Generalized Fermat primes p Such That 3 is a Primitive Root Modulo p

Predrag Terzich

Abstract : We explore property of generalized Fermat primes of the form :
$F_n(2\cdot q)=(2\cdot q)^{2^n}+1$, where $n>1$ and q is an odd prime number

1. Introduction

Generalized Fermat numbers of the form $a^{2^n}+1$ with $a>2$,see [1] are generalization of usual Fermat numbers $2^{2^n}+1$ .Generalized Fermat numbers can be prime only for even a, because if a is odd then every generalized Fermat number will be divisible by 2 . Many of the known largest prime numbers are generalized Fermat primes . A primitive root of a prime p is an integer g such that g (mod p) has modulo order p−1 , see [2] . It is known that 3 is a primitive root modulo p for every Fermat prime F_n with $n>0$. In this paper we explore for which conditions 3 is a primitive root modulo p for generalized Fermat primes $F_n(a)$ with $n>1$.

2. Conjectures

2.1. **Definition** : Let p be a prime number of the form $p=F_2(2\cdot q)=(2\cdot q)^{2^2}+1$, where q is an odd prime number .

Consider the output of the following Maple code :

```
with(numtheory):
i:=0:
n:=2:
for q from 1 to 8640200 do
if isprime(q) then
if isprime((2*q)^(2^n)+1) then
i:=i+1:
if not(primroot((2*q)^(2^n)+1)=3) then
print(q);
end if;
end if;
end if;
end do;
i;
```

3
48997

2.1. **Conjecture** : If q is a greater than 3 then 3 is a primitive root modulo p for all p .

2.2. **Definition** : Let p be a prime number of the form $p = \mathcal{F}_3(2\cdot q) = (2\cdot q)^{2^3} + 1$, where q is an odd prime number .

Consider the output of the following Maple code :

```
with(numtheory):
i:=0:
n:=3:
for q from 1 to 8640200 do
if isprime(q) then
if isprime((2*q)^(2^n)+1) then
i:=i+1:
if not(primroot((2*q)^(2^n)+1)=3) then
print(q);
end if;
end if;
end if;
end do;
i;
```

271
18206

2.2. **Conjecture** : If q is a greater than 271 then 3 is a primitive root modulo p for all p

2.3. **Definition** : Let p be a prime number of the form $p = \mathfrak{F}_4(2\cdot q) = (2\cdot q)^{2^4} + 1$, where q is an odd prime number .

Consider the output of the following Maple code :

```
with(numtheory):
i:=0:
n:=4:
for q from 1 to 640200 do
if isprime(q) then
if isprime((2*q)^(2^n)+1) then
i:=i+1:
if not(primroot((2*q)^(2^n)+1)=3) then
print(q);
end if;
end if;
end if;
end do;
i;
```

1901

2.3. **Conjecture** : 3 is a primitive root modulo p for all p .

2.4. **Definition** : Let p be a prime number of the form $p = \mathfrak{F}_5(2\cdot q)^{2^5} + 1$, where q is an odd prime number .

Consider the output of the following Maple code :

```
with(numtheory):
i:=0:
n:=5:
for q from 1 to 840200 do
if isprime(q) then
if isprime((2*q)^(2^n)+1) then
i:=i+1:
if not(primroot((2*q)^(2^n)+1)=3) then
print(q);
end if;
end if;
end if;
end do;
i;
```

1194

2.4. **Conjecture** : 3 is a primitive root modulo p for all p .

References

1. Definition of generalized Fermat number available at :
   http : //mathworld.wolfram.com/GeneralizedFermatNumber.html
2. Definition of primitive root modulo p available at :
   http : //en.wikipedia.org/wiki/Primitive_root_modulo_n

E−mail adress : tersit26@gmail.com