial step is now taken to interpret the above field ℓ as the link *distance* of the piecewise linear manifold. The conditions are thus set for the introduction of a Regge calculus lattice with metric $g_{\mu\nu}^{\mathrm{RC}}$ given by the previously defined link lengths.

Traditionally, the primary idea of Regge calculus is to provide a piecewise linear approximation to the spacetime manifolds of general relativity by means of the gluing together of four-dimensional simplices, with curvature concentrated only in their two-dimensional faces. In this approach, however, the approximation goes in the opposite direction, since simplicial gravity is now seen as more fundamental, having the smooth manifolds of general relativity as a large-scale limit. Moreover, whereas the basic concept of the Regge calculus consists in the link lengths, which once given are enough to determine the geometry of the lattice structure, Lehto et al. also introduced the number of vertices n as a dynamical variable. This conferred on the lattice vertices a quality resembling that of a free gas, in the sense that any pair of vertices can with high probability have any mutual distance, helping thus to avoid the rise of long-range correlations typical of fixed-vertex lattices.

A quantization of the Regge calculus lattice is subsequently introduced by means

[17] Causal sets can also be classified as graph-theoretical, considering that they can be thought of as consisting of a locally-finite, loopless, directed graph without circuits, in the sense that it is not possible to come back to a starting vertex while following the allowed directions in it.

of the Euclidean path-integral formalism. A pregeometric partition function Z summing over ASC, fields ℓ, and vertices n is first proposed as a formal abstraction, to be geometrically realized by a corresponding Regge calculus partition function Z^{RC} summing over piecewise linear manifolds and link lengths ℓ. Indeed, quantization can be viewed as one of the reasons why the notion of length must be introduced in order to move forwards.

Two reasons can be offered for the choice of the starting point of this framework. On the one hand, being concerned with the production of a genuine pregeometry, an abstract simplicial complex was chosen on the grounds that, as such, it is abstract enough to be free from geometric notions intrinsic to it. On the other hand, an abstract simplicial complex with a variable set of elements was seen fit to provide an appropriate setting in which to study further and from a different point of view the character of diffeomorphism invariance, itself a crucial element for a possible richer understanding of the physical existence of spacetime. This is because the requirement of diffeomorphism symmetry of the functional integral measure $\prod \mathrm{d}\ell \exp(-S_{\mathrm{pg}})$, where S_{pg} is a Euclidean pregeometric action, with respect to a displacement of vertices results in a free-gas behaviour of the latter as elements of a geometric lattice.

In one sense, this work is no exception among all those analyzed in this section. As it was seen, geometric concepts such as link length had to be assumed to realize or materialize the abstract simplicial complex as a geometric object, a Regge calculus lattice, bringing it thus closer to the more familiar geometric world and rendering it fit for quantization. However, despite the assumption of geometry, we would nevertheless like to lay emphasis on a distinctive feature of this work—it recognizes clearly pregeometric and geometric realms and, even though it cannot do without the latter, it keeps them clearly distinguished from one another rather than, as is more common, soon losing sight of all differences between them.

4.3.2 Causal sets by Bombelli et al.

Bombelli, Lee, Meyer, and Sorkin (1987) proposed that at the smallest scales spacetime is a causal set: a locally finite set of elements endowed with a partial order corresponding to the macroscopic relation that defines past and future. This partial-order relation is required to have transitivity and irreflexivity properties of causality between point-events.

In this framework, causal order is viewed as prior to metric and not the other way around; in more detail, the differential structure and the conformal metric of a manifold are derived from a causal order. Because only the conformal metric (it has no associated measure of length) can be obtained in this way, the transition is made to a non-continuous spacetime consisting of a finite but large number of ordered elements, a causal set C. In such a space, size can be measured by counting. The transition to the classical limit is expressed as the possibility of a faithful embedding of C in M, or of a coarse grained version C' of C, arising from the assumption that small-scale fluctuations would render C non-embeddable.

What considerations have led to the choice of a causal set? In this respect, Sorkin explained:

> The insight underlying these proposals is that, in passing from the continuous to the discrete, one actually *gains* certain information, because "volume" can now be assessed (as Riemann said) *by counting*; and with both order *and* volume information present, we have enough to recover geometry. (Sorkin, 2005, p. 5[18])

This is a valid recognition resting on the fact that, given that general relativity's local metric is fixed by causal structure and conformal factor, a discrete causal set will be able to reproduce these features and also provide a way for the continuous manifold to emerge. However, we note that Sorkin is not here following a principle in the sense of Section 3.3 that naturally leads him to the conclusion that a causal set structure underlies spacetime. Furthermore, volume is already a geometric concept, so that "to recover geometry" above must mean "to recover a continuous, metric spacetime," not "geometry" in the stricter sense of the word.

Finally, the use of further geometric concepts in the formulation of this picture becomes clear in (Sorkin, 1991). The correspondence principle between a manifold $(M, \mathbf{g})$ and a causal set C from which it is said to emerge, for example, reads:

> The manifold (M, g) "emerges from" the causal set C iff C "could have come from sprinkling points into M at unit density, and endowing the sprinkled points with the order they inherit from the light-cone structure of g." (Sorkin, 1991, p. 156)

The above-mentioned unit density means that there exists a *fundamental unit of volume*, expected to be the Planck volume, 10^{-105} m^3. Furthermore, Sorkin also considered the *distance* between two causal set elements x and y as the number of elements in the longest chain joining them (p. 17), thus turning the ordering relations into lines and, possibly, x and y into points. Thus, if a causal set is to be seen as a pregeometric framework, due to its geometric assumptions, it makes again a questionable case of it. However, we do not know whether its proponents have staked any such claim.

4.4 Number-theoretical pictures

Some investigators have identified the key to an advancement in the problems of spacetime structure as lying in the number fields used in current theories or to be used in future ones. Butterfield and Isham (2000, pp. 84–85), for example, arrived at the conclusion that the use of real and complex numbers in quantum mechanics presupposes that space is a continuum. According to this view, standard quantum mechanics could not be used in the construction of any theory that attempts to go beyond such a characterization of space.

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[18] Page number refers to the online preprint gr-qc/0309009 v1.

4.4.1 Hill's discrete spacetime

Hill (1955) assumed that the structure of space is determined by its allowed symmetry transformations. Instead of requiring invariance of his theory under the full continuous group of Lorentz transformations L, he selected the subgroup of L named L_r in which translations and Lorentz transformations have rational, rather than real, coefficients. This subgroup is a group with respect to successive transformations and, in addition, it is dense in the full group L. From the latter property it follows that the deviation of Hill's picture from the continuous treatment of special relativity leaves very little room for its disproof, as he himself stated.

As the energy-momentum space is first discretized by restricting the values of energy and momentum variables to a certain set of rational numbers, quantum-mechanical wave functions become a special case of almost periodical functions, having space and time as continuous variables. When the change to spacetime points with only rational coordinates is made, the desired invariance of the background is realized, but the essence of the wave functions is obscure since the energy-momentum space lacks a unique interpretation.

Thus, guided by the Lorentz invariance of special relativity and the concept of wave function in quantum mechanics, Hill presented a picture in which some degree of success is achieved as regards the unification of ideas from the two theories; compared with some earlier lattice models, the problems of breaking the Lorentz invariance and of having an impossibly large minimum velocity (such as in Schild's) are solved. However, as Hill admitted, the implications of his model—especially experimentally testable ones—are not well-known.

Finally, since Hill assumed the structure of traditional quantum mechanics and of special relativity, the role of geometry is ubiquitous in his theory. However, since he did not endeavour to go further and apply his ideas to the construction of some form or other of pregeometry, his work does not attempt to go a great deal beyond today's received theories; therefore, the usual criticism made in this section as to the surreptitious use of geometric notions does not apply.

4.4.2 Rational-number spacetime by Horzela et al.

Horzela, Kapuścik, Kempczyński, and Uzes (1992) criticized those discrete representations of spacetime that assume an elementary length, and which may furthermore violate relativistic invariance, on the grounds that the former is experimentally not observed and the latter is, in fact, experimentally verified. They proposed to start their analysis by studying the actual experimental method used to measure spacetime coordinates, the radio-location method.

Their basic claim is that the measured values of the coordinates $t = (1/2)(t_1+t_2)$ and $x = (c/2)(t_2 - t_1)$ of an event in any system of reference are always rational (rather than real) numbers, since both the time of emission t_1 and reception t_2 of the radio signal are crude, straightforward measurements made with a clock.[19]

[19] For this idea to work, it is also essential that the value c of the speed of light be understood

Subsequently, they showed that the property of being rational is preserved when calculating the coordinates of an event in another system, as well as in the calculation of the relative velocity of this system. Therefore, they maintained that Lorentz invariance holds in a spacetime pictured in terms of rational-number coordinates. In addition, since the rational numbers are dense in the real numbers, such a spacetime frees itself from any notion of elementary distance. Thus, if spacetime must be described in a discrete manner, the rational number field is then seen as the key to a better understanding of it.

While Horzela et al. suggested that this line of investigation should be furthered by means of algebraic mathematical methods, they admitted not knowing at the time of their writing of any guiding physical principle to do this. As a consequence, since they do not propose to go as far as building a more complete spacetime picture based on their preliminary findings, the usual criticisms being made in this section do not apply. Further physical guiding principles are lacking, but then this has been clearly expressed and action was taken in accord with it; geometric ideas are of course present, but then no real attempt to transcend the framework of relativity was made after all.

4.4.3 Volovich's number theory

Volovich (1987) argued that, since at the Planck scales the usual notion of spacetime is suspected to lose its meaning, the building blocks of the universe cannot be particles or fields or strings for these are defined on such a background; in their place, he proposed that the numbers be considered as the basic entities. Moreover, Volovich suggested that for lengths less than the Planck length, the Archimedean axiom[20] may not hold. But why—he asked—should one construct a non-Archimedean geometry over the field of real numbers and not over some other field? In this he considered the field of rational numbers $\mathbb{Q}$ and a finite Galois field, since these two are contained as subfields of any other field.

The question above is inspired in Volovich's proposed general principle that all physical parameters undergo quantum fluctuations, including the number field on which a theory is developed (p. 14). He maintained that this means that the usual field of real numbers is apparently only realized with some probability but that, in principle, there also exists the probability of the occurrence of any other field. His programme therefore consists in reducing "all physics" to geometry over arbitrary number fields. Volovich furthermore speculated that the fundamental physical laws should be invariant under a change of the number field, in analogy with Einstein's principle of general invariance (p. 14).

In his view, "number theory and the corresponding branches of algebraic geometry are nothing else than the ultimate and unified physical theory" (p. 15).

as the result of a "crude" measurement.

[20]The Archimedean axiom states that any given segment of a straight line can be eventually surpassed by adding arbitrarily small segments of the same line.

We judge such a statement not only too quick but also as staking a claim about a physical theory that is more definitive than any such theory could ever be.

Furthermore, it is difficult to see what principle led Volovich to propose this programme; it appears that only analogies have guided his choices. Finally, the role of geometry in this framework is crystal-clear: instead of attempting to reduce "all physics" to Archimedean geometry over the real numbers, it is suggested that one should rather attempt to reduce "all physics" to non-Archimedean *geometry* over an arbitrary number field. For this reason, although Volovich did not so claim, his scheme could, again, hardly be considered a true instance of pregeometry.

4.5 Relational or process-based pictures

Having found inspiration in Leibniz's relational conception of space, some researchers have proposed that spacetime is not a thing itself but a resultant of a complex of relations among basic things. Others, on a more philosophical strand, have found inspiration in Heraclitus's principle that "all is flux" and have put forward pregeometric pictures in which, roughly speaking, processes are considered to be more fundamental than things. This view is sometimes linked with the currently much-disseminated idea of information, especially in its quantum-mechanical form.

4.5.1 Axiomatic pregeometry by Perez Bergliaffa et al.

Perez Bergliaffa, Romero, and Vucetich (1998) presented an axiomatic framework that, via rules of correspondence, is to serve as a "pregeometry of space-time." This framework assumes the objective existence of basic physical entities called things and sees spacetime not as a thing itself but as a resultant of relations among those entities.

In order to derive from this more basic substratum the topological and metric properties of Minkowskian spacetime, a long list of concepts and axioms is presented. But for the said general requirements of objectivity and relationality, these "ontological presuppositions" (p. 2283) appear arbitrary rather than stemming from some physical principle concerning spacetime.

Among these concepts, we would like to centre our attention on something called ontic space E_{o}. Although its exact meaning cannot be conveyed without reviewing a great deal of the contents of the article in question, it could be intuitively taken to be a form of space connected with the basic entities, which is prior (in axiomatic development) to the more familiar geometric space E_{G} of physics—Minkowskian space, in this case. Perez Bergliaffa et al. proved (pp. 2290–2291) that the ontic space E_{o} is metrizable and gave an explicit metric $d(x, y)$ for it, a *distance* between things, turning, again, relations into lines and, possibly, things into points. Finally, by means of an isomorphism between E_{o} and a subspace dense in a complete space, the geometric space E_{G} is obtained as this complete space itself, while at the same time it inherits the metric of E_{o}.

One notices once again the stealthy introduction of a metric for pregeometry, with the ensuing geometric objectification of earlier abstract concepts, in order to derive from it further geometric notions. This procedure must be criticized in view that this approach explicitly claims to be an instance of pregeometry.

4.5.2 Cahill and Klinger's bootstrap universe

Cahill and Klinger (1997) proposed to give an account of what they rather questionably called "the 'ultimate' modelling of reality" (p. 2). This, they claimed, could only be done by means of pregeometric concepts that do not assume any notions of things but only those of processes since, according to them, the former notions (but not the latter) suffer from the problem of their always being capable of further explanation in terms of new things. One must wonder for what mysterious reason certain processes cannot be explained in terms of others.

As an embodiment of the concepts above, these authors used a "bootstrapped self-referential system" (p. 3) characterized by a certain iterative map that relies on the notion of relational information B_{ij} between monads i and j. In order to avoid the already rejected idea of things, these monads are said to have no independent meaning but for the one they acquire via the relational information B_{ij}.

Due to reasons not to be reviewed here, the iterative map will allow the persistence of large B_{ij}. These, it is argued, will give rise to a tree-graph—with monads as nodes and B_{ij} as links—as the most probable structure and as seen from the perspective of monad i. Subsequently, so as to obtain (by means of probabilistic mathematical tools) a persistent background structure to be associated with some form of three-dimensional space, a *distance* between any two monads is defined as the smallest number of links connecting them in the graph.

"This emergent 3-space," Cahill and Klinger argued, "...does not arise within any *a priori* geometrical background structure" (p. 6). Such a contention is clearly mistaken for a graph with its defined idea of distance is certainly a geometric background for the reasons we have pointed out earlier. Thus, yet another pregeometric scheme falls prey to geometry.

4.6 Quantum-cosmological pictures

Quantum-cosmological approaches tackle the problem of spacetime structure from the perspective of the universe as a whole and seen as a necessarily closed quantum system. For them, the problem of the constitution of spacetime is bound up with the problem of the constitution of the cosmos, customarily dealing as well with the problem of how they interdependently originated.

4.6.1 Eakins and Jaroszkiewicz's quantum universe

Jaroszkiewicz (2001) and Eakins and Jaroszkiewicz (2002, 2003) presented a theoretical picture for the quantum structure and running of the universe. Its first class of basic elements are event states; these may or may not be factored out as yet more fundamental event states, depending on whether they are direct products (classicity) of yet more elementary event states $|\psi\rangle$, or whether they are entangled (non-separability) event states $|\Psi\rangle$. The second class of basic elements are the tests acting on the event states. These tests Σ, represented by Hermitian operators $\hat{\Sigma}$, provide the topological relationships between them, ultimately endowing the structure of states with an evolution (irreversible acquisition of information) via "state collapse" and a quantum arrow of time. A third component is a certain information content of the universe—sometimes, although not necessarily, represented by the semi-classical observer—at each given step in its evolution (stage), which, together with the present state of the universe, determines the tests which are to follow. Thus, the universe is a self-testing machine, a quantum automaton, in which the traditional quantum-mechanical observer is explained but totally dispensed with.

In (Eakins & Jaroszkiewicz, 2003, p. 1), these authors found a connection between causal set theory and their own framework. In the latter, two different causal set structures arise: one in connection with the separation and entanglement of states, seen to be related to the transmission of quantum-mechanical information which does not respect Einstein's locality; the other in connection with the separation and entanglement of operators, seen to be related to the transmission of classical information.

Eakins and Jaroszkiewicz (2002, p. 5), like Requardt but after their own fashion, also attempted to understand the present structure of spacetime as a consequence of primordial events, which in this case can be traced back to what they call the "quantum big bang."[21] Hence the classification of this approach.

The subject of pregeometry as pioneered by Wheeler is also touched upon (Eakins & Jaroszkiewicz, 2003, p. 2). The view that the characteristic feature of pregeometric approaches consists in "avoiding any assumption of a pre-existing manifold" (p. 2) is also stated there, although it is nowhere mentioned that the notion of distance, according to Wheeler, should be avoided as well. In any case, these authors explained that their task is to reconcile pregeometric, bottom-up approaches to quantum gravity with other holistic, top-bottom ones as in quantum cosmology.

Since the traditional machinery of quantum mechanics is assumed in the approach, so is therefore geometry (see Section 2.2). Again, this fact would undermine the status of this programme as a genuine representative of pregeometry. (See Table 1.)

Of all the above approaches, this one is perhaps the most physically inspired one in the sense that it gives clear explanations about the nature of its building blocks and puts forward judicious physical principles and mechanisms—as contrasted to purely mathematically inspired ones—by means of which the traditional notions of spacetime and quantum non-locality may emerge.

[21] In this case, the emergence of spacetime is more sophisticated than in Requardt's case, since here it happens in the non-parameter, intrinsic time given by state vector collapse.

Author	Geometric objects	Geometric magnitudes
Dadić & Pisk	vertices, edges	overlap, length, distance
Antonsen	idem	length, distance
Requardt	idem	idem
Lehto et al.	idem	idem
Bombelli et al.	idem	length, distance, volume
Hill	as in sp. rel. & q. mech.	as in sp. rel. & q. mech.
Horzela et al.	as in sp. rel.	as in sp. rel.
Volovich	as in mod. phys.	as in mod. phys.
Perez Bergliaffa et al.	points, lines	length, distance
Cahill & Klinger	idem	idem
Eakins & Jaroszkiewicz	as in q. mech.	as in q. mech.
Nagels	points, lines	length, distance
Stuckey & Silberstein	idem	idem

Table 1: Summary of the use of geometric concepts in the pregeometric approaches analyzed, including those of Section 5.3.1. Abbreviations: Special relativity (sp. rel.), quantum mechanics (q. mech.), modern physics (mod. phys.).

4.7 The inexorability of geometric understanding?

Finally, we would like to take note of certain views expressed by Anandan, as they constitute the absolute epitome of pregeometry's mode of working described so far. Anandan is led to suggest

> a philosophical principle which may be schematically expressed as
>
> Ontology = Geometry = Physics.
>
> The last equality has not been achieved yet by physicists because we do not have a quantum gravity. But it is here proposed as a philosophical principle which should ultimately be satisfied by a physical theory. (Anandan, 1997, p. 51)

Anandan appears to have taken the geometric essence of the present description of spacetime as a ground to argue ahead that, correspondingly, any successful future description of it must of necessity be geometric. Such is his conviction that he raises this idea to the status of a principle that any physical theory should "ultimately" follow. But does it ensue from the fact that physics has so far described Nature by means of geometry that it will continue to do so in the future? Physics *must* describe Nature geometrically come what may—*whence such an inevitability?* Moreover, it is not clear in what way the first part of the equality has, as implicitly stated, already been achieved. Is this a suggestion that reality itself is geometric? It is hard to know what to make of such a cryptic notion.

Thus, one witnesses here an overstated version of the cases analyzed through this section. If in the latter the use of geometry was, so to speak, accidental or non-intentional, in Anandan's statement one sees the explicit claim materialized that geometry *must* be the mode of description of physical science, in particular, concerning a solution to the problem of the structure of spacetime. No stronger grip of geometry on physics could possibly be conceived. This view clashes head-on with our own as to what the role of geometry could be in these matters.

4.8 Appraisal

The connection with the first part of this article now comes to the fore. From the mists of antiquity to the forefront of twenty-first century theoretical physics, *geometry remains the enduring, irreplaceable tool by means of which humans portray the world*. Ironically enough, even in a field that sets itself the task of explaining the origin of geometry in physics and names itself pregeometry, one is to find the most explicit displays of geometric understanding. Some reasons suspected to have a bearing on this peculiar state of affairs will be offered in the following section.

5 Beyond geometry

It is the cherished realm of geometry that, some have proposed, might need to be transcended in the search for a novel theory of spacetime structure—or more profoundly still, in the search for a deeper layer of the nature of things. In this section, some of the views espoused by Clifford in 1875, by Eddington in 1920, and by Wheeler in the decades between 1960 and 1980 will be explored as examples of this sort of proposal. In Wheeler's case, the reason for such a requirement will be seen to stem directly from the general views of Section 3.3, namely, that progress in the physical world picture comes after providing explanations of Nature of a fundamentally different kind than those currently in force. Clifford and Eddington do not make an explicit case for this point, but their ideas can be seen to rest on similar foundations.

5.1 Clifford's elements of feeling

Views that seem to advocate the provisional character of geometric explanations in physical theories date at least as far back as 1875. For then, not only did Clifford express his better-known belief that matter and its motion are in fact nothing but the curvature of space—and thus reducible to geometry—but also lesser-known ideas about matter and its motion—and so perhaps, indirectly, geometry—being, in turn, only a partial aspect of "the complex thing we call feeling."

Within the context of an investigation to which we do not wish to commit ourselves, but from which we would nevertheless like to rescue an interesting idea, Clifford (1886, pp. 172–173) asked about the existence of something that is not part of the "material or phenomenal world" such as matter and its motion, but that is its "non-phenomenal counterpart." After a linguistic reinterpretation of Clifford, this issue can be understood to be connected with the search for a new brand of theoretical entities, on a deeper level than the presently accepted ones, and on the basis of which to provide a new explication of Nature. In this regard, Clifford wrote:

> The answer to this question is to be found in the theory of sensation; which tells us not merely that there is a non-phenomenal world, but also in some measure what it is made of. Namely, the reality corresponding to our perception of the motion of matter is an element of the complex thing we call feeling. What we might perceive as a plexus of nerve-disturbances is really in itself a feeling; and the succession of feelings which constitutes a man's consciousness is the reality which produces in our minds the perception of the motions of his brain. These elements of feeling have relations of nextness or contiguity in space, which are exemplified by the sight-perceptions of contiguous points; and relations of succession in time which are exemplified by all perceptions. Out of these two relations the future theorist has to build up the world as best he may. (Clifford, 1886, p. 173)

Allowing for 129 years of history, it could be said that Clifford pursued after this particular fashion the conviction that matter and geometry do not lie at the bottom of things, but that, on the contrary, they are themselves an aspect of deeper entities—for him, feelings—and their non-geometric relations, contiguity and succession. It is in these objects beyond matter and geometry that, Clifford believed, one can catch a glimpse of a deeper layer of the constitution of the world.

5.2 Eddington's nature of things

Forty-five years later, Eddington (1920, pp. 180–201) expressed similar views as to what he called the nature of things. In analyzing general relativity, he identified its basic elements as the point-events, and the interval as an elementary relation between them. As to the nature of the latter, Eddington wrote:

> Its [the interval's] geometrical properties... can only represent one aspect of the relation. It may have other aspects associated with features of the world outside the scope of physics. But in physics we are concerned not with the nature of the relation but with the number assigned to express its intensity; and this suggests a graphical representation, leading to a geometrical theory of the world of physics. (Eddington, 1920, p. 187)

Along these lines (which accord with our views of Section 2), he suggested that the *individual* intervals between point-events probably escape today's scales and clocks, these being too rudimentary to capture them. As a consequence, in general relativity one only deals with macroscopic values composed out of many individual intervals. “Perhaps,” he ventured further in an allusion to transcending geometric magnitudes,

> even the primitive interval is not quantitative, but simply 1 for certain pairs of point-events and 0 for others. The formula given [$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$] is just an average summary which suffices for our coarse methods of investigation, and holds true only statistically. (Eddington, 1920, p. 188)

One must not be confused by the association of the numbers 1 and 0 to the primitive intervals and think that such numbers represent their lengths. In the passage, the *non-quantitativeness* of the primitive intervals is clearly stated. What the 1 and 0 represent is merely whether the intervals exist or not, any other general designation having been equally satisfactory for the purpose. Eddington’s insight into the need to transcend geometric notions is reinforced by his further remarks: “[W]e can scarcely hope to build up a theory of the nature of things if we take a scale and a clock as the simplest unanalysable concepts” (p. 191). It is nothing short of remarkable to find expressed such deep views, on the one hand, inspired in the general theory of relativity, while on the other, in a sense contrary to its spirit, only five years after the publication of the latter. We note, however, that Eddington’s avowal only seems to be for the overthrow of geometric magnitudes but not of geometric objects, since he does not seem to find fault with the concepts of point-event and interval.

Eddington also stressed the role of the human mind in the construction of physical theories. He argued that, out of the above primitive intervals (taken as elements of reality and not of a theory of it) between point-events, a vast number of more complicated qualities can arise; as a matter of fact, however, only certain qualities of all the possible ones do arise. Which qualities are to become apparent (in one’s theories) and which not depends, according to Eddington, on which aspects of these elementary constituents of the world (and not of a theory of it) the mind singles out for recognition. “Mind filters out matter from the meaningless jumble of qualities,” he said,

> as the prism filters out the colours of the rainbow from the chaotic pulsations of white light. Mind exalts the permanent and ignores the transitory... Is it too much to say that mind’s search for permanence has created the world of physics? So that the world we perceive around us could scarcely have been other than it is? (Eddington, 1920, p. 198)

At this point, a clarification is in order in connection with the parenthetical additions above and with Eddington’s remarks. He talked about the mind creating the world of physics, but one finds such a thing a daring exploit, for how could the mind create the physical world? Or to put it in the words of Devitt and Sterelny (1987), “how could we, literally, have made the stars?” (p. 200). Rather, what the mind creates is *theories of the world* and into these inventions it puts, naturally, all its partialities and prejudices.

This being said, one feels tempted to add to Eddington's observations, as related ideas promptly suggest themselves. Matter is but one of a whole range of entities that physical theories find to be conserved. Paraphrasing Eddington, one could therefore ask: is it too much to say that mind's search for conservation laws and symmetries has created the current theories of the physical world? What is more, after our analysis of geometry and pregeometry, along the same lines one could furthermore ask: is it too much to say that *mind's geometric instinct* has created all the current theories of the physical world?

5.3 Wheeler's pregeometry

Wheeler (1964, 1980), Misner et al. (1973), and Patton and Wheeler (1975) expressed pioneering ideas on what they called pregeometry already four decades ago. Since we understand Wheeler to be by far the main contributor to this idea, the kind of pregeometry to be analyzed in this section will be correspondingly called Wheeler's pregeometry. In order to avoid confusion, it must be remarked here yet again—in addition to the early comments of Section 4—that Wheeler's pregeometry is not really what later on came to be known by the same name, i.e. pre-manifold physics, but the more radical stance of going beyond geometry in a proper sense.

His basic demand amounts to the rejection of geometric concepts in order to explain geometric structure. Indeed, Wheeler (1980) advocated "a concept of pregeometry that breaks loose at the start from all mention of geometry and distance;" he was wary of schemes in which "too much geometric structure is presupposed to lead to a believable theory of geometric structure;" and he clearly recognized that "to admit distance at all is to give up on the search for pregeometry" (pp. 3–4). This means that one surprisingly finds the very inventor of pregeometry generally invalidating all the schemes analyzed in Section 4, as far as they are to be an expression of his original pregeometry.

What are the grounds for these strong pronouncements of Wheeler's? Firstly, he envisaged (Patton & Wheeler, 1975, p. 547; Misner et al., 1973, p. 1201), among other things, spacetime collapse in the form of the big bang, a possible big crunch, black holes, and a supposed foam-like structure of spacetime on small scales, as indicators that spacetime cannot be a continuous manifold, since it has the ability to become singular. Confronted thus by the need to rethink spacetime, he posed the question:

> If the elastic medium is built out of electrons and nuclei and nothing more, if cloth is built out of thread and nothing more, we are led to ask out of what "pregeometry" the geometry of space and spacetime are built. (Wheeler, 1980, p. 1)

The reason why Wheeler seeks a "pregeometric building material" (Misner et al., 1973, p. 1203) in order to account for the spacetime continuum is that he believes that genuine explanations about the nature of something do not come about by explicating a concept in terms of similar ones, but by reducing it to a different, more basic kind of object. This is evidenced in the passage: "And must not this something, this 'pregeometry,' be as far removed from geometry as the quantum mechanics of electrons is far removed from elasticity?" (p. 1207)

Having stated Wheeler's motives for going beyond geometry, what were his proposals to implement this programme? His attempts include pregeometry as a Borel set or bucket of dust (see Section 5.3.1), pregeometry as binary-choice logic, and pregeometry as a self-referential universe.[22]

An early attempt at pregeometry based on binary choice was Wheeler's "sewing machine," into which rings or loops of space were fed and connected or left unconnected by the machine according to encoded binary information (yes or no) for each possible pair of rings (Misner et al., 1973, pp. 1209–1210). Wheeler considered the coded instructions to be given from without either deterministically or probabilistically, and then asked whether fabrics of different dimensionality would arise, and whether there would be a higher weight for any one dimensionality to appear. We note that, as a design for pregeometry, this idea succeeds in staying clear of geometric magnitudes but fails, at least superficially, to avoid geometric objects—Wheeler's loops of space have no size and yet are *rings*. We say this failure is superficial because these rings or loops, so long as they have no size, could as well be called abstract elements. If nothing else, the mention of rings is witness to the fact that geometric modes of thought are always lurking in the background.

A subsequent idea still based on binary-choice logic was pregeometry as the calculus of propositions (Misner et al., 1973, pp. 1209, 1211–1212). This conception was much less picturesque and rather more abstract. Wheeler exploited the isomorphism between the truth-values of a proposition and the state of a switching circuit to toy with the idea that " 'physics' automatically emerges from the statistics of very long propositions, and very many propositions," (Patton & Wheeler, 1975, p. 598) in thermodynamical analogy. In this reference and in (Wheeler, 1980), however, the expectation that pure mathematical logic *alone* could have anything whatsoever to do with providing a foundation for physics was sensibly acknowledged. In this case, we note that, perhaps due to its abstractness and total divorcement with physics, this conception of pregeometry is truly free of all geometry in a strict sense. For all this scheme was worth, it is at least rewarding to find it has this feature.

Later on, inspired in self-referential propositions, Wheeler conceived of the idea of pregeometry as a self-referential universe. As with the previous approaches, Wheeler (1980) admitted to having no more than a vision of how this understanding of geometry in terms of (his) pregeometry might come about. Two basic ingredients in this vision appear to be (i) events consisting of a primitive form of quantum principle: investigate Nature and create reality in so doing (see below) and, more intelligibly, (ii) stochastic processes among these events producing form and dependability out of randomness. His vision reads thus:

(1) Law without law with no before before the big bang and no after

[22]See (Demaret, Heller, & Lambert, 1997, pp. 157–161) for an independent review of Wheeler's pregeometry.

after collapse. The universe and the laws that guide it could not have existed from everlasting to everlasting. Law must have come into being... Moreover, there could have been no message engraved in advance on a tablet of stone to tell them how to come into being. They had to come into being in a higgledy-piggledy way, as the order of genera and species came into being by the blind accidents of billions upon billions of mutations, and as the second law of thermodynamics with all its dependability and precision comes into being out of the blind accidents of motion of molecules who would have laughed at the second law if they had ever heard of it. (2) Individual events. Events beyond law. Events so numerous and so uncoordinated that flaunting their freedom from formula, they yet fabricate firm form. (3) These events, not of some new kind, but the elementary act of question to nature and a probability guided answer given by nature, the familiar everyday elementary quantum act of observer-participancy. (4) Billions upon billions of such acts giving rise, via an overpowering statistics, to the regularities of physical law and to the appearance of continuous spacetime. (Wheeler, 1980, pp. 5–6)

Earlier, Patton and Wheeler (1975, pp. 556–562) had elaborated further on this. They considered two alternatives to the problem of spacetime collapse, namely: (a) taking into account quantum mechanics, the event of universal collapse can be viewed as a "probabilistic scattering in superspace." In this way, the universe is reprocessed at each cycle, but the spacetime manifold retains its fundamental mode of description in physics. This alternative was not favoured by these authors, but instead (b) considering as an overarching guiding principle that the universe must have a way to come into being (cosmogony), they suggested that such a requirement can be fulfilled by a "quantum principle" of "observer-participator."[23] According to it, the universe goes through one cycle only, but only if in it a "communicating community" arises that can give meaning to it. This universe is thus self-referential, which was the standpoint favoured by these authors.

We cannot agree with the latter proposal. For although it is correct to say that only a communicating community of beings— which furthermore engage in scientific activities—can give meaning to the displays of Nature, it is beyond any reasonable doubt that the universe exists and moves on independently of any consciousness that it might give rise to. Finally, as far as pregeometric ideas are concerned, this vision of self-referential cosmology succeeds in eschewing all kinds of geometric concepts again due to its being conceived in such broad and general terms. Regardless of its worth and in the general context of paradoxical geometric pregeometry, let this be a welcome feature of Wheeler's effort.

Regardless of Wheeler's actual implementations of his pregeometry and any criticisms thereof, the imperative demand that the foundations of such a theory should be completely free of geometric concepts remains unaltered. The fact that

[23]This principle is tantamount to the first ingredient (i) above, and perhaps helps to clarify it.

Author	Basic objects	Basic relations
Clifford	feelings	nextness, succession
Eddington	*points*, *intervals*	statistical interaction
Wheeler		
Sewing machine	*rings*	nextness (prob. amp.)
Borel set	*points*	idem
Calculus of propositions	propositions	statistical interaction
Self-referential universe	acts of observation	idem

Table 2: Summary of the use of basic objects and relations in the non-geometric or partly non-geometric approaches analyzed. Geometric concepts appear in italics. Abbreviations: Probability amplitude (prob. amp.).

so many (including, to the smallest extent, Wheeler himself) have misconstrued his idea of a programme for pregeometry is perhaps a tell-tale sign of something beyond simple carelessness in the reading of his views. Indeed, the large-scale misinterpretation here documented may just as well stem from the lack of all background for thought that one encounters as soon as one attempts to dispense with geometry. Physics knows of no other mode of working than geometry, and thus physicists have reinterpreted Wheeler's revolutionary new idea in the only way that made sense to them—*setting out to find geometry in pregeometry.*

5.3.1 More buckets of dust

As two final claimants to pregeometry to be analyzed, we turn to the works of Nagels and of Stuckey and Silberstein in the light of their close connection with Wheeler. In these works, their authors independently attempt again to build "space as a bucket of dust" after Wheeler's (1964, pp. 495–499) earlier effort. This earlier effort, clearly connected with the above-mentioned "sewing machine" in character and purpose, consisted in starting with a Borel set of points without any relations to each other whatsoever and assembling them into structures of different dimensionality on the basis of different quantum-mechanical probability amplitudes attributed to the relations of nearest neighbour between the points. Wheeler dismissed his own trial because, among other reasons, the same quantum-mechanical principles used to define adjacency rendered the idea untenable since points that had once been neighbours would remain correlated after departing from each other (p. 498). We observe once more that this idea, in its use of the concept of points (cf. earlier rings), is not free from geometric objects, but that this flaw is minimized by Wheeler's avoidance of the introduction of any quantitative geometry. As for the mentioned quantum amplitudes, these need not have geometric connotations as long as they do not arise from an inner product of state vectors. (See Table 2.)

Nagels' (1985) attempt started off with a scheme similar to Wheeler's, where "the only structure imposed on individual points is a uniform probability of adjacency between two arbitrarily chosen points" (p. 545). He only assumed that these probabilities are very small and that the total number of points is very large (and possibly infinite). He thus believed his proposal to satisfy the always desired requirement of "a bare minimum of assumptions" and very natural ones at that. However, much too soon he fell prey to quantitative geometry. Being a pregeometric framework directly inspired in Wheeler, it is startling to come across the following early remarks:

> Without a background geometry, the simplest way to introduce distance is to specify whether or not two given points are "adjacent." [...] We may then say that two "adjacent" points are a distance of 1 unit of length apart. (Nagels, 1985, p. 546)

For his part, Stuckey introduced in his attempt

> a pregeometry that provides a metric and dimensionality over a Borel set (Wheeler's "bucket of dust") without assuming probability amplitudes for adjacency. Rather, a non-trivial metric is produced over a Borel set X per a uniformity base generated via the discrete topological group structures over X. We show that entourage multiplication in this uniformity base mirrors the underlying group structure. One may exploit this fact to create an entourage sequence of maximal length whence a fine metric structure. (Stuckey, 2001, p. 1)

Thus, both authors managed to create pregeometric metrics, i.e. metrics that will give a *notion of distance for pregeometry* and by means of which they hoped to obtain the usual spacetime metric. This assertion is well supported by Stuckey and Silberstein's (2000) remarks: "our pregeometric notion of distance" (p. 9) and "the process by which this metric yields a spacetime metric with Lorentz signature must be obtained" (p. 13). Stuckey (2001) moreover asserted that the pregeometries of Nagels, Antonsen, and Requardt have, arguably, overcome the difficulties represented by the presupposition of "too much geometric structure" as previously brought up by Wheeler, since their assumptions are minimal and "the notion of length per graph theory is virtually innate" (p. 2). Arguably, indeed, because the assumption of length is totally contrary to the spirit of pregeometry according to its very inventor, and its introduction acts to geometrically objectify what could have otherwise been pregeometric schemes.

Alas, what has become of Wheeler's reasonable dictum: "to admit distance at all is to give up on the search for pregeometry," or his demands for "break[ing] loose at the start from all mention of geometry and distance" and for pregeometry to be "as far removed from geometry as the quantum mechanics of electrons is far removed from elasticity?" Or yet to quote Wheeler in two other early, illuminating passages:

> One might also wish to accept to begin with the idea of a distance, or edge length, associated with a pair of these points, even though this

> idea is already a very great leap, and one that one can conceive of later supplying with a natural foundation of its own.
>
> [...] [T]he use of the concept of distance between pairs of points seems unreasonable... [L]ength is anyway not a natural idea with which to start. The subject of analysis here is "pregeometry", so the concept of length should be derived, not assumed *ab initio.* (Wheeler, 1964, pp. 497, 499)

Has all this simply gone into oblivion?

The question, indeed, remains—why the need of a metric for pregeometry? Should not pregeometry produce the traditional spacetime metric by non-geometric means alone? Should it not go even "beyond Wheeler" and do without any sort of geometric objects too? This state of affairs comes to show most clearly to what extent the craving of the human mind for geometric explanations goes; at the same time, one catches a glimpse of the difficulties that might be encountered in any attempt that genuinely attempts to go beyond such explanations, so deeply rooted in the mind.

After having spelt out Wheeler's vision for (his) pregeometry, another reason supporting this venture is presented next, as we near the end of this exploration.

5.4 The influence of language on thought

Another context—supplementary to our observations of Section 3.3—also exists for the belief that going beyond geometry may be a necessary step for theoretical physics to take. Such a context comes from the writings of linguist B. L. Whorf on his now infamous principle of linguistic relativity. After a judicious interpretation of it, one will be armed with quite a straightforward idea about the influence of language on thought to be applied in connection with the topic of this article. Whorf wrote:

> It was found that the background linguistic system... of each language is not merely a reproducing instrument for voicing ideas but rather is itself the shaper of ideas... We dissect nature along lines laid down by our native languages. The categories and types that we isolate from the world of phenomena we do not find there because they stare every observer in the face; on the contrary, the world is presented in a kaleidoscopic flux of impressions which has to be organized by our minds—and this means largely by the linguistic systems in our minds.
>
> This fact is very significant for modern science, for it means that no individual is free to describe nature with absolute impartiality but is constrained to certain modes of interpretation even while he thinks himself most free... We are thus introduced to a new principle of relativity, which holds that all observers are not led by the same physical evidence to the same picture of the universe, unless their linguistic backgrounds

are similar, or can in some way be calibrated. (Whorf, 1956, pp. 212–214)

One ought to be careful as to what to make of Whorf's remarks, as their significance could go from (i) an implication of an *immutable constraint* of language upon thought with the consequent fabrication of an unavoidable world picture to which one is led by the language used, to (ii) milder insinuations about language being "only" a *shaper of ideas* rather than a tyrannical master. How far is the influence of language on thought to be taken to go?

In this, we will argue along with the ideas of Devitt and Sterelny's. In their book, they clearly explained the circular—although not viciously so—process in which thought and language interact. On this issue, they wrote:

> We feel a pressing need to understand our environment in order to manipulate and control it. This drive led our early ancestors, in time, to express a primitive thought or two. They grunted or gestured, *meaning something by* such actions. There was speaker meaning without conventional meaning. Over time the grunts and gestures caught on: linguistic conventions were born. As a result of this trail blazing it is much easier for others to have those primitive thoughts, for they can learn to have them from the conventional ways of expressing them. Further, they have available an easy way of representing the world, a way based on those conventional gestures and grunts. They borrow their capacity to think about things from those who created the conventions. With primitive thought made easy, the drive to understand leads to more complicated thoughts, hence more complicated speaker meanings, hence more complicated conventions. (Devitt & Sterelny, 1987, p. 127)

In the first place, this means that, as one could have already guessed, thought must precede any form of language, or else how could the latter have come into being? Secondly, it shows that thought, as a source of linguistic conventions, is not in any way restricted by language, although the characteristics of the latter do, in fact, *facilitate* certain forms of thought by making them readily available, in the sense that the thinking processes of many can now benefit from existing concepts already made by a few others. This is especially the case in science, which abounds in instances of this kind. For example, thought about complex numbers is facilitated by the already invented imaginary-number language convention "$i = \sqrt{-1}$," just like the thought of a particle being at different places at the same time is facilitated by the already invented state-vector-related conventions of quantum mechanics.

At the same time, an existing language can also *discourage* certain thoughts by making them abstruse and recondite to express (Devitt & Sterelny, 1987, p. 174). This does not mean that certain things cannot be thought—they eventually can be—but only that their expression does not come easily as it is not straightforwardly supported by the language. As an example of this instance, we can mention the reverse cases of the above examples. That is to say, the evident difficulty of conjuring up any thoughts about complex numbers and delocalized particles *before* the formal concepts above supporting these thoughts were introduced into the language of science by some specific individuals. The fact, however, that such linguistic conventions were created eventually shows that thought beyond linguistic conventions is possible—essential, moreover, for the evolution of science.

One is thus led to the view that the picture that physics makes of Nature is not only guided by the physicist's imagination, but also by the physicist's language—with its particular vices and virtues. This language, due to its very constitution attained after a natural development, favours geometric modes of thinking, while it appears to dissuade any sort of non-geometric contemplation. Pregeometric schemes have shown this fact very clearly: imagination was able to produce varied creative frameworks, but they all spoke the same geometric language. Only against this *language-favoured background* did the physicists' minds roam effortlessly.

The reason why humans have naturally developed primitive geometric thoughts, leading to geometric modes of expression, in turn reinforcing more geometric-like thinking, and so forth—in conclusion, developing a geometric understanding—can only be guessed at. Perhaps, it lies in the intricacies of their evolutionary origins on Earth, and the resulting character and dispositions of their brains.

The prospect of dispensing with geometric thinking is not encouraging, for when geometry is lost, much is lost with it. However, we feel this endeavour must be pursued. In this respect, Misner et al. issued the following remarks:

> And is not the source of any dismay the apparent loss of guidance that one experiences in giving up geometrodynamics—and not only geometrodynamics but geometry itself—as a crutch to lean on as one hobbles forward? Yet there is so much chance that this view of nature is right that one must take it seriously and explore its consequences. Never more than today does one have the incentive to explore [Wheeler's] pregeometry. (Misner et al., 1973, p. 1208)

6 Conclusion

A long way has so far been travelled. Starting off in the realm of geometry, its foundations in the form of geometric objects and magnitudes were laid down and its use in all of present-day physics was highlighted. It became evident that geometry and, in particular, quantitative geometry is not only introduced into theories to link them to experience via measurements, but also in order to satisfy a more basic need seemingly possessed by the human mind—a predilection to conceptualize the world after a geometric fashion.

The abode of pregeometry, one of the latest attempts to throw new light on the problem of space and time, was visited next. Although judging by its name and goals pregeometry was the last place in which one—for better or worse—would have expected to find geometry, the former nevertheless fell prey to the latter. Pregeometry, in its need for its own geometric means to explain the origin of geometry in physics, cannot but raise the disquieting question: is the human mind so dependent upon geometric means of description that they cannot be avoided? Or has pregeometry not tried hard enough? In any case, it must be concluded that pregeometry has failed to live up to the semantic connotations of its name and to the original intentions of its creator, as well as to the intentions of its present-day practitioners, only to become a considerable incongruity.

Next the vistas of a land farther beyond were surveyed. Different older suggestions for the need to overcome geometry in physics were analyzed, including those views pertaining to the father of pregeometry proper. In general, this proposal stemmed from the desire to reach into yet another layer in the nature of things.

On a whole, different related reasons were put forth for the need to transcend geometry at the point where theoretical physics comes to confront the unresolved problem of empty spacetime. Essentially, the investigation of empty spacetime was seen to demand ridding it of all its contents, including its metric field, i.e. its quantitative geometry. Moreover, a look at the lessons of history and a consideration of the mazelike nature of current philosophical controversies suggested that understanding spacetime anew demands going beyond its intrinsic geometric description *entirely*, for which the use of non-geometric methods is required, as well as the guidance of a physical principle directly relevant to the existence of spacetime. This conclusion was supplemented by an investigation of the influence of language on thought, which revealed that the language of physics may be discouraging physicists' creative activities from effortlessly straying into non-geometric realms.

The journey ends, for the time being, at a junction in the road at which one must pause in order to evaluate what has been here expounded—and make a decision. Geometry has furthered human knowledge by a long way indeed, but has it come today to the limits of its use? If the task of geometry has been completed, new means should be devised that allow one to *think* about Nature—stimulated by a facilitation to *talk* about Nature—in a different, non-geometric way.

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