The local Doppler effect due to Earth's rotation and the CNGS neutrino anomaly?

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In this brief paper, we show if the neutrino velocity discrepancy obtained in [1] may be due to the local Doppler effect between a local clock attached to a given detector at Gran Sasso, say C_G, and the respective instantaneous clock crossing C_G, say C_C, being this latter at rest in the instantaneous inertial frame having got the velocity of rotation of CERN about Earth's axis in relation to the fixed stars.

1 Definitions and Solution

Firstly, the effect investigated here is not the same one that was investigated in [2], but, throughout this paper, we will use some useful configurations defined in [2]. The relative velocity between Gran Sasso and CERN due to the Earth daily rotation may be written:

$$\vec{v}_G - \vec{v}_C = 2\omega R \sin\alpha \hat{e}_z, \tag{1}$$

where $\hat{e}_z$ is a convenient unitary vector, the same used in [2], ω is the norm of the Earth angular velocity vector about its daily rotation axis, being R given by:

$$R_E = \frac{R}{\cos\lambda}, \tag{2}$$

where R_E is the radius of the Earth, its averaged value $R_E = 6.37 \times 10^6 m$, and α given by:

$$\alpha = \frac{1}{2}\left(\alpha_G - \alpha_C\right), \tag{3}$$

where α_C and α_G are, respectively, CERN's and Gran Sasso's longitudes ($\leftarrow WE \rightarrow$). Consider the inertial (in relation to the fixed stars) reference frame $O_C x_C y_C z_C \equiv Oxyz$ in [2]. This is the lab reference frame and consider this frame with its local clocks at each spatial position as being ideally synchronized, viz., under an ideal situation of synchronicity between the clocks of $O_C x_C y_C z_C \equiv Oxyz$. This situation is the expected ideal situation for the OPERA collaboration regarding synchronicity in the instantaneous lab (CERN) frame.

Now, consider an interaction between a single neutrino and a local detector at Gran Sasso. This event occurs at a given spacetime point $(t_\nu, x_\nu, y_\nu, z_\nu)$ in $O_C x_C y_C z_C \equiv Oxyz$. The interaction instant t_ν is measured by a local clock C_C at rest at (x_ν, y_ν, z_ν) in the lab frame, viz., in the $O_C x_C y_C z_C \equiv Oxyz$ frame. But, under gedanken, at this instant t_ν, according to $O_C x_C y_C z_C \equiv Oxyz$, there is a clock C_G attached to the detector at Gran Sasso that crosses the point $(t_\nu, x_\nu, y_\nu, z_\nu)$ with velocity given by Eq. (1). Since C_G crosses C_C, the Doppler effect between the proper tic-tac rates measured at each location of C_C and C_G, viz., measured at their respective locations in their respective reference frames (the reference frame of C_G is the $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ in [2], also inertial in relation to the fixed stars), regarding a gedanken control tic-tac rate continuosly sent by C_C, say via electromagnetic pulses from C_C, is not transverse. Since the points at which C_C and C_G are at rest in their respective reference frames will instantaneously coincide, better saying, will instantaneously intersect, at t_ν accordingly to C_C, they must be previously approximating, shortening their mutual distance during the interval $t_\nu - \delta t_\nu << t_\nu$ along the line passing through these clocks as described in the C_C world.

Suppose C_C sends $\mathcal{N}$ electromagnetic pulses to C_G. During the C_C proper time interval $(t_\nu - \delta t_\nu) - 0 = t_\nu - \delta t_\nu$ * within which C_C emits the N electromagnetic pulses, the first emitted pulse travels the distance $c\left(t_\nu - \delta t_\nu\right)$ and reaches the clock C_G, as described by C_C. Within this distance, there are $\mathcal{N}$ equally spaced distances between consecutive pulses as described in the C_C world, say λ_C:

$$\mathcal{N}\lambda_C = c\left(t_\nu - \delta t_\nu\right). \tag{4}$$

Also, since the clocks C_C and C_G will intersect at t_ν, as described in $O_C x_C y_C z_C \equiv Oxyz$, during the interval δt_ν, the clock C_G must travel the distance $2\omega R \sin\alpha\, \delta t_\nu$ in the C_C world to accomplish the matching spatial intersection at the instant t_ν, hence the clock C_G travels the $2\omega R \sin\alpha\, \delta t_\nu$ in the C_C world, viz., as described by C_C in $O_C x_C y_C z_C \equiv Oxyz$:

$$\mathcal{N}\lambda_C = 2\omega R \sin\alpha\, \delta t_\nu \Rightarrow \delta t_\nu = \mathcal{N}\frac{\lambda_C}{2\omega R \sin\alpha}. \tag{5}$$

*The initial instant C_C starts to emit the electromagnetic pulses is set to zero in both the frames $O_C x_C y_C z_C \equiv Oxyz$ and $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$; zero also is the instant the neutrino starts the travel to Gran Sasso in $O_C x_C y_C z_C \equiv Oxyz$; hence the instant the neutrino starts the travel to Gran Sasso and the emission of the first pulse by C_C are simultaneous events in $O_C x_C y_C z_C \equiv Oxyz$. These events are simultaneous in $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ too, since they have got the same spatial coordinate $z_c = z = 0$ along the $O_C z_C \equiv Oz$ direction as defined in [2]. The relative motion between CERN and Gran Sasso is parallel to this direction. The only one difference between these events is the difference in their $x_C = x$ coordinates, being $x_C = 0$ for the neutrino departure and $x_C = L = 7.3 \times 10^5\, m$ for C_C, being these locations perpendicularly located in relation to the relative velocity given by the Eq, (1).

Solving for t_ν, from the Eqs. (4) and (5), one reaches:

$$t_\nu = \frac{\mathcal{N}\lambda_C}{c}\left(1 + \frac{c}{2\omega R\sin\alpha}\right). \tag{6}$$

Now, from the perspective of C_G, in $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$, there must be $\mathcal{N}$ electromagnetic pulses covering the distance:

$$c\left(t_\nu^G - \delta t_\nu^G\right) - 2\omega R\sin\alpha\left(t_\nu^G - \delta t_\nu^G\right), \tag{7}$$

where $t_\nu^G - \delta t_\nu^G$ is the time interval between the non-proper instants $t^G = t_\nu = 0$, at which the C_C clock send the first pulse, and the instant $t_\nu^G - \delta t_\nu^G$, at which this first pulse reaches C_G, as described by C_G in its world $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$. Within this time interval, $t_\nu^G - \delta t_\nu^G$, C_G describes, in its $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ world, the clock C_C approximating the distance:

$$2\omega R\sin\alpha\left(t_\nu^G - \delta t_\nu^G\right), \tag{8}$$

with the first pulse traveling:

$$c\left(t_\nu^G - \delta t_\nu^G\right), \tag{9}$$

giving the distance within which there must be $\mathcal{N}$ equally spaced pulses, say, spaced by λ_G, as described by C_G in its $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ world:

$$\mathcal{N}\lambda_G = (c - 2\omega R\sin\alpha)\left(t_\nu^G - \delta t_\nu^G\right). \tag{10}$$

With similar reasoning that led to the Eq. (5), now in the $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ C_G world, prior to the spatial matching intersection between C_C and C_G, the C_C clock must travel the distance $\mathcal{N}\lambda_G$ during the time interval δt_ν^G, with the C_C approximation velocity $2\omega R\sin\alpha$:

$$\mathcal{N}\lambda_G = 2\omega R\sin\alpha\,\delta t_\nu^G \Rightarrow \delta t_\nu^G = \mathcal{N}\frac{\lambda_G}{2\omega R\sin\alpha}. \tag{11}$$

From Eqs. (10) and (11), we solve for t_ν^G:

$$t_\nu^G = \mathcal{N}\frac{\lambda_G}{2\omega R\sin\alpha}\frac{1}{[1-(2\omega R\sin\alpha)/c]}. \tag{12}$$

From the Eqs. (6) and (12), we have got the relation between the neutrino arrival instant t_ν as measured by the CERN reference frame, $O_C x_C y_C z_C \equiv Oxyz$, and the neutrino arrival instant t_ν^G as measured by the Gran Sasso reference frame, $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$, at the exact location of the interaction at an interation location within the Gran Sasso block of detectors, provided the effect of the Earth daily rotation under the assumptions we are taking in relation to the intantaneous movements of these locations in relation to the fixed stars as previously discussed:

$$\frac{t_\nu^G}{t_\nu} = \frac{\lambda_G}{\lambda_C}\left[1-(2\omega R\sin\alpha)^2/c^2\right]^{-1} = \gamma^2\frac{\lambda_G}{\lambda_C}, \tag{13}$$

where $\gamma \geq 1$ is the usual relativity factor as defined above.

Now, λ_G/λ_C is simply the ratio between the spatial displacement between our consecutive gedanken control pulses, being these displacements defined through our previous paragraphs, leading to the Eqs. (4) and (10). Of course, this ratio is simply given by the relativistic Doppler effect under an approximation case in which C_C is the source and C_G the detector. The ratio between the Eqs. (10) and (4) gives:

$$\frac{\lambda_G}{\lambda_C} = [1-(2\omega R\sin\alpha)/c]\frac{\left(t_\nu^G - \delta t_\nu^G\right)}{(t_\nu - \delta t_\nu)}. \tag{14}$$

But the time interval $(t_\nu - \delta t_\nu)$ is a proper time interval measured by the source clock C_C, as previously discussed. It accounts for the time interval between the first pulse sent and the last pulse sent as locally described by C_C is its $O_C x_C y_C z_C \equiv Oxyz$ world. These two events accur at different spatial locations in the C_G detector clock world $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$, since C_C is approximating to C_G is this latter world. Hence, $t_\nu - \delta t_\nu$ is the Lorentz time contraction of $t_\nu^G - \delta t_\nu^G$, viz.:

$$t_\nu - \delta t_\nu = \gamma^{-1}\left(t_\nu^G - \delta t_\nu^G\right) \quad \therefore$$

$$\frac{\left(t_\nu^G - \delta t_\nu^G\right)}{t_\nu - \delta t_\nu} = \gamma = \left[1-(2\omega R\sin\alpha)^2/c^2\right]^{-1/2}. \tag{15}$$

With the Eqs. (14) and (15), one reaches the usual relativistic Doppler effect expression for the approximation case:

$$\frac{\lambda_G}{\lambda_C} = \sqrt{\frac{1-(2\omega R\sin\alpha)/c}{1+(2\omega R\sin\alpha)/c}}. \tag{16}$$

With the Eq. (16), the Eq. (13) reads:

$$\frac{t_\nu^G}{t_\nu} = \left[1-(2\omega R\sin\alpha)^2/c^2\right]^{-1/2}[1+(2\omega R\sin\alpha)/c]^{-1} =$$

$$= \frac{\gamma}{[1+(2\omega R\sin\alpha)/c]}. \tag{17}$$

Since $(2\omega R\sin\alpha)/c << 1$, we may apply an approximation for the Eq. (17), viz.:

$$\gamma \approx 1 + \frac{1}{2}\frac{(2\omega R\sin\alpha)^2}{c^2}, \tag{18}$$

and:

$$[1+(2\omega R\sin\alpha)/c]^{-1} \approx 1-(2\omega R\sin\alpha)/c, \tag{19}$$

from which, neglecting the higher order terms, the Eq. (17) reads:

$$\frac{t_\nu^G}{t_\nu} \approx 1 - \frac{2\omega R\sin\alpha}{c} \quad \therefore \tag{20}$$

$$t_\nu^G - t_\nu = -\frac{2\omega R\sin\alpha}{c}t_\nu. \tag{21}$$

From this result, the clock that tag the arrival interaction instant t_ν^G in Gran Sasso turns out to measure an arrival time that is shorter than the one measured in the CERN reference frame $O_C x_C y_C z_C \equiv Oxyz$ in [2], this latter given by t_ν. The discrepancy under the phenomenon we are investigating here, ϵ, turns out to be easy to calculate, since t_ν is simply given by the time interval between a mutual synchronicity at $t_C = t_G = 0$ and the instant a given time tagging in Gran Sasso occurs. One verifies this discrepancy depends on t_ν, from which, if t_ν is large and includes the process of a neutrino travel within it, the measured discrepancy may also be relatively large. Such effect, to be kept small, requires a constant synchronicity between the frames of CERN and Gran Sasso defined in [2]. Hence, the discrepancy between $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ and $O_C x_C y_C z_C \equiv Oxyz$ per elapsed time at a baseline located in $O_C x_C y_C z_C \equiv Oxyz$ that crosses the $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$ clock that is located at the position of a neutrino detection event (neutrinos are, actually, detected in $O_G x_G y_G z_G \equiv \tilde{O}\tilde{x}\tilde{y}\tilde{z}$) per event detection reads*:

$$\frac{\epsilon}{t_\nu} = \frac{t_\nu^G - t_\nu}{t_\nu} = -\frac{2\omega R \sin\alpha}{c} \ \textbf{per event detection from } 0, \tag{22}$$

viz., **per event detection from a mutual synchronicity** $t_\nu = t_G = 0$. With the values† $\omega = 7.3 \times 10^{-5} s^{-1}$, $R = R_E \cos\lambda \approx 6.4 \times 10^6 m \times \cos(\pi/4) = 4.5 \times 10^6 m$, $\sin\alpha \approx \sin(7\pi/180) = 1.2 \times 10^{-1}$, $c = 3.0 \times 10^8 ms^{-1}$ and $L = 7.3 \times 10^5 m$, the relative discrepancy ϵ/t_ν, given by the Eq. (22), reads:

$$\frac{\epsilon}{t_\nu} = -2\bar{6}3\,\frac{ns}{s}. \tag{23}$$

If the clocks were perfectly synchronized at the neutrino departure event, the discrepancy would be (t_ν exactly equals L/c, without residual discrepancy):

$$\epsilon = -0.64\,ns, \tag{24}$$

which is small in relation to the $\approx 60\,ns$ measured by the OPERA Collaboration, but, asseverating, Eq. (24) is due to no residual synchronicity discrepancy accumulated due to the Doppler effect discussed here, at the neutrino departure event.

Generating a $64\,ns$ discrepancy...

If one supposes the synchronicity between the CERN and Gran Sasso reference frames were verified $100L/c - L/c$ before the neutrino departure instant in the CERN reference frame, which seems to be very unlikely, the interval to an event at Gran Sasso reads $100L/c$ in the CERN frame, viz., $t_\nu = 100L/c$ in the Eq. (22). Of course, under this situation, one obtains a $64\,ns$ discrepancy from this same Eq. (22).

*It must be asseverated that t_ν must obey the condition $t_\nu/T << 1$, where T is the period of the Earth rotation about its axis, thus quite instantaneous in relation to the Earth daily kinematics, as discussed in [2].

†See the Eqs. (2) and (3). The latitudes of CERN and Gran Sasso are, respectively: $46^{\text{deg}}14^{\text{min}}3^{\text{sec}}$(N) and $42^{\text{deg}}28^{\text{min}}12^{\text{sec}}$(N). The longitudes of CERN and Gran Sasso are, respectively: $6^{\text{deg}}3^{\text{min}}19^{\text{sec}}$(E) and $13^{\text{deg}}33^{\text{min}}0^{\text{sec}}$(E).

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References

1. The OPERA collaboration: T. Adam et al. Measurement of the neutrino velocity with the OPERA detector in the CNGS beam http://arxiv.org/abs/1109.4897 *arXiv:1109.4897*, 2011.
2. Armando V.D.B. Assis. Prog. Phys. 4(4), 85 to 90 (2011).