The complete Doppler formula

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Let f be the observed frequency of an electromagnetic wave emitted by a source that is moving at a relative speed with respect to the observer. We know that frequency f times λ gives c, the speed of light, where λ is the observed wavelength, thus

$$f = \frac{c}{\lambda} \tag{1}$$

Now if we increase f with a differential df, then c is increased with a differential dv as λ remains constant,

$$f + df = \frac{c + dv}{\lambda} \tag{2}$$

$$f + df = \frac{c}{\lambda} + \frac{dv}{\lambda} \tag{3}$$

$$f + df = f + \frac{dv}{\lambda} \tag{4}$$

and dividing both sides by f it yields

$$\frac{df}{f} = \frac{dv}{f\lambda} \tag{5}$$

$$\frac{df}{f} = \frac{dv}{c} \tag{6}$$

So integrating we get

$$\ln \frac{f}{f_0} = \frac{v}{c} \tag{7}$$

$$f = f_0 \exp(\frac{v}{c}) \tag{8}$$

where f_0 is the original frequency in the light source.