Accelerating universe and the expanding atom

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Abstract: In the cosmic Euclidean volume, inverse of the fine structure ratio is equal to the natural logarithm of ratio of number of (electrons or positrons) and the Avogadro number. Bohr radius of hydrogen atom, quanta of the angular momentum and the strong interaction range - are connected with the large scale structure of the massive universe. In the accelerating universe, as the space expands, in hydrogen atom, distance between proton and electron increases and is directly proportional to the size of the universe. Obtained value of the present Hubble constant is 70.75 Km/sec/Mpc. 'Rate of decrease in fine structure ratio' is a measure of cosmic rate of expansion. Considering the integral nature of number of protons (of any nucleus), integral nature of 'hbar' can be understood.

Keywords: Hubble's constant; present universe mass, electron rest mass; proton rest mass; strong interaction range; reduced planck's constant; fine structure ratio; Bohr radius;

1 Introduction

Considering and comparing the ratio of characteristic size of the universe and classical radius of electron with the electromagnetic and gravitational force ratio of electron and proton, Dirac in his large number hypothesis [1,2] suggested that, magnitude of the gravitational constant G inversely varies with the cosmic time. In supporting of this till today no such data is reported [3]. Considering the characteristic mass of the universe, in this paper an attempt is made to understand the mystery of the origin of the fine structure ratio, integral quantum constant, Bohr radius and the strong interaction range.

1.1 Hubble's law

Hubble's law is the name for the astronomical observation in physical cosmology that:

1. all objects observed in deep space (interstellar space) are found to have a doppler shift observable relative velocity to Earth, and to each other; and
2. this doppler-shift-measured velocity, of various galaxies receding from the Earth, is proportional to their distance from the Earth and all other interstellar bodies.

In effect, the space-time volume of the observable universe is expanding and Hubble's law is the direct physical observation of this process [4,5]. It is considered the first observational basis for the expanding space paradigm and today serves as one of the pieces of evidence most often cited in support of the Big Bang model [6,7]. Although widely attributed to Edwin Hubble, the law was first derived from the General Relativity equations by Georges Lemaitre in a 1927 article [8] where he proposed that the Universe is expanding and suggested an estimated value of the rate of expansion, now called the Hubble constant. Two years later Edwin Hubble confirmed the existence of that law and determined a more accurate value for the constant that now bears his name. The law is often expressed by the equation

$$v = H_0 D, \tag{1}$$

with H_0 the constant of proportionality (the Hubble constant), D is the galaxy distance and v is the recession velocity of the galaxy. The SI unit of H_0 is sec^{-1} but it is most frequently quoted in $Km/s/Mpc$.

1.2 Magnitude of the Hubble's constant

The value of the Hubble constant H_0 is estimated by measuring the redshift of distant galaxies [9] and then determining the distances to the same galaxies (by some other method than Hubble's law). The Hubble Key Project [10] used the Hubble space telescope to establish the most precise optical determination in May 2001 of 72 ± 8 Km/s/Mpc, consistent with a measurement of H_0 based upon Sunyaev-Zel'dovich effect observations of many galaxy clusters having a similar accuracy. The most precise cosmic microwave background radiation determinations by WMAP for the seven year release in 2010 found 71.0 ± 2.5 Km/s/Mpc. Most accurate value is $70.4^{+1.3}_{-1.4}$ Km/s/Mpc [11]. Thus in this paper it is taken as $H_0 \cong 70.4$ Km/s/Mpc.

1.3 Physical constants and their fundamental ratios

Characteristic size of the universe is

$$R_0 \cong \frac{c}{H_0} \cong 1.314147 \times 10^{26} \text{ m} \tag{2}$$

Classical radius of electron of mass m_e is

$$R_e \cong \frac{e^2}{4\pi\varepsilon_0 m_e c^2} \cong 2.8794 \times 10^{-15} \text{ m} \tag{3}$$

Ratio of R_0 and R_e is

$$X_1 \cong \frac{R_0}{R_e} \cong \frac{4\pi\varepsilon_0 m_e c^3}{e^2 H_0} \cong 4.6635 \times 10^{40} \tag{4}$$

Electromagnetic and gravitational force ratio of electron of mass m_e and proton of mass m_p is

$$X_2 \cong \frac{e^2}{4\pi\varepsilon_0 G m_p m_e} \cong 2.26867 \times 10^{39} \tag{5}$$

Ratio of X_1 and X_2 is

$$\frac{X_1}{X_2} \cong \frac{4.6635 \times 10^{40}}{2.26867 \times 10^{39}} \cong 20.5561 \tag{6}$$

1.4 Characteristic mass of the present universe

Let the cosmic closure density is,

$$\rho_0 \cong \frac{3H_0^2}{8\pi G} \tag{7}$$

Volume of the universe in a Euclidean sphere of radius $\left(\frac{c}{H_0}\right)$ is equal to

$$v_0 \cong \frac{4\pi}{3}\left(\frac{c}{H_0}\right)^3 \tag{8}$$

Mass of the universe in a Euclidean sphere is

$$M_0 \cong \rho_0 \cdot v_0 \cong \frac{c^3}{2GH_0} \cong 8.84811 \times 10^{52} \text{ Kg} \tag{9}$$

If m_n is the mass of nucleon, number of nucleons in a Euclidean volume of size $\frac{c}{H_0}$ is

$$X_3 \cong \frac{M_0}{m_n} \cong \frac{c^3}{2GH_0 m_n} \cong 5.286322 \times 10^{79} \tag{10}$$

From these ratios it is noticed that,

$$X_1 \approx \sqrt{X_3} \approx X_2 \tag{11}$$

J. V. Narlikar says [12]:*Reactions among physicists have varied as to the significance of all these numbers. Some dismiss it as a coincidence with the rejoinder 'So what' ? Others have read deep significance into these relations. The later class includes such distinguished physicists as A. S. Eddington and P. A. M. Dirac.*

Dirac pointed out in 1937 that the relationships (3) to (11) contain the Hubble constant H_0 and therefore the magnitudes computed in these formulae vary with the epoch in the standard Friedmann model. Finally

Dirac made a distinction between e, m_e, and m_p on one side and G on the other in the sense that the former are atomic quantities where as G has macroscopic significance. In the Machian cosmologies, G is in fact related to the large scale structure of the universe. Dirac therefore assumed that, if we use 'atomic units' that always maintain fixed values for atomic quantities, then G varies with cosmic time t as $G \,\alpha\, t^{-1}$.

2 Cosmological estimation of the Avogadro like number

In strong (nuclear) gravity [13-21] the strong or atomic gravitational constant is the supposed physical constant of strong gravitation, involved in the calculation of the gravitational attraction at the level of elementary particles and atoms. The idea of strong gravity originally referred specifically to mathematical approach of Abdus Salam of unification of gravity and quantum chromo-dynamics, but is now often used for any particle level gravity approach. In literature one can refer the works of Abdus Salam, C. Sivaram, Sabbata, A. H. Chamseddine, J. Strathdee, Usha Raut, K. P. Sinha, J. J. Perng, E. Recami, R. L. Oldershaw, K. Tennakone, S. I Fisenko and S. G. Fedosion.

In the published papers [22-29], authors proposed that, ratio of atomic gravitational constant G_A and the classical gravitational constant G is close to the squared Avogadro like number.

$$\frac{G_A}{G} \cong N^2 \tag{12}$$

where N is the Avogadro like number. If $d_s \cong 1.21$ to 1.22 fm is the minimum scattering distance between electron and nucleus it is noticed that,

$$d_s \cong \frac{1}{N^2}\left(\frac{\hbar c}{Gm_e^2}\right)^2 \frac{2Gm_e}{c^2} \cong \frac{1}{N^2}\left(\frac{m_p}{m_e}\right)^2 \frac{c}{H_0} \tag{13}$$

It can also be considered as the strong interaction range [30]. As the universe is accelerating, space expands and the minimum scattering distance between electron and the nucleus increases and is proportional to the size of the expanding universe. In a ratio form above relation can be expressed as

$$N^2 \cong \frac{c}{H_0 d_s} \cdot \left(\frac{m_p}{m_e}\right)^2 \tag{14}$$

At present if $H_0 \cong 70.4$ Km/sec/Mpc and $d_s \cong 1.22$ fm, $N \cong 6.0263 \times 10^{23}$. In the expanding universe, N^2 seems to be a constant. By measuring the values of (H_0, d_s, c, m_p and $\mathrm{m_e}$) magnitudes of N^2 and N can be estimated.

2.1 The cosmic variable physical constants

2.1.1 The reduced Planck's constant

Considering relations (9 and 13) it is noticed that

$$\hbar \cong \frac{Gm_p\sqrt{m_e M_0}}{c} \cong \sqrt{\frac{Gm_p^2 m_e c}{2H_0}} \cong 1.0572 \times 10^{-34} \text{ J.sec} \tag{15}$$

where M_0 is the characteristic mass of the present universe. This is a striking, astounding and accurate coincidence! This is a multi-purpose expression also. Any value of the atomic constant can be estimated with this expression. Qualitatively it suggests that, $\hbar$ is a cosmic variable but not a constant. This relation suggests that as the universe is accelerating, magnitude of H_0 decreases and magnitude of $\hbar$ increases. Number of electrons in the Euclidean volume of the universe can be expressed as

$$\frac{M_0}{m_e} \cong \frac{c^3}{2GH_0 m_e} \cong \left(\frac{\hbar c}{Gm_e m_p}\right)^2 \tag{16}$$

2.1.2 The fine structure ratio

Considering $(N/2)$ electrons and $(N/2)$ positrons and from relations (15 and 16) it is noticed that,

$$\frac{1}{\alpha} \cong \ln\left(\frac{M_0}{(N/2)\, m_e}\right) \cong \ln\left(\frac{c^3}{NGH_0 m_e}\right) \cong 137.0237 \tag{17}$$

Considering the strong gravity concept $G_A \cong N^2 G$, this relation can also be expressed as

$$\frac{1}{\alpha} \cong \ln\left(\frac{c^3}{NGH_0 m_e}\right) \cong \ln\left(\frac{c^3}{\sqrt{GG_A}H_0 m_e}\right) \tag{18}$$

This relation can be interpreted in the following way.

1. In the cosmic Euclidean volume, natural logarithm of ratio of number of (electrons or positrons) and the Avogadro number is a measure of the strength of electromagnetic interaction.
2. Alternatively it can also be expressed as natural logarithm of geometric mean of number of electrons in the cosmic Euclidean volume and number of electrons in strong gravity (where $G_A \cong N^2 G$) is a measure of the strength of electromagnetic interaction.
3. Factor 2 can be related with $(N/2)$ electrons or $(N/2)$ positrons.

Thus, $\frac{1}{\alpha}$ can be expressed as

$$\frac{1}{\alpha} \cong \ln\sqrt{\frac{c^3}{GH_0 m_e} \cdot \frac{c^3}{G_A H_0 m_e}} \tag{19}$$

Qualitatively this relation also suggests that as the universe is accelerating, magnitude of H_0 decreases and magnitude of α decreases. Interesting thing is that, relations (17 and 18) suggests that, fine structure ratio is independent of the proton rest mass and depends only on H_0, G, G_A & m_e.

3 The reduced Planck's constant - a strange coincidence

David Gross [31] says: *After sometime in the late 1920s Einstein became more and more isolated from the mainstream of fundamental physics. To a large extent this was due to his attitude towards quantum mechanics, the field to which he had made so many revolutionary contributions. Einstein, who* *understood better than most the implications of the emerging interpretations of quantum mechanics, could never accept it as a final theory of physics. He had no doubt that it worked, that it was a successful interim theory of physics, but he was convinced that it would be eventually replaced by a deeper, deterministic theory. His main hope in this regard seems to have been the hope that by demanding singularity free solutions of the nonlinear equations of general relativity one would get an overdetermined system of equations that would lead to quantization conditions.* These words clearly suggests that, at fundamental level there exists some interconnection in between quantum mechanics and gravity [32]. Writing the relation (15) in a ratio form,

$$X_4 \cong \frac{\hbar c}{Gm_p\sqrt{M_0 m_e}} \cong 1 \tag{20}$$

How to interpret this ratio? Compared to the above ratios $X_1, X_2,$ and X_3 this ratio is close to unity. Giving a primary significance to the existence of m_e, m_p, G & c, and considering the Machian concept of the distance cosmic back ground [33,34,35], $\hbar$ can be considered as the compound physical constant. From the atomic structure point of view also this idea can be strengthened. If electron is revolving round the nucleus, naturally m_p and m_e both are the characteristic physical inputs. By considering the origin of the Bohr radius of Hydrogen atom this proposal can be given a chance. If so: in the expanding universe 'quanta' increases with increasing mass of the universe. Any how this is a very sensitive problem.

Considering the 'integral nature' of number of protons (of any nucleus), integral nature of $n \cdot \hbar$ can be understood. Considering any two successive integers n and $(n+1)$, their geometric state is $\sqrt{n\,(n+1)} \cdot \hbar$. If this logic is true, it can be suggested that $\hbar$ is a compound physical constant and is connected with the large scale structure of the universe. The cosmological fine structure ratio can be given as

$$\alpha \cong \frac{e^2}{4\pi\varepsilon_0 G m_p \sqrt{m_e M_0}} \tag{21}$$

It is the strength of electromagnetic interaction and is an intrinsic property of nature. Several different types of astrophysical observations [36,37], have established the evidence that the expansion of the universe entered a phase of acceleration. Cosmic acceleration and dark energy constitute one of the most important and challenging of current problems in cosmology and other areas of physics. By any chance if the noticed relation (15) is found to be true and valid, and if universe is really accelerating and its mass is increasing, then 'rate of increase in $\hbar$' or 'rate of decrease in α' will be a measure of cosmic rate of expansion[38,39]. With reference to relation (15), magnitude of the Hubble's constant can be fitted as

$$H_0 \cong \frac{G m_p^2 m_e c}{2\hbar^2} \cong 70.74955 \text{ Km/sec/Mpc} \tag{22}$$

3.1 Bohr radius of the Hydrogen atom

In hydrogen atom, potential energy of electron in Bohr radius [40,41] can be expressed as

$$E_P \cong -\frac{e^2}{4\pi\varepsilon_0 G m_p M_0} \times \frac{e^2 c^2}{4\pi\varepsilon_0 G m_p} \tag{23}$$

Total energy of electron in Bohr radius can be expressed as

$$E_P \cong -\frac{e^2}{4\pi\varepsilon_0 G m_p M_0} \times \frac{e^2 c^2}{8\pi\varepsilon_0 G m_p} \tag{24}$$

Considering the integral nature of number of protons (of any nucleus), above relation can be expressed as

$$E_T \cong -\frac{e^2}{4\pi\varepsilon_0 G \left(n \cdot m_p\right) M_0} \times \frac{e^2 c^2}{8\pi\varepsilon_0 G \left(n \cdot m_p\right)} \tag{25}$$

where $n = 1, 2, 3, ..$ Thus in a discrete form this relation can be expressed as

$$E_T \cong -\frac{1}{n^2} \times \frac{e^2}{4\pi\varepsilon_0 G m_p M_0} \times \frac{e^2 c^2}{8\pi\varepsilon_0 G m_p} \tag{26}$$

Thus Bohr radius of hydrogen atom can be expressed as

$$a_0 \cong \frac{4\pi\varepsilon_0 G m_p M_0}{e^2} \cdot \frac{G m_p}{c^2} \cong \frac{1}{2}\left(\frac{4\pi\varepsilon_0 G m_p^2}{e^2}\right) \cdot \frac{c}{H_0} \tag{27}$$

This is a very simple and natural fit. The real beauty of the Mach's principle can be seen here. Surprisingly, it indicates that, 'Bohr radius' is independent of the rest mass of electron! $\frac{Gm_p}{c^2}$ is the characteristic black hole size of the proton !! $\frac{e^2}{4\pi\varepsilon_0 G m_p M_0}$ is nothing but the electromagnetic and gravitational force ratio of proton and the expanding universe !!! Considering this relation (27) as a fundamental and characteristic assumption in the Machian cosmology, equation (15) can be obtained and can be confirmed. Thus

$$a_0 \propto M_0 \propto \frac{c}{H_0} \tag{28}$$

In the expanding universe, as the space expands, in hydrogen atom, distance between proton and electron increases and is directly proportional to the size of the expanding universe.

3.2 Alternative to the Planck scale

If $\hbar$ is a cosmic variable, then what about the validity of 'Planck mass' and 'Planck scale'? Answer is very simple. $\sqrt{\frac{\hbar c}{G}}$ can be replaced with $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$. It can be called as the 'Coulomb mass'. Its corresponding rest energy is $\sqrt{\frac{e^2 c^4}{4\pi\varepsilon_0 G}}$. It can be called as the 'Coulomb energy'. Planck energy can be replaced with the 'Coulomb energy'.

$$M_C \cong \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}} \cong 1.859211 \times 10^{-9}\ \text{Kg} \tag{29}$$

$$M_C c^2 \cong \sqrt{\frac{e^2 c^4}{4\pi\varepsilon_0 G}} \cong 1.042941 \times 10^{18}\ \text{GeV} \tag{30}$$

Coulomb size can be expressed as

$$R_C \cong \sqrt{\frac{e^2 G}{4\pi\varepsilon_0 c^4}} \cong 1.38068 \times 10^{-36}\ \text{m} \tag{31}$$

Clearly speaking e, c and G play a vital role in fundamental physics. With these 3 constants space-time curvature concepts at a charged particle surface can be studied.

Conclusion

Large dimensionless constants and compound physical constants reflects an intrinsic property of nature. Whether to consider them or discard them depends on physical interpretations, experiments and observations. Cosmic acceleration can be confirmed by measuring the 'rate of decrease' in the fine structure ratio. The mystery can be resolved only with further research and analysis.

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